Recent results and prospects from NA48/2: $K^{\pm}\rightarrow\pi l\nu$ and $K^{\pm}\rightarrow\pi^{+}\pi^{-}e^{\pm}\nu$, $K^{\pm}\rightarrow\pi^{0}\pi^{0}e^{\pm}\nu$ decays

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On behalf of the *NA48/2 collaboration*:
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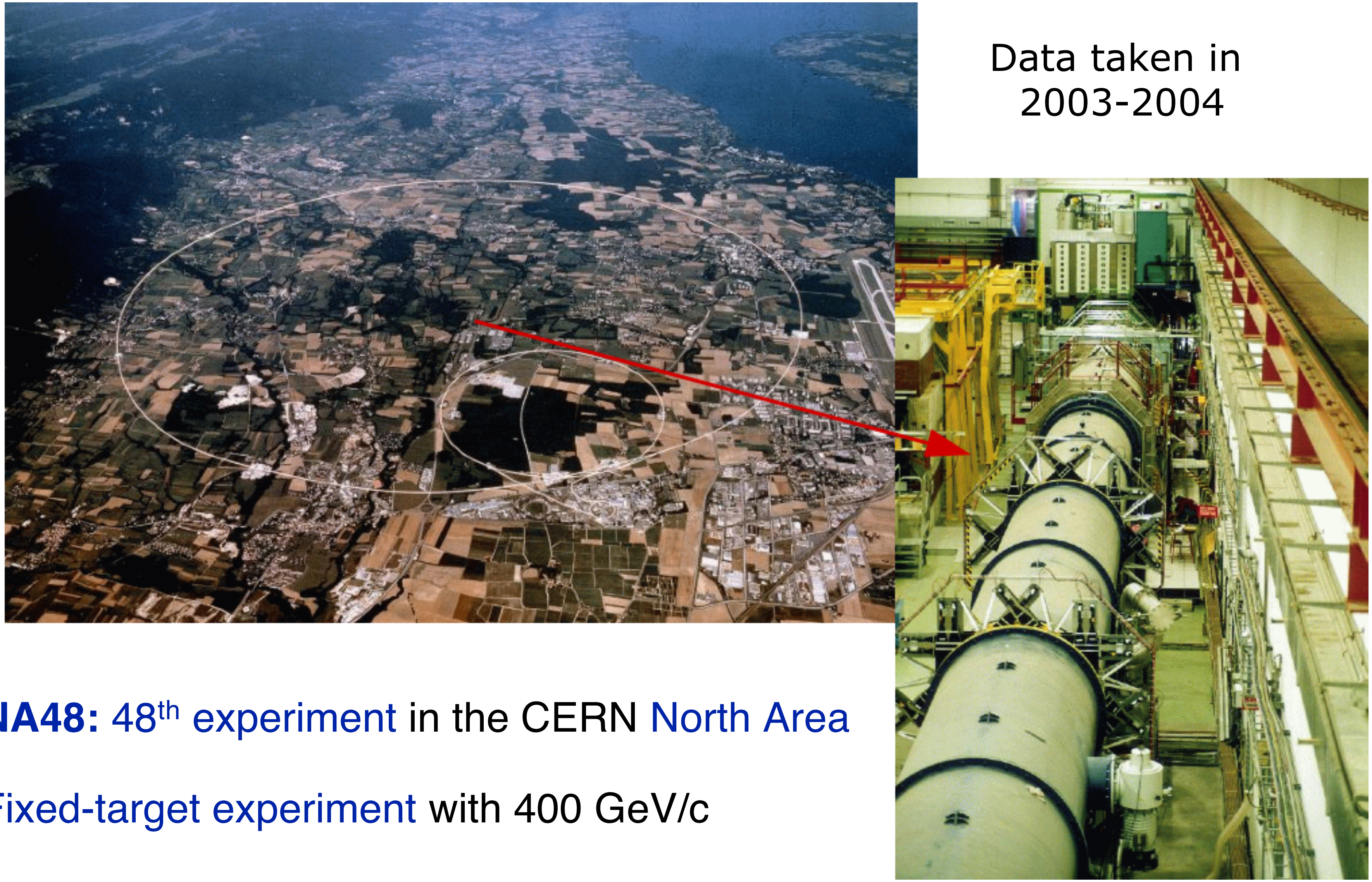
Outline

- Kaon physics at CERN
  - ★ the NA48/2 experiment
- The semileptonic K_{l3} decays $\mathbf{K^{\pm} \rightarrow \pi^0 e^{\pm} \nu}$ and $\mathbf{K^{\pm} \rightarrow \pi^0 \mu^{\pm} \nu}$
  - ★ Introduction & parametrization
  - ★ Preliminary Form Factors results
- The K_{e4} decays $\mathbf{K^{\pm} \rightarrow e^{\pm} \nu \pi^{+} \pi^{-}}$ and $\mathbf{K^{\pm} \rightarrow e^{\pm} \nu \pi^0 \pi^0}$
  - ★ Interest of the decays in the ChPT framework
  - ★ Preliminary Branching Ratio measurements

preliminary

preliminary

The NA48/2-Experiment at CERN

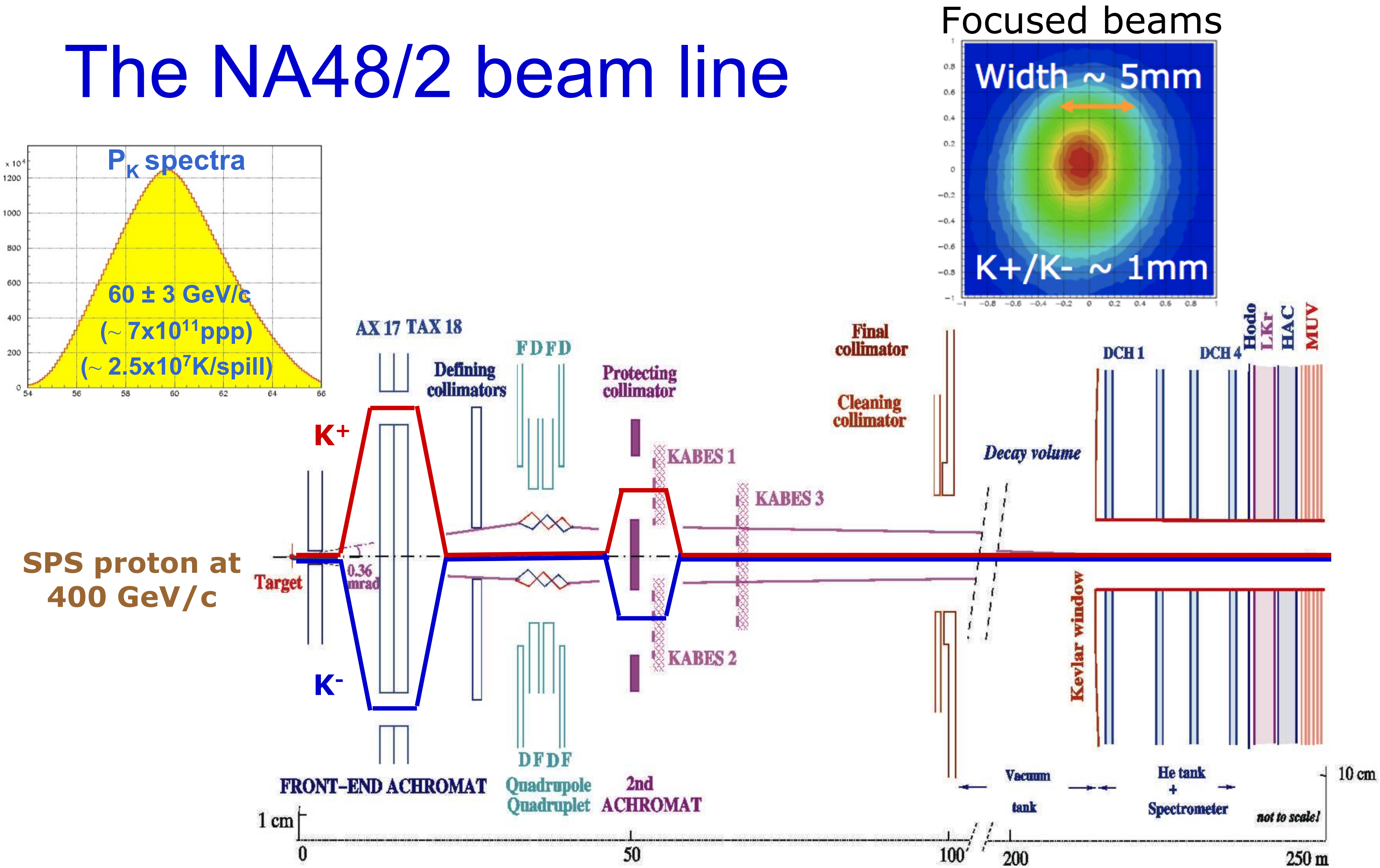


Data taken in 2003-2004

NA48: 48^{th} experiment in the CERN North Area

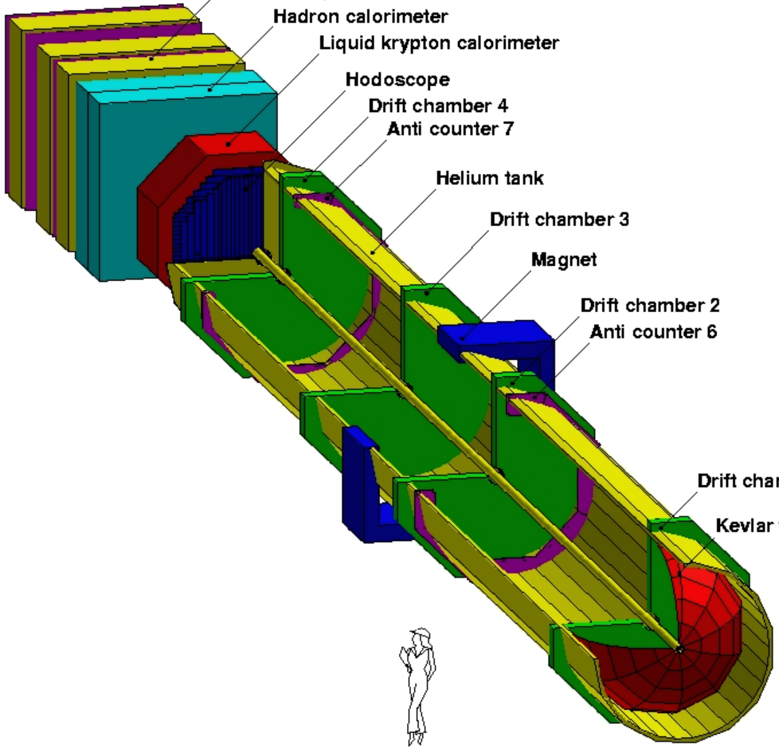
Fixed-target experiment with 400 GeV/c proton beam from the SPS

The NA48/2 beam line



- fixed target experiment at CERN-SPS
- Kaon decays in vacuum tank: 22%
- 6.3×10^7 particles per pulse in decay region
- Simultaneous, unseparated, focused beams
- Similar acceptance for K^+ and K^- decays
- $K^+/K^- \sim 1.8$

The NA48/2 Detector



Magnetic spectrometer:

$\sigma_p/p = (1.0 \oplus 0.044\ p)\%$ (p in GeV/c)

$\sigma(M_{3\pi\pm}) = 1.7\ \mathrm{MeV}/c^2$

Hodoscope:

σ_t=150ps

LKr electromagnetic calorimeter:

$\sigma_E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV)

$\sigma_x = \sigma_y \sim 1.5$mm for E=10 GeV

$\sigma(M_{\pi\pi^0\pi^0}) = 1.4\ \mathrm{MeV}/c^2$

E/p ratio used for e/π discrimination

- ~100 m long decay region in vacuum
- Triggers based on LKr peaks, CHOD hits and DCH multiplicity
- Similar acceptance between K^+ and K^- beams checked reversing magnetics fields
- Pion decay products, from the hadronic beam, remain into the beam pipe

V_{us} extraction from K_{l3} Decays

$$\Gamma(K_{l3(\gamma)}) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW} |V_{us}|^2 |f_+(0)|^2 I_K^l (1 + 2\delta_{SU(2)}^l + 2\delta_{EM}^l)$$

$C_K^2 = 1$ for K^0, $= \frac{1}{2}$ for $K^\pm$.

