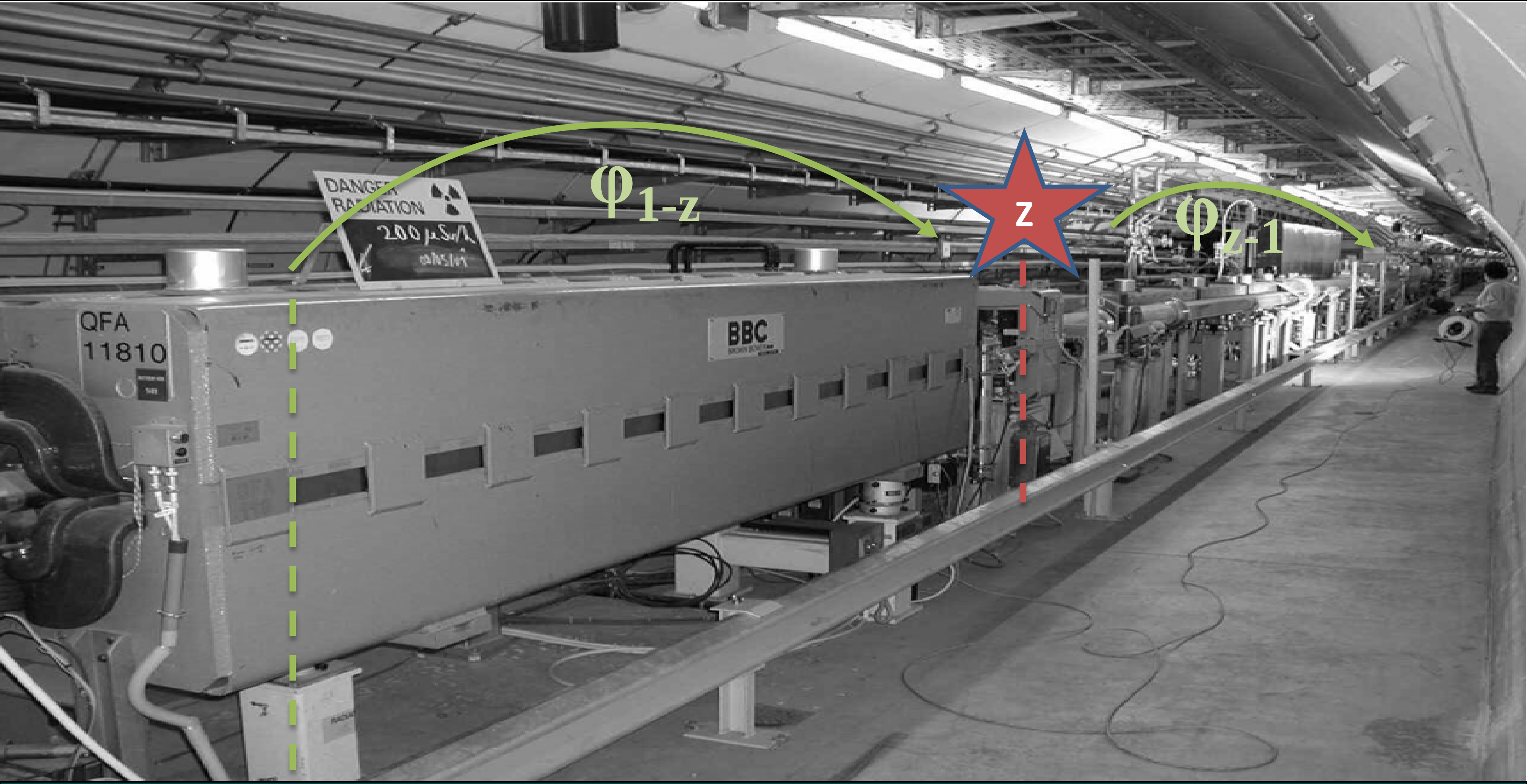
# Dependence of impedance Vs phase advance in HEADTAIL

