ASYMPTOTICALLY SAFE ELECTROWEAK INTERACTIONS

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INTRODUCTION

The Nonlinear Sigma Model with values in a coset space *G/H* arises whenever a symmetry group *G* is spontaneously broken to a subgroup *H*. In a minimal realization of the Standard Model, the NLSM geometry describes the dynamics of the Goldstone bosons responsible for the electroweak symmetry breaking of *SU(2) x U(1)* to *U(1)*.

- We consider the possibility that the theory is renormalizable in a nonperturbative sense, namely at a nontrivial ultraviolet (UV) Fixed Point (FP) of the Renormalization Group (RG) flow [1,2].
- A FP is defined by the requirement that the beta functions of the dimensionless couplings have a zero $\beta(g_i) = 0$.
- A quantum field theory is said to be A*symptotically Safe (AS)* if it has a FP with a finite number of UV-attractive directions. Observable quantities such as decay rates and cross sections can be expressed as functions of the couplings and if the couplings are finite, also the observable quantities will be finite. The good UV limit is obtained by requiring that out theory lies on a renormalization group trajectory for which all couplings remain finite when the energy goes to infinity.
- If the FP has a finite number *n* of attractive directions, then the set of all points belonging to these trajectories is a *n*-dimensional surface called the *"UV-critical surface"*. In an asymptotically safe theory we would have to perform *n* experiments at some given energy scale k_0 in order to pin down completely our position in theory space. Everything else could then be computed, at least in principle, and would constitute genuine predictions that could be verified experimentally.
- The main tool used for study the scale dependence of our theory is the Exact Renormalization Group Equation (ERGE)[3]:

$$k\frac{d\Gamma_k[\varphi]}{dk} = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2\Gamma_k[\varphi]}{\delta\varphi\delta\varphi} + R_k\right)^{-1} k\frac{dR_k}{dk}\,. \qquad (1)$$

This equation applies to the Effective Average Action Γ_k which is a generalization of the standard effective action, obtained by implementing an Infrared (IR) cutoff R_k in its functional definition. The ERGE is well defined both in the IR and UV and we can thus forget about regularization.

THE ELECTROWEAK MODEL

We consider a Nonlinear Sigma Model where the Goldstone bosons are coupled to the electroweak gauge bosons. We use a geometrical description where the Golstone bosons are coordinates $\varphi^\alpha(x)$ of a field *U(x)* taking values in *SU(2) x U(1) / U(1) ~ SU(2)*. The lowest order terms in the Euclidean action are

$$\Gamma_k = \frac{1}{2f^2}\int d^4x\, h_{\alpha\beta} D_\mu\varphi^\alpha D^\mu\varphi^\beta + \frac{1}{4g^2}\int d^4x\, W^i_{\mu\nu} W_i^{\mu\nu} + \frac{1}{4g'^2}\int d^4x\, B_{\mu\nu}B^{\mu\nu}\,, \qquad (2)$$

where W^i are the $SU(2)_L$ gauge fields and *B* is the $U(1)_R$ gauge filed, *g* and *g'* are the dimensionless gauge couplings and ***f = 2/υ*** is the Goldstone boson coupling with mass dimension *-1 (υ=EW VEV)*. The covariant derivative is:

$$D_\mu\varphi^\alpha = \partial_\mu\varphi^\alpha + W^i_\mu R_i^\alpha - B_\mu L_3^\alpha\,.$$

The right-invariant vectorfields R_i generate $SU(2)_L$ while the left-invariant L_3 generates $U(1)_R$. Gauge invariance of the SM demands that the metric $h_{\alpha\beta}$ be invariant under the action of these vectorfields, but not necessarily under the full $SU(2)_R$ generated by L_i. The most general metric of this type is of the form

$$h_{\alpha\beta} = L^1_\alpha L^1_\beta + L^2_\alpha L^2_\beta + (1-2a_0)L^3_\alpha L^3_\beta\,,$$

where the parameter $\boldsymbol{a_0}$ measures the violation of the "custodial" symmetry and vanishes in the bare SM Lagrangian. It is therefore customary to assume that the metric $h_{\alpha\beta}$ is bi-invariant and to consider the $SU(2)_R$ breaking as due to a separate term in the effective action. Among all the possible further terms of the action we shall be interested in the following ones:

$$\Gamma'_k = -\frac{a_0}{f^2}\int d^4x\, L^3_\alpha L^3_\beta D_\mu\varphi^\alpha D^\mu\varphi^\beta - \frac{a_1}{2}\int d^4x\, B_{\mu\nu}W_i^{\mu\nu}R^i_\alpha L^\alpha_3\,.$$