$S_{EW} = 1.0232$: short-distance EW correction.

To be measured by experiments:
- $\Gamma(K)$ Inclusive of radiative corrections
- I_K integral of form factors over phase space

To be determined by theory:
- $f(0)$ hadronic matrix element at $q^2=0$
- $\delta_{SU(2)}$, δ_{EM} form factors corrections for SU(2) breaking and long-distance EM corrections

Form Factors in K_{l3} Decays

Two form factors in K_{l3} decays: $f_+(t)$, $f_-(t)$ (with $t = q^2$, *the* squared 4-momentum transfer to the l-ν system)

$$M = \frac{G_F}{2} V_{us}(f_+(t)(P_K + P_\pi)^\mu \bar{u}_l \gamma_\mu (1+\gamma_5) u_\nu + f_-(t) m_l \bar{u}_l (1+\gamma_5) u_\nu)$$

$f_+(t)$ = **vector** form factor $f_0(t)$ = **scalar** form factor

($f_-(t)$ can only be measured with μ decays)

$$f_0(t) = f_+(t) + \frac{t}{(m_K^2 - m_\pi^2)} f_-(t)$$

- $f_+(0)$ cannot be measured directly,
  needs to be given by theory (lattice QCD, χPT);
  ratio of form factors accessible by experiments

$$\bar{f}_+(t) = \frac{f_+(t)}{f_+(0)} \qquad \bar{f}_0(t) = \frac{f_0(t)}{f_+(0)}$$

Form Factors parameterization

- **Pole Parametrization:** assume the exchange of vector (1^-) or scalar (0^+) resonances with mass m_V/m_S. $f_+(t)$ can be described by K*(892), for $f_0(t)$ no obvious dominance is seen

$$f_{+,0}(t) = \frac{m^2_{V,S}}{m^2_{V,S} - t}$$

- **Linear and quadratic parametrization:** Taylor expansion in the momentum transfer without a direct physical meaning

$$\overline{f_{+,0}}(t) = \left(1 + \lambda_{+,0} \frac{t}{m^2_\pi} \right)$$

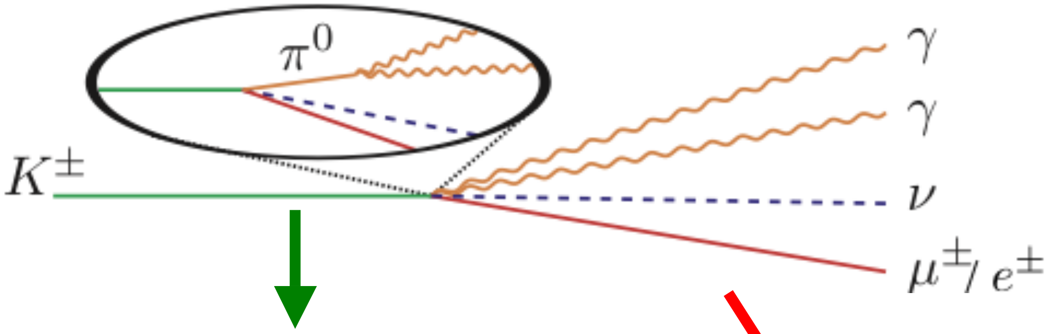
linear

$$\overline{f_{+,0}}(t) = \left(1 + \lambda'_{+,0} \frac{t}{m^2_\pi} + \frac{1}{2} \lambda''_{+,0} \left(\frac{t}{m^2_\pi}\right)^2 \right)$$

quadratic

Large correlation between parameters for quadratic

$K^{\pm}{}_{l3}$ Selection

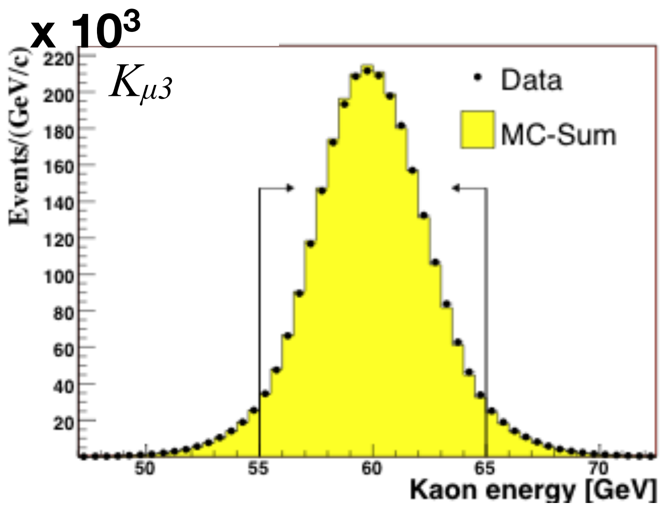


One good event:

photons & track in-time

$m^2{}_{\text{miss}} < 10\ (\text{MeV}/c^2)^2$

$55 < E_{K\pm} < 65$ GeV (under assumption of missing neutrino)



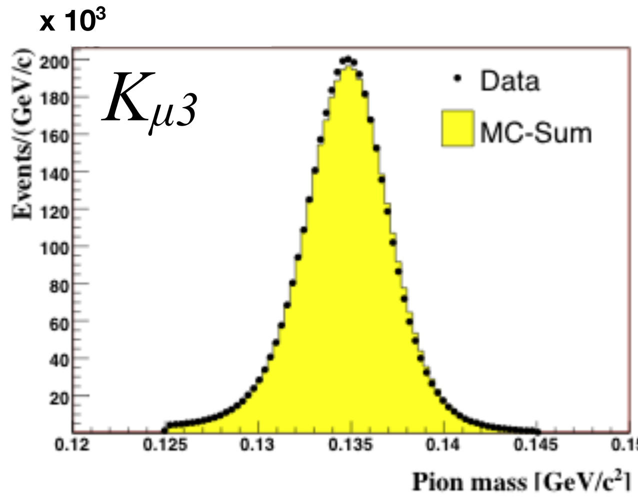
One good track:

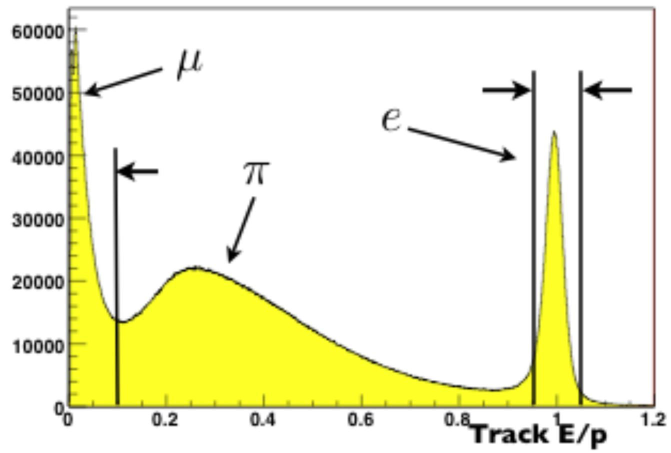
$\mu^{\pm}$ ID: μ veto & E/p

$e^{\pm}$ ID: E/p

One good $\pi^0 \to \gamma\gamma$:

π^0 mass: $|m_{\gamma\gamma} - m_{\pi 0}| < 10$ MeV/c^2





3-days data taking,
Minimum bias trigger

$K^{\pm}_{l3}$ Background Suppression

Main background: $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}$ with $\pi^{\pm} \rightarrow \mu^{\pm}$ decay or mis-ID

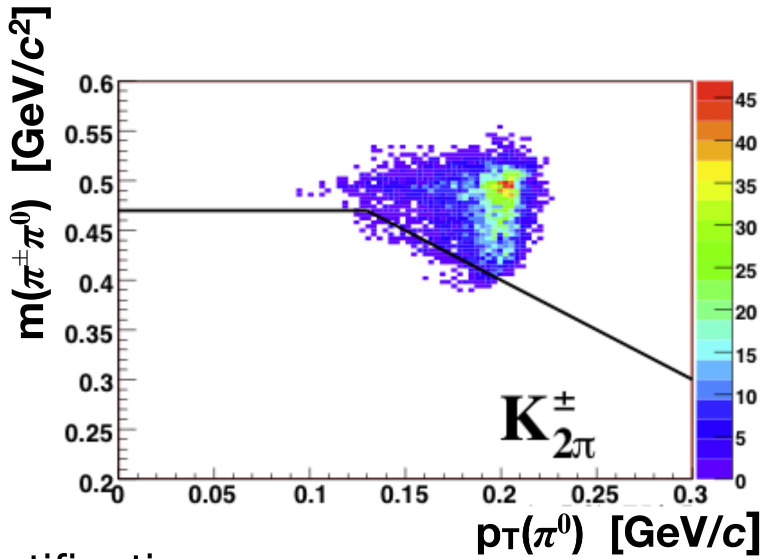
Without suppression, background at the 20% level

Use 2-body vs. 3-body decay: $p_T(\pi^0)$

Use π vs. μ: $m(\pi^{\pm}\pi^{0}) \approx m_K$

Background contamination ≈ 0.5%

Acceptance loss ≈ 24%



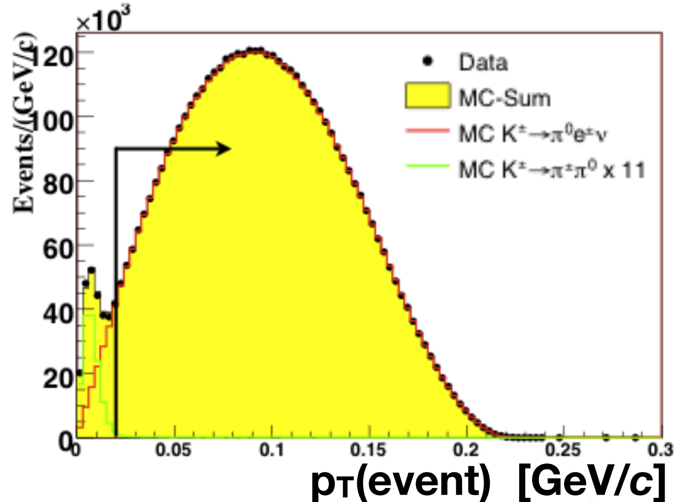
Main background: $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}$ with $\pi^{\pm} \rightarrow e^{\pm}$ mis-identification

$\pi^{\pm}$ with E/p > 0.95 can fake a $K^{\pm}_{e3}$ decay

Cut on transverse momentum of the event

Background contamination < 0.1%

Acceptance loss ≈ 3%



2.5 10^6 $K^{\pm}_{\mu 3}$ candidates selected

4.0 10^6 $K^{\pm}_{e3}$ candidates selected

Radiative Effects

K_{l3} decay rate with first order radiative corrections:

$$\Gamma_{K_{l3}} = \Gamma^0_{K_{l3}} + \Gamma^1_{K_{l3}} = \Gamma^0_{K_{l3}}(1 + 2\delta^{Kl}_{EM})$$

Simulation code provided by KLOE (C.Gatti, EPJ C45 (2006) 417).