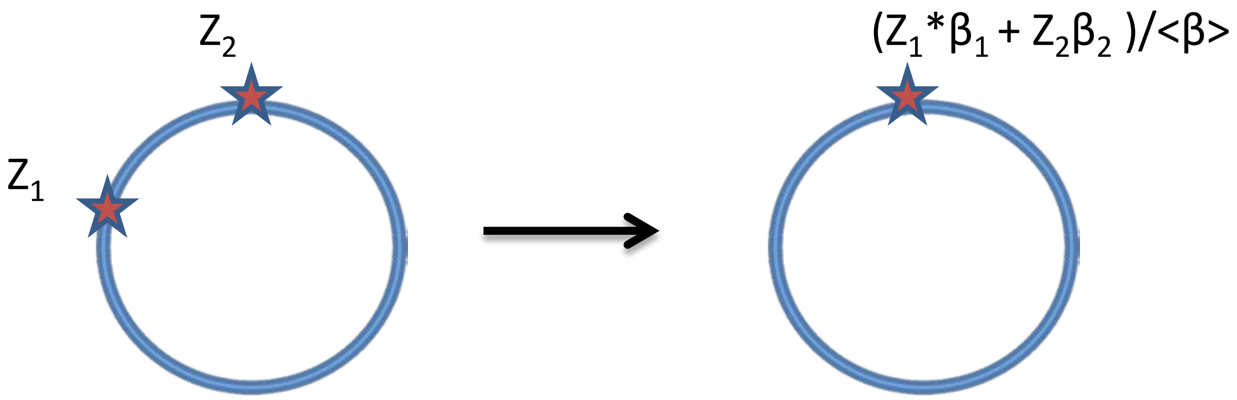
N.Biancacci, B.Salvant




**Acknowledgement: H. Bartosik, G.Rumolo**

# Lumped kick Vs distributed kick

Aim of this presentation is to study the reliability of our common procedure of lumping all the impedance in one point of the lattice *weighting the impedance by the beta function* in the points they are located.



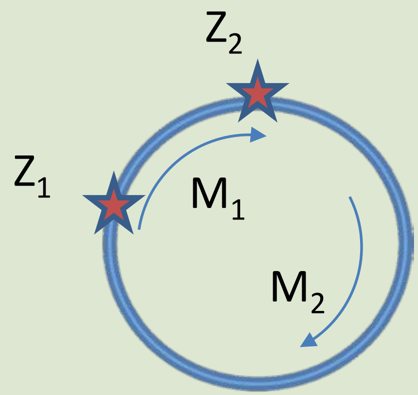

We’ll try to answer some questions:

1- Is the phase advance between the impedances playing any rule?

2- How the effective impedance depends on different optics (Q26 Vs Q20)?

# 1- Is the phase advance between the impedances playing any rule?

In order to answer the first question we first set up the simulation code we are going to use:

| Single kick (Nicolas ver.) | Multi kick (Diego's ver.) | Theory (Sacherer) |
|---|---|---|
| $Z=(Z_1*\beta_1 + Z_2\beta_2)/\langle\beta\rangle$   |   | $\left(Z_x^{eff}\right)_{m,n} = \frac{\sum_{k=-\infty}^{k=+\infty} Z_x\left(\omega_k^x\right) h_{m,n}\left(\omega_k^x - \omega_{\xi_x}\right)}{\sum_{k=-\infty}^{k=+\infty} h_{m,m}\left(\omega_k^x - \omega_{\xi_x}\right)}$ |
| All the impedance are lumped in the same point. The beam is tracked for 1024 turns via One Turn Map and each turn the Z interaction is calculated. | The beam is tracked along the MAD-X lattice creating matrices $M_1$, $M_2$.. $M_i$ between the points where the Zi interaction is calculated. *No need of any normalization on beta functions.* | The effective impedance is calculated the overlapping integral between the impedance and the bunch spectrum. |

# 1- Is the phase advance between the impedances playing any rule?

**Simple case:** only one impedance. Here already is reasonable that the starting point of the impedance would not play any rule.
**Note:** the *impedance is always the same* coming from a kicker (MKPA.11936). It was just artificially placed in different positions in the lattice as dipoles (MBB/MBA), quads (QD/QF) or kickers (MKs..) in order to study the effect, if one.

| | Single kick ($Z_{xeff}/Z_{yeff}$) [kΩ] | Multi kick ($Z_{xeff}/Z_{yeff}$) [kΩ] | Theory (Sacherer) ($Z_{xeff}/Z_{yeff}$) [kΩ] |
|---|---|---|---|
| **MBB.10870** | -138/485 | -134/479 | -138/482 |
| **MBA.10430** | -283/217 | -274/214 | -284/217 |
| **MBA.10230** | -281/217 | -274/214 | -284/218 |
| **MKPA.11936** | -95/685 | -95/670 | -92/668 |
| **QD.10310** | -65/965 | -63/939 | -65/919 |
| **QF.10210** | -343/171 | -332/168 | -345/171 |

Agreement within 5%. The impedance is different due to the beta functions. The dispersion is not playing a relevant role. Ok, but this was easy…

# 1- Is the phase advance between the impedances playing any rule?

**Two impedance case:** now we place two impedance sources with different phase advances and compute the *Zeff* in both cases (theory as reference).

