

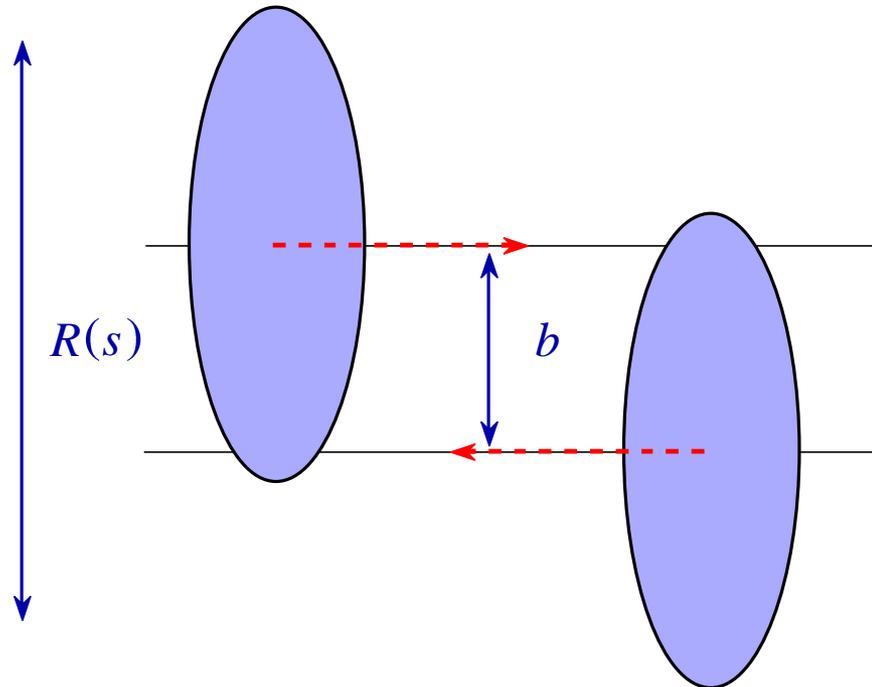
# Geometrical scaling in hadronic collisions (new look at GS)

Michał Praszalowicz  
Jagellonian University  
Kraków, Poland

# „Geometrical Scaling”

J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;  
A.J. Buras, J. Dias de Deus, Nucl.Phys. B 71 (1974) 481;  
J. Dias de Deus, P. Kroll, J. Phys. G 9 (1983) L81;  
J. Diasde Deus, Acta Phys. Polon. B 6 (1975) 613.

$$A(b,s) = A(b/R(s))$$



# „Geometrical Scaling”

J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;  
A.J. Buras, J. Dias de Deus, Nucl.Phys. B 71 (1974) 481;  
J. Dias de Deus, P. Kroll, J. Phys. G 9 (1983) L81;  
J. Diasde Deus, Acta Phys. Polon. B 6 (1975) 613.

*Geometric scaling for the total  $\gamma^*$  p cross-section in the low  $x$  region.*

A.M. Stasto, K. J. Golec-Biernat , J. Kwiecinski  
Phys.Rev.Lett. 86 (2001) 596-599

$$\sigma_{\gamma^*p} \sim \frac{F_2(x, Q^2)}{Q^2} = \sigma_0 \mathcal{F} \left( \frac{Q^2}{Q_{\text{sat}}^2(x)} \right)$$

# „Geometrical Scaling”

J. Dias de Deus, Nucl. Phys. B 59 (1973) 231;  
A.J. Buras, J. Dias de Deus, Nucl.Phys. B 71 (1974) 481;  
J. Dias de Deus, P. Kroll, J. Phys. G 9 (1983) L81;  
J. Diasde Deus, Acta Phys. Polon. B 6 (1975) 613.

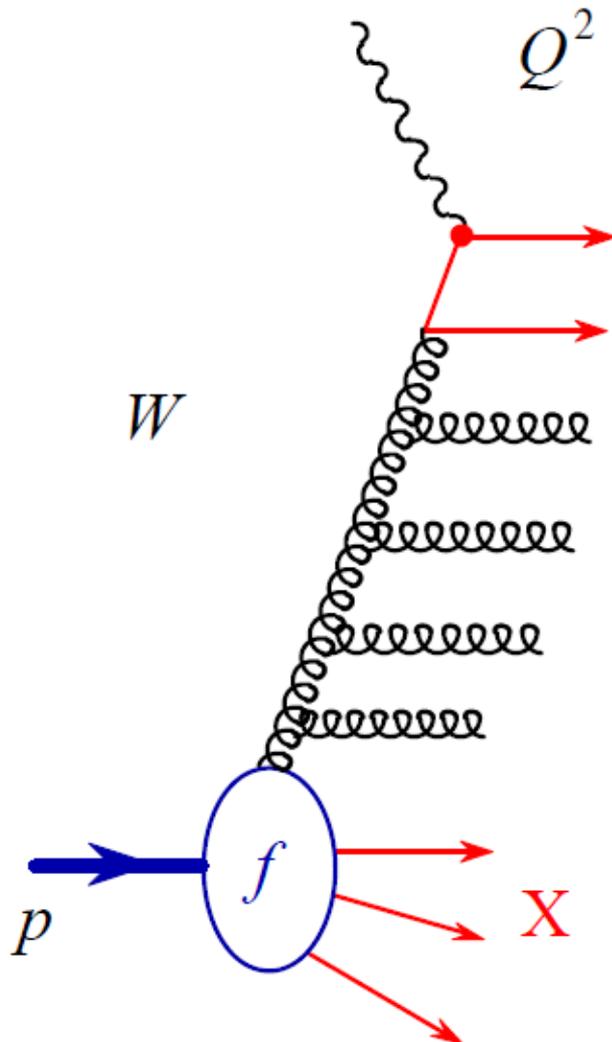
*Geometric scaling for the total  $\gamma^*$  p cross-section in the low  $x$  region.*

A.M. Stasto, K. J. Golec-Biernat , J. Kwiecinski  
Phys.Rev.Lett. 86 (2001) 596-599

L. McLerran, M. Praszalowicz: Acta Phys.Polon.B41:1917,2010, B42:99,2011  
M. Praszalowicz: Phys.Rev.Lett.106:142002,2011  
M. Praszalowicz: Acta Phys.Polon. B42 (2011) 1557-1566 (Cracow Epiphany  
Conference 2011)  
M. Praszalowicz, T. Stebel in preparation

$$\frac{dN_{\text{ch}}}{d\eta dp_{\text{T}}^2}(s, p_{\text{T}}) = \frac{1}{Q_0^2} \mathcal{F} \left( \frac{p_{\text{T}}^2}{Q_{\text{sat}}^2(s)} \right)$$

# Geometrical scaling in DIS at low $x$



$$dP \sim \frac{\alpha_s C_R}{\pi^2} \frac{d^2 k_T}{k_T^2} \frac{d\xi}{\xi}$$

Resumations:

$$\int \frac{d^2 k_T}{k_T^2} \rightarrow \ln Q^2$$

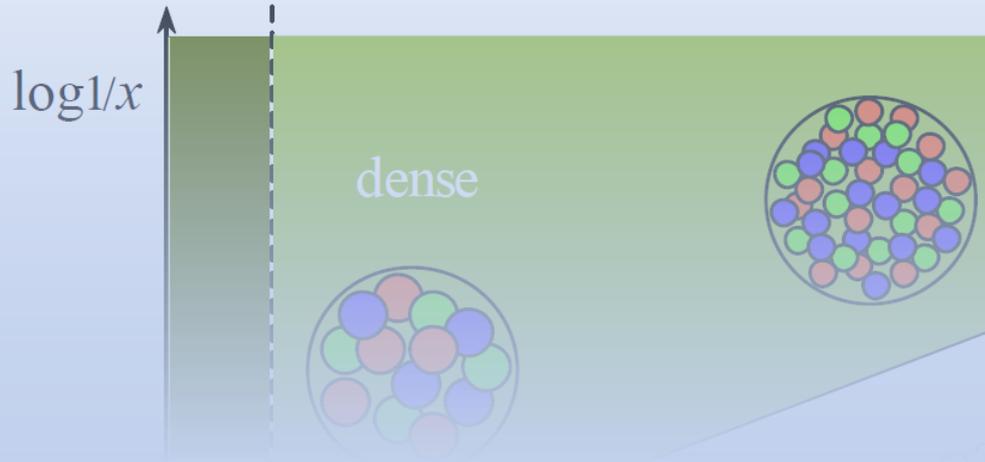
$$\sum \alpha_s^n \ln^n Q^2 \rightarrow \text{DGLAP}$$

