PARAMETERS OF THE NEUTRINO SECTOR IN TAU DECAYS

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1 Introduction

The Standard Model includes neutrinos as massless particles, but neutrino oscillations showed, that neutrinos are not massless. A simple extension of adding gauge singlet fermions to the particle spectrum allows normal Yukawa mass terms for neutrinos. The smallness of the neutrino masses can be well understood within the seesaw mechanism (type I). After spontaneous symmetry breaking of the Standard Model gauge group one obtains a $(n_L+n_R)\times(n_L+n_R)$ Majorana mass matrix M_ν for neutrinos. The mixing between the n_R "right-handed" singlet fermions and the neutral parts of the n_L lepton doublets gives masses for the neutrinos which are of the size expected from neutrino oscillations.
The diagonalization of the mass matrix gives rise to a split spectrum consisting of heavy and light states of neutrinos given by $U^T M_\nu U = \mathrm{diag}(m^{light}_{n_L}, m^{heavy}_{n_R})$. For the case $n_R = 1$ we diagonalize M_ν with a rotation matrix determined by two angles, two masses, and Majorana phases. For the case $n_R = 2$ we diagonalize the mass matrix with a unitary matrix determined by complex parameters, four masses, and Majorana phases. In both cases we take $n_L = 3$.
We calculate the one-loop radiative corrections to the mass parameters which produce mass terms for the neutral leptons. In both cases we numerically analyse light neutrino masses as functions of the heavy neutrinos masses. Parameters of the model are varied to find light neutrino masses that are compatible with experimental data of solar $\Delta m^2_\odot$ and atmospheric $\Delta m^2_{\mathrm{atm}}$ neutrino oscillations.

2 Description Of The Model

2.1 The Tree Level

The Yukawa Lagrangian of the leptons is given by

$$\mathcal{L}_{\mathrm{Y}} = -\frac{\sqrt{2}}{v}\left(\phi^\dagger \bar{\ell}_R M_\ell + \tilde{\phi}^\dagger \bar{\nu}_R M_D\right) D_L + \mathrm{H.c.} \tag{1}$$

in a vector and matrix notation. ℓ_R, ν_R, and $D_L = (\nu_L\ \ell_L)^T$ are the vectors of the right-handed charged leptons, of the right-handed neutrino singlets, and of the left-handed lepton doublets, respectively. The charged-lepton mass matrix M_ℓ is $n_L \times n_L$, while the Dirac neutrino mass matrix M_D is $n_R \times n_L$. The vacuum expectation value of the neutral component of ϕ is $v/\sqrt{2}$. The mass terms for the neutrinos are

$$-\bar{\nu}_R M_D \nu_L - \frac{1}{2}\bar{\nu}_R C M_R \bar{\nu}_R^T + \mathrm{H.c.}\,, \tag{2}$$

where C is the charge-conjugation matrix and M_R is non-singular and symmetric. Equation (2) can be written in a compact form as a mass term with an $(n_L+n_R)\times(n_L+n_R)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & \hat{M}_R \end{pmatrix}, \tag{3}$$

where the hat indicates that $\hat{M}_R$ is a diagonal matrix. M_ν can be diagonalized as

$$U^T M_\nu U = \hat{m} = \mathrm{diag}\left(m_1, m_2, \ldots, m_{n_L+n_R}\right), \tag{4}$$

where the m_i are real and non-negative. In order to implement the seesaw mechanism [1, 2] we assume that the elements of M_D are of order m_D and those of M_R are of order m_R, with $m_D \ll m_R$. Then, the neutrino masses m_i with $i = 1, 2, \ldots, n_L$ are of order m_D^2/m_R, while those with $i = n_L+1, \ldots, n_L+n_R$ are of order m_R. It is useful to decompose the $(n_L+n_R)\times(n_L+n_R)$ unitary matrix U as [3, 4]

$$U = \begin{pmatrix} U_L \\ U_R^* \end{pmatrix}, \tag{5}$$

where the submatrix U_L is $n_L \times (n_L+n_R)$ and the submatrix U_R is $n_R \times (n_L+n_R)$. With these submatrices, the left- and right-handed neutrinos are written as linear superpositions of the n_L+n_R physical Majorana neutrino fields χ_i:

$$\nu_L = U_L P_L \chi \quad \text{and} \quad \nu_R = U_R P_R \chi\,, \tag{6}$$

where P_L and P_R are the projectors of chirality.

