$\mathcal{B}$ Physics: Theory Overview

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Introduction

A great deal of work on $B$ physics was done over the past 15-20 years, mostly in the context of the $B$ factories: finding methods for measuring the SM parameters, examining ways of looking for NP, analysing the results, etc. Unfortunately, most of the measurements agreed with the SM. Although there were several hints of NP, mostly in $\bar{b} \to \bar{s}$ transitions, there were no statistically-significant signals.

CDF and DØ then demonstrated that $B$ physics can be done at hadron colliders. They made a number of measurements involving $B^0_s$ mesons, in particular $B^0_s - \bar{B}^0_s$ mixing.

The LHC will continue this exploration of $B$ physics. They will focus mainly on $B^0_s$ mesons, but may well be able to repeat (and improve upon?) some of the measurements made at BaBar and Belle. As always, the hope is to find a signal of NP. It seems clear now that very large signals are ruled out. But the LHC may well have the precision to detect small deviations from the SM. In this talk, I will discuss a number of $B$-physics measurements to be made at the LHC which have the potential for revealing NP.
$B^0_s$-$\bar{B}^0_s$ Mixing

Formalism: mass eigenstates $B_L$ and $B_H$ ($L, H$: light and heavy states) are admixtures of the flavour eigenstates $B^0_s$ and $\bar{B}^0_s$:

$$|B_L\rangle = p |B^0_s\rangle + q |\bar{B}^0_s\rangle ,$$
$$|B_H\rangle = p |B^0_s\rangle - q |\bar{B}^0_s\rangle ,$$

with $|p|^2 + |q|^2 = 1$. Initial flavour eigenstates oscillate into one another according to the Schrödinger equation with $H = M^s - i\Gamma^s/2$ ($M^s$ and $\Gamma^s$ are the dispersive and absorptive parts of the mass matrix). The off-diagonal elements $M^s_{12}$ and $\Gamma^s_{12}$ are generated by $B^0_s$-$\bar{B}^0_s$ mixing.

Defining $\Delta M_s \equiv M_H - M_L$ and $\Delta \Gamma_s \equiv \Gamma_L - \Gamma_H$, we have

$$\Delta M_s = 2|M^s_{12}| , \quad \Delta \Gamma_s = 2|\Gamma^s_{12}| \cos \theta_s , \quad \frac{q}{p} = e^{-2i\beta_s} ,$$

where $\theta_s \equiv \arg(-M^s_{12}/\Gamma^s_{12})$ is the CP phase in $\Delta B = 2$ transitions.
Weak phases $\theta_s$ and $2\beta_s$ independent – SM predicts $\theta_s$ and $2\beta_s \simeq 0$ (but $\theta_s \neq -2\beta_s$!). Note: in many papers, one writes $\phi_s$ instead of $\theta_s$. But LHCb uses $\phi_s$ for what is measured via indirect CPV (combination of $-2\beta_s$, $\theta_s$, $\delta_{NP}$) $\implies$ notation confusion. Points: $\Delta \Gamma_s$ is sizeable and is $> 0$ in SM; in presence of NP, can have $2\beta_s \neq 0$ and $\Delta \Gamma_s < 0$.

$J/\psi\phi$: In 2008 the CDF and DØ collaborations measured the CP asymmetry in $B^0_s \to J/\psi\phi$, and found a hint for indirect CP violation. The 2011 update gives (at 68% C.L.)

$$2\beta^{\psi\phi}_s \in [2.3^\circ, 59.6^\circ] \cup [123.8^\circ, 177.6^\circ], \quad \text{CDF},$$

$$\in [9.7^\circ, 52.1^\circ] \cup [127.9^\circ, 170.3^\circ], \quad \text{DØ}.$$

Note that the measurement is insensitive to the transformation $(2\beta^{\psi\phi}_s, \Delta \Gamma_s) \leftrightarrow (\pi - 2\beta^{\psi\phi}_s, -\Delta \Gamma_s) \implies 2\beta^{\psi\phi}_s$ has twofold ambiguity.