$$\int \frac{d\xi}{\xi} \rightarrow \ln W$$

$$\sum \alpha_s^n \ln^n W \rightarrow \text{BFKL}$$

# Saturation

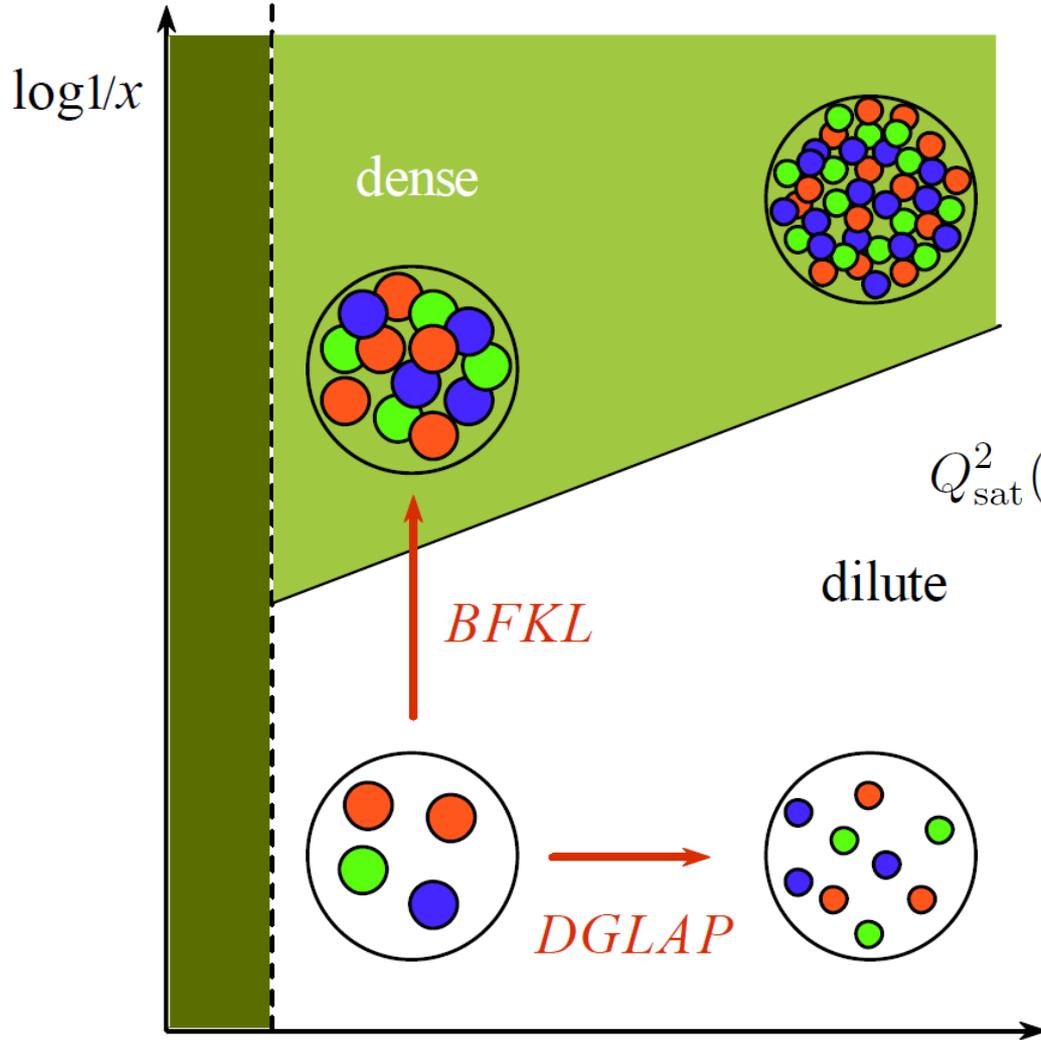
small  $x$   
large  $W$



In case you have never not seen this plot

# Saturation

small  $x$   
large  $W$



large  $x$   
small  $W$

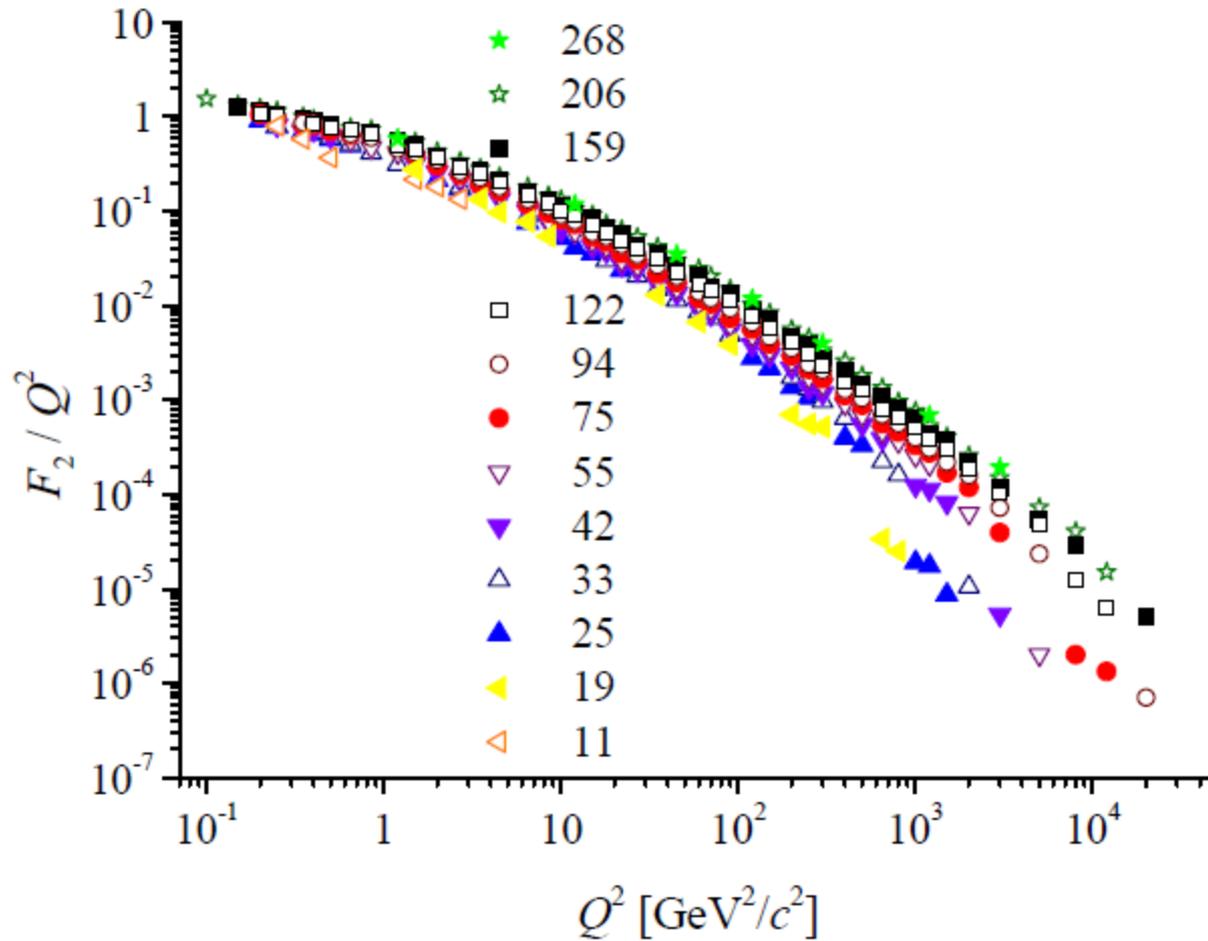
# Geometrical Scaling

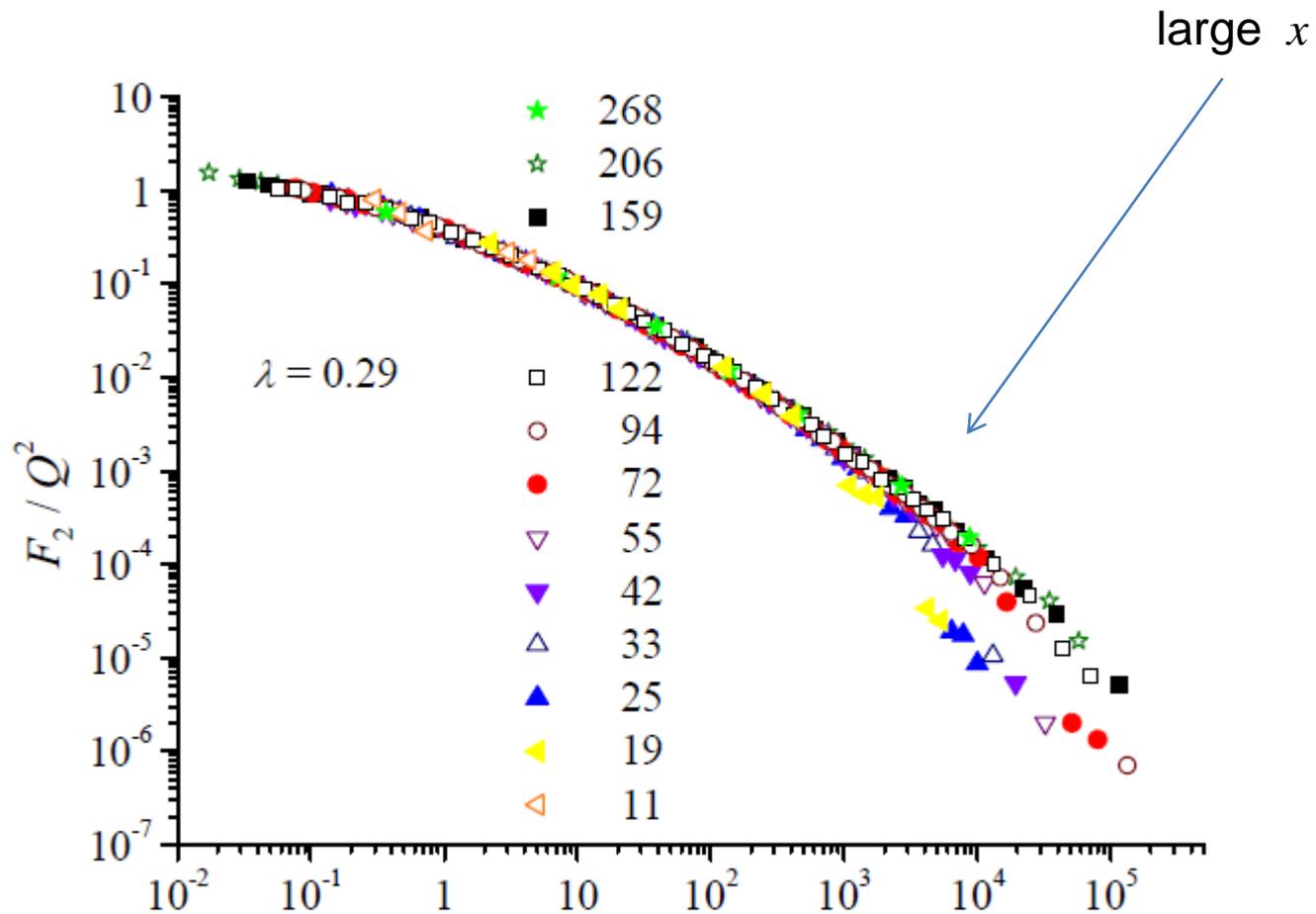
$$\sigma_{\gamma^*p} = \int dr^2 |\psi(r, Q^2)|^2 \sigma_{dP}(r^2 Q_s^2(x))$$