The leptonic charged-current Lagrangian is

$$\mathcal{L}_{\mathrm{cc}} = \frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}\gamma^\mu P_L U_L \chi + \mathrm{H.c.}\,, \tag{7}$$

where g is the SU(2) gauge coupling constant. Three neutral particles are coupling to neutrinos. The interaction of the Z boson with the neutrinos is given by

$$\mathcal{L}^{(\nu)}_{\mathrm{nc}} = \frac{g}{4c_w} Z_\mu \bar{\chi}\gamma^\mu\left[P_L\left(U_L^\dagger U_L\right) - P_R\left(U_L^T U_L^*\right)\right]\chi\,, \tag{8}$$

where c_w is the cosine of the Weinberg angle.

The Yukawa couplings of the Higgs boson h^0 to the neutrinos are given by

$$\mathcal{L}^{(\nu)}_{\mathrm{Y}}\left(h^0\right) = \frac{g}{4m_W} h^0\, \bar{\chi}\left[\left(U_R^\dagger M_D U_L + U_L^T M_D^T U_R^*\right)P_L + \left(U_L^\dagger M_D^\dagger U_R + U_R^T M_D^* U_L^*\right)P_R\right]\chi\,. \tag{9}$$

The coupling of the neutrinos and the Goldstone boson G^0 is similar to that of the Higgs boson h^0. The right-hand side expression in (9) gets multiplied by a factor $(-i)$.

2.2 One-loop Corrections

In the standard seesaw, one-loop corrections to the mass matrix , i.e. the self energies, are determined by the neutrino interactions with the Z boson, the neutral Goldstone boson G^0, and the Higgs boson h^0 [5]. In the calculation of the self energies the neutrino couplings to the Z boson as well as the Higgs and Goldstone bosons are determined by eqs. (8) and (9). Each diagram contains a divergent piece but when summing up the three contributions the result turns out to be finite [6].

Once the one-loop corrections are taken into account the neutral fermion mass matrix is given by

$$M_\nu^{(1)} = \begin{pmatrix} \delta M_L & M_D^T + \delta M_D^T \\ M_D + \delta M_D & \hat{M}_R + \delta M_R \end{pmatrix} \approx \begin{pmatrix} \delta M_L & M_D^T \\ M_D & \hat{M}_R \end{pmatrix} \tag{10}$$

where the $0_{3\times3}$ matrix appearing at tree level (3) is replaced by the contribution δM_L. This correction a symmetric matrix, it dominates among all the sub-matrices of corrections.

Neglecting the sub-dominant pieces in (10), one-loop corrections to the neutrino masses originate via the relation

$$\delta M_L = U_L^* \Sigma_L^S(p^2) U_L^\dagger = U_L^* \Sigma_L^S(0) U_L^\dagger\,, \tag{11}$$

where we evaluate the one-loop neutrino self-energy Σ_L^S at zero external momentum squared. After contractions of similar terms the final expression for the finite one-loop correction is given by

$$\delta M_L = \delta M_L^{(Z)} + \delta M_L^{(h^0)}, \tag{12}$$

where

$$\delta M_L^{(Z)} = \frac{3g^2}{64\pi^2 m_W^2} U_L^* \hat{m}^3 \left(\frac{\hat{m}^2}{m_Z^2} - \mathbb{1}\right)^{-1} \ln\left(\frac{\hat{m}^2}{m_Z^2}\right) U_L^\dagger\,; \tag{13}$$

$$\delta M_L^{(h^0)} = \frac{g^2}{64\pi^2 m_W^2} U_L^* \hat{m}^3 \left(\frac{\hat{m}^2}{m_{h^0}^2} - \mathbb{1}\right)^{-1} \ln\left(\frac{\hat{m}^2}{m_{h^0}^2}\right) U_L^\dagger\,. \tag{14}$$