Following this, there were many papers looking at NP in $B^0_s-\bar{B}^0_s$ mixing. It was also pointed out that NP in the decay $\bar{b} \to \bar{s}c\bar{c}$ is possible

[C. -W. Chiang+, JHEP 1004, 031 (2010)].
LHCb has greatly improved this result. First, they remove the twofold ambiguity by measuring $\Delta \Gamma_s > 0$: $\Delta \Gamma_s = 0.120 \pm 0.028 \text{ ps}^{-1}$ [R. Aaij+, 1202.4717]. This is done via the decay $B^0_s \rightarrow J/\psi \phi (\rightarrow K^+ K^-)$, looking at the interference between the s- and p-wave $K^+ K^-$ angular momentum states [Y. Xie+, JHEP 0909, 074 (2009)].

Second, they find [LHCb Collaboration, CERN-LHCb-CONF-2012-002]

$$2 \beta^{J/\psi \phi}_s = (-0.06 \pm 5.77 \text{ (stat)} \pm 1.54 \text{ (syst)})^\circ,$$

in agreement with the SM.

To completely search for NP, LHCb has to measure $B^0_s - \bar{B}^0_s$ mixing in as many different decays as possible. This has already begun:

$J/\psi f_0(980)$: LHCb measures $\beta^{J/\psi f_0}_s = (-25.2 \pm 25.2 \pm 1.1)^\circ$ [R. Aaij+, PLB707, 497 (2012)]. Advantage of decay: because $f_0(980)$ is a scalar, no angular analysis is needed. Disadvantage: $f_0(980)$ not a pure $s\bar{s}$ state, so there are possibly other contributions to the decay, and this leads to hadronic uncertainties [R. Fleischer+, EPJ C71, 1832 (2011)].
$J/\psi\pi^+\pi^-$: LHCb measures $\beta_{J/\psi\pi^+\pi^-} = (-1.09^{+9.91}_{-9.97}+0.23)\degree$ [LHCb collaboration, 1204.5675]. A-priori, since this is a 3-body state, its CP can be $+$ or $-$, and so it cannot be used to cleanly extract weak-phase information. However, it has been shown that the $J/\psi\pi^+\pi^-$ state is almost purely CP $-$ [LHCb collaboration, 1204.5643], so that there is little error due to the CP $+$ state.

Other decays which are potentially of interest are (i) $B_s^0(\bar{B}_s^0) \to D_s^\pm K^\mp$ [R. Fleischer, NPB671, 459 (2003)] – measures $(2\beta_s + \gamma)$, (ii) $B_s^0 \to D_s^+D_s^-$ [R. Fleischer, EPJ C10, 299 (1999), EPJ C51, 849 (2007)] – have to deal with penguin pollution, (iii) $B_s^0(\bar{B}_s^0) \to D_{CP}^0 K\bar{K}$ [S. Nandi+, 1108.5769], (iv) certain other 3-body decays.

$B_s^0 \to K^0\bar{K}^0$ is a pure $\bar{b} \to \bar{s}$ penguin decay, $A = V_{tb}^*V_{ts}P_{tc}^{'} + V_{ub}^*V_{us}P_{uc}^{'}$. SM: indirect CPV measures $|\beta_{s}^{eff}| \leq 14\degree$ [B. Bhattacharyya+, 1203.3435]. If a larger value of $|\beta_{s}^{eff}|$ is measured $\implies$ NP. The decay modes in which one or both of the final-state particles are vectors can also be used.
Like-sign Dimuon Asymmetry

The DØ Collaboration has reported an anomalously large CP-violating like-sign dimuon charge asymmetry in the $B$ system. The updated measurement is [V. M. Abazov+, PRD84, 052007 (2011)]

$$A_{sl}^b \equiv \frac{N_{b}^{++} - N_{b}^{--}}{N_{b}^{++} + N_{b}^{--}} = -(7.87 \pm 1.72 \pm 0.93) \times 10^{-3},$$

a $3.9\sigma$ deviation from the SM prediction, $A_{sl}^{b,SM} = (-2.3^{+0.5}_{-0.6}) \times 10^{-4}$.