$$\sigma_{\gamma^*p} = \sigma_{\gamma^*p} \left( \frac{Q_s(x)}{Q} \right)$$

GS does not depend on the particular form of the dipole cross-section

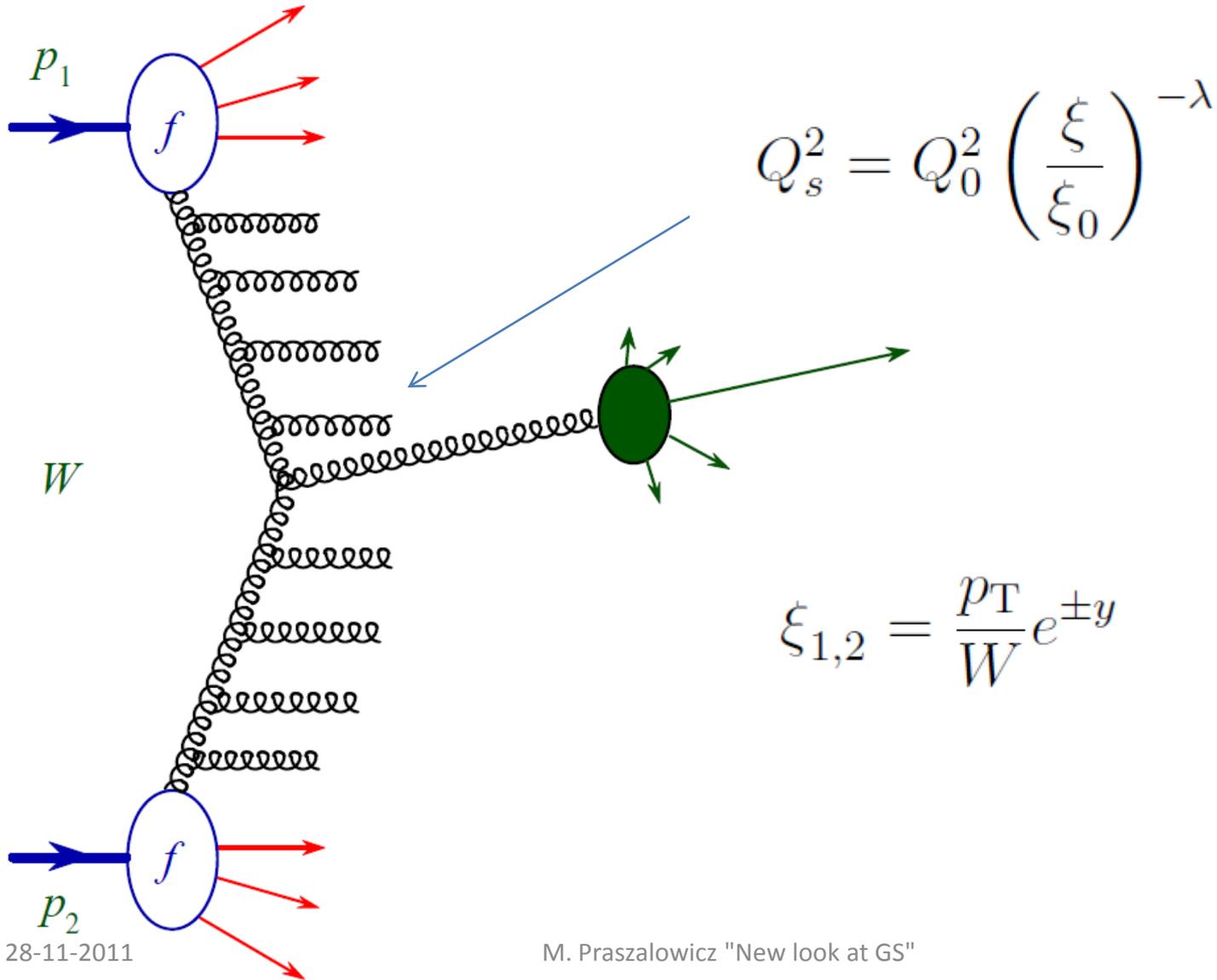
# Combined HERA data 2009 for $e^+$ (with T. Stebel)





$$\tau = \frac{Q^2}{Q_{\text{sat}}^2(x)} \quad Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{x}{x_0} \right)^{-\lambda} \quad \lambda = 0.29$$

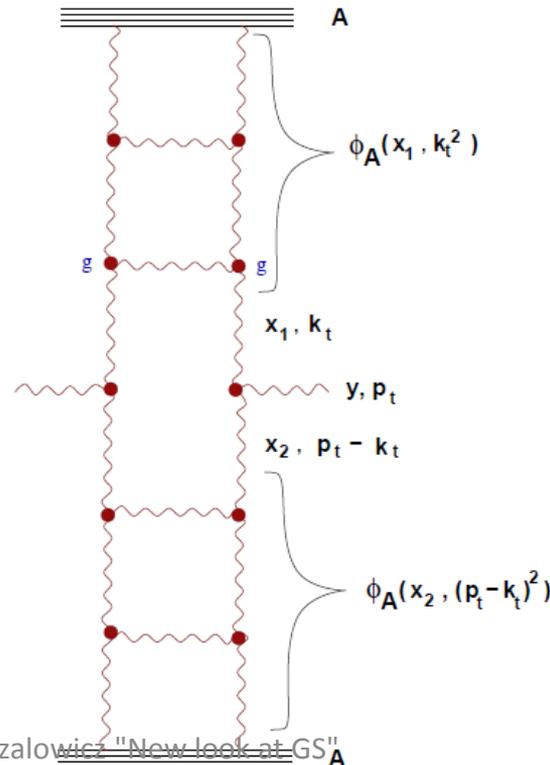
# p-p at the LHC



# Gribov, Levin & Ryskin formula

High P(T) Hadrons In The Pionization Region In QCD.  
 Published in **Phys.Lett.B100:173-176,1981.**

$$\frac{dN}{d\eta d^2 p_T} = \text{const} \frac{1}{p_T^2} \int dk_T^2 \alpha_s(k_T) \varphi_1(x_1, k_T^2) \varphi_2(x_2, (k - p)_T^2)$$



Khazzev, Levin  
 Phys.Lett.B523:79-87,2001.

M. Praszalowicz "New look at GS" A

# Geometrical scaling of $p_T$ distributions

$$\frac{dN_{\text{ch}}}{dy dp_T^2}(s, p_T) = \frac{1}{Q_0^2} F(\tau)$$

multiplicity distribution  
is a universal function  
of scaling variable  $\tau$

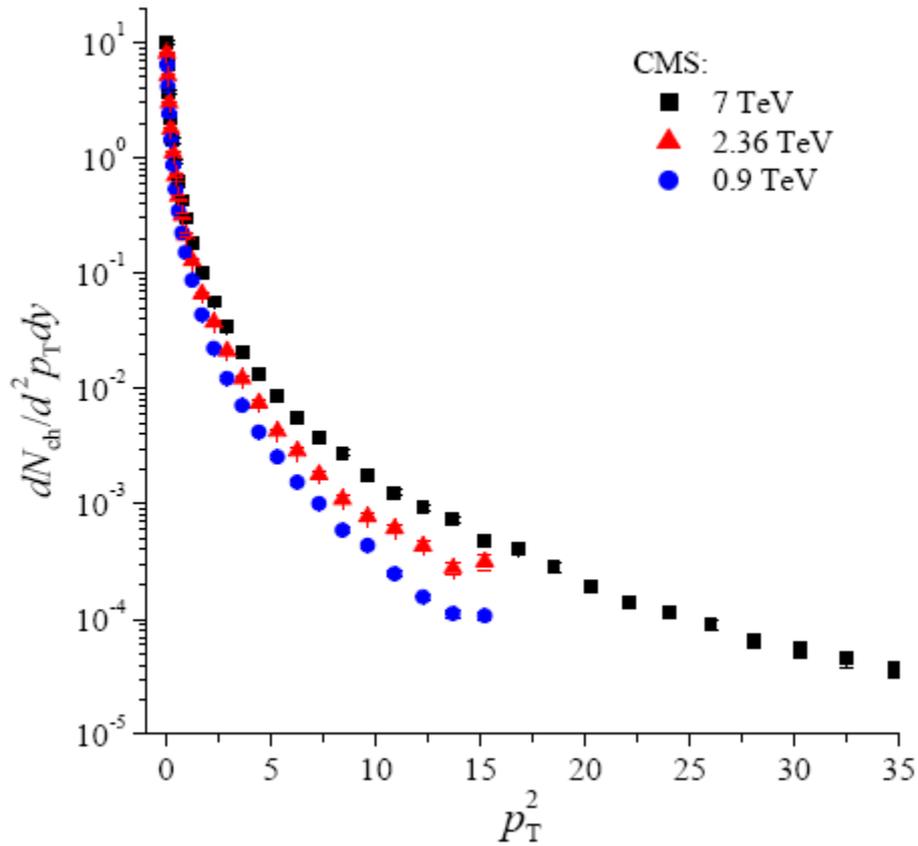
$$Q_{\text{sat}}^2(x) = Q_0^2 \left( \frac{1}{x_0} \frac{p_T}{\sqrt{s}} \right)^{-\lambda}$$



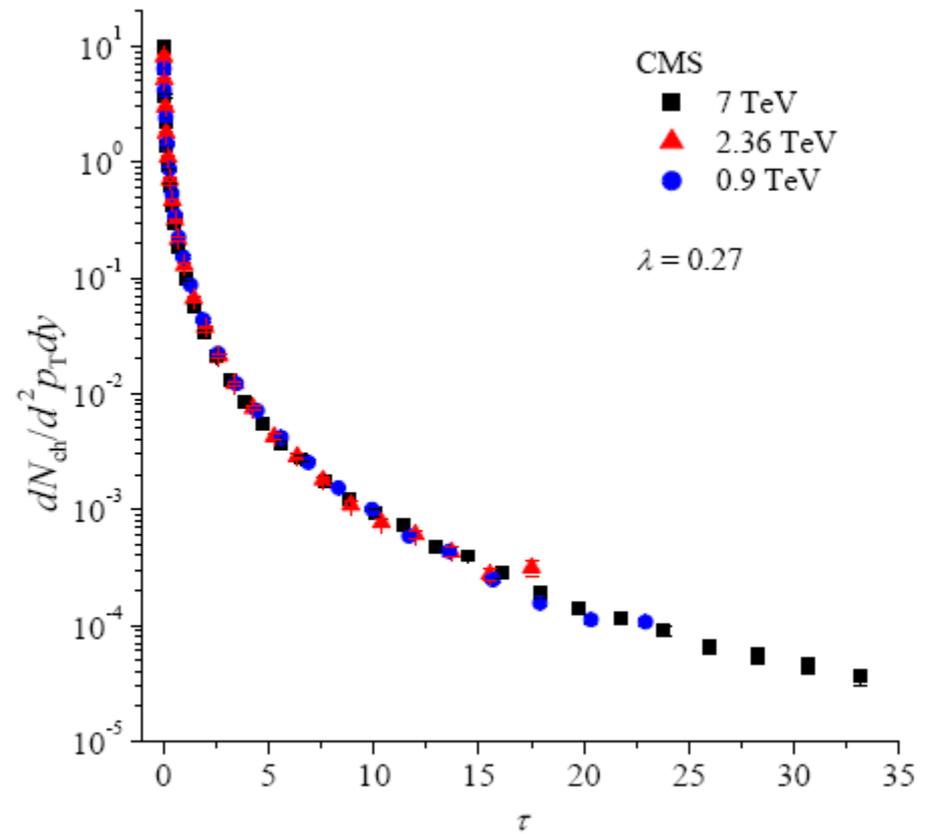
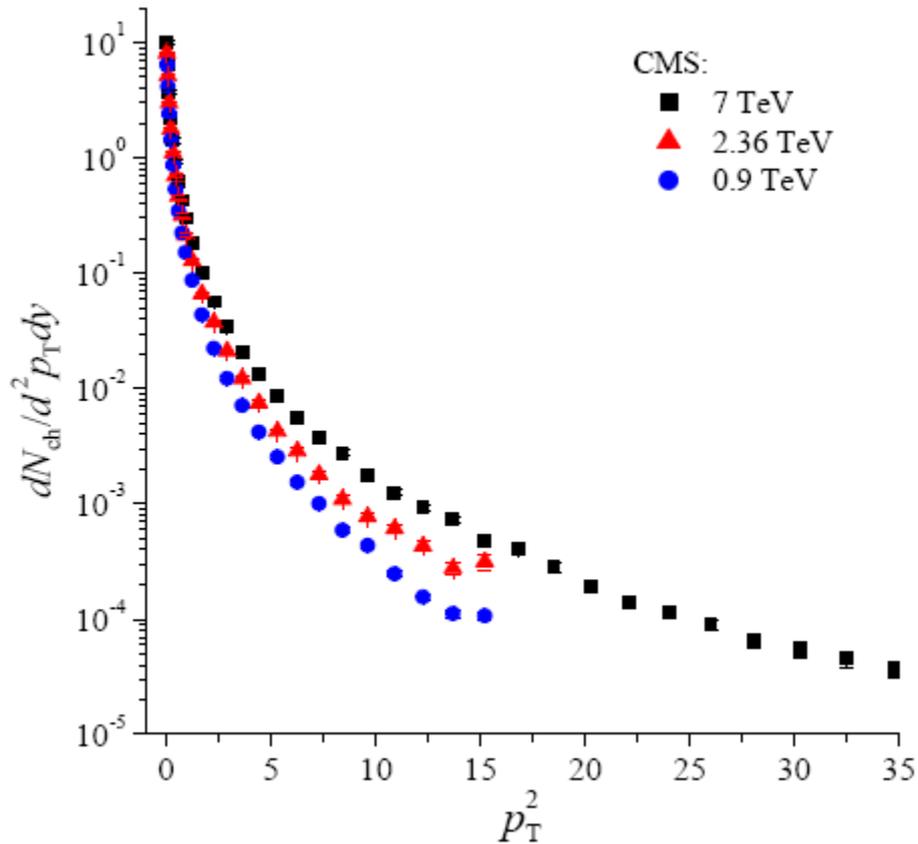
$$\tau = \frac{p_T^2}{Q_{\text{sat}}^2(p_T/\sqrt{s})} = \frac{p_T^2}{1 \text{ GeV}^2} \left( \frac{p_T}{\sqrt{s} \times 10^{-3}} \right)^\lambda$$

