Multi-Vector Boson Production via Gluon-Gluon Fusion

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Introduction

- LHC is now running for quite a while. The data does not seem to require us to go beyond the standard model. Various extensions of the standard model are getting seriously constrained.

- Even the window for the Higgs boson mass is narrowing alarmingly. (Of course, there seems to be some suggestions for the existence of the Higgs boson in this narrow range.) Possibility of a heavy Higgs boson would remain.

- It would appear that to test the model and look for the directions for the extension, one may need to probe as many standard model processes as possible.

- In particular, one would be looking for processes that have many particles in the final state, or have small cross-section.
Introduction

- At LHC and proposed hadron colliders such as HE-LHC, one of the features is large gluon luminosity.
- This has already been seen for a number of processes. The processes with one or two gluons become more important as one goes from Tevatron energies to LHC energies and beyond.
- Therefore, processes with gluon-gluon scattering would be important and observable at LHC. They can also contribute to the backgrounds to the BSM physics scenarios.
- We are interested in a particular set of processes $pp \rightarrow VV'g/\gamma X$. Here $V, V' = W, Z, \gamma$. We are particularly interested in the contribution from the gluon-gluon scattering. Since $W, Z, \gamma$ do not couple to the gluons directly, these processes take place at the one-loop.
Introduction

- A selection of the processes of interest are:

  \[ gg \rightarrow \gamma\gamma g \]
  \[ gg \rightarrow \gamma Z g \]
  \[ gg \rightarrow \gamma Z \gamma \]
  \[ gg \rightarrow ZZ g \]
  \[ gg \rightarrow ZZ \gamma \]
  \[ gg \rightarrow ZZZ \]
  \[ gg \rightarrow WW g \]
  \[ gg \rightarrow WW \gamma \]
  \[ gg \rightarrow WWZ \]

- The first process has already been examined long ago. Our primary focus would be on the second process \( gg \rightarrow \gamma Z g \), with some comments on the first and third process. The work is in progress for other processes also.
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These generic processes take place at one-loop. There is no contribution at the tree level. Therefore, one-loop order is the leading order.

The one loop diagrams that make contribution are box-type and pentagon-type diagrams. For the processes involving W-bosons, there are also triangle diagrams.

These diagrams have a quark loop. We have also considered the possibility of a heavy quark in the loop, i.e., the top quark.

We will now look at the first three processes in some detail.

For each quark flavour, there are 24 pentagon-type and 18 box-type diagrams. Only half of these diagrams are independent.
Processes

Figure: Typical diagrams contributing to the processes.
Processes

- If we look at the processes $gg \rightarrow \gamma Zg/\gamma$, the amplitude can be written as

$$M_{P,B}^i = M_{P,B}^{i,V} + M_{P,B}^{i,A}$$

The amplitude for both box-type and pentagon-type diagrams is sum of vector coupling and axial-vector coupling part. This is because, the Z-boson has both of these couplings.

- In the case of box-type diagrams, the axial-vector coupling pieces add up to zero (Furry’s theorem). So the box-type diagrams contribute only through vector coupling of the Z-boson.

- For the process $gg \rightarrow \gamma Z\gamma$, the box diagrams do not exist. It gets contribution from only pentagon-type diagrams. Furthermore, here vector coupling contributions add up to zero. So there is contribution from axial-vector coupling of the Z-boson only.
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Our calculation proceeds as follows:

- For each class of diagrams, we write down the amplitude for a prototype diagram using the standard model Feynman rules.
- The amplitude for the rest of diagrams is generated by appropriate permutations of the external legs. One has to be a bit careful due the presence of $\gamma_5$ in the amplitude.
- The trace of $\gamma$ matrices is computed in $d$ dimensions using FORM. The amplitude is now written in terms of tensor integrals.
- This is a brute-force evaluation of the amplitude, so the expression is quite huge. It is to be evaluated numerically.
Calculation

• The most complicated tensor integrals that appear in the calculations are:

\[
E^{\mu\nu\rho\sigma\delta} = \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu k^\rho k^\sigma k^\delta}{N_0 N_1 N_2 N_3 N_4},
\]  

\[
D^{\mu\nu\rho\sigma} = \int \frac{d^d k}{(2\pi)^d} \frac{k^\mu k^\nu k^\rho k^\sigma}{N_0 N_1 N_2 N_3}.
\]  

Here, \( N_i = k_i^2 - m_q^2 + i\epsilon \) and \( k_i \) is the momentum of the \( i^{th} \) internal line in the corresponding scalar integrals; \( d = (4 - 2\epsilon) \) and \( m_q \) is the mass of the quark in the loop.

• We also examine the effect of non-zero \( m_q \).
Calculation

- The tensor integrals are reduced to scalar integrals using the techniques of Oldenborgh and Vermaseren.
- For massless quarks in the loop, we have computed the scalar integrals and checked with existing results. These integrals are used to make certain checks on our calculation.
- For the massive quarks in the loop, we use OneLoop library (vanHameren) for the bubble, triangle, and box scalar integrals.
- For the pentagon scalar integrals, we use the result of vanNeerven. Using this technique, one can write pentagon scalar integrals in terms of box scalar integrals,

\[ E_0 = \sum_{i=0}^{4} c_i D_0^{(i)}. \]  

(3)
Checks

We have made a number of checks on our calculation. These are listed below.

