

Heavy-ion collisions: a point-of-view from non-equilibrium statistical physics

Robi Peschanski

Institut de Physique Théorique, Saclay

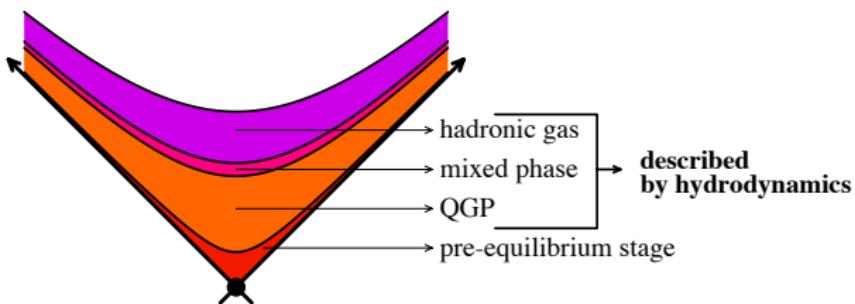
Krakow QCD Workshop

(November 27-30, 2011)

Why non-equilibrium Statistical Physics?

Far-from-equilibrium processes in heavy-ion collisions

Aim: Use models and tools from Statistical Physics



(from F.Gélis)

- Initial → Pre-equilibrium state \equiv non-thermal → non-thermal
- Pre-equilibrium → Hydrodynamics \equiv non-thermal → thermal
- Hydrodynamics → Hadronic \equiv thermal → non-thermal

1 Introduction: Non-equilibrium work identities

2 Thermal → Non-Thermal state

3 Non-Thermal → Non-Thermal state.

4 Non-Thermal → Thermal state.

5 Conclusions and Outlook

1 Introduction: Non-equilibrium work identities

2 Thermal → Non-Thermal state

3 Non-Thermal → Non-Thermal state.

4 Non-Thermal → Thermal state.

5 Conclusions and Outlook

- 1 **Introduction: Non-equilibrium work identities**
- 2 **Thermal → Non-Thermal state**
- 3 **Non-Thermal → Non-Thermal state.**
- 4 **Non-Thermal → Thermal state.**
- 5 **Conclusions and Outlook**

- 1 **Introduction: Non-equilibrium work identities**
- 2 **Thermal → Non-Thermal state**
- 3 **Non-Thermal → Non-Thermal state.**
- 4 **Non-Thermal → Thermal state.**
- 5 **Conclusions and Outlook**

- 1 **Introduction: Non-equilibrium work identities**
- 2 **Thermal → Non-Thermal state**
- 3 **Non-Thermal → Non-Thermal state.**
- 4 **Non-Thermal → Thermal state.**
- 5 **Conclusions and Outlook**

Hints in non-equilibrium statistical physics

Non-equilibrium Work identities

- Entropy growth vs. Jarzynski identity

Jarzynski (1997)

$$\langle \Delta S \rangle = \frac{1}{T} (\langle \mathcal{W} \rangle - \{ \langle \mathcal{W} \rangle - T \Delta S \}) \geq 0 \Rightarrow \langle \mathcal{W} \rangle \geq \Delta F.$$

$$\left\langle e^{-\mathcal{W}/T} \right\rangle = e^{-\Delta F/T}$$

- "Reverse process" \Rightarrow Crooks identity (for probabilities)

Crooks (1998)

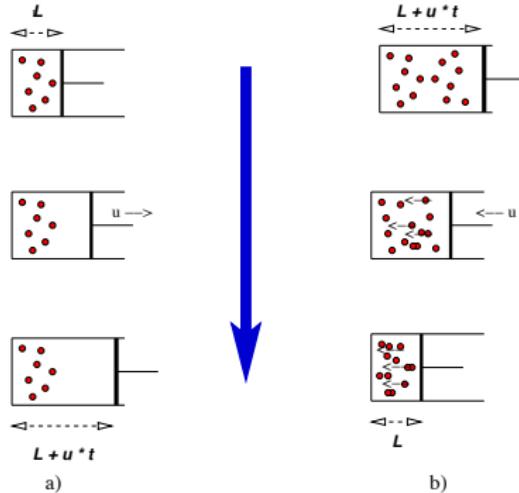
$$\mathcal{P}^R(-\mathcal{W}) = e^{-(\mathcal{W} + \Delta F)/T} \mathcal{P}(\mathcal{W})$$

- Crooks identity \Rightarrow Jarzynski identity

$$\begin{aligned} \left\langle e^{-(\mathcal{W} - \Delta F)/T} \right\rangle &\equiv \int_{-\infty}^{\infty} \mathcal{P}(\mathcal{W}) e^{-(\mathcal{W} - \Delta F)/T} d\mathcal{W} \\ &= \int_{-\infty}^{\infty} \mathcal{P}^R(-\mathcal{W}) d(-\mathcal{W}) = 1 = \left\langle e^{-(\mathcal{W} - \Delta F)/T} \right\rangle^R \end{aligned}$$