3 Case $\mathbf{n_R = 1}$

First we consider the minimal extension of the standard model adding only one right-handed field ν_R to the three left-handed fields contained in ν_L.

We use the parametrization of $M_D = m_D \vec{a}^T$ with $|\vec{a}| = 1$. Diagonalization of the symmetric mass matrix M_ν (3) in block form is:

$$U^T M_\nu U = U^T \begin{pmatrix} 0 & m_D\vec{a} \\ m_D\vec{a}^T & \hat{M}_R \end{pmatrix} U = \begin{pmatrix} \hat{M}_l & 0 \\ 0 & \hat{M}_h \end{pmatrix}. \tag{15}$$

The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined analytically by finding eigenvalues of the hermitian matrix $M_\nu M_\nu^\dagger$. These eigenvalues are the squares of the masses of the neutrinos $\hat{M}_l = \mathrm{diag}(0, 0, m_l)$ and $\hat{M}_h = m_h$. Solutions $m_D^2 = m_h m_l$ and $m_R^2 = (m_h - m_l)^2 \sim m_h^2$ correspond to the seesaw mechanism.

We can construct the diagonalization matrix U for the tree level from two diagonal matrices of phases and three rotation matrices $U_{\mathrm{tree}} = U_\phi(\phi_i)U_{12}(\alpha_1)U_{23}(\alpha_2)U_{34}(\beta)U_i$, where the angle β is determined by the masses m_l and m_h. The values of ϕ_i and α_i can be chosen to cover variations in M_D.

Diagonalization of the mass matrix after calculation of one-loop corrections is performed with a unitary matrix $U_{\mathrm{loop}} = U_{\mathrm{egv}}U_\varphi(\varphi_1, \varphi_2, \varphi_3)$, where U_{egv} is an eigenmatrix of $M_\nu^{(1)} M_\nu^{(1)\dagger}$ and U_φ is a phase matrix. The second light neutrino obtains its mass from radiative corrections. The third light neutrino remains massless.

It is possible to estimate masses of the light neutrinos from experimental data of solar and atmospheric neutrino oscillations ($\Delta m^2_\odot = 7.59\times10^{-23}\,\mathrm{GeV}^2$, $\Delta m^2_{\mathrm{atm}} = 2.43\times10^{-21}\,\mathrm{GeV}^2$) assuming that the lightest $m_{l_3} = 0$ and considering the normal ordering of the light neutrinos:

$$\begin{aligned} m_{l_1} &= 5.0\times10^{-11}\,\mathrm{GeV}, \\ m_{l_2} &= 8.7\times10^{-12}\,\mathrm{GeV}. \end{aligned} \tag{16}$$

However the numerical analysis shows that we can reach those values only for a heavy singlet with the mass near to the Plank scale, see Fig. 1.