| Q26 | Single kick ($Z_{xeff}/Z_{yeff}$) [kΩ] | Multi kick ($Z_{xeff}/Z_{yeff}$) [kΩ] | Theory (Sacherer) ($Z_{xeff}/Z_{yeff}$) [kΩ] |
|---|---|---|---|
| QF.10210 → QF.10610 $\Delta\varphi_y \approx 172^o$ | -685/345 | -655/340 | -690/344 |
| QF.10210 → QD.10510 $\Delta\varphi_y \approx 129^o$ | -407/1157 | -394/1128 | -410/1088 |
| QF.10210 → QF.10410 $\Delta\varphi_y \approx 86^o$ | -686/344 | -657/340 | -530/413 |
| QF.10210 → QD.10310 $\Delta\varphi_y \approx 43^o$ | -408/1160 | -393/1134 | -410/1091 |
| QF.10210 → MBB.10290 $\Delta\varphi_y \approx 39^o$ | -435/888 | -420/870 | -437/858 |
| QF.10210 → MBA.10250 $\Delta\varphi_y \approx 25^o$ | -543/503 | -523/497 | -547/499 |

Still, agreement within 5%, no phase advance effect.

# 1- Is the phase advance between the impedances playing any rule?

**Two impedance case:** now we place two impedance sources with different phase advances and compute the *Zeff* in both cases (theory as reference).

| Q20 | Single kick ($Z_{xeff}/Z_{yeff}$) [kΩ] | Multi kick ($Z_{xeff}/Z_{yeff}$) [kΩ] | Theory (Sacherer) ($Z_{xeff}/Z_{yeff}$) [kΩ] |
|---|---|---|---|
| QF.10210 → QF.10610 $\Delta\varphi_v \approx 133^o$ | -527/417 | -496/409 | -533/411 |
| QF.10210 → QD.10510 $\Delta\varphi_v \approx 100^o$ | -343/988 | -326/955 | -344/914 |
| QF.10210 → QF.10410 $\Delta\varphi_v \approx 68^o$ | -328/418 | -497/410 | -530/413 |
| QF.10210 → QD.10310 $\Delta\varphi_v \approx 32^o$ | -343/1007 | -325/954 | -343/911 |
| QF.10210 → MBB.10290 $\Delta\varphi_v \approx 28^o$ | -368/807 | -346/785 | -366/765 |
| QF.10210 → MBA.10250 $\Delta\varphi_v \approx 18^o$ | -436/538 | -416/530 | -442/529 |

Also here agreement within 5%, no phase advance effect. The impedances are different. We'll look closer to that later…

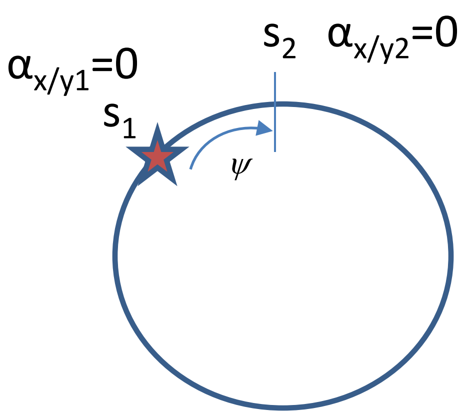
# 1- Is the phase advance between the impedances playing any rule?

**Conclusion(?):** the impedance from theory, single kick model and distributed kick one, is the same within a 5%. The differences are only due to the different beta functions in the place of the kicks as could be verified.

**But..** A previous work from B.salvant, N.Mounet and S.White found the equivalent kick from impedance in function of the phase advance difference between points 1 and 2 in the hyp of twiss functions $\alpha_x$, $\alpha_y$=0

Impedance source in $s_1$

Impedance kick $\Delta x'_1 = W x_1$ transported to $s_2$ in Headtail




$$\Delta x'_2 = W \frac{\beta_1}{\beta_2}\left(x_2 \cos^2\psi - \beta_2 x'_2 \sin\psi \cos\psi\right)$$

$$\Delta x_2 = W\beta_1\left(x_2 \cos\psi \sin\psi - \beta_2 x'_2 \sin^2\psi\right)$$

*B.salvant, N.Mounet and S.White*

# 1- Is the phase advance between the impedances playing any rule?

For **φ=nπ** the equivalent kick is the usual angle kick dependent on particle's position:

$$\Delta x'_2 = W \frac{\beta_1}{\beta_2} x_2 \qquad \Delta x_2 = 0$$

For **φ=π/2** the equivalent kick is the reverted: a position kick dependent on angle!

$$\Delta x_2 = -\beta_2 \beta_1 W x'_2 \qquad \Delta x'_2 = 0$$

If this were true the effect of the impedance should be very different. But our model reveals an independence on phase advance.

This is because the effective impedance is coming from a tune shift that's an averaged observable.