note that for  $\lambda = 0$  scaling variable  $\tau = p_T^2$

# Geometrical scaling of $p_T$ distributions

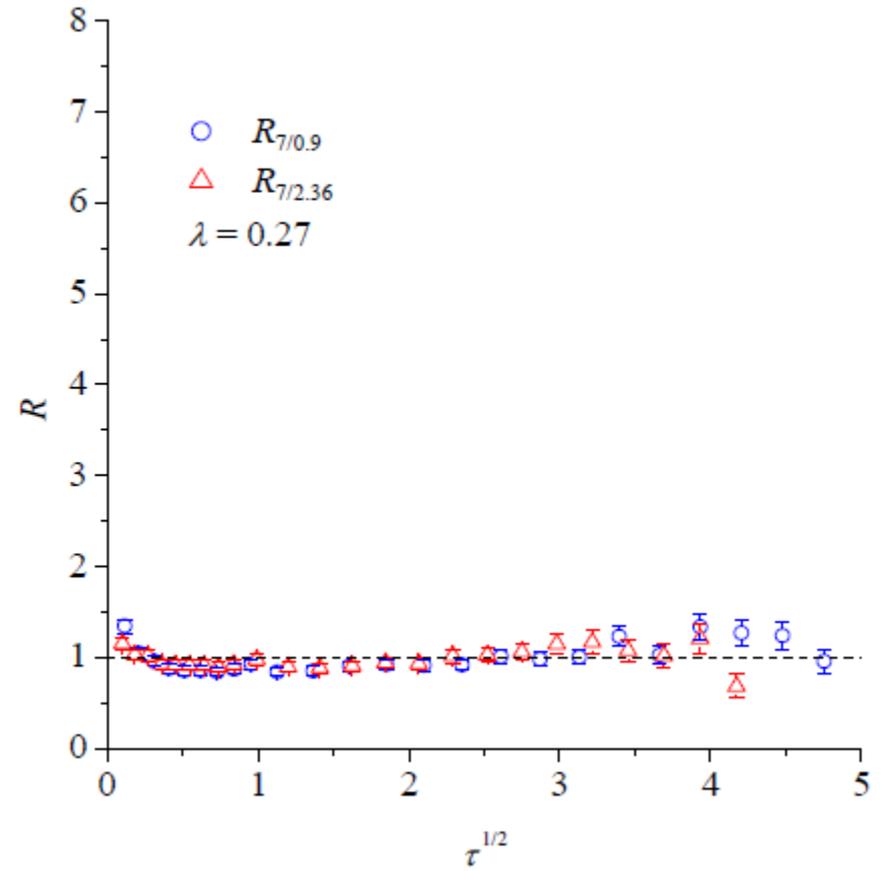
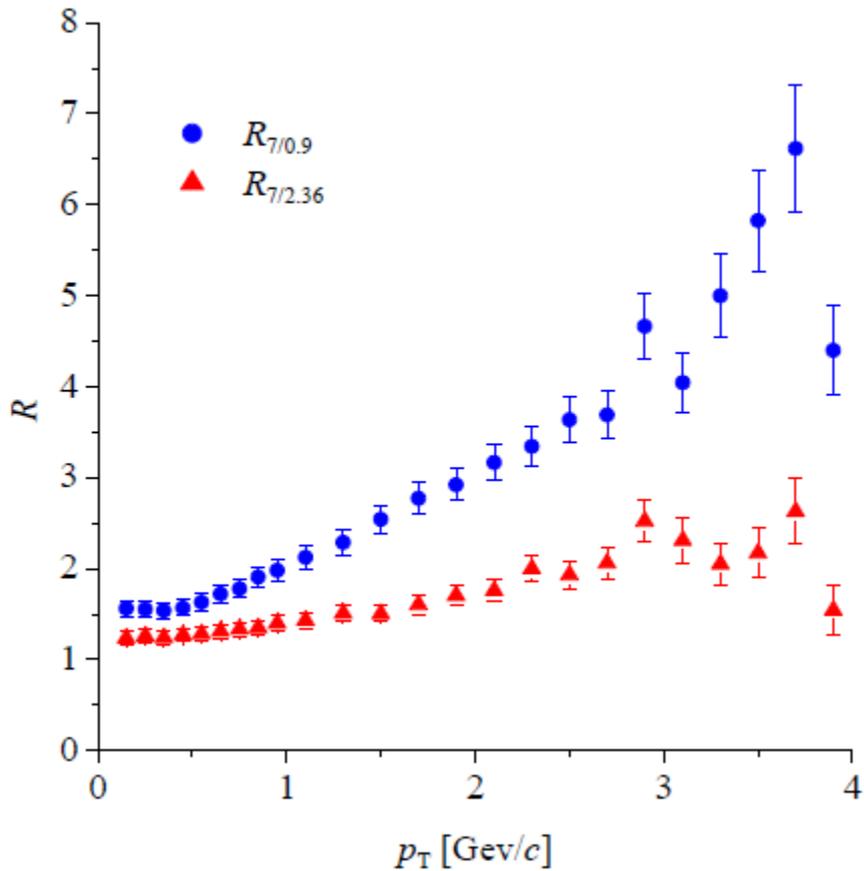


# Geometrical scaling of $p_T$ distributions



# Ratios of $p_T$ spectra

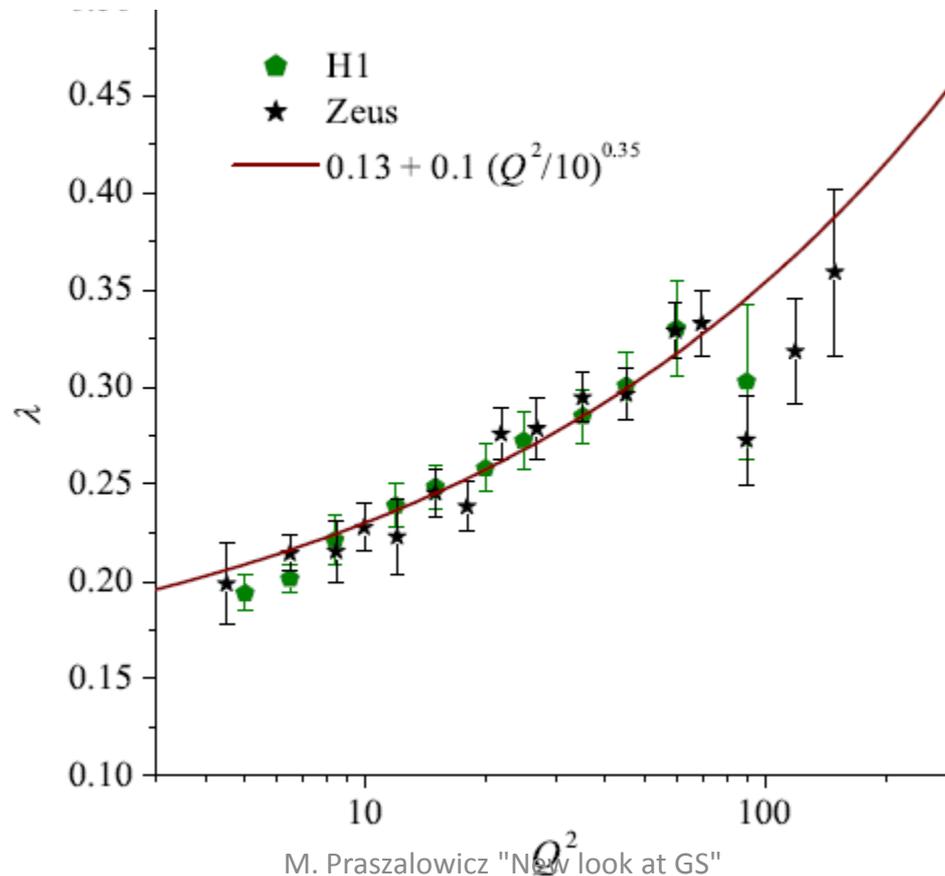
quality of GS can be examined by looking at the ratios:



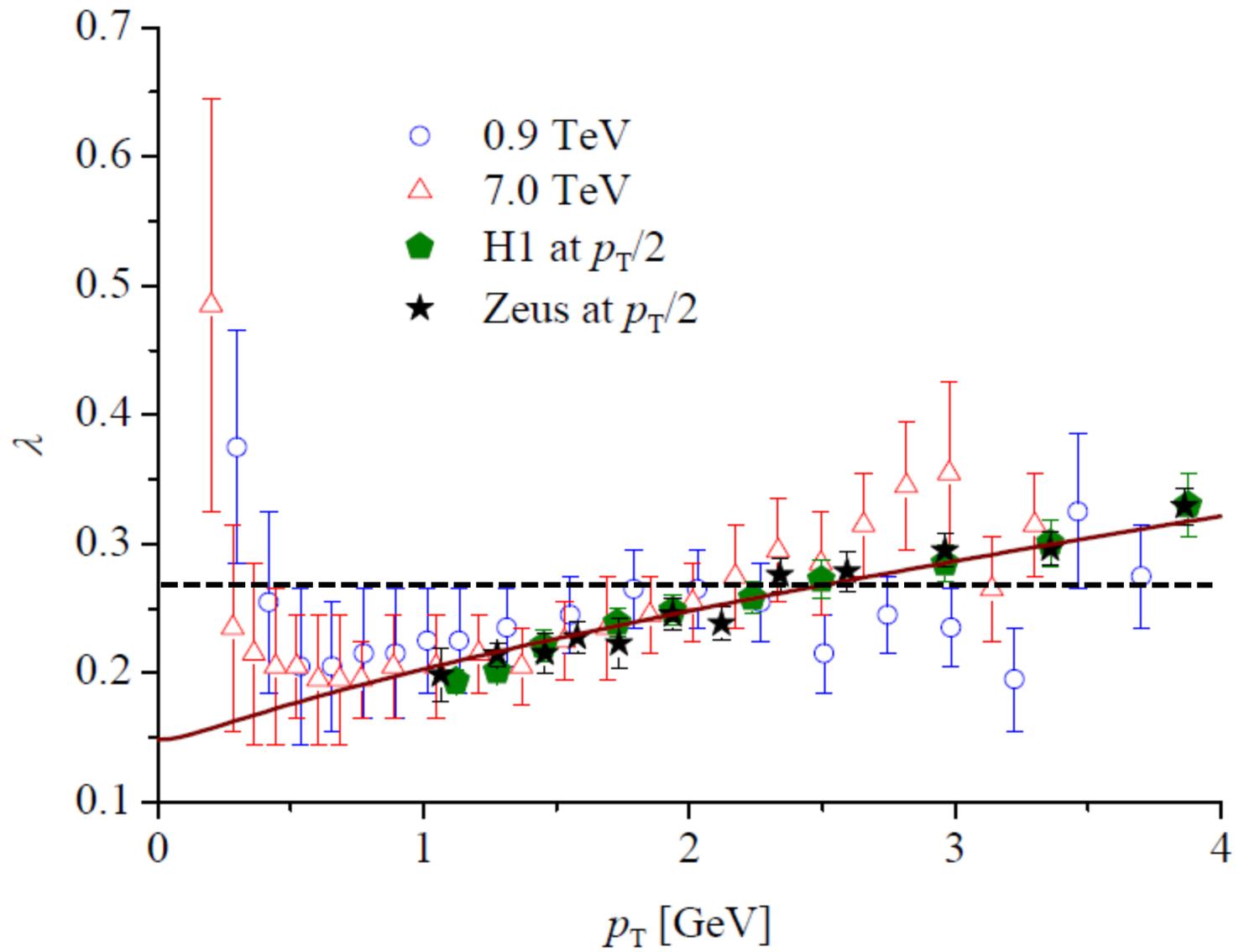
small increase with  $\tau$

# Geometrical Scaling with $\lambda(Q^2)$

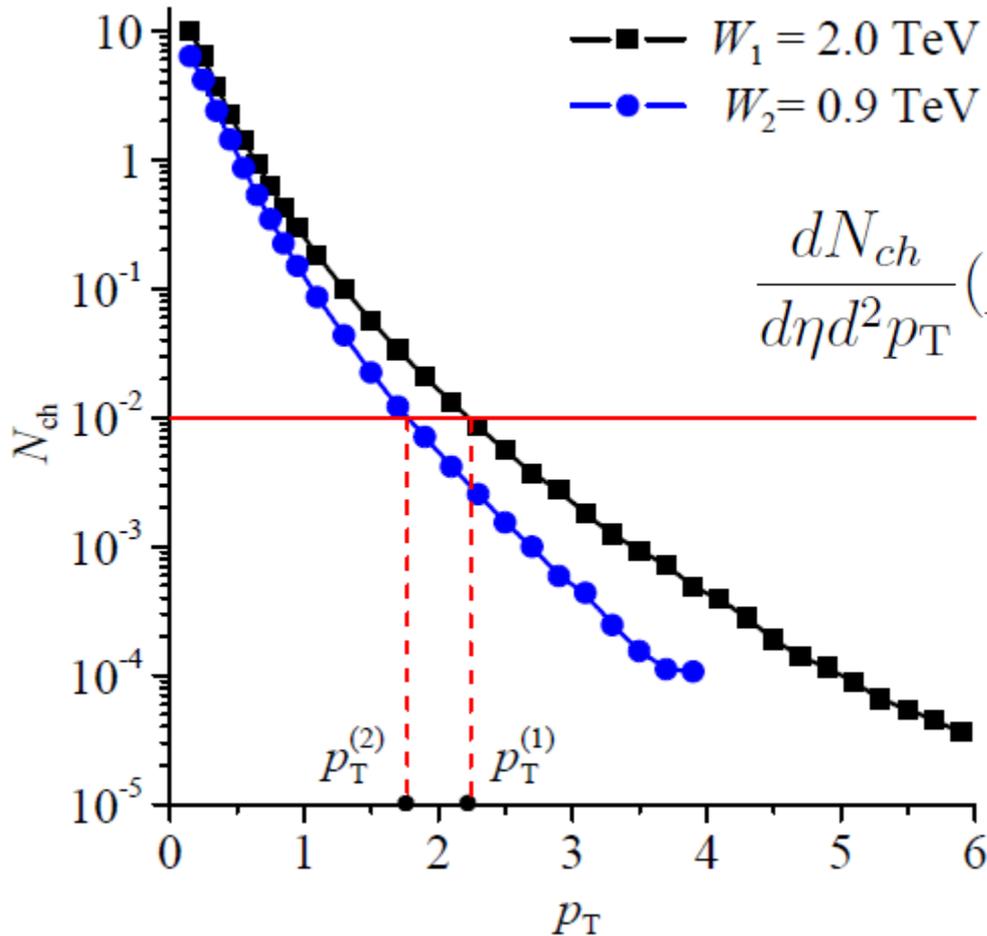
$$F_2(x, Q^2) \sim \sigma_0 Q_{\text{sat}}^2 \sim \frac{1}{x^{\lambda(Q^2)}}$$