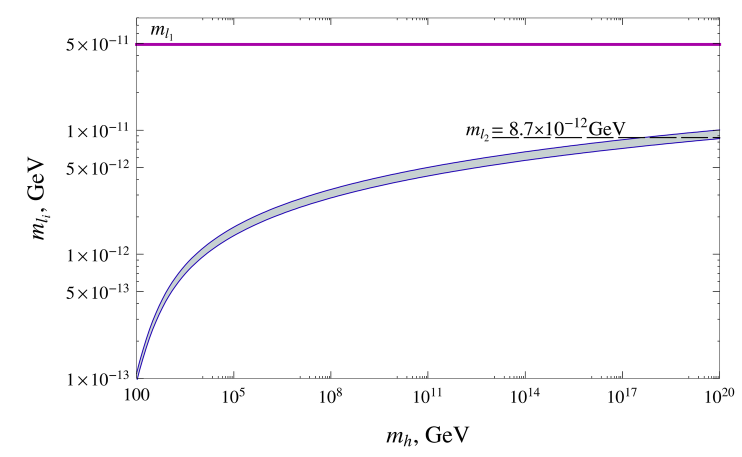

Figure 1: Calculated masses of two light neutrinos as a function of the heavy neutrino mass m_h. The mass of the third light neutrino is zero, when $n_R = 1$. Solid lines show the boundaries of allowed neutrino mass ranges when the model parameters are constrained by the experimental data on neutrino oscillations. Due to the scale, the band of the allowed m_{l_1} values appears as a line. The dashed line indicates the estimated experimental mass of the second light neutrino for the case $n_R = 1$.

4 Case $\mathbf{n_R = 2}$

If we add two singlet fields ν_R to the three left-handed fields ν_L, the radiative corrections give masses to all three light neutrinos.

Now we parametrize

$$M_D = \begin{pmatrix} m_{D_2}\vec{a}^T \\ m_{D_1}\vec{b}^T \end{pmatrix} \tag{17}$$

with $|\vec{a}| = 1$ and $|\vec{b}| = 1$. Diagonalizing the symmetric mass matrix M_ν (3) in block form we write:

$$U^T M_\nu U = U^T \begin{pmatrix} 0_{3\times3} & m_{D_2}\vec{a}\ \ m_{D_1}\vec{b} \\ \begin{matrix} m_{D_2}\vec{a}^T \\ m_{D_1}\vec{b}^T \end{matrix} & \hat{M}_R \end{pmatrix} U = \begin{pmatrix} \hat{M}_l & 0 \\ 0 & \hat{M}_h \end{pmatrix}. \tag{18}$$

The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined by the seesaw mechanism: $m^2_{D_i} \approx m_{h_i} m_{l_i}$ and $m^2_{R_i} \approx m^2_{h_i}$, $i = 1, 2$. Here we use $m_1 > m_2 > m_3$ ordering of masses. The third light neutrino is massless at tree level.

The diagonalization matrix for tree level $U_{\mathrm{tree}} = U_{12}(\alpha_1, \alpha_2)U_{\mathrm{egv}}(\beta_i)U_\phi(\phi_i)$ is composed of a rotation matrix, an eigenmatrix of $M_\nu M_\nu^\dagger$ and a diagonal phase matrix, respectively.

Diagonalization of the mass matrix including the one-loop correction is performed with a unitary matrix $U_{\mathrm{loop}} = U_{\mathrm{egv}}U_\varphi(\varphi_i)$, where U_{egv} is the eigenmatrix of $M_\nu^{(1)} M_\nu^{(1)\dagger}$ and U_φ is a phase matrix.

In numerical calculations the model parameters as well as the derived masses of the light neutrinos are obtained in several steps. First, the diagonal mass matrix for tree level is constructed. The lightest neutrino is massless, and the masses of other two light neutrinos are estimated from experimental data on solar and atmospheric neutrino oscillations.

The masses of the heavy neutrinos are input parameters. This diagonal matrix is used to constrain the parameters α_i and ϕ_i that enter the tree-level mass matrix M_ν and its diagonalization matrix U_{tree}. Then the diagonalization matrix is used to evaluate one-loop corrections to the mass matrix. Diagonalization of the corrected mass matrix yields masses for three light neutrinos. If the calculated mass difference is compatible with the experimental neutrino mass difference, the parameter set is kept. Otherwise, another set of the parameters is generated. Figures 2-4 illustrate the obtained results.