Parameters used for the normalisation (JHEP 11 (2008) 006).

Mode	$\delta^{K_{e3}}_{EM}(\%)$
K^0_{e3}	0.495 ± 0.110
$K^{\pm}_{e3}$	0.050 ± 0.125

About **1% effect** on Dalitz plot slope for $K_{\mu 3}$ and **10%** for K_{e3}

Fit to the Dalitz Plot Density

$$\rho(E_l^*, E_\pi^*) = \frac{d^2N(E_l^*, E_\pi^*)}{dE_l^* dE_\pi^*} \propto Af_+^2(t) + Bf_+(t)(f_0 - f_+)\frac{m_K^2 - m_\pi^2}{t} + C\left[(f_0 - f_+)\frac{m_K^2 - m_\pi^2}{t})\right]^2$$

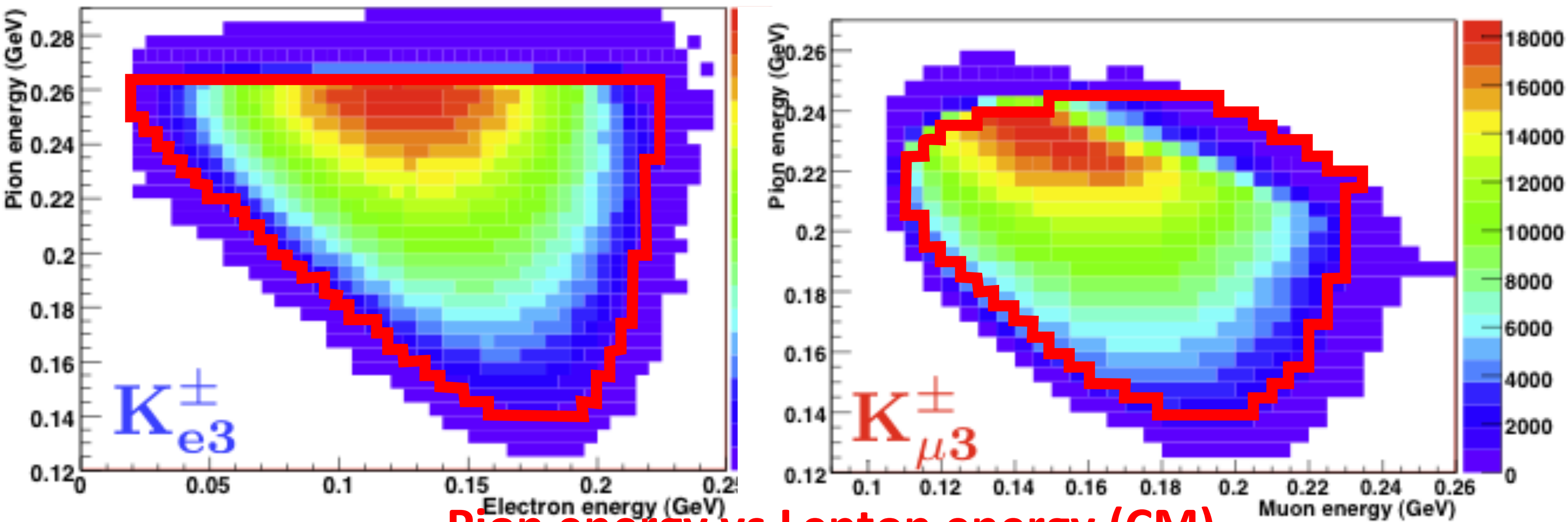
E_l^*, E_π^* = energies of $l^\pm$, π^0 in $K^\pm$ rest frame

A, B, C = known kinematical terms

Fit with form factor-weighted MC performed in 5 x 5 MeV^2 bins in (E_l^*, E_π^*)

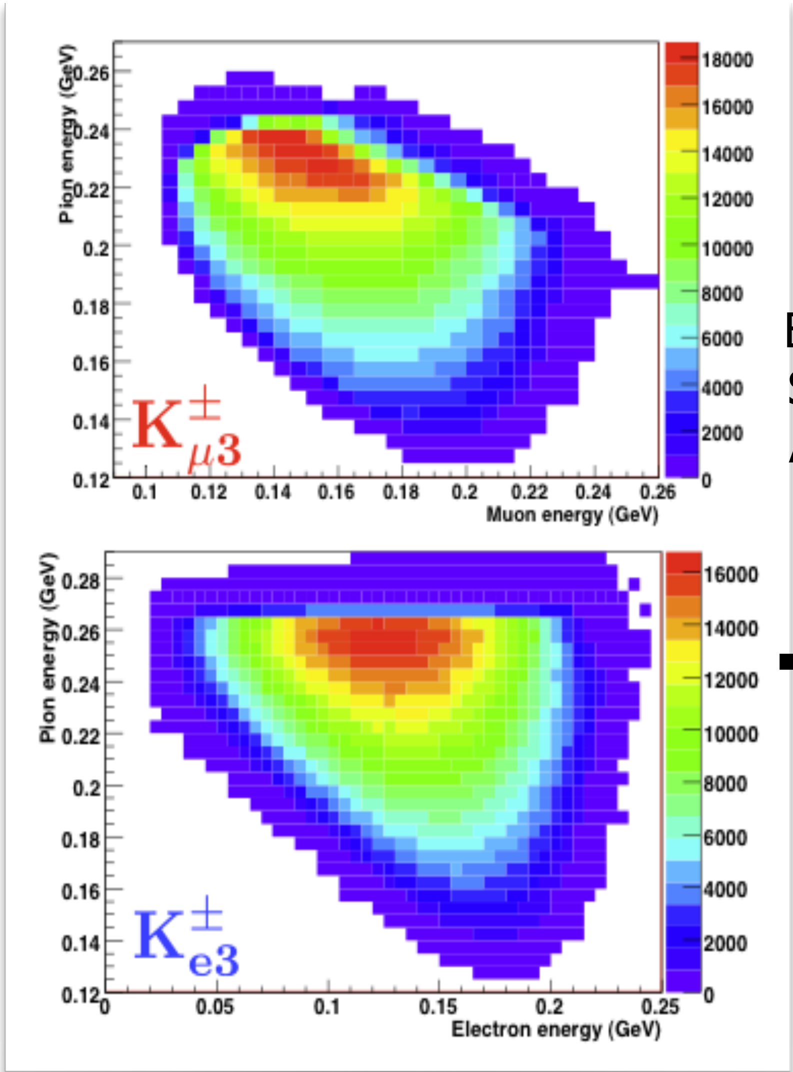
Cells outside or crossing the border of the physical region are not fitted

Background subtraction, acceptance correction, reweighting for radiative

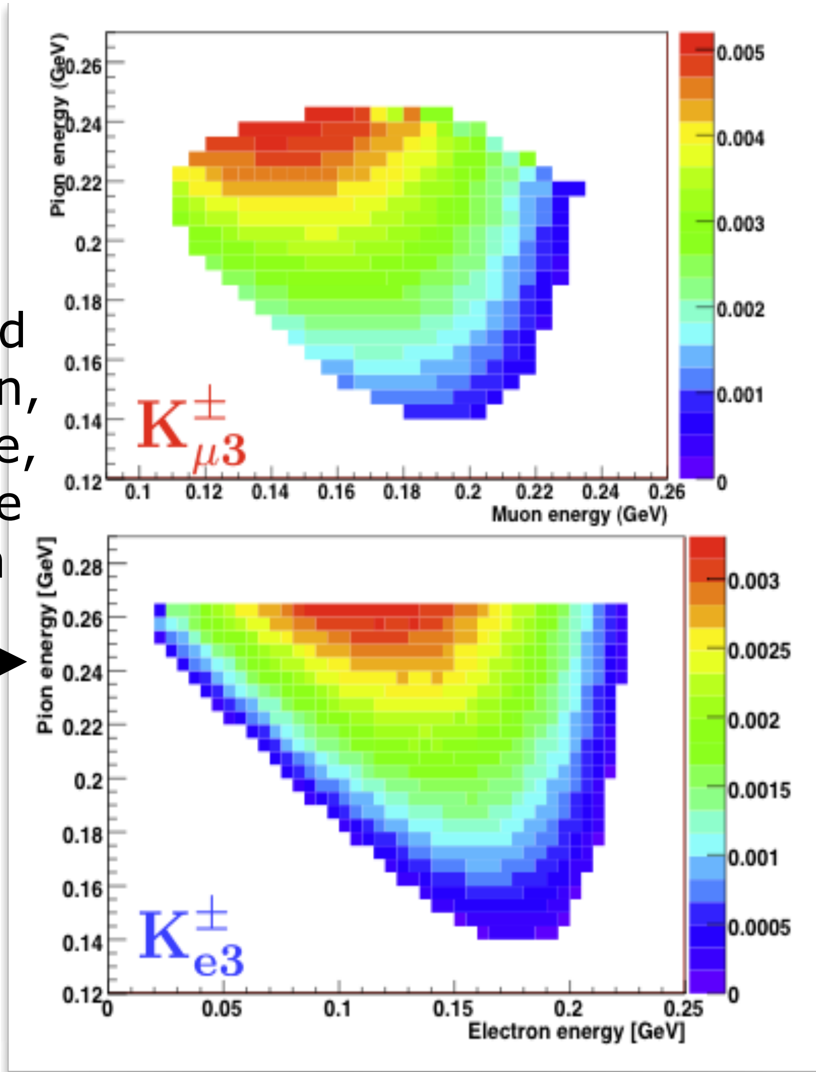


Pion energy vs Lepton energy (CM)