A quadrupolar error in s1 is provoking a tune shift independently of its local position equal to:

$$\Delta \nu_1 = \frac{1}{4\pi} \beta_1 \Delta k_1$$

moving it to a position s2 where we have a $\beta_2$ function we just scale the wake so that the effect will be the same:

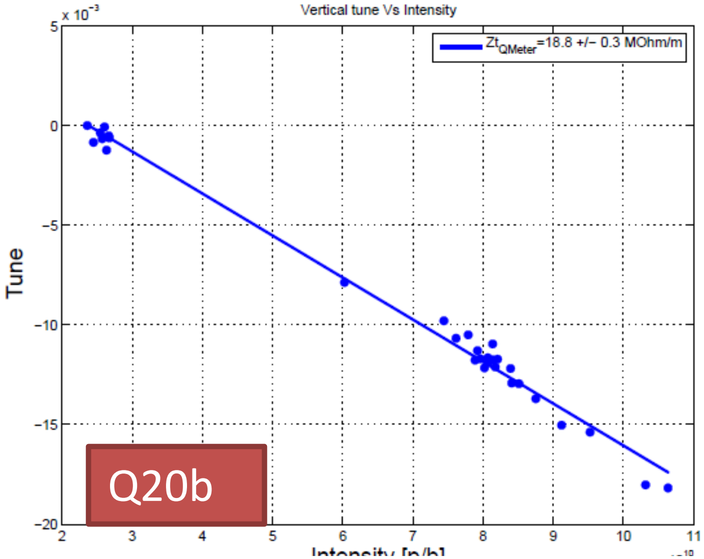
$$\Delta \nu_1 = \frac{1}{4\pi} \beta_2 \left[ \left( \frac{\beta_1}{\beta_2} \right) \Delta k_1 \right] = \frac{1}{4\pi} \beta_2 \Delta k_2$$

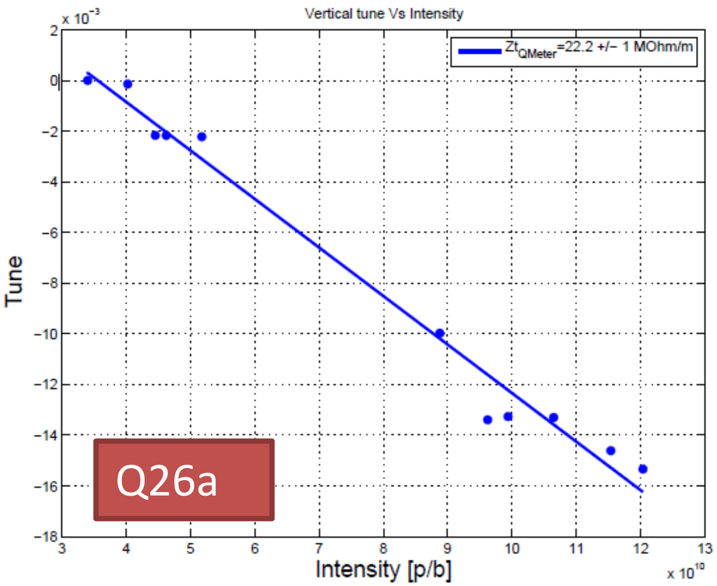
# 1- Is the phase advance between the impedances playing any rule?

**Conclusion:** the effect of scaling the wake makes the tune shift equivalent to the one provoked by the impedance in the original position. From the given formula point of view, it's not wrong to accept the case of $\phi=n\pi$: the effect over the tune is the same, what is wrong will be the induced beta beating that depends on the phase position of the applied perturbation. Applying the correct formula will lead to match also the beating.

**Question**(B.Salvant): The tune is the same for both distributed and lumped impedance model. Would it be the same for the rise time of the instability?

# 2- how the effective impedance depends on different optics (Q26 Vs Q20)?



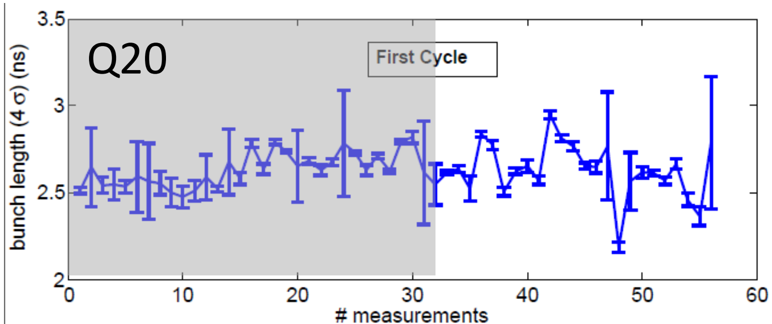



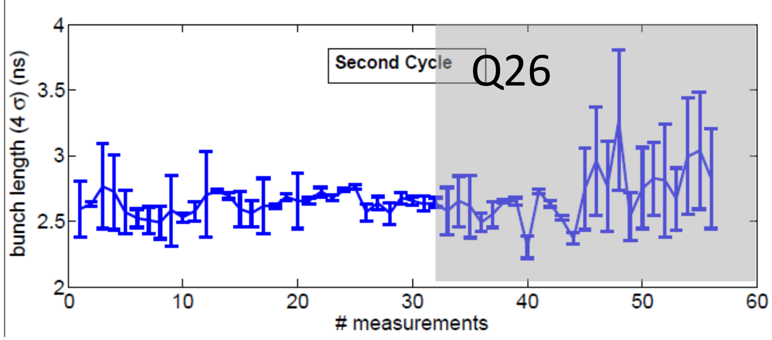

Slope_Q20: -0.021 (*10e-11)