Now, it has been shown that, if this anomaly is real, it implies NP in $B_{s}^{0}$-$\bar{B}_{s}^{0}$ mixing. Such NP effects can appear in $M_{12}^{s}$ and/or $\Gamma_{12}^{s}$. In fact, it has been argued that NP in $\Gamma_{12}^{s}$ should be considered as the main explanation for the above result [C. Bobeth+, 1109.1826].

In the SM, the dominant contribution to $\Gamma_{12}^{s}$ comes from $\bar{b} \rightarrow \bar{s} c \bar{c}$. Significant NP contributions, i.e. comparable to the SM, can come mainly from $\bar{b} \rightarrow \bar{s} \tau^{+} \tau^{-}$. This is straightforward to detect. For example, if $\mathcal{B}(B_{s}^{0} \rightarrow \tau^{+} \tau^{-})$ is observed to be at the percent level, this will be a clear indication of NP (in the SM, $\mathcal{B}(B_{s}^{0} \rightarrow \tau^{+} \tau^{-}) = 7.9 \times 10^{-7}$). Thus, this is one decay that LHCb should try to measure.
$B^0_s \rightarrow VV$ Decays

$B^0_s \rightarrow V_1 V_2$: 3 decays. $V_1$ and $V_2$ can have relative orbital angular momentum $l = 0$ (s wave), $l = 1$ (p wave), or $l = 2$ (d wave).

Equivalently, one decomposes the decay amplitude into components in which the polarizations of the final-state vector mesons are either longitudinal ($A_0$), or transverse to their directions of motion and parallel ($A_{||}$) or perpendicular ($A_\perp$) to one another.

(1) Triple Product (TP): In the $B$ rest frame, the TP takes the form $\vec{q} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$, where $\vec{q}$ is the difference of the two final momenta, and $\vec{\varepsilon}_1$ and $\vec{\varepsilon}_2$ are the polarizations of $V_1$ and $V_2$. The TP is odd under both P and T, and thus constitutes a potential signal of CPV. In $B^0_s \rightarrow V_1 V_2$, there are two TP's: $A^{(1)}_T \propto \text{Im}(A_\perp A_0^*)$ and $A^{(2)}_T \propto \text{Im}(A_\perp A_{||}^*)$.

“TP’s are a signal of CP violation:” not quite accurate. In general the $A_i$ ($i = 0, ||, \perp$) possess both weak (CP-odd) and strong (CP-even) phases. Thus, $\text{Im}(A_\perp A_0^*)$ and $\text{Im}(A_\perp A_{||}^*)$ can both be nonzero even if the weak phases vanish. In order to obtain a true signal of CP violation, one has to compare the $B$ and $\bar{B}$ decays.
The TP’s for the $\bar{B}$ decay are $-\text{Im}(\bar{A}_\perp \bar{A}_0^*)$ and $-\text{Im}(\bar{A}_\perp \bar{A}_\parallel^*)$. The true (CP-violating) TP’s are then given by $\frac{1}{2}[\text{Im}(A_\perp A_0^*) + \text{Im}(\bar{A}_\perp \bar{A}_0^*)]$ and $\frac{1}{2}[\text{Im}(A_\perp A_\parallel^*) + \text{Im}(\bar{A}_\perp \bar{A}_\parallel^*)]$. But there are also fake (CP-conserving) TP’s, due only to strong phases of the the $A_i$’s. These are given by $\frac{1}{2}[\text{Im}(A_\perp A_0^*) - \text{Im}(\bar{A}_\perp \bar{A}_0^*)]$ and $\frac{1}{2}[\text{Im}(A_\perp A_\parallel^*) - \text{Im}(\bar{A}_\perp \bar{A}_\parallel^*)]$. Thus, for the fake TP’s, it is necessary to distinguish $B$ and $\bar{B}$.