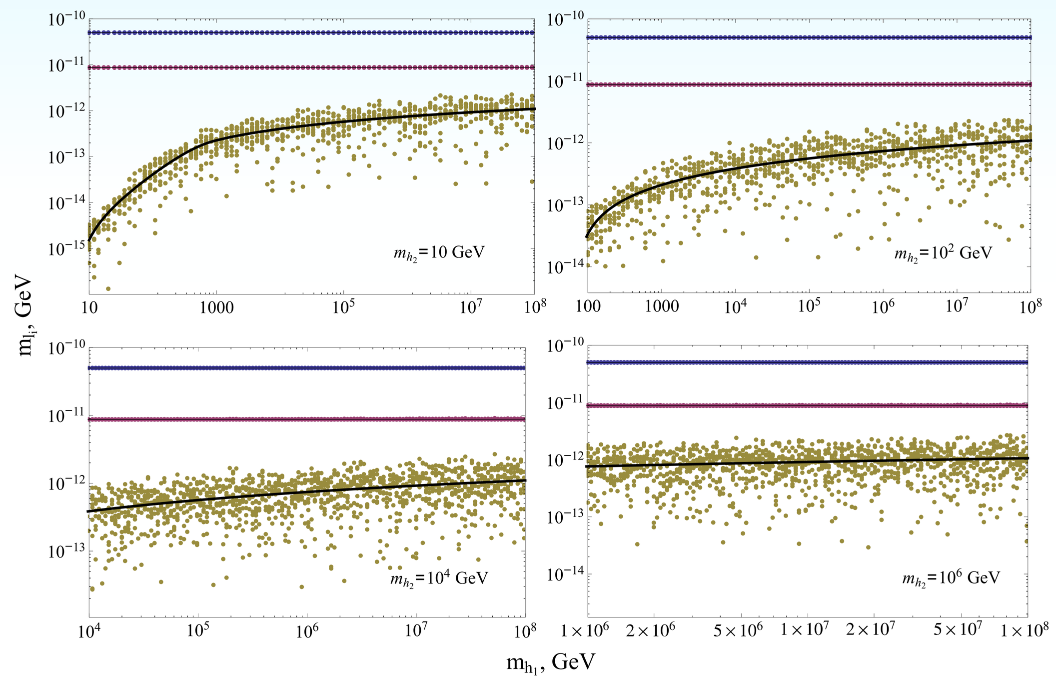

Figure 2: The masses m_{l_i} of the light neutrinos as functions of the heaviest right-handed neutrino mass m_{h_1}, for the case $n_R = 2$. The value of m_{h_2} is shown in the plots. The black solid lines indicate the mean values of the scatter data.