Slope_Q26: -0.028 (*10e-11)

**Q20: (18.8 +/- 0.3) MΩ/m**

**Q26: (22.2 +/- 1.0) MΩ/m**

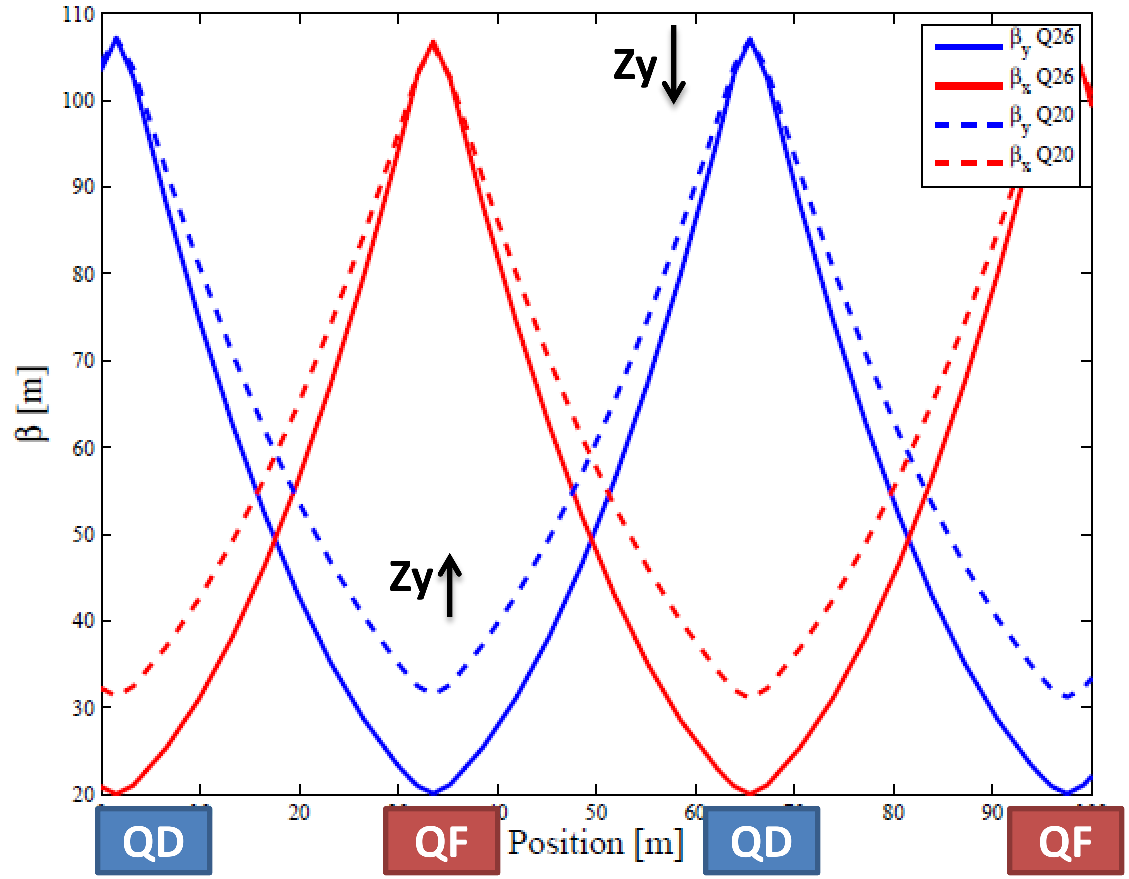



T.Argyropoulos and J.Muller

# 2- how the effective impedance depends on different optics (Q26 Vs Q20)?

The measured impedances from the two optics in SPS reveals a mismatch of almost 3MΩ/m. This is due to the different beta function that is sampling the impedance.




Near QFs and QDs the two optics have the almost the same value.