CPV due to interference of two amplitudes. Common way to look for CPV – non-zero rate difference between the decay and its CP-conjugate decay (direct CPV). The direct CP asymmetry is proportional to $\sin \phi \sin \delta$, where $\phi$ and $\delta$ are the relative weak and strong phases of the two amplitudes. IOW, direct CPV requires a non-zero strong-phase difference. OTOH, the true TP is proportional to $\sin \phi \cos \delta$, so no strong-phase difference is necessary. Helps in search for NP. Also, in SM, true TP’s are generally small (or zero) [A. Datta+, IJMP A19, 2505 (2004)] $\Longrightarrow$ good way to find NP.
CDF and LHCb have measured the true TP asymmetries in $B^0_s \rightarrow \phi\phi$ (no flavour tagging needed) [T. Aaltonen+, 1107.4999; LHCb Collaboration, LHCb-CONF-2011-052]:

$$A_U (\perp \parallel) = -0.007 \pm 0.064 \text{ (stat)} \pm 0.018 \text{ (syst)} \quad \text{CDF},$$
$$= -0.064 \pm 0.057 \text{ (stat)} \pm 0.014 \text{ (syst)} \quad \text{LHCb},$$
$$A_V (\perp 0) = -0.120 \pm 0.064 \text{ (stat)} \pm 0.016 \text{ (syst)} \quad \text{CDF},$$
$$= -0.070 \pm 0.057 \text{ (stat)} \pm 0.014 \text{ (syst)} \quad \text{LHCb}.$$  

Agrees with SM prediction ($A_U = A_V = 0$).

In SM, certain fake TP’s are very small [A. Datta+, PLB701, 357 (2011)] can partially distinguish the SM from NP through the measurement of the fake $A_T^{(2)}$ TP. This applies to $B \rightarrow \phi K^*$ and $B^0_s \rightarrow \phi\phi$.

If the time-dependent angular analysis of a pure-penguin $\bar{b} \rightarrow \bar{s}$ $B^0_s \rightarrow VV$ decay, such as $B^0_s \rightarrow \phi\phi$ or $B^0_s \rightarrow K^{*0}\bar{K}^{*0}$, can be performed, there are many tests of NP in the decay, see D. London+, Europhys. Lett. 67, 579 (2004), PRD69, 114013 (2004).
(2) \( f_T, f_L \): naively, one expects \( f_T \ll f_L \), where \( f_T (f_L) \) is the fraction of transverse (longitudinal) decays. However, it was observed that \( f_T / f_L \simeq 1 \) in \( B \to \phi K^* \). One explanation of this “polarization puzzle” is that \( 1/m_B \) penguin annihilation (PA) contributions are important [A. L. Kagan, PLB601, 151 (2004)]. PA can be sizeable within QCDF.

\( \exists \) two penguin decay pairs whose amplitudes are the same under flavour SU(3), and for which there is a good estimate of SU(3) breaking (QCDF): \( (B_s^0 \to \phi \phi, B_d^0 \to \phi K^{0*}) \) and \( (B_s^0 \to \phi \bar{K}^{0*}, B_d^0 \to \bar{K}^{0*} K^{0*}) \) [A. Datta+, EPJ C60, 279 (2009)]. Given the polarization in the \( B_d^0 \) decay, can predict the polarization in the \( B_s^0 \) decay \( \implies \) test PA.

Partially done – \( B_s^0 \to \phi \phi \) measured:

\[
\text{predict : } \frac{f_T(B_s^0 \to \phi \phi)}{f_T(B_d^0 \to \phi K^{0*})} = 1.36 \pm 0.59 ,
\]

\[
\text{expt : } \frac{f_T(B_s^0 \to \phi \phi)}{f_T(B_d^0 \to \phi K^{0*})} = 1.25 \pm 0.11 .
\]

Theoretical error large, but reasonable agreement.
Measuring U-spin/SU(3) breaking

Consider charmless $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ decays whose amplitudes are equal under U spin ($d \leftrightarrow s$). In general, there are four observables in these processes: the CP-averaged $\bar{b} \to \bar{d}$ and $\bar{b} \to \bar{s}$ decay rates $B_d$ and $B_s$, and the direct CP asymmetries $A_d$ and $A_s$. In the U-spin limit, $X = 1$, where $X \equiv -(A_s/A_d)(B_s/B_d)$. Thus, by measuring the four observables, and computing the deviation of $X$ from 1, one can measure U-spin breaking [M. Imbeault+, PRD84, 056002 (2011)].