Thermal \rightarrow Non-Thermal

The Piston model ($L \gg ut \gg 1$, $n \sim 1$ shock in expansion)

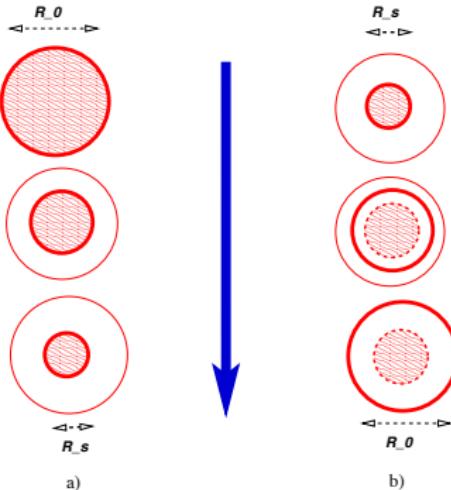


Lua & Grosberg (2008)

- Process a: Fast Expansion
- $\langle \mathcal{W} \rangle = -\frac{8}{\pi} \frac{e^{-\frac{1}{2}(ut)^2}}{L(ut)^2} \approx 0 > -\log \langle \exp -\mathcal{W} \rangle = \Delta F \equiv -\log \{(L + ut)/L\}$
- Process b: Fast Compression
- $\langle \mathcal{W} \rangle^{(\mathcal{R})} \approx \frac{1}{(1+\frac{ut}{L})} \cdot \frac{(ut)^3}{\pi L} \gg -\log \langle \exp -\mathcal{W} \rangle^{(\mathcal{R})} = \Delta F \equiv -\log \{L/(L + ut)\}$

A Cell model

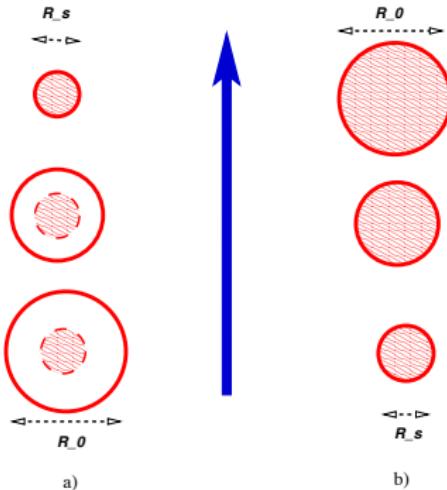
Fast variation of the QCD color correlation length



- a): Fast Compression \Rightarrow gluon distribution ($k \sim \mathcal{W}; R_s$) near saturation
- $\mathcal{P}^C(\mathcal{W})d\mathcal{W} \equiv \frac{\phi(k;R)}{\int_{k_0} \phi(k;R)dk} dk \theta(k-k_0) \sim \sqrt{\frac{8}{\pi}} R_s^2 k^2 e^{-R_s^2 k^2} R_s dk \theta(k-k_0)$
- b): Fast Expansion of a thermal state: use Crooks identity ($k \sim -\mathcal{W}; R_s$)
- $\mathcal{P}^E(k)dk \sim (Q_0 R_s)^3 e^{-k/T} \times \sqrt{\frac{8}{\pi}} R_s^2 k^2 e^{-R_s^2 k^2} R_s dk \theta(k-k_0)$

A Cell model

Dominant (rare) events vs. typical (frequent) events



- a): "Dominant" Rare event in Fast Compression
- $\langle \mathcal{W} \rangle^{(C)} = \sqrt{\frac{2}{\pi}} (Q_s - Q_0) \gg -\log \langle \exp -\mathcal{W}/T \rangle = \Delta F \sim 3T \log \frac{Q_s}{Q_0}$
- b): "Dominant" Rare event in Fast Expansion
- $\langle \mathcal{W} \rangle^{(E)} \approx -T \frac{Q_0^6}{Q_s^6} > \Delta F \sim -3T \log \frac{Q_s}{Q_0}$

Non-equilibrium, Non-thermal identities

- Sasa identity for a stationary dynamic state

Hatano,Sasa (2001)

$$\mathcal{P}_\lambda^{\text{Stat}}(z) = e^{-\Psi_\lambda(z)} \quad \lambda = \text{dynamics} \quad z = \text{phase-space}$$

$$\left\langle \exp - \int_{t_i}^{t_f} dt \dot{\lambda} \frac{\partial}{\partial \lambda} \{ \log \mathcal{P}_\lambda^{\text{Stat}}(z) \} \right\rangle_{(z; \lambda_i)} = 1$$