The averages are:
$<\beta>_{Q20}$=54 m
$<\beta>_{Q26}$=42 m

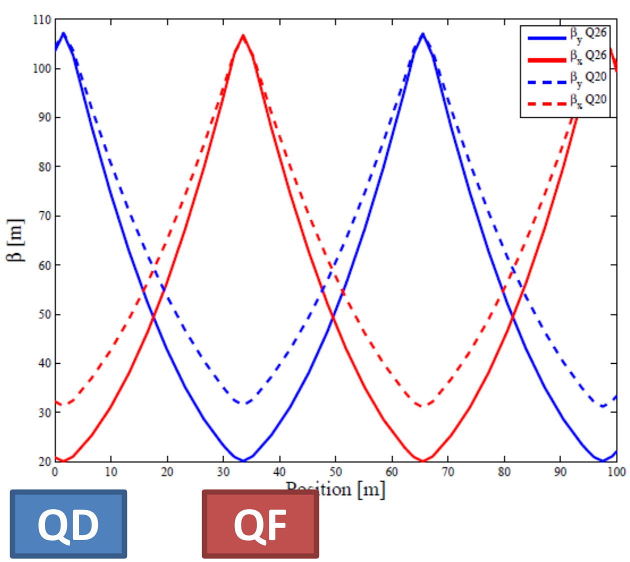
**Near QDs Zy is reduced;**
**Near QFs Zy is increased;**

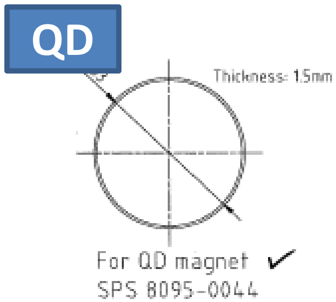
# 2- how the effective impedance depends on different optics (Q26 Vs Q20)?

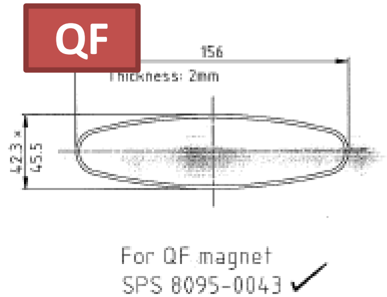
The measured impedances from the two optics in SPS reveals a mismatch of almost 3MΩ/m. This is due to the different beta function that is sampling the impedance. Since we observe a reduction of the impedance there should be some high impedance source near QDs that is reduced.

**ReWall?**

The resistive wall is spread all over the accelerator. It is true also that the beam pipe is quite different from QDs to QFs…