Fit to the Dalitz Plot Density



Background Subtraction, Acceptance, Final State Radiation



Pion energy vs Lepton energy (CM)

Systematics

preliminary

$K^{\pm}_{e3}$	$\Delta\lambda'_+$ ($\times 10^{-3}$)	$\Delta\lambda''_+$ ($\times 10^{-3}$)	Δm_V (MeV/c^2)
Kaon Energy	± 0.3	± 0.1	± 6
Vertex	± 0.2	± 0.1	± 0
Bin size	± 0.0	± 0.1	± 2
Energy scale	± 0.1	± 0.0	± 0
Acceptance	± 0.2	± 0.0	± 3
2nd Analysis	± 0.9	± 0.4	± 1
FF input	± 0.4	± 0.0	± 1
Systematic	± 1.1	± 0.4	± 7
Statistical	± 0.7	± 0.3	± 3

➜ systematics somewhat larger than statistical uncertainty

$K^{\pm}_{\mu 3}$	$\Delta\lambda'_+$ ($\times 10^{-3}$)	$\Delta\lambda''_+$ ($\times 10^{-3}$)	$\Delta\lambda_0$ ($\times 10^{-3}$)	Δm_V (MeV/c^2)	Δm_S (MeV/c^2)
Kaon Energy	± 0.1	± 0.0	± 0.3	± 1	± 8
Vertex	± 1.0	± 0.5	± 0.1	± 2	± 7
Bin size	± 0.8	± 0.4	± 0.7	± 3	± 10
Energy scale	± 0.3	± 0.1	± 0.1	± 0	± 1
Acceptance	± 0.2	± 0.1	± 0.3	± 2	± 5
$K_{2\pi}$ background	± 1.7	± 0.5	± 0.6	± 3	± 0
2nd Analysis	± 0.1	± 0.1	± 0.2	± 2	± 5
FF input	± 0.3	± 0.8	± 0.1	± 7	± 3
Systematic	± 2.2	± 1.1	± 1.0	± 9	± 16
Statistical	± 3.0	± 1.1	± 1.4	± 8	± 31

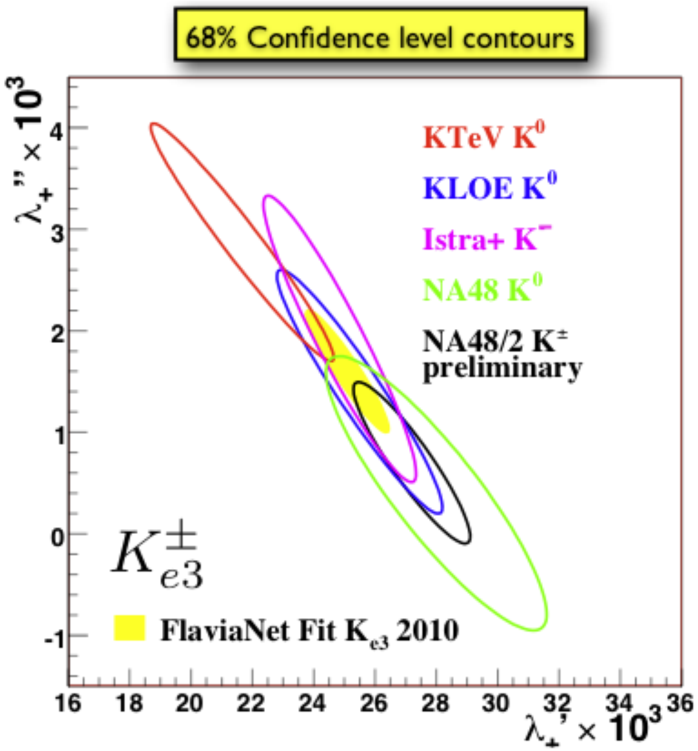
→ $K_{2\pi}$ background

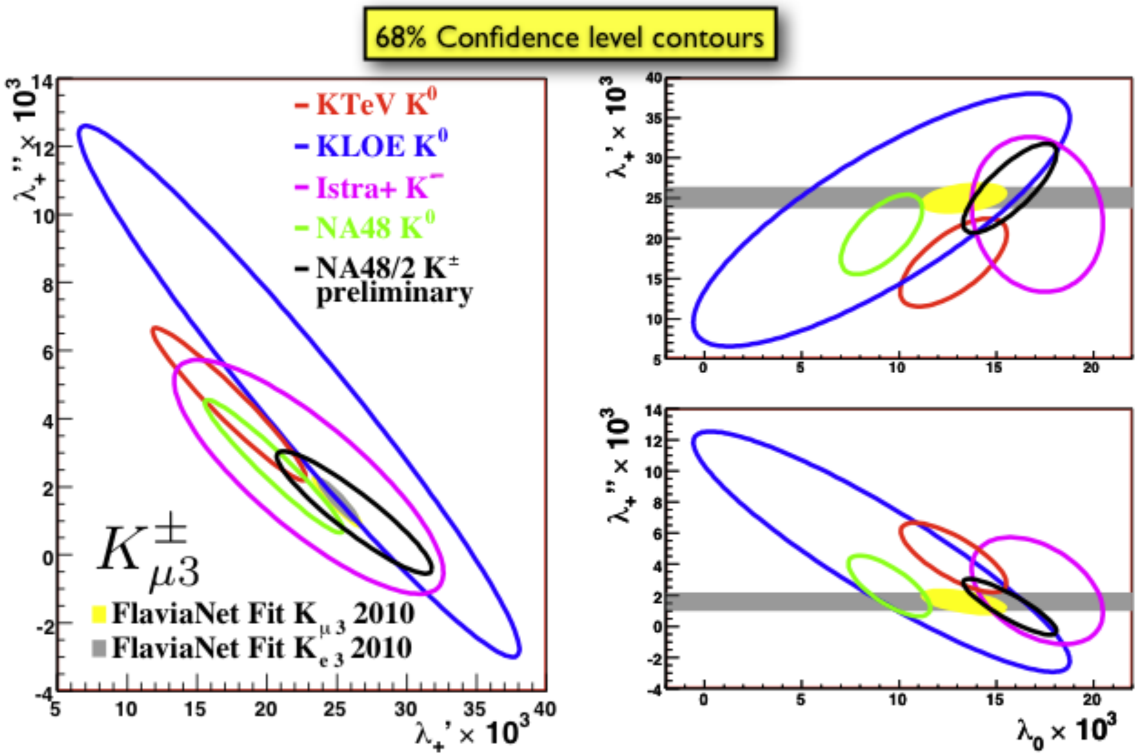
➜ dominated by statistics

Results for $K^{\pm}{}_{e3}$ and $K^{\pm}{}_{\mu3}$

preliminary

Quadratic ($\times 10^{-3}$)	λ'_{+}	λ''_{+}	λ_0
$K^{\pm}_{\mu3}$	$26.3 \pm 3.0_{\text{stat}} \pm 2.2_{\text{syst}}$	$1.2 \pm 1.1_{\text{stat}} \pm 1.1_{\text{syst}}$	$15.7 \pm 1.4_{\text{stat}} \pm 1.0_{\text{syst}}$
$K^{\pm}_{e3}$	$27.2 \pm 0.7_{\text{stat}} \pm 1.1_{\text{syst}}$	$0.7 \pm 0.3_{\text{stat}} \pm 0.4_{\text{syst}}$	
Pole ($\mathrm{MeV/c^2}$)	m_V		m_S
$K^{\pm}_{\mu3}$	$873 \pm 8_{\text{stat}} \pm 9_{\text{syst}}$		$1183 \pm 31_{\text{stat}} \pm 16_{\text{syst}}$
$K^{\pm}_{e3}$	$879 \pm 3_{\text{stat}} \pm 7_{\text{syst}}$		