- Crooks-Sasa identity

cf. K. Mallick:

$$\mathcal{P}^C(z)/\mathcal{P}^E(z) = \exp - \int_{t_i}^{t_f} dt \dot{\lambda} \frac{\partial}{\partial \lambda} \{ \log \mathcal{P}_\lambda^{\text{Stat}}(z; \lambda) \}$$

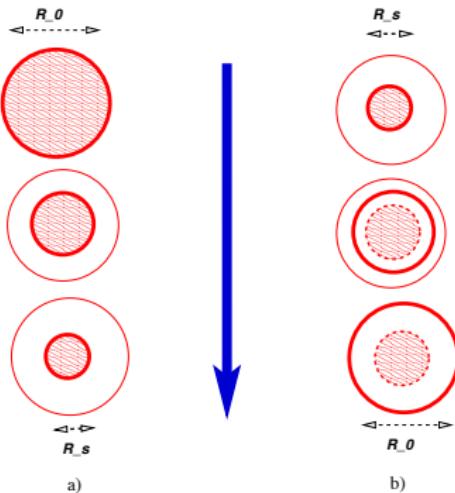
- Geometric Scaling ⇒ Sasa Identity

$$\mathcal{P}_{R_s}^{\text{Stat}}(k) \equiv \frac{\phi(kR_s)}{\int \phi(kR_s) dk} \quad R_s = \text{saturation scale} \quad k = \text{momentum}$$

$$\frac{\mathcal{P}^C(k)}{\mathcal{P}^E(k)} = \frac{\mathcal{P}_{R_s}^{\text{Stat}}(k)}{\mathcal{P}_{R_0}^{\text{Stat}}(k)} \xrightarrow{\text{Geometric Scaling}} \left\langle \exp - \int_{t_i}^{t_f} dt \dot{R} \frac{\partial}{\partial R} \{ \log \mathcal{P}_R^{\text{Stat}}(k) \} \right\rangle_{(k; R_0)} = 1$$

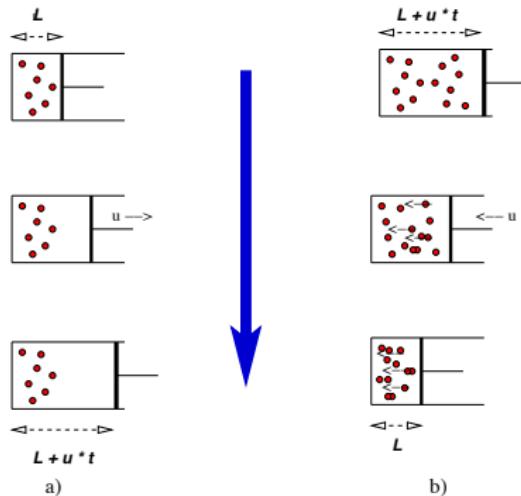
“Glasma Formation”: Toy Model

Fast shrinking of color correlation length



- a): Fast Non-Thermal Compression with Saturation
- $\langle \mathcal{W}_{Diss} \rangle^{(\mathcal{C})} = \Delta S^{(\mathcal{C})} \equiv \left[\langle \mathcal{W} \rangle^{(\mathcal{C})} \right] - \Delta F^{(\mathcal{C})} \sim \left[\frac{R_0^2}{R_s^2} - 1 \right] - \log \frac{R_0^2}{R_s^2}$
- b): Fast Non-Thermal Compression with de-Saturation
- $\langle \mathcal{W}_{Diss}^{(\mathcal{E})} \rangle = \Delta S^{(\mathcal{E})} \equiv \langle \mathcal{W} \rangle^{(\mathcal{E})} - \Delta F^{(\mathcal{E})} \sim - \left[1 - \frac{R_s^2}{R_0^2} \right] + \log \frac{R_s^2}{R_0^2}$

Thermalization Problem



- Process a): After Expansion and Many Shocks

Palmieri & Ronis (2007)

$$\bullet \quad T \rightarrow T \left[\frac{L}{L+ut} \right]^2$$

- Process b): Thermalization After Fast Compression: Generalized Entropy

$$\bullet \quad \Delta S^{(C)} = \left[\frac{R_0^2}{R_s^2} - 1 \right] - \log \frac{R_0^2}{R_s^2} \approx \frac{Q_s^2}{Q_0^2}$$

- **Hints from non-equilibrium statistical physics**

New Tools: Non-Equilibrium Work Identities

- **Thermal → Non-Thermal Expansion Process**

Superposition of the **soft** thermal spectrum with the **hard** perturbative tail

- **Non-Thermal → Non-Thermal Compression Process**

Entropy increase with Q_s^2

Comparison with Kutak (2001)?

- **Non-Thermal → Thermal Process**

New tools available: solvable toy models, generalized entropy, etc...

- **Outlook:** Comparison with microscopic descriptions : instabilities, AdS/CFT