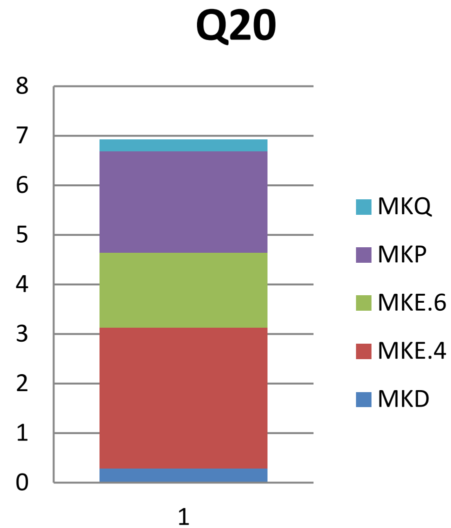



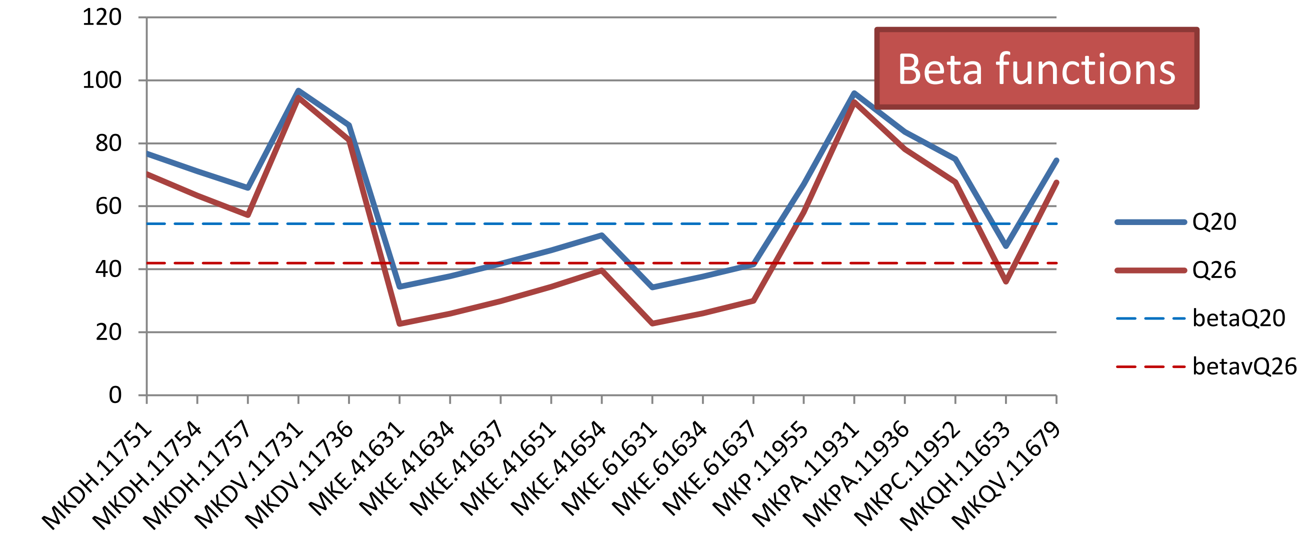



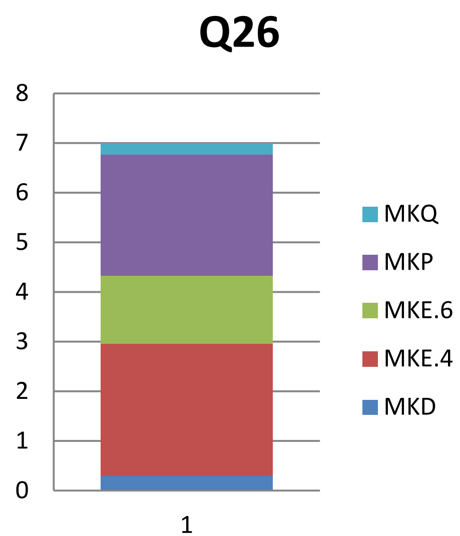

…but the impedance from QDs pipe is less than the one from QFs (closer to the beam). This push in the opposite direction the impedance amount.

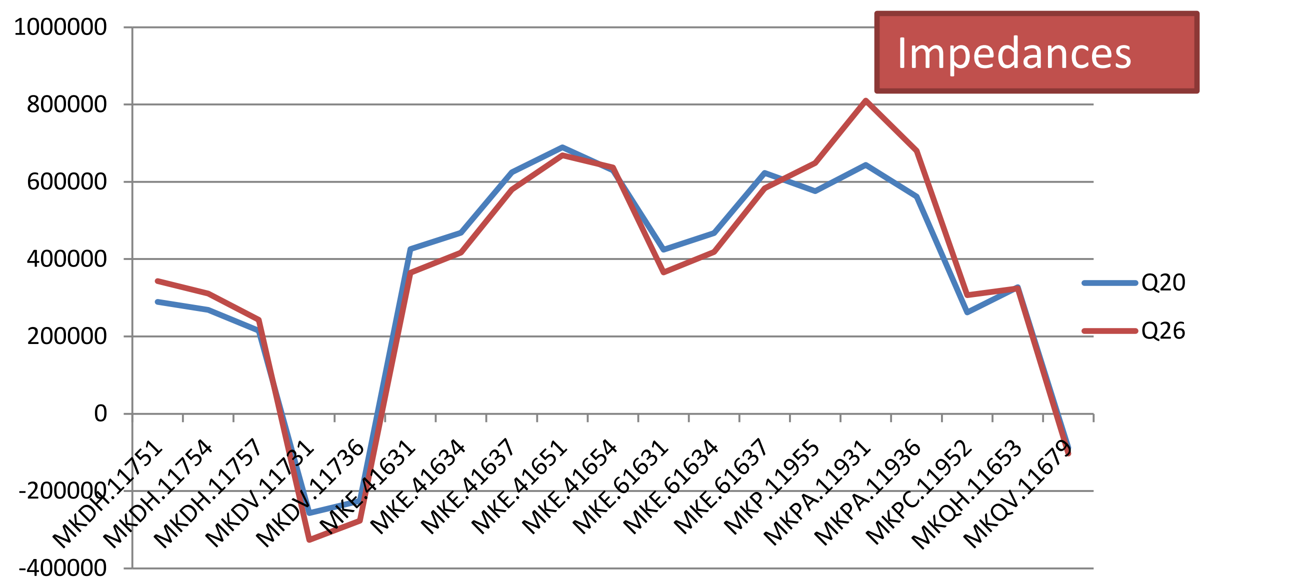
# 2- how the effective impedance depends on different optics (Q26 Vs Q20)?