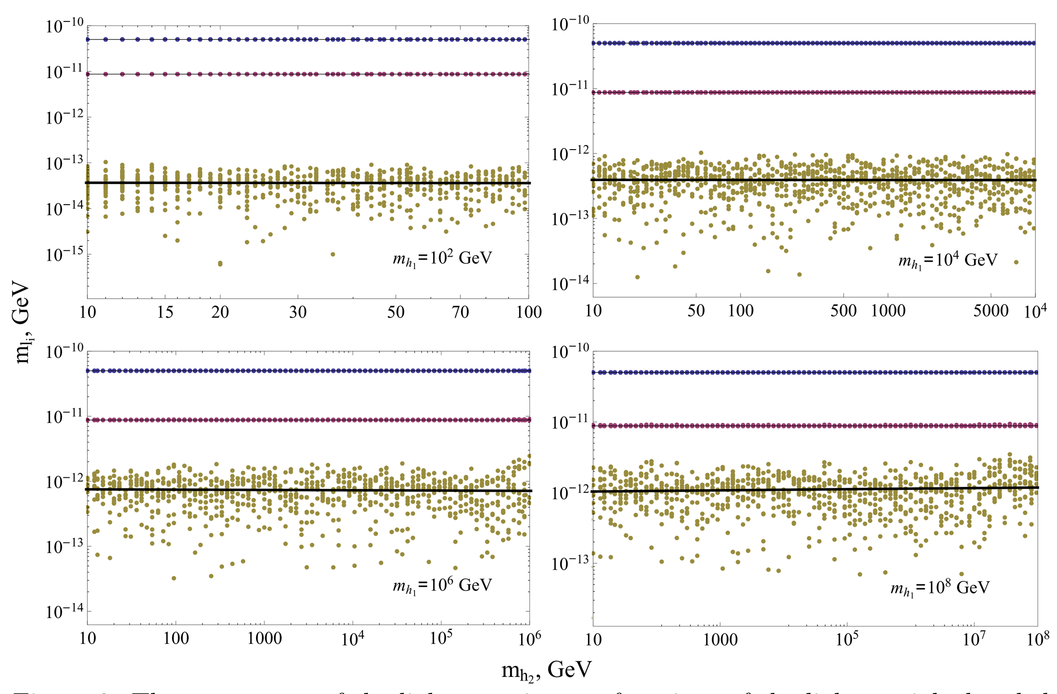

Figure 3: The masses m_{l_i} of the light neutrinos as functions of the lightest right-handed neutrino mass m_{h_2}, for the case $n_R = 2$. The value of m_{h_1} is shown in the plots. The black solid lines indicate the mean values of scatter data.

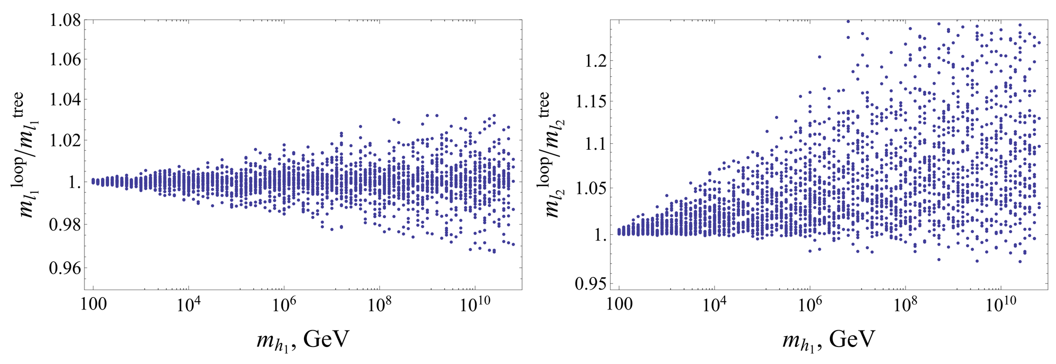

Figure 4: The ratio between the one-loop and the tree-level result for the masses of the light neutrinos ν_{L_i} $(i = 1, 2)$ with $m_{l_i}^{\mathrm{loop}} = m_{l_i}^{\mathrm{tree}} + \delta m_{l_i}^{\mathrm{loop}}$ as a function of the heaviest right-handed neutrino mass m_{h_1}, with $m_{h_2} = 100$ GeV fixed.

5 Conclusions

1. For the case $n_R = 1$ we can match the differences of the calculated light neutrino masses to $\Delta m^2_\odot$ and $\Delta m^2_{\mathrm{atm}}$ only for a mass of the heavy singlet of order 10^{17} GeV. Only normal ordering of neutrino masses is possible.
2. In the case $n_R = 2$ we obtain three non vanishing masses of light neutrinos. The numerical analysis shows that the values of light neutrino masses (especially of the lightest mass) depend on the choice of the heavy neutrinos masses. The radiative corrections generate the lightest neutrino mass and have a big impact on the second lightest neutrino mass.
3. In future we plan to apply our parametrization to study the τ polarization coming from the decay of a W boson in the data of the CMS experiment at LHC and thus determine restrictions to the parameters of the neutrino sector.

6 References

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