H. Kowalski, L.N. Lipatov, D.A. Ross,  
G. Watt, Eur.Phys.J.C70:983-998,2010.



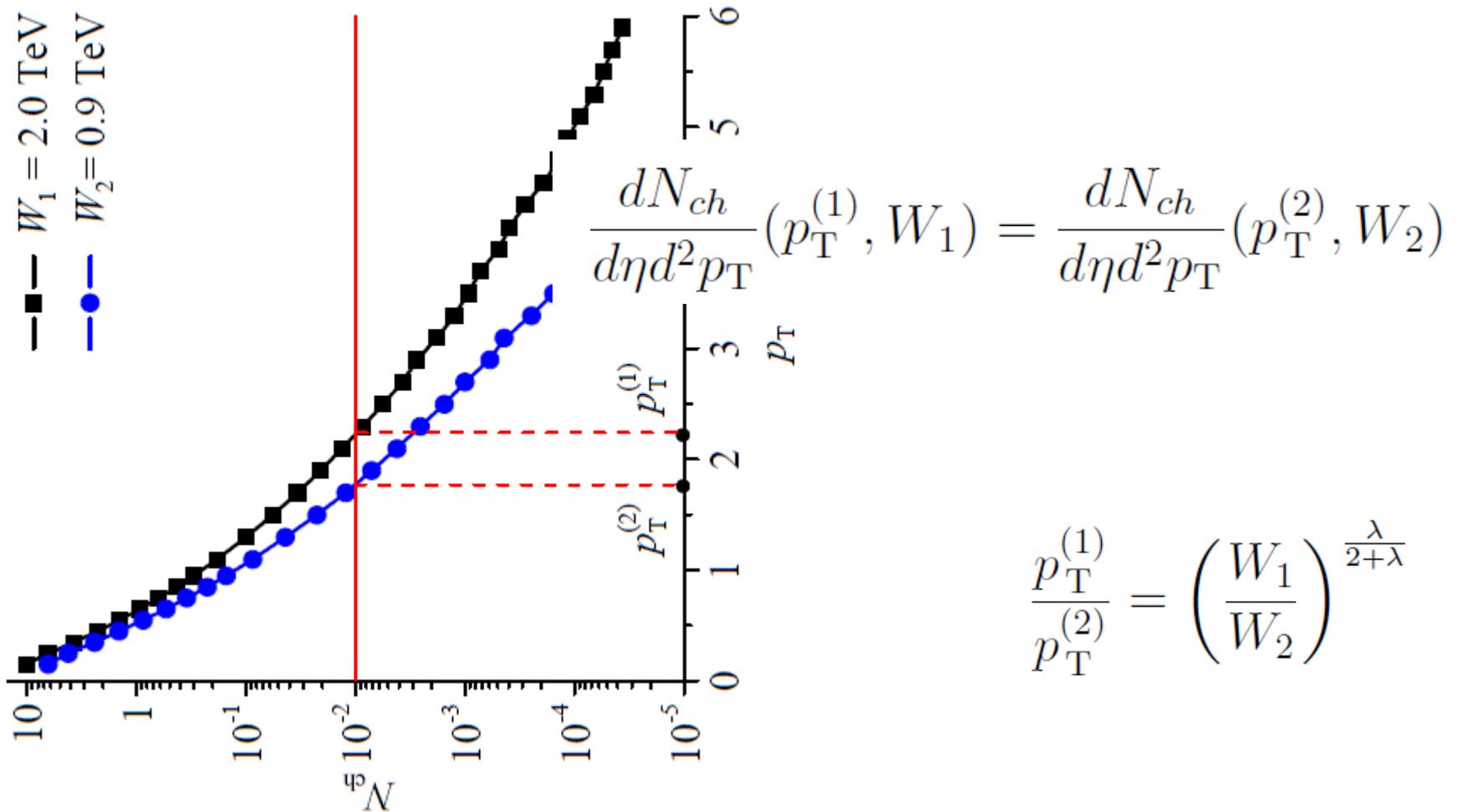
# Yet another look at GS



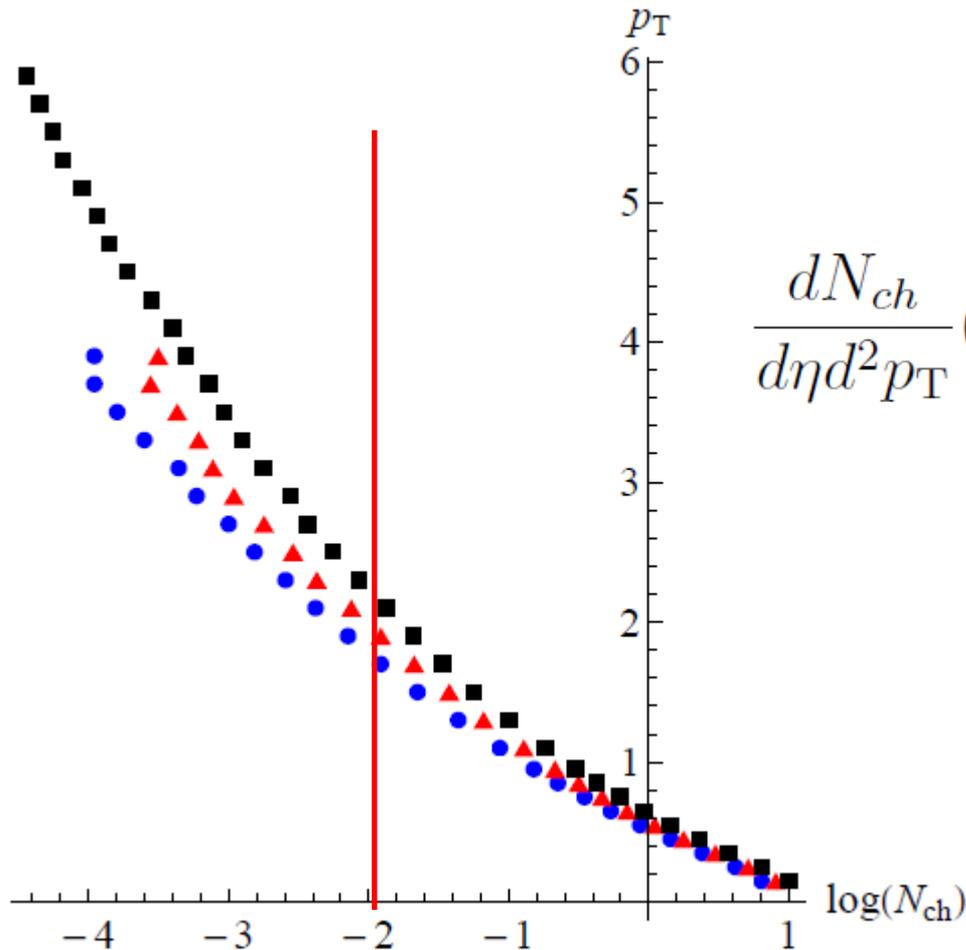
$$\frac{dN_{ch}}{d\eta d^2p_T}(p_T^{(1)}, W_1) = \frac{dN_{ch}}{d\eta d^2p_T}(p_T^{(2)}, W_2)$$

$$\frac{p_T^{(1)}}{p_T^{(2)}} = \left( \frac{W_1}{W_2} \right)^{\frac{\lambda}{2+\lambda}}$$

# Yet another look at GS



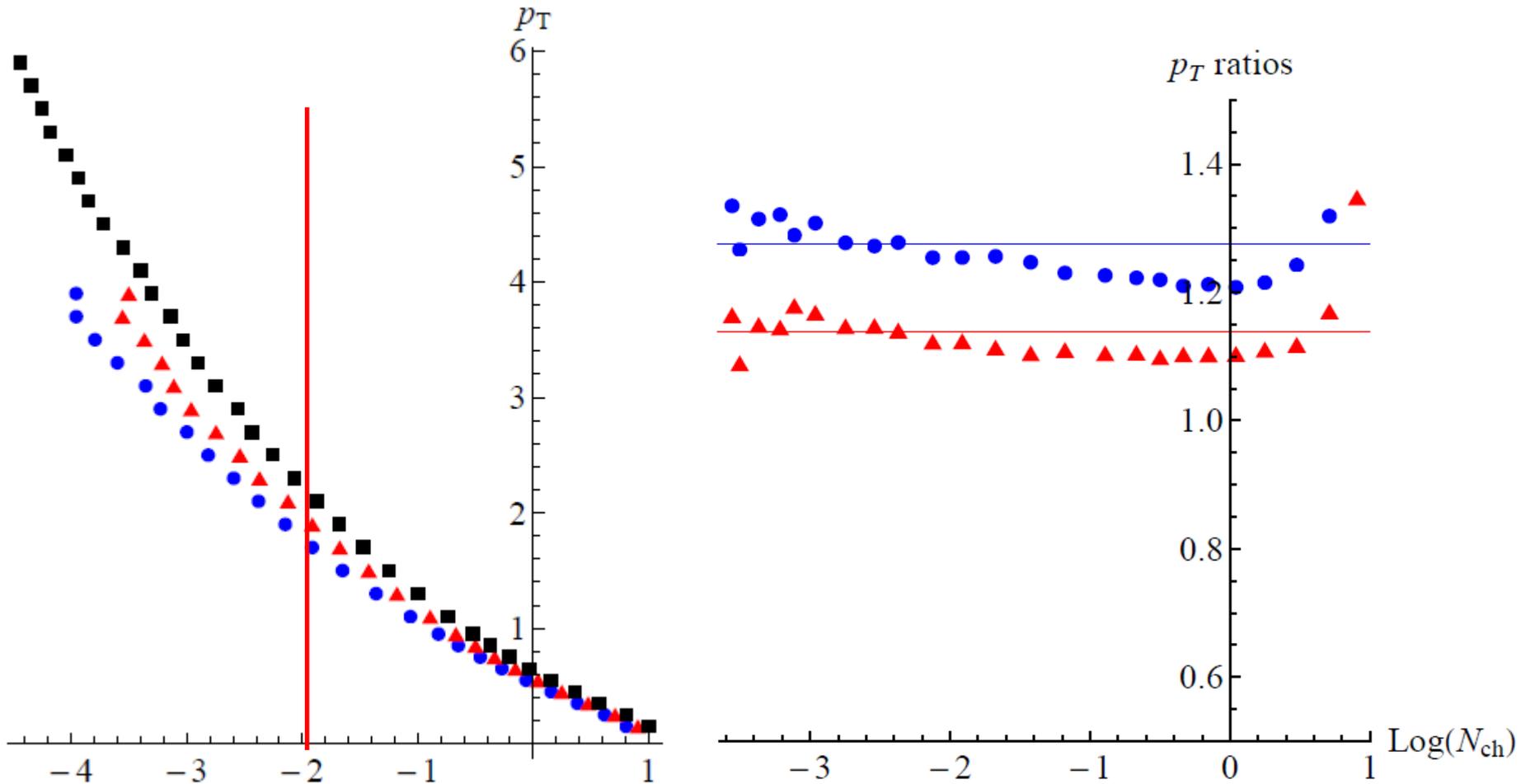
# Yet another look at GS



$$\frac{dN_{ch}}{d\eta d^2p_T}(p_T^{(1)}, W_1) = \frac{dN_{ch}}{d\eta d^2p_T}(p_T^{(2)}, W_2)$$

$$\frac{p_T^{(1)}}{p_T^{(2)}} = \left( \frac{W_1}{W_2} \right)^{\frac{\lambda}{2+\lambda}}$$

# Yet another look at GS



# To Do list

- check quality of GS for (coming soon) different LHC energies
- check rapidity dependence (so far ignored):  $x = e^{\pm y} p_T / \sqrt{s}$
- check GS for identified particles (mass effects)
- find universal shape  $F(\tau)$
- connection with unintegrated glue, connection with DIS
- construct models – difficult (no factorization)
- include  $b$  dependence
- study breaking of GS

# Geometrical scaling in HI collisions

much richer data than  
in pp allow to study different  
aspects of geometrical scaling:

- energy dependence (like pp)
- $A$  dependence
- centrality dependence

RHIC energies are probably  
too low, but let's try....

# Geometrical scaling in HI collisions

much richer data than  
in pp allow to study different  
aspects of geometrical scaling:

## But why should it work

- energy dependence (like pp)
- $A$  dependence
- centrality dependence

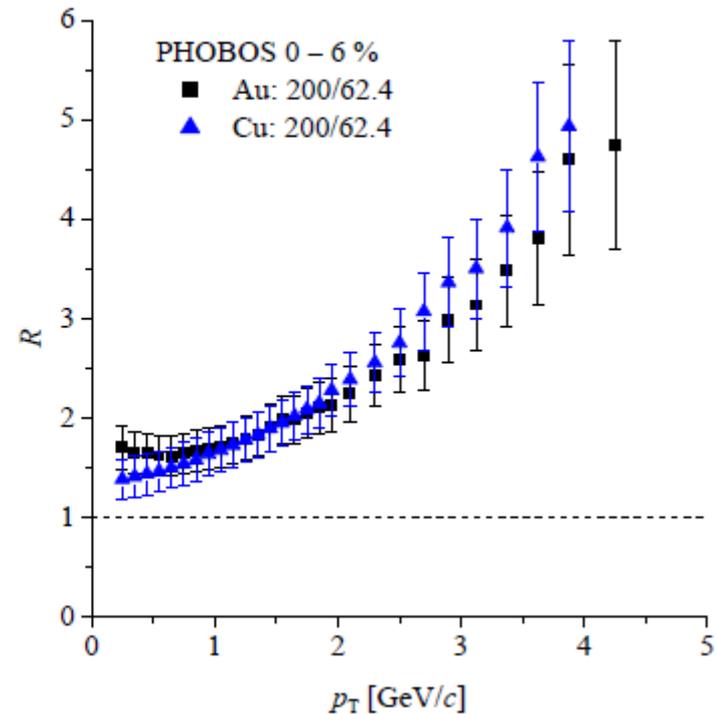
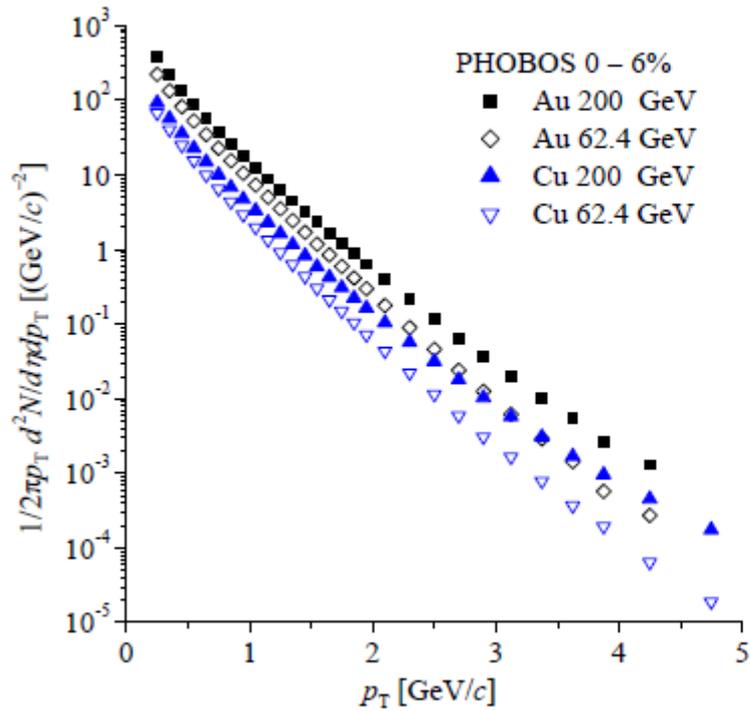
at all ?