The dimensionless couplings $\boldsymbol{a_0}$ and $\boldsymbol{a_1}$ enter in the computation of the oblique parameters ***S*** and ***T***:

$$S = -16\pi a_1(m_Z) + \frac{1}{6\pi}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right]\,,$$

$$T = \frac{2}{\alpha}a_0(m_Z) - \frac{3}{8\pi\cos^2\theta_W}\left[\frac{5}{12} - \log\left(\frac{m_H}{m_Z}\right)\right]\,. \qquad (3)$$

Beta functions

Solving the beta functional equation (1) we obtain a set of coupled equations for the beta functions for the dimensionless couplings [4]: $(\tilde f^2 = k^2 f^2)$

$$k\frac{d\tilde f^2}{dk} = 2\tilde f^2 - \frac{1}{2}\frac{\tilde f^2}{(4\pi)^2}\left(\tilde f^2(1+2a_0) + 6g^2 + 3g'^2\right)\,,$$

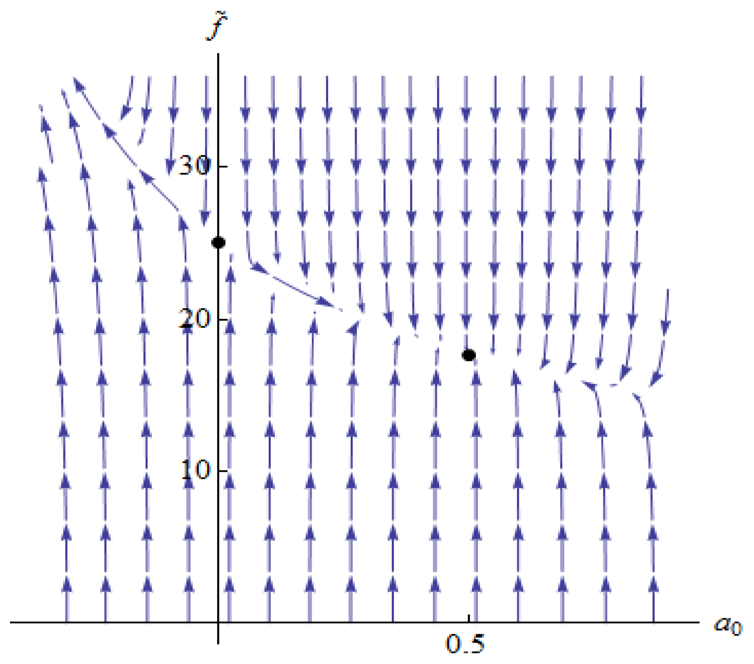
$$k\frac{dg^2}{dk} = -\frac{29}{2}\frac{g^4}{(4\pi)^2}\,, \quad k\frac{dg'^2}{dk} = \frac{1}{6}\frac{g'^4}{(4\pi)^2}\,,$$

$$k\frac{da_0}{dk} = \frac{1}{2}\frac{1}{(4\pi)^2}\left(\tilde f^2 a_0(1-2a_0) + \frac{3}{2}g'^2\right)\,,$$

$$k\frac{da_1}{dk} = \frac{1}{(4\pi)^2}\left(\tilde f^2 a_1 + \frac{1}{6}\right)\,.$$

RESULTS

The running of the gauge couplings is very slow and for our purposes is a good approximation to treat them as constants. Setting *g=0.65* and *g'=0.35* we find two physically acceptable UV FP's:



FPI 1 UV-attractive direction

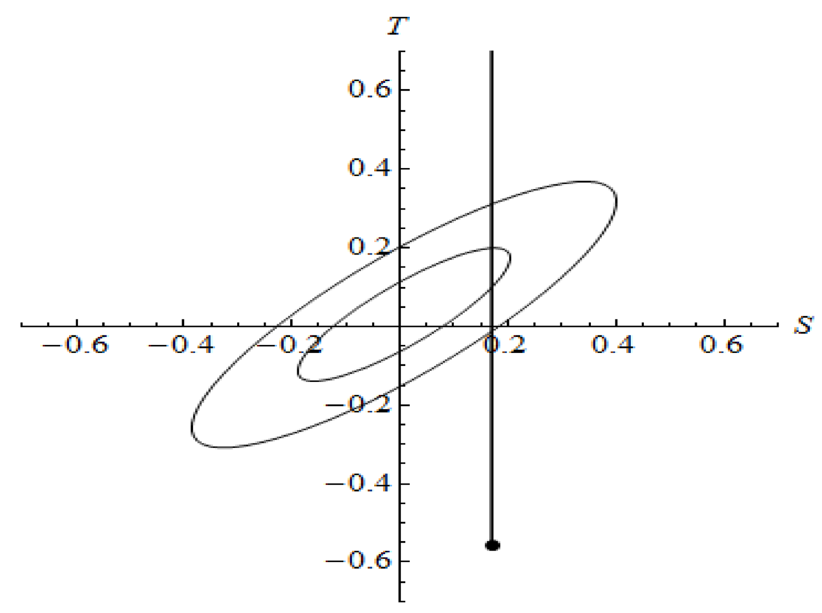
$\tilde f_* = 25.1 \quad a_{0*} = -0.000292 \quad a_{1*} = -0.000265$