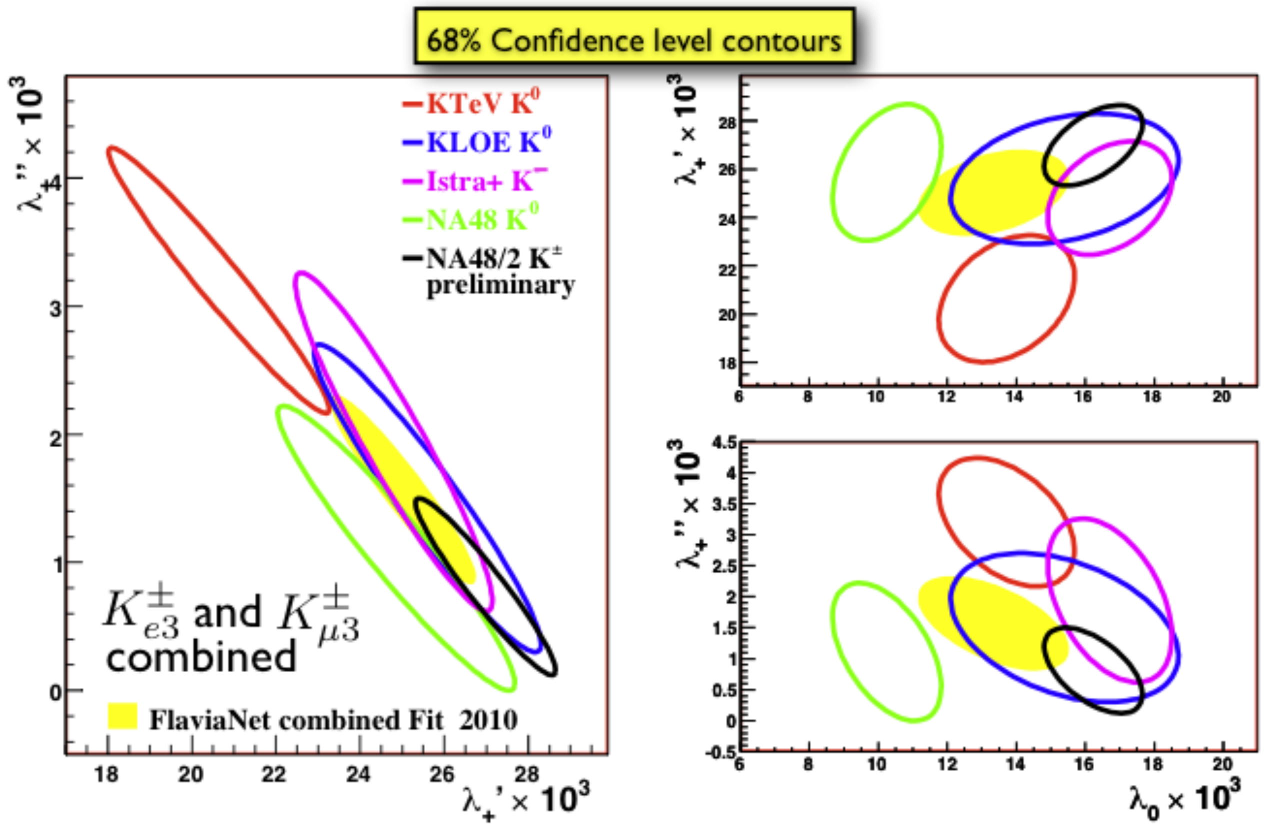




Combined Result

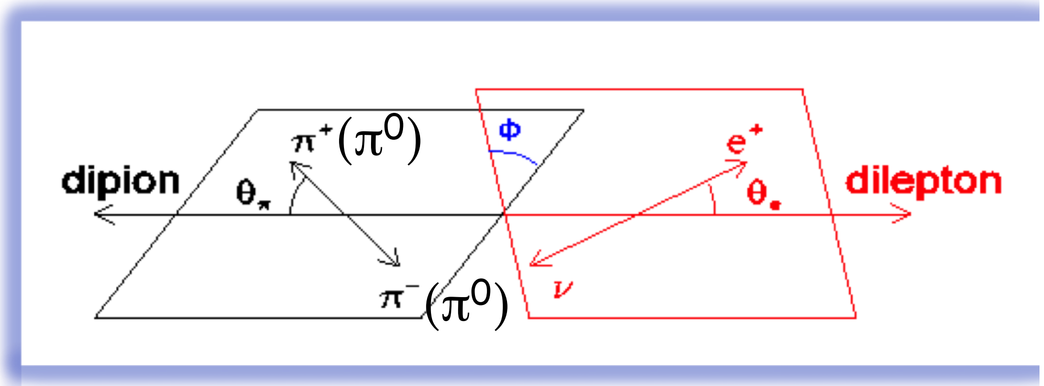
preliminary

Quadratic ($\times 10^{-3}$)	λ'_+	λ''_+	λ_0
$K^{\pm}_{\mu 3} K^{\pm}_{e3}$ combined	26.98 ± 1.11	0.81 ± 0.46	16.23 ± 0.95
Pole (MeV/c^2)	m_V		m_S
$K^{\pm}_{\mu 3} K^{\pm}_{e3}$ combined	877 ± 6		1176 ± 31



- Results for $K^{\pm}_{e3}$ and $K^{\pm}_{\mu 3}$ from NA48/2 in good agreement
- High precision preliminary results, competitive with other measurements. Smallest error in the combined result.

Ke4: $K^{\pm}\rightarrow\pi^{+}\pi^{-}e^{\pm}\nu$ and $K^{\pm}\rightarrow\pi^{0}\pi^{0}e^{\pm}\nu$



The ± kinematics is fully described by 5 variables: $M^2_{\pi\pi}$, $M^2_{e\nu}$, θ_{π} , θ_e , ϕ (Cabibbo-Maksymovicz 1965)

Reduced to 3 variables for 00: $M^2_{\pi\pi}$, $M^2_{e\nu}$, θ_e

Partial Wave expansion of the amplitude (Pais-Treiman 1968)

F,G= 2 Axial Form Factors

$F=F_s e^{i\delta_s}+F_p e^{i\delta_p}\cos\theta_{\pi}$+d-wave term…

$G=G_p e^{i\delta_g}$+d-wave term…

H=1 Vector Form Factor

$H=H_p e^{i\delta_h}$+d-wave term…

F (F_p,F_s), G, H and $\delta=\delta_p-\delta_s$ for ± and only F_s for 00 used as fit parameters

q^2 dependence can be studied expanding fitted form factors: (Amoros-Bijnens 1999)

$F_s=f_s+f'_s q^2+f''_s q^4+f_e(M^2_{e\nu}/4m^2_{\pi})+\ldots$

$F_p=f_p+f'_p q^2+\ldots$

$G_p=g_p+g'_p q^2+\ldots$

$H_p=h_p+h'_p q^2+\ldots$

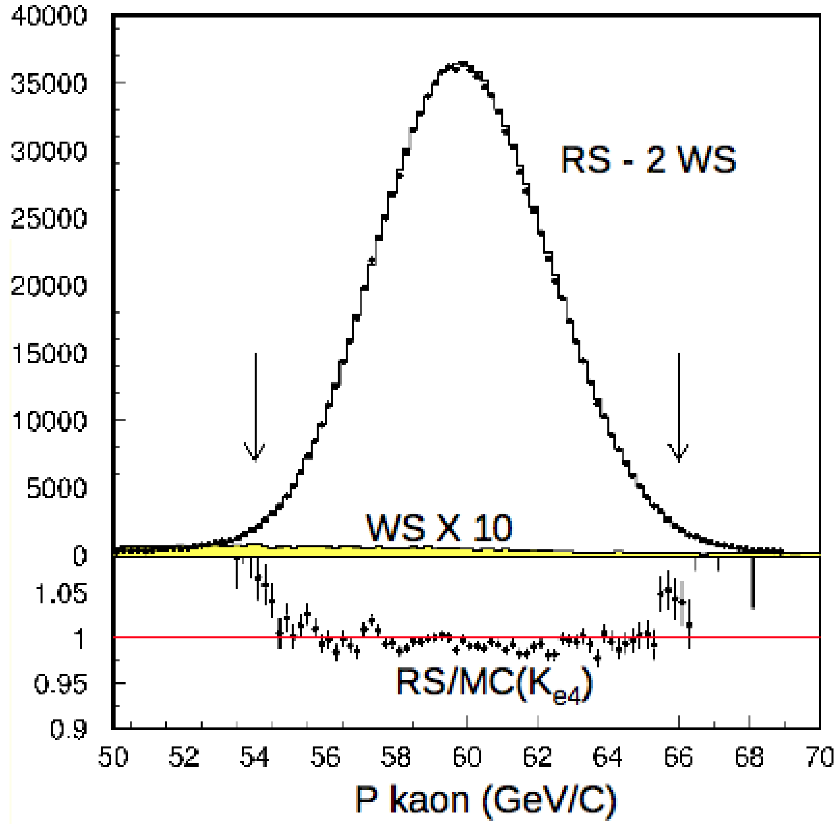
$q^2=(M^2_{\pi\pi}/4m^2_{\pi})-1$

(this Taylor expansion is valid in the Isospin symmetry limit)

$K^{\pm}\rightarrow\pi^{+}\pi^{-}e^{\pm}\nu$ Selection

Signal selection:

- three tracks, total charge ±1
- two opposite sign pions
- E/p for e and π ID



Background main sources:

- $K^{\pm}\rightarrow\pi^{+}\pi^{-}\pi^{\pm}$
  ↳ $e^{\pm}\ \nu$ or misidentified as $e^{\pm}$
- $K^{\pm}\rightarrow\pi^{0}(\pi^{0})\pi^{\pm}$
  ↳ $e^{+}e^{-}\gamma$ +1e misidentified as π and γ(s) undetected

Total (2003+2004 data)

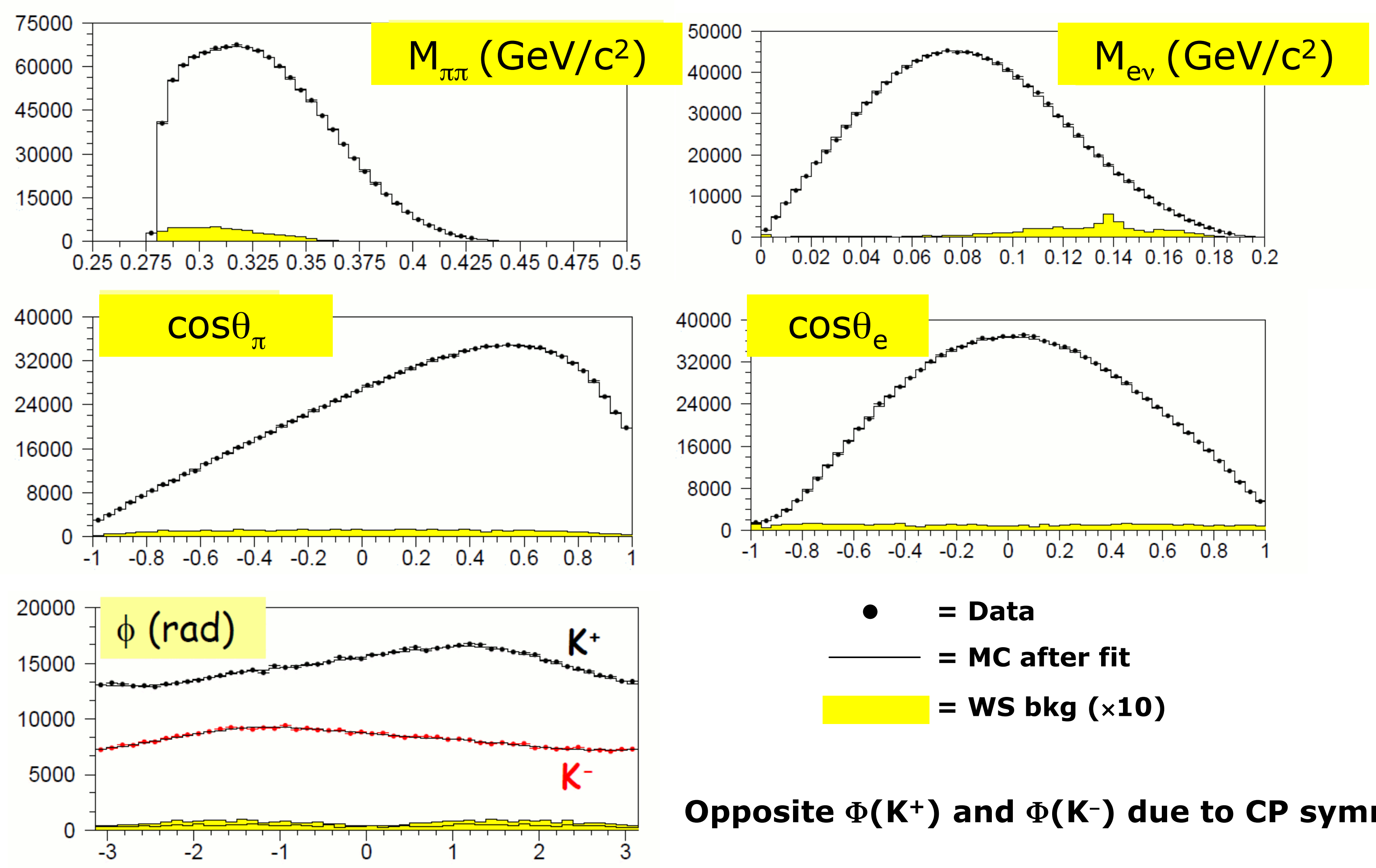
selected events: $1.15\cdot10^{6}$

The background is studied using the electron *"wrong"* sign (WS) events with same sign pions (we assume $\Delta Q=\Delta S$ and total charge ±1) and cross check with MC.

RS/WS=2 for $K_{3\pi}$

The total bkg is at level of 1%.