**Kickers?** (Tsutsui's model)

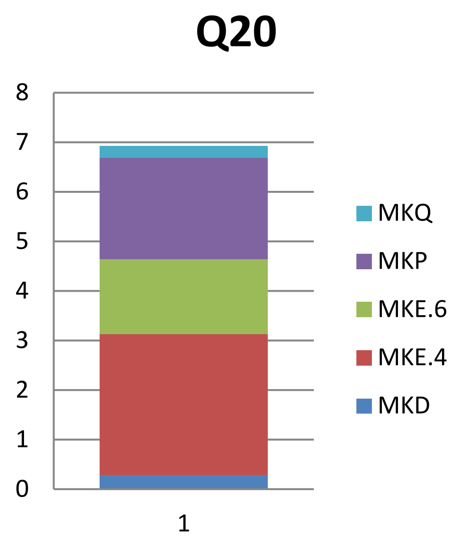


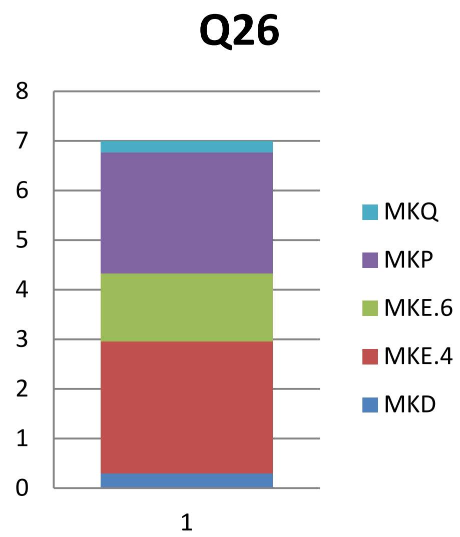



# 2- how the effective impedance depends on different optics (Q26 Vs Q20)?




- The total amount of impedance coming from Kickers is, in the frame of Tsutsui's theory, of almost 7 MΩ/m for both optics.
- In Q26 is 1-2% more than Q20.
- From measurement the impedance in Q26 seems to be almost 3M Ω/m higher, over the accuracy limit.
- MKPs present a bigger impedance somehow compensated by MKDVs (negative contribution).




Any other big sources?

# Conclusions

We answered the two question about interplay of impedance and phase advance:
1- The phase advance is not playing a role if we are interested in effective impedance measurements since the tune is an averaged concept. If we'll be interested in the impedance beating, the full lattice HEADTAIL model has to be used.
2- Passing from Q20 to Q26 do not necessarily means that the impedance will be the same. The MD of august revealed already a big reduction of the impedance.

**Outlook**

1- The tune is averaged, but the rise-time? there could be a discrepancy between the lumped and distributed HT model. To be checked!
2- Playing with the beta function is giving us many hints about impedance localization. How can we modify the optics in order to reveal it?

Dear daddy Noel, I wrote 20k lines of Matlab code… what about sending me an impedance for Christmas?

**Buon Natale!!!**

# Beam parameters

**<u>HEADTAIL beam parameters</u>**

```
sigma_z=0.149705;
int=1e10:2e10:9e10;
e=1.602176487*10^-19;
c= 299792458; % [cm/s] units
mp=1.672621637*10^-27;
gamma=27.7286; @injection
beta=1/sqrt(1+1/gamma^2);
v=beta*c;
circ=6911.5038;
f_rev=v/circ;
T0=1/f_rev;
tunex0=20.13;
tuney0=20.18;
betav_Y=circ/(2*pi*tuney0);
betav_X=circ/(2*pi*tunex0);
```

# Twiss function for selected elements

| | βx / βy | αx / αy | $D_x$ / $D_y$ |
|---|---|---|---|
| **MBB.10870** | 43.01 / 56.21 | 1.28 / -1.56 | 3.8/0 |
| **MBA.10430** | 88.06 / 25.36 | 2.1 / -0.74 | 1.6/0 |
| **MBA.10230** | 87.96 / 25.40 | 2.19 / -0.75 | 5/0 |
| **MKPA.11936** | 29.55 / 78.09 | -0.9 / 1.995 | 0.9/0 |
| **QD.10310** | 20.12 / 107.07 | ~0 / ~0 | 0 |
| **QF.10210** | 106.97 / 20.08 | ~0 / ~0 | 0 |

# Twiss function for selected elements

| Q26 | βx / βy | $\alpha_x$ / $\alpha_y$ | $\mu_x$ / $\mu_y$ |
|---|---|---|---|
| QF.10210 | 106.97/20.08 | 0/0 | 0.24/0.25 |
| QD.10310 | 31.20/107.08 | 0/0 | 0.36/0.37 |
| QF.10410 | 107.08/20.08 | 0/0 | 0.48/0.49 |
| MBB.10290 | 28.65/79.94 | 0.8/-1.9 | 0.32/0.36 |
| MBA.10250 | 62.72/38.16 | 1.6/-1.1 | 0.26/0.32 |
| QD.10510 | 20.14/106.76 | 0/0 | 0.6/0.61 |
| QF.10610 | 106.98/20.07 | 0/0 | 0.72/0.73 |

| Q20 | βx / βy | $\alpha_x$ / $\alpha_y$ | $\mu_x$ / $\mu_y$ |
|---|---|---|---|
| QF.10210 | 107.06/31.05 | 0/0 | 0.19/0.19 |
| QD.10310 | 31.20/106.67 | 0/0 | 0.28/0.28 |
| QF.10410 | 106.4/31.42 | 0/0 | 0.37/0.38 |
| MBB.10290 | 40.25/84.06 | 1/2 | 0.25/0.27 |
| MBA.10250 | 70.66/48.99 | 1/1 | 0.21/0.24 |
| QD.10510 | 31.35/107.14 | 0/0 | 0.46/0.47 |
| QF.10610 | 107.43/31.16 | 0/0 | 0.56/0.56 |