FPII 2 UV-attractive directions

$\tilde f_* = 17.7 \quad a_{0*} = 0.501 \quad a_{1*} = -0.000530$

Oblique parameters

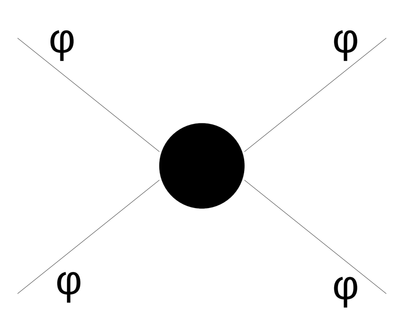
When one evolves the flow toward higher energies, the couplings will generally diverge. However there may be trajectories that hit a FP in the UV, such trajectories are said to be AS trajectories. Requiring that the world be described by a renormalizable trajectory leads to predictions for low energy values of the couplings, this allows to predict the values of the ***S*** and ***T*** parameters using (3):



The dot corresponds to the prediction obtained flowing down from by FPI, the half-line represents the values predicted flowing down from FPII. The ellipses show the 1 and 2 σ experimental bounds for *S=0.01±0.10* and *T=0.03±0.11* [5].

Goldstone boson scattering amplitude

In previous calculations of the RG flow of the NLSM we have kept the full field dependence of the effective action but expanded to second order in momenta. By contrast in order to compute the scattering amplitude of two Goldstone bosons to two Goldstone bosons we need terms of fourth order in the field, but we have to retain the full momentum dependence. We use the running of the Goldstone coupling *f* to solve iteratively the functional equation (1) in order to obtain the full quantum effective action Γ_0. This procedure eliminates all divergences and produces a formula for the improved four-point amplitude, which has a much better high energy behavior with respect to the usual leading perturbative result $A(s,t,u) \sim s/\upsilon^2$ $(s = E^2_{cm})$. In the limit $s \to \infty$ we have (*PRELIMINARY RESULT*):



$$A(s,t,u) \sim \log\frac{s}{8\pi^2\upsilon^2}$$

COMMENTS

- The requirement of AS makes this minimal model, despite the absence of a fundamental Higgs boson, consistent with electroweak precision tests and with unitarity constraints *(PRELIMINARY RESULT)*.
- For realistic theories, it is important to study what happens to the AS picture when fermions are coupled to the Goldstone bosons. To preserve the AS picture it is necessary to introduce four-fermion interactions [6]:

$$-\frac{2h}{f}\int d^4x\,(\bar\psi_L^{ia}U_{ij}\psi^{aj} + h.c.) + \lambda_1\int d^4x\,\bar\psi_L^{ia}\psi_R^{ja}\bar\psi_R^{jb}\psi_L^{ib} + \cdots$$

- In case of discovery of an "Higgs" particle, the theory can be extended introducing this new scalar degree of freedom:

$$\Gamma_k = \frac{1}{2f^2}\int d^4x\, h_{\alpha\beta}D_\mu\varphi^\alpha D^\mu\varphi^\beta(1 + 2aHf + bH^2f^2 + \cdots)$$

The study of such a model in the AS perspective can, in principle, give predictions about the relevant couplings of the Higgs interactions with the other SM particles. Moreover, the Higgs potential can be predicted.

References:

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[2] S. Weinberg, ``General Relativity: An Einstein centenary survey``, ed. S. W. Hawking and W. Israel, pp.790--831, Cambridge University Press (1979); M. Niedermaier and M. Reuter, Living Rev. Relativity 9, 5 (2006); R. Percacci, ``Asymptotic Safety``, in ``Approaches to Quantum Gravity: Towards a New Understanding of Space, Time and Matter``, Cambridge University Press (2009).

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[5] K. Nakamura *et al.* (Particle Data Group), J. Phys. G 37, 075021 (2010).

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