$K^{\pm}\rightarrow\pi^{+}\pi^{-}e^{\pm}\nu$ Data/MC Comparison



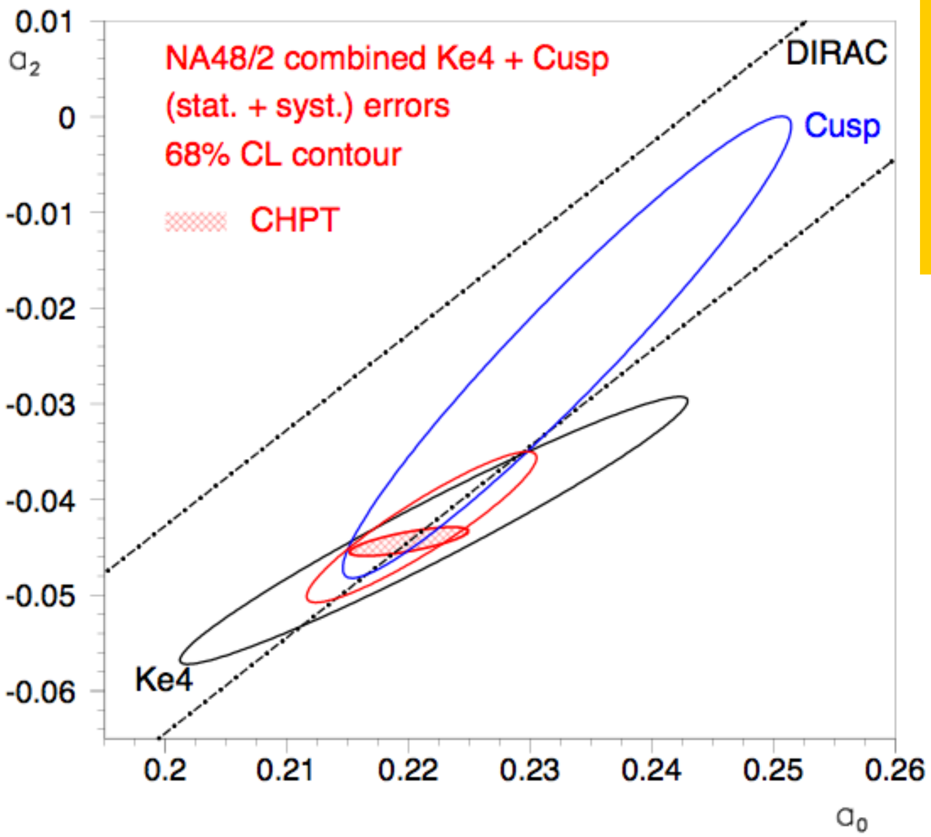
● = Data

—— = MC after fit

= WS bkg (×10)

Opposite $\Phi(K^+)$ and $\Phi(K^-)$ due to CP symmetry

Scattering lengths measured from phase shift



Two statistically independent measurements by NA48: from K_{e4} form factors and from $\pi^+\pi^- \rightarrow \pi^0\pi^0$ scattering length in $K^\pm \rightarrow \pi^\pm\pi^+\pi^-$ decays

✓Different systematics: calorimeter and trigger vs. electron misID and background

✓ Different theoretical inputs: rescattering in final state and ChPT expansion vs.Roy equation and Isospin breaking connection

Precise ChPT prediction constraint:
[CGL NPB 603(2001), PRL86(2001)]

$$a_0 - a_2 = 0.265 \pm 0.004$$
$$a_0 = 0.220 \pm 0.005$$
$$a_2 = -0.0444 \pm 0.0010$$

Using ChiPT constraint:

$a_0 = 0.2196 \pm 0.0028 \pm 0.0020$ and
$a_2 = -0.0444 \pm 0.0007 \pm 0.0005 \pm 0.0008$
or $(a0 - a2) = 0.2640 \pm 0.0021 \pm 0.0015$

Total error $\Delta a_2 = \pm 0.0012$ $\Delta a_0 = \pm 0.0034$
$\Delta(a_2 - a_2) = \pm 0.0026$

Published in :
Eur. Phys. J C70 (2010) 635
Eur. Phys. J C64 (2009) 589

$K^\pm \to \pi^+\pi^- e^\pm \nu$ Form Factor Results

Form factors (series expansion in q^2) normalised to f_s :

$f'_s/f_s = 0.152 \pm 0.007 \pm 0.005$

$f''_s/f_s = -0.073 \pm 0.007 \pm 0.006$

$f'_e/f_s = 0.068 \pm 0.006 \pm 0.007$

$f_p/f_s = -0.048 \pm 0.003 \pm 0.004$

$g_p/f_s = 0.868 \pm 0.010 \pm 0.010$

$g'p/f_s = 0.089 \pm 0.017 \pm 0.013$

$h_p/f_s = -0.398 \pm 0.015 \pm 0.008$

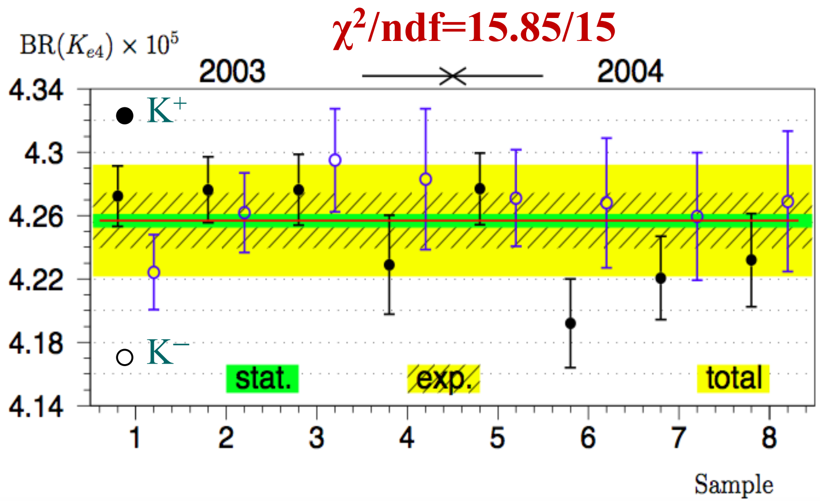
(stat+syst error quoted) stat syst

Branching ratio value will fix the absolute values of the form factors

$K^{\pm}\rightarrow\pi^{+}\pi^{-}e^{\pm}\nu$ Branching fraction

Source	Correction %	Uncertainty %
Acceptance stability		0.18
Muon veto efficiency		0.16
Accidental	0.12	0.21
Particle Identification		0.09
Background		0.07
Radiative Correction		0.08
Trigger efficiency		0.11
MC statistics		0.05
Total systematic	0.12	0.37
External		0.72

- $K_{3\pi}$ decays (1.9×10^9) as normalization
- (1.11×10^6) signal events
- background = 0.95% of K_{e4}



PDG 2010: $(4.09\pm0.10)10^{-5}$

$$BR(K^{+}_{e4}) = (4.255 \pm 0.008) \times 10^{-5};\quad BR(K^{-}_{e4}) = (4.261 \pm 0.011) \times 10^{-5}$$

$$BR(K^{\pm}_{e4}(+-)) = (4.257 \pm 0.004_{stat} \pm 0.016_{syst} \pm 0.031_{ext}) \times 10^{-5}$$

CERN-PH-EP-2012-185 *and* *http://arxiv.org/pdf/1206.7065.pdf*

$BR(K_{3\pi})=(5.59\pm0.04)\%$
Source of external error

$K^{\pm}\to\pi^{+}\pi^{-}e^{\pm}\nu$ absolute form factors

$BR(K_{e4}) = \tau_K\,(|V_{us}|\,f_s)^2 \times I$ where I is the integral over phase space that depends on normalised form factors

The overall form factor normalization:

$f_s = 5.705 \pm 0.003_{stat} \pm 0.017_{exp} \pm 0.031_{ext}$

$= 5.705 \pm 0.035_{norm}$

New

$f_s' = 0.867 \pm 0.040_{stat} \pm 0.029_{syst} \pm 0.005_{norm}$

$f_s'' = -0.416 \pm 0.040_{stat} \pm 0.034_{syst} \pm 0.003_{norm}$

$f_e' = 0.388 \pm 0.034_{stat} \pm 0.040_{syst} \pm 0.002_{norm}$

$f_p = -0.274 \pm 0.017_{stat} \pm 0.023_{syst} \pm 0.002_{norm}$

$g_p = 4.952 \pm 0.057_{stat} \pm 0.057_{syst} \pm 0.031_{norm}$

$g_p' = 0.508 \pm 0.097_{stat} \pm 0.074_{syst} \pm 0.003_{norm}$

$h_p = -2.271 \pm 0.086_{stat} \pm 0.046_{syst} \pm 0.014_{norm}$

Normalised errors are fully correlated

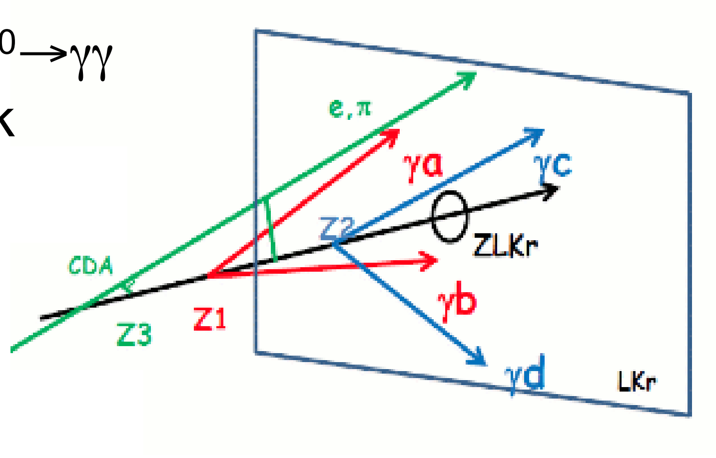
Large anti-correlations in f'_s, f''_s and g_p, g'_p - omitted to be conservative

S118 $f_s \times |V_{us}| = 1.23 \pm 0.03$ (corresponding to 5.59 x 0.22 from PDG 1976)
E865 $f_s \times |V_{us}| = 1.282 \pm 0.018$ (5.75 x 0.2229 from PDG 2002)

NA48/2 $f_s \times |V_{us}| = 1.285 \pm 0.001_{stat} \pm 0.004_{syst} \pm 0.005_{ext}$
Corresponding to $f_s = 5.705 \pm 0.003_{stat} \pm 0.017_{syst} \pm 0.031_{ext}$
using $|V_{us}| = 0.2252 \pm 0.0009$ from PDG 2012