- **UV Finiteness**
  The process is expected to be UV finite. Pentagram diagrams are obviously UV finite. But individual box diagram is not. However, when we add box diagrams, the UV divergences cancel. We have checked numerically that the amplitude is UV finite.

- **IR Finiteness**
  The process has mass singularities due to small light quark mass. These singularities show up as $\log^2(m_q)$ and $\log(m_q)$. These large logarithms must cancel. We have checked this cancellation for both types of mass singularities. There is no soft IR divergence, as we will be making $p_T$ cuts on the jets.
• **Gauge Invariance**
  We also expect gauge invariance with respect to the gauge particles. To check this, we replace the polarization vector of $g/\gamma/Z$ bosons with the corresponding momentum vector. We find that the amplitude vanishes when we make this replacement for the photon, gluon and suitably for the $Z$-boson.
  
  For the process $gg \rightarrow \gamma Z g$, the axial part of the amplitude does not interfere with the vector part of the amplitude due to the color factor structure. This factor is symmetric for the vector part of the amplitude, while it is antisymmetric for the axial-vector part. We have checked that these parts are separately gauge invariant.
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As we discussed we have made numerous checks to confirm the reliability of our numerical results.

We compute the amplitude numerically using the real polarization basis for the vector bosons.

We use a PVM implementation of the VEGAS algorithm (AMCI) to do the integration and use a cluster of machines to get the numerical results.

We are presenting results for 14 TeV LHC. The numbers will be about a factor of 4 lower for the 7-8 TeV LHC. We apply generic cuts.
Numerical Results

- Plot 1 displays cross section as a function of centre-of-mass energy. These results include following kinematic cuts:

\[ P_T^{\gamma,Z,j} > 50 \text{ GeV}, \ |\eta^{\gamma,Z,j}| < 2.5, \ R(\gamma,j) > 0.6. \]

We have also chosen factorization and renormalization scales as \( \mu_f = \mu_R = p_T^Z \).

- Plot 2 shows the cross section as a function of \( p_T^{\text{min}} \). Here CM energy is 14 TeV.

- Plots 3 and 4 display the \( p_T \) and \( \eta \) for the photon.

- Then there are similar plots for the gluon-jet and Z-boson.
Numerical Results

Figure: (i) Centre of mass energy (ii) $p_T^{\text{min}}$ dependence of the cross-section for $gg \rightarrow \gamma Z g$. 
Numerical Results

Figure: (i) $p_T$ and (ii) $\eta$ distributions of the photon.
Numerical Results

Figure: (i) $p_T$ and (ii) $\eta$ distributions of the gluon.
Numerical Results

Figure: (i) $p_T$ and (ii) $\eta$ distributions of the Z-boson.
Numerical Results

- We observe that few hundred such events have already been produced at the LHC by now. Of course, one will have to dig it out of other processes.

- We note that the photon and Z-boson $p_T$ distributions are harder, as compared to the gluon-jet distribution. It is not surprising since the photon and Z-boson are emitted from the quark loop, while the gluon can be emitted from another gluon (in the box diagram). The rapidity distribution of the gluon-jet is also more spread out. The virtue of this is that a cut on $p_T$ and rapidity of a photon can be used to discriminate from the processes where, it is emitted directly as bremsstrahlung. It would happen in quark initiated processes.
Numerical Results

- We find that the top-quark makes negligible contribution to the process. It appears that this decoupling of a quark occurs starting around $m_q = 100 \text{GeV}$. Given the scales in the process, it appears to be a low value. There is no propagator enhancement even at the LHC energies, unlike some other processes.

- The results for the process $gg \rightarrow \gamma\gamma g$ are already in literature for massless quark in the loop. We included heavy-quark, top-quark, in the calculation and found that it made negligible contribution. The typical cross-section for $p_T^{min} = 30 \text{ GeV}$, at 14 TeV LHC, is about 642 fb.

- The cross-section for the process $gg \rightarrow \gamma\gamma Z$ is quite small, as expected. It is about 0.05 fb.
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- We have presented the results for the processes $gg \rightarrow \gamma Zg, \gamma Z\gamma$. These processes occur at one loop through pentagon and box-type diagrams.
- The process $gg \rightarrow \gamma Z\gamma$ does not get contribution from the vector coupling of the Z-boson. So it probes the axial vector coupling of the Z-boson.
- The process $gg \rightarrow \gamma Zg$ gets contribution from both vector and axial vector coupling of the Z boson. However, both contributions are separately gauge invariant.
- We find that there is decoupling of the top-quark. So its contribution is negligible.
- These are standard model processes and cross-sections are large enough so that these processes could be observable at the LHC.