RHIC energies are probably  
too low, but let's try....

# GS in HI: A dependence

B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **94** (2005) 082304 [arXiv:nucl-ex/0405003].

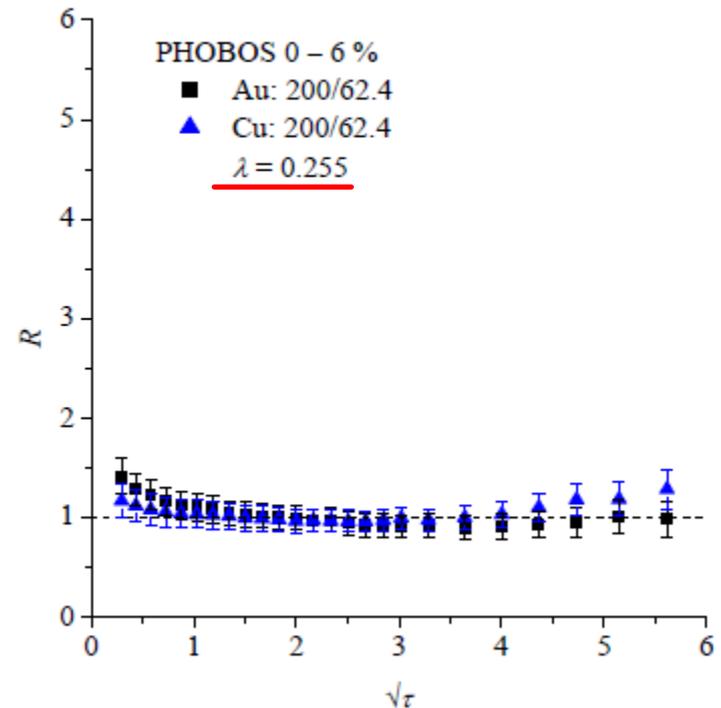
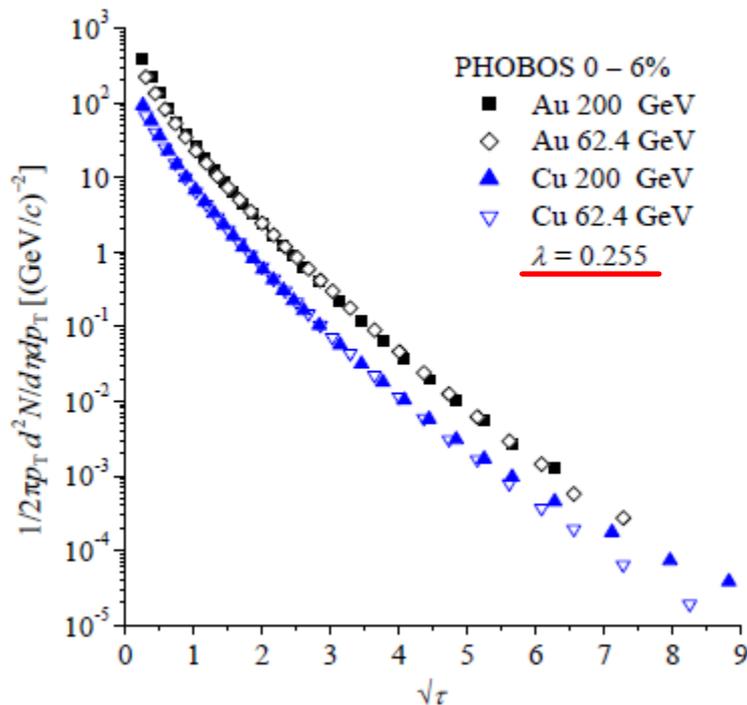
B. Alver *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **96** (2006) 212301 [arXiv:nucl-ex/0512016].



# GS in HI: A dependence

B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **94** (2005) 082304 [arXiv:nucl-ex/0405003].

B. Alver *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **96** (2006) 212301 [arXiv:nucl-ex/0512016].



# GS in HI: A dependence

$$\text{transverse parton size } s \sim \frac{\pi}{Q^2}$$

$$\text{cross section } \sigma \sim \alpha(Q^2) \frac{\pi}{Q^2}$$

$$\text{nucleus transverse size } S_A \sim \pi R_A^2$$

$$\text{critical \# partons} \sim \frac{S_A}{\sigma} \sim \frac{Q^2 R_A^2}{\alpha(Q^2)}$$

Saturation starts when # of partons in the nucleus  $N_A$  is equal to the critical #  $S_A/\sigma$

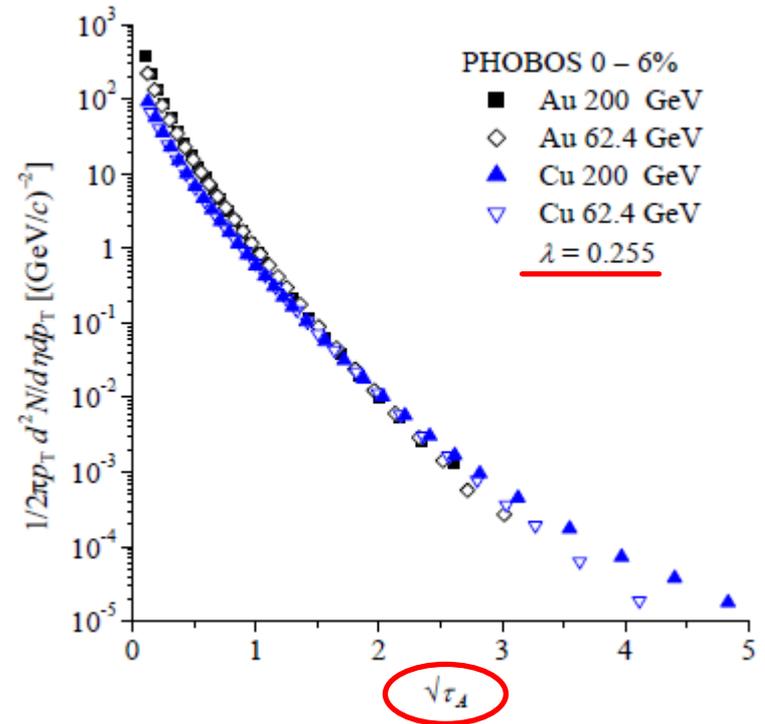
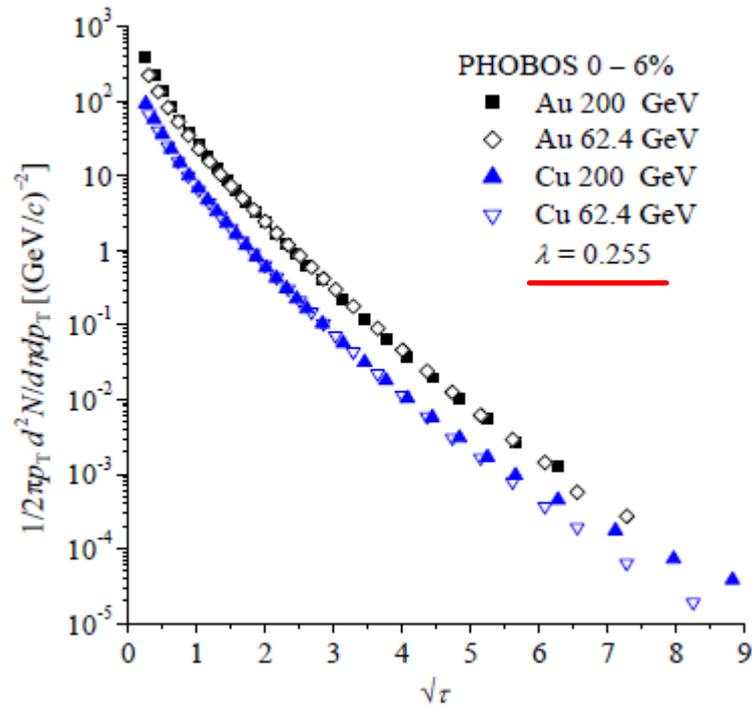
$$N_A \sim \frac{S_A}{\sigma} \implies Q_{\text{sat}}^2 \sim \alpha(Q^2) \frac{N_A}{R_A^2} \sim \frac{A}{A^{2/3}} \sim A^{1/3}$$

$$Q_{A \text{ sat}}^2 \sim A^{1/3} Q_{\text{sat}}^2$$

# GS in HI: A dependence

B. B. Back *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **94** (2005) 082304 [arXiv:nucl-ex/0405003].

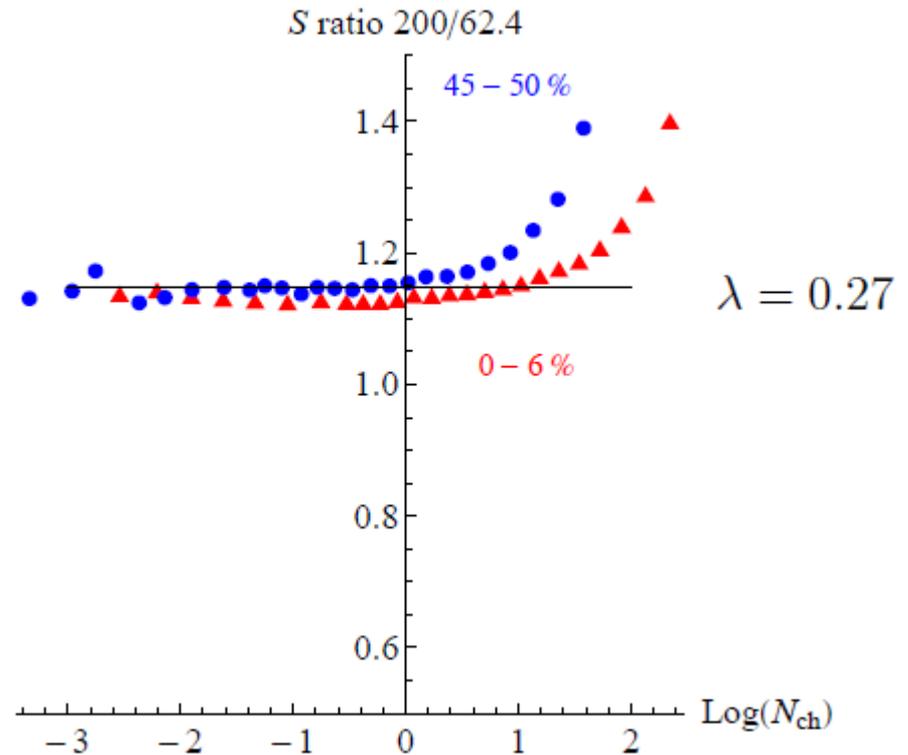
B. Alver *et al.* [PHOBOS Collaboration], Phys. Rev. Lett. **96** (2006) 212301 [arXiv:nucl-ex/0512016].