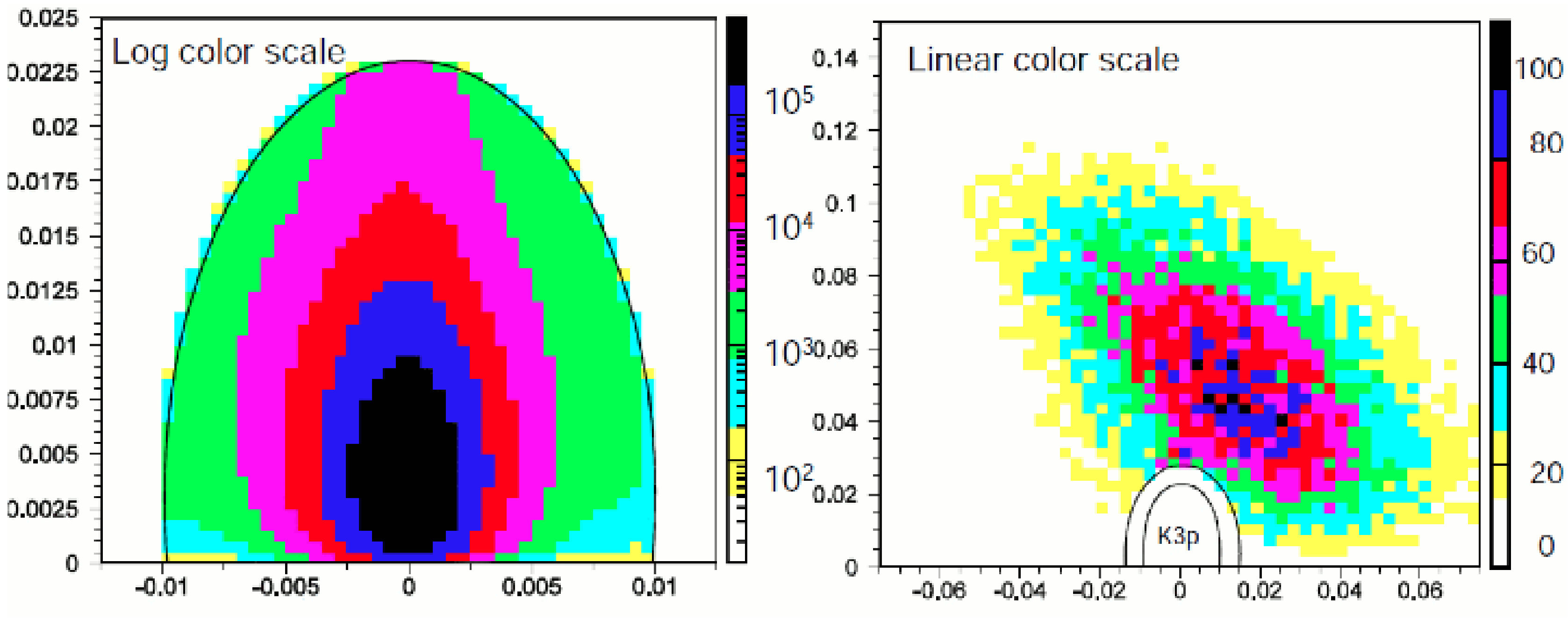
$K^\pm \to \pi^0\pi^0 e^\pm \nu$ Branching fraction

Decay position: average assuming $\pi^0 \to \gamma\gamma$ and combined with charged track



Assign m_π to the charged track, plot Pt to beam vs inv. Mass

Elliptic cut separates $K_{3\pi}$ from K_{e4}

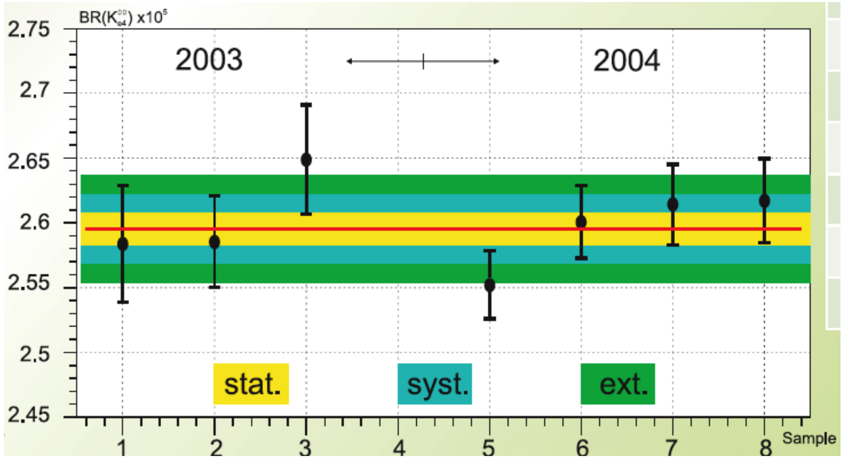


P_t(GeV/c) vs ($M_{3\pi}$ — M_K) (GeV/c^2)

$K^\pm \rightarrow \pi^0\pi^0 e^\pm \nu$ Branching fraction

Systematic Uncertainty	**%**
Background	0.35
Simulation statistics	0.12
Form Factors dependence	0.20
Radiative effects	0.23
Trigger efficiency	0.80
Particle ID	0.10
Beam geometry	0.10

- $K_{3\pi}$ decays (71×10^6) as normalization
- 44909 signal events
- background = 1.3% of K_{e4}
- $BR(K^\pm \rightarrow \pi^0\pi^0\pi^\pm) = (1.761 \pm 0.022)\%$



$$\mathrm{PDG}: (2.2 \pm 0.4) \times 10^{-5}$$

Preliminary Results

$$BR(K^\pm_{e4}(00)) = (2.595 \pm 0.012_{stat} \pm 0.024_{syst} \pm 0.032_{ext}) \times 10^{-5}$$

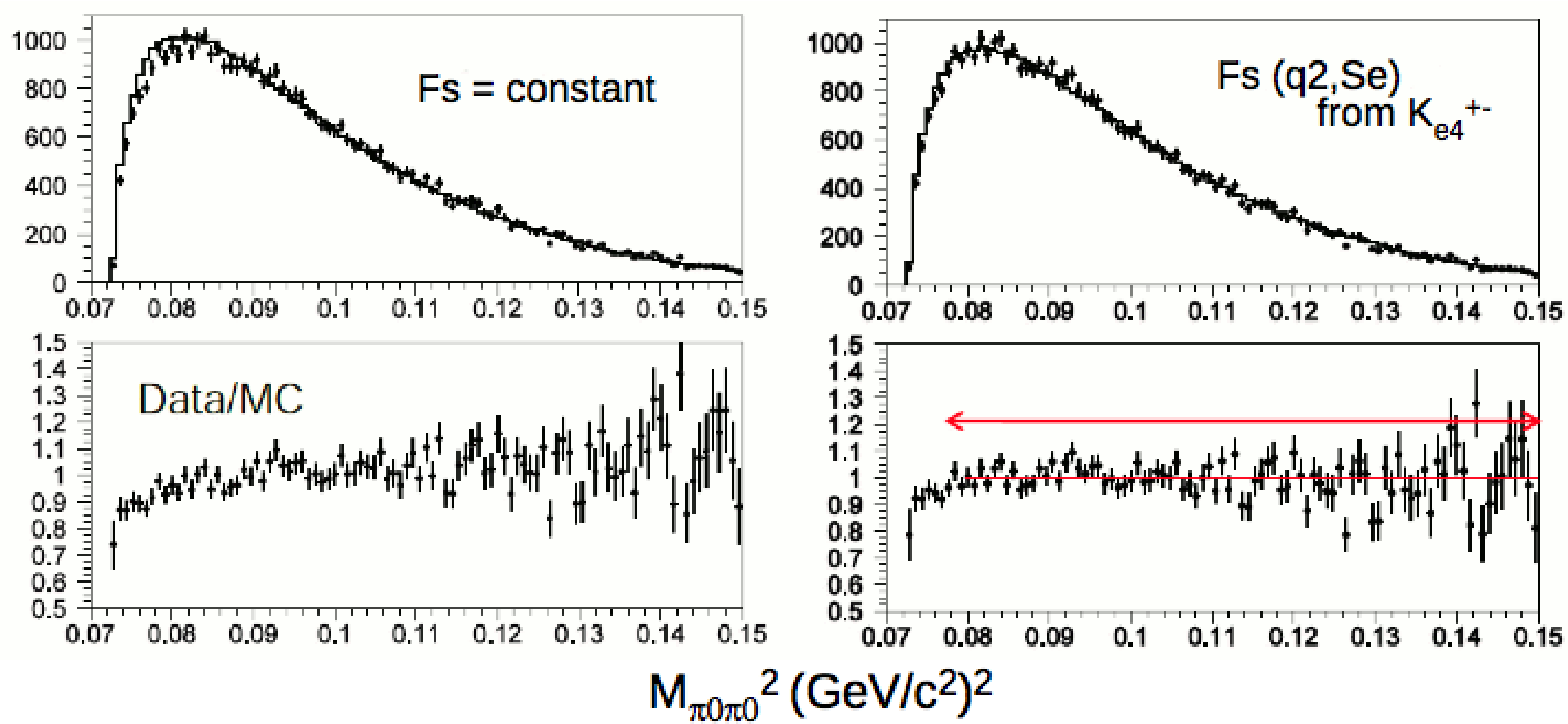
$K^{\pm} \rightarrow \pi^0 \pi^0 e^{\pm} \nu$ Form factor

preliminary

Compare $M(\pi^0\pi^0)^2$ distribution with MC simulation with :

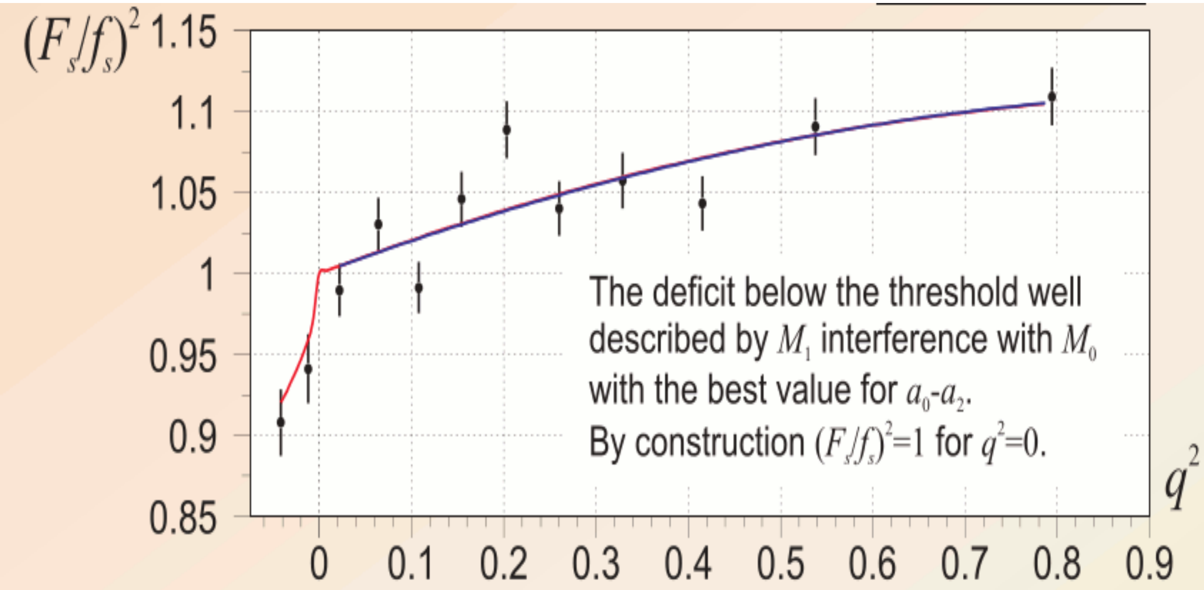
Constant form factor

Form factor from $K^{\pm} \rightarrow \pi^+\pi^- e^{\pm} \nu$: good agreement + $\pi\pi$ rescattering



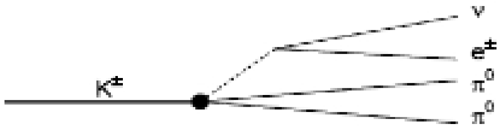
$K^\pm \to \pi^0\pi^0 e^\pm \nu$ Form factor

preliminary

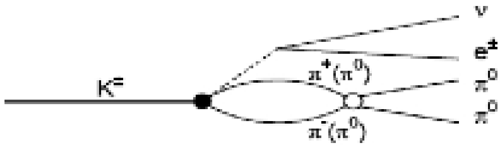


One loop with negative interference (charge exchange scattering +- →00)

1-loop calculation for 3π decays: Cabibbo, PRL 93(2004)121801



Tree level M0



1-loop M1

Above threshold: $|M|^2 = |M0 + i\ M1|^2 = M0^2 + M1^2$
Below threshold : $|M|^2 = |M0 + M1|^2 = M0^2 + M1^2 + 2\ M0\ M1$
$q2 = S\pi/4m\pi{+}^2 - 1 \quad \sigma\pi = \sqrt{(4m\pi{+}^2/S\pi - 1)} = \sqrt{(|q2|/(1+q2))}$

M0 = unperturbed amplitude: $Fs = fs\ (1 + a\ q2 + b\ q4 + c\ Se/4m\pi{+}^2)$
M1 = scattering amplitude: $-\ 2/3\ (a0-a2)\ fs \sqrt{(|q2|/(1+q2))}$

Summary and Outlook (I)

NA48/2:

4 million $K^{\pm}_{e3}$ and **2.5 million** $K^{\pm}_{\mu3}$ events with very small background analysed

- Very precise preliminary results on $K^{\pm}_{e3}$ and $K^{\pm}_{\mu3}$ **form factors**, competitive with the current world averages
- First measurement for both K^{+} and K^{-} **decays**

NA62: (NA48 successor)

2007/08 data for measurement of $\Gamma(K\to e\nu)/\Gamma(K\to\mu\nu)$

- Huge $K^{\pm}_{e3}$ and $K^{\pm}_{\mu3}$ statistics of O***(10^7) events*** on tape

Also special run with **neutral beam**

- O***(10^6) events*** of each $K^{0}_{L,e3}$ and $K^{0}_{L,\mu3}$ on tape

Summary and Outlook (II)

<u>NA48/2:</u>

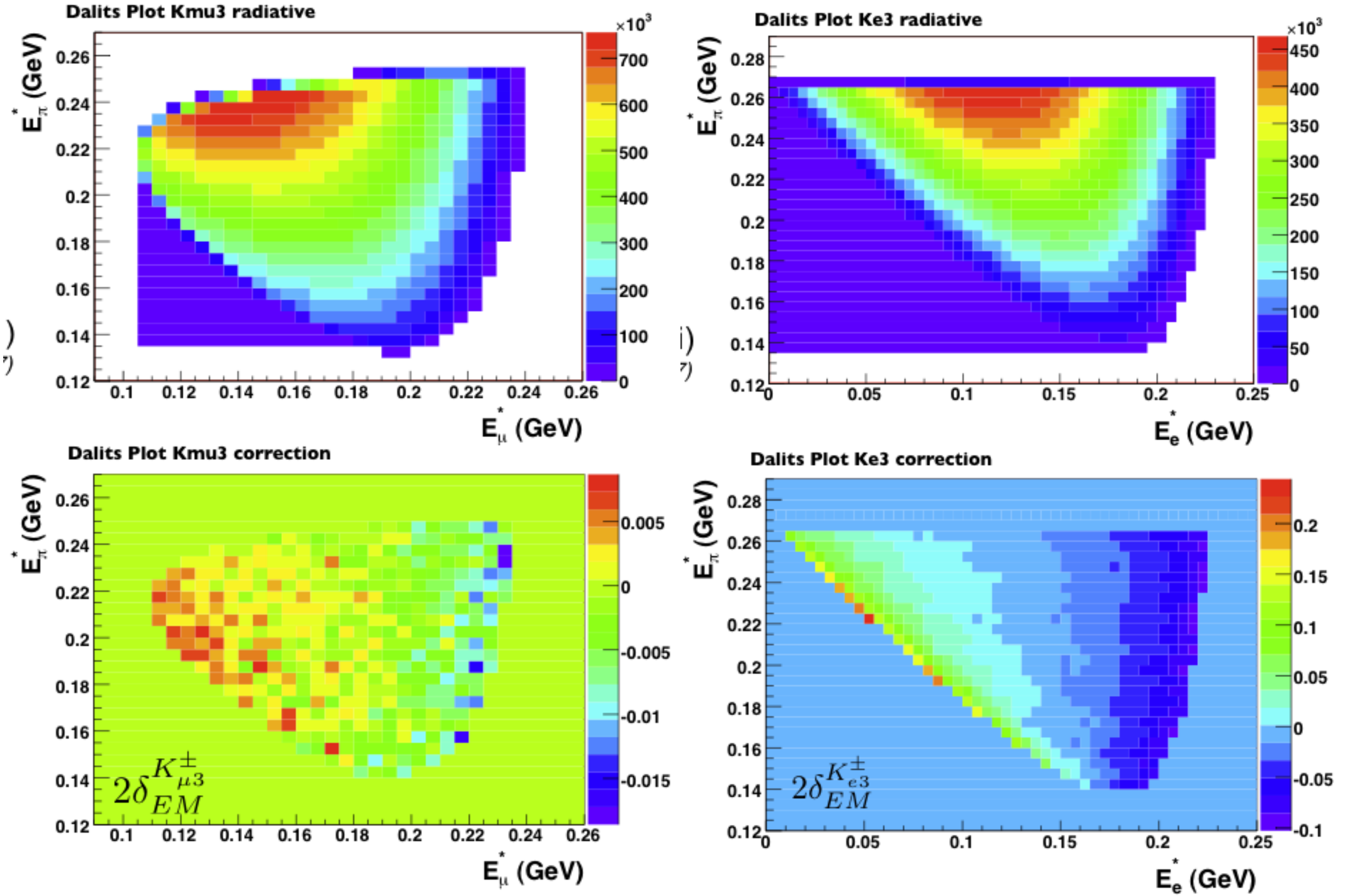
1.11 million $K^{\pm}\to\pi^{+}\pi^{-}e^{\pm}\nu$ and **45000** $K^{\pm}\to\pi^{0}\pi^{0}e^{\pm}\nu$ events with very small background analysed

- Improved branching fractions, 3 times and 10 times better than PDG
- First results on Fs $K^{\pm}\to\pi^{0}\pi^{0}e^{\pm}\nu$ form factor are consistent with $K^{\pm}\to\pi^{+}\pi^{-}e^{\pm}\nu$
- Expected observation of several thousands decays in similar muonic modes ($K^{\pm}\to\pi^{0}\pi^{0}\mu^{\pm}\nu$ never observed, $K^{\pm}\to\pi^{+}\pi^{-}e^{\pm}\nu$ 7 events observed)

Accepted for publication in Phys. Lett. B

And many more events expected in **<u>NA62</u>** in the near future

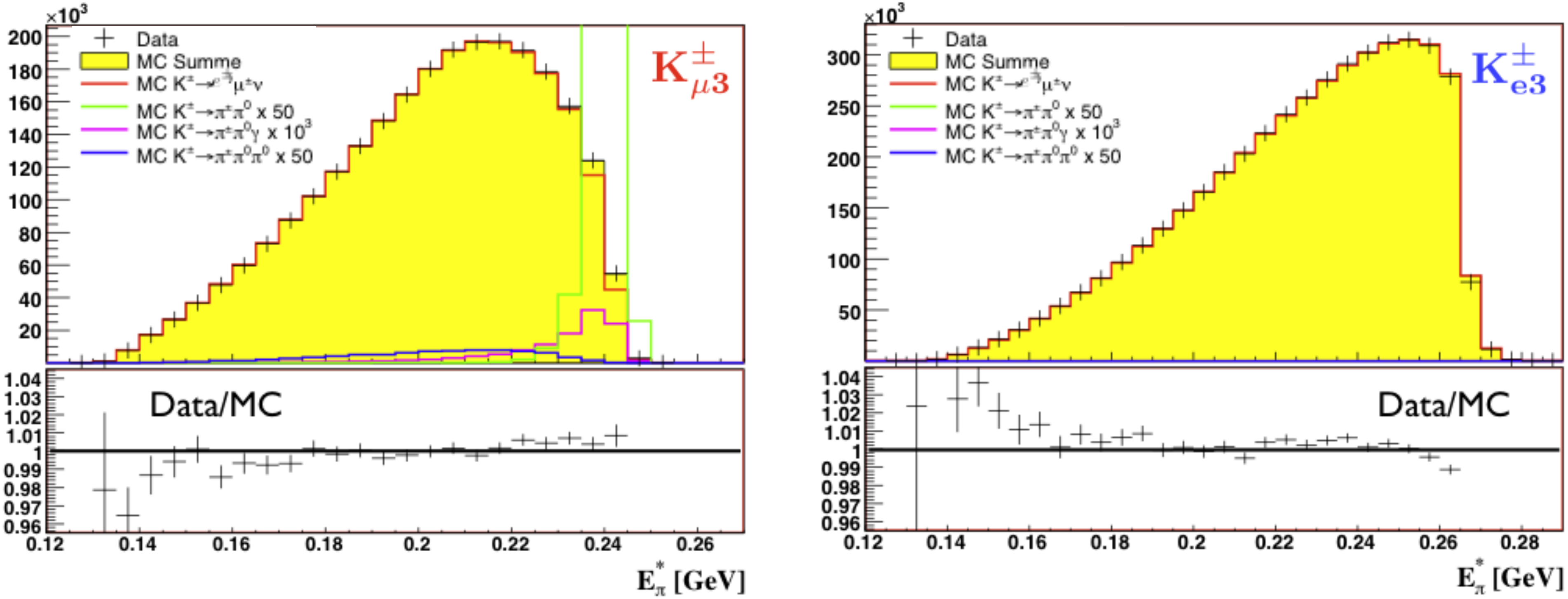