This can be applied to decay pairs involving $B_s^0$ decays:

1. $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$,
2. $B_s^0 \to \pi^+K^-$ and $B_d^0 \to \pi^-K^+$,
3. $B_d^0 \to K^0\bar{K}^0$ and $B_s^0 \to \bar{K}^0K^0$,
4. $B_d^0 \to K^+K^-$ and $B_s^0 \to \pi^+\pi^-$.

The first (second) decay is $\bar{b} \to \bar{d}$ ($\bar{b} \to \bar{s}$).

If one neglects annihilation- and exchange-type diagrams, there are 12 additional pairs of decays to which this analysis can be applied. These are not related by U spin, but are instead related by SU(3).
$B^0_s \rightarrow \mu^+ \mu^-$: SM prediction is $B(B^0_s \rightarrow \mu^+ \mu^-) = (3.35 \pm 0.32) \times 10^{-9}$. In July, 2011, the CDF Collaboration (7 fb$^{-1}$) reported the measurement of $B(B^0_s \rightarrow \mu^+ \mu^-) = (1.8^{+1.1}_{-0.9}) \times 10^{-8}$ [T. Aaltonen+, PRL107, 191801 (2011)]. However, the latest LHCb does not confirm this result: $B(B^0_s \rightarrow \mu^+ \mu^-) \leq 4.5 \times 10^{-9}$ (95% C.L.) [R. Aaij+, 1112.1600]

$B^0_d \rightarrow \bar{K}^* \mu^+ \mu^-$: Several slight discrepancies observed by Belle, BaBar and CDF in the exclusive CP-conserving decay $B \rightarrow K^* \mu^+ \mu^-$. Most striking anomaly was seen in lepton forward-backward asymmetry ($A_{FB}$) at low $q^2$ (dilepton invariant mass), and involves sign and size of $A_{FB}$, position of zero crossing. LHCb has measured various observables in $B^0_d \rightarrow \bar{K}^* \mu^+ \mu^-$ [R. Aaij+, 1112.3515]. LHCb does not confirm the Belle/BaBar/CDF result of a large $A_{FB}$ in the low-$q^2$ region.

$B \rightarrow \pi K$: In the latest update of the $\pi K$ puzzle, it was seen that, although NP was hinted at in $B \rightarrow \pi K$ decays, it could be argued that the SM can explain the data [S. Baek+, PLB675, 59 (2009)]. That is, the $\pi K$ puzzle is now only a discrepancy of 1-2$\sigma$. LHCb has measured the direct CPA in $B^0_d \rightarrow K^+ \pi^-$, and finds $-0.088 \pm 0.011 \pm 0.008$ [R. Aaij+, 1202.6251], to be compared with HFAG: $-0.098^{+0.012}_{-0.011}$. 
Conclusions

In the past, there were a number of hints of NP in some $B$ decays, usually in $\bar{b} \to \bar{s}$ transitions. Unfortunately, with recent LHCb measurements, most of these have gone away. This suggests that, if NP is present, very large signals are unlikely.

Still, the LHC has the precision to detect some deviations from the SM predictions. To this end, it is best to make measurements of as many different processes as possible. In this talk, I have mentioned several different possibilities (some of which have already been measured):

- $B_s^0 - \bar{B}_s^0$ mixing in $B_s^0 \to J/\psi \phi$, $J/\psi f_0(980)$, $J/\psi \pi^+ \pi^-$, $D_s^\pm K^\mp$, $D_s^+ D_s^-$, $D_{CP}^0 K \bar{K}$ and other 3-body decays,
- indirect CPV in $B_s^0 \to K^0 \bar{K}^0$,
- $B_s^0 \to \tau^+ \tau^-$,
- TP’s (true and fake) and longitudinal/transverse polarizations in $B_s^0 \to VV$ decays (e.g. $B_s^0 \to \phi \phi$, $B_s^0 \to \phi K^{0*}$, $B_s^0 \to K^{*0} \bar{K}^{*0}$),
- U-spin/SU(3) breaking, etc.

Hopefully we will see a sign of NP.