# Yet another look at GS in HI centrality dependence

preliminary

$$S_{W_1/W_2}^{p_T} = \frac{p_T^{(1)}}{p_T^{(2)}} = \left( \frac{W_1}{W_2} \right)^{\frac{\lambda}{2+\lambda}}$$

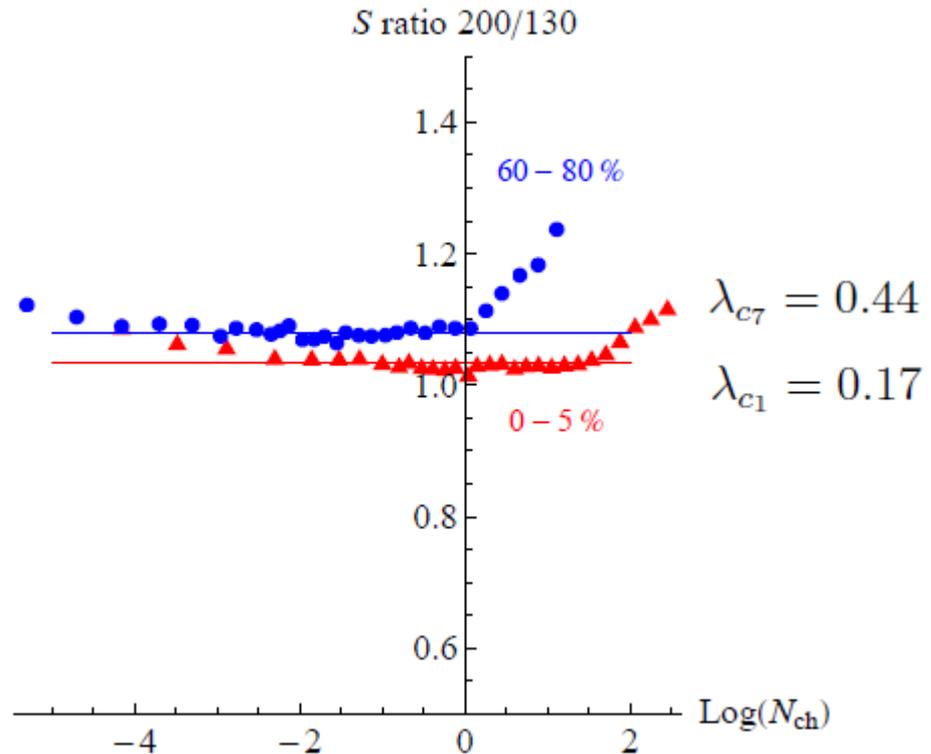


PHOBOS for two centrality classes: 0 – 5% (red triangles) and 45 – 50%

# Yet another look at GS in HI centrality dependence

preliminary

$$S_{W_1/W_2}^{p_T} = \frac{p_T^{(1)}}{p_T^{(2)}} = \left( \frac{W_1}{W_2} \right)^{\frac{\lambda}{2+\lambda}}$$



STAR for two centrality classes:  $c_1 = 0-5\%$  (red triangles) and  $c_7 = 60-80\%$

# Saturation scale in HI

D. Kharzeev and M. Nardi, Phys. Lett. B **507** (2001) 121.

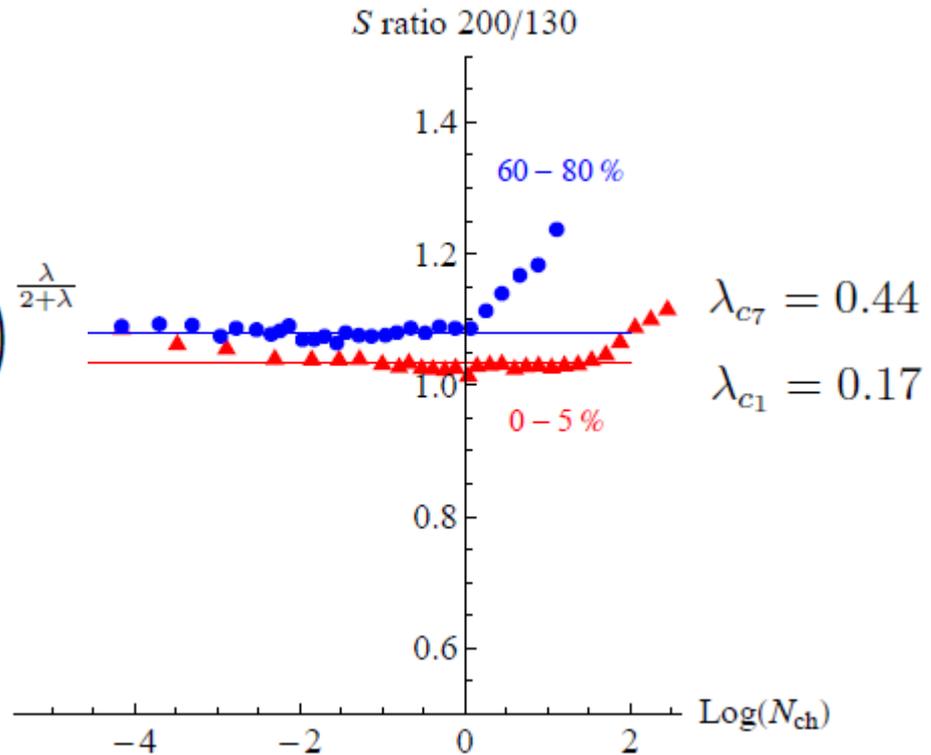
$$Q_{A,cs}^2 = A^{1/3} Q_s^2 \tilde{\rho}(c, W)$$

$A$  is total number of nucleons in the nucleus,  $Q_s^2$  is a "nucleon" saturation scale and  $\tilde{\rho}(c, W)$  is the (reduced) participant density in the transverse plane.

# Yet another look at GS in HI centrality dependence

preliminary

$$S_{W_1/W_2}^{p_T} = \frac{p_T^{(1)}}{p_T^{(2)}} = \left( \frac{\tilde{\rho}_1}{\tilde{\rho}_2} \right)^{\frac{1}{2+\lambda}} \left( \frac{W_1}{W_2} \right)^{\frac{\lambda}{2+\lambda}}$$



STAR for two centrality classes:  $c_1 = 0-5\%$  (red triangles) and  $c_7 = 60-80\%$

# Yet another look at GS in HI centrality dependence

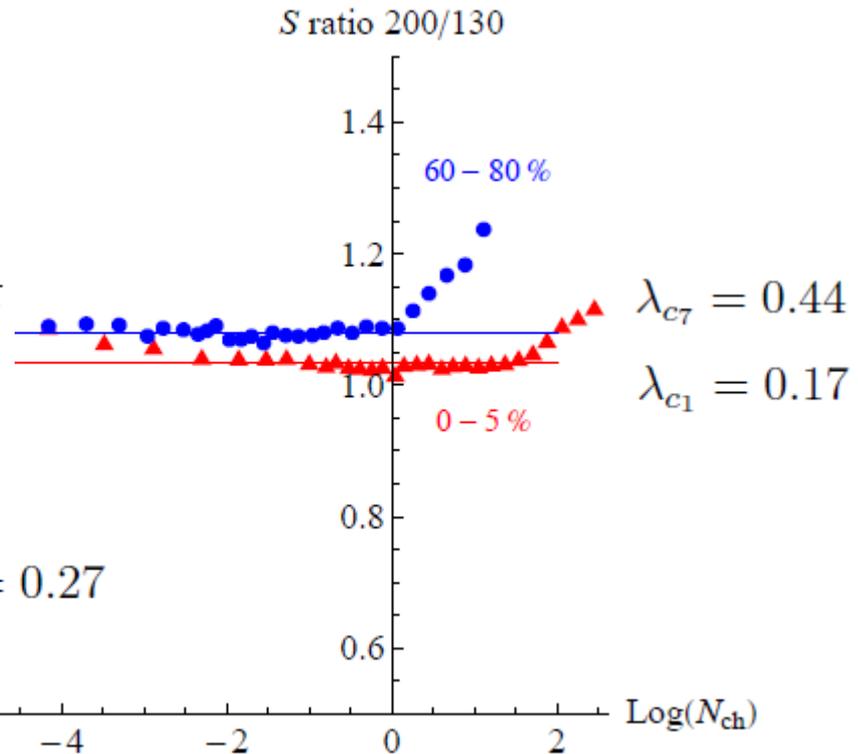
preliminary

$$S_{W_1/W_2}^{PT} = \frac{p_T^{(1)}}{p_T^{(2)}} = \left( \frac{\tilde{\rho}_1}{\tilde{\rho}_2} \right)^{\frac{1}{2+\lambda}} \left( \frac{W_1}{W_2} \right)^{\frac{\lambda}{2+\lambda}}$$

$$\frac{\tilde{\rho}(0 - 5\%, 200 \text{ GeV})}{\tilde{\rho}(0 - 5\%, 130 \text{ GeV})} = 0.96$$

$$\frac{\tilde{\rho}(60 - 80\%, 200 \text{ GeV})}{\tilde{\rho}(60 - 80\%, 130 \text{ GeV})} = 1.08$$

$$\lambda = 0.27$$



STAR for two centrality classes:  $c_1 = 0-5\%$  (red triangles) and  $c_7 = 60-80\%$

# STAR vs. PHOBOS

## STAR Au

- rapidity

$$|\eta| < 0.5$$

- energy: 200 & 130
- centralities up to 60-80 %

## PHOBOS Au (& Cu)

- rapidity

$$0.2 < \eta < 1.4$$

- energy 200 & 64.2
- centralities up to 45-50%

# To Do list for Heavy Ions

- same as for pp (waiting for higher energies than at RHIC)  
importance of  $y$  dependence!

plus

- centrality dependence
- dependence on  $A$
- comparison with thermal distributions

# Summary

1. Geometrical scaling in variable  $\tau = \frac{p_T^2}{Q_0^2} \left( \frac{p_T^2}{W^2} \right)^{\lambda/2}$  works very well for CMS (and also UA1)  $p_T$  spectra with  $\lambda = 0.27$
2. As a consequence  $dN_{\text{ch}}/dy \sim W^\lambda \langle p_T \rangle \sim W^{\lambda_{\text{eff}}/2}$  where  $\lambda_{\text{eff}} = \frac{2\lambda}{2 + \lambda} < \lambda$ .
3.  $p_T$  dependent  $\lambda$  improves GS
5. GS in HI works qualitatively at RHIC energies
6. Why it does work? (fragmentation, FSI, in HI hydro...)
7. LHC will provide further insight into GS:
  - energy dependence,
  - rapidity dependence
  - HI:  $A$  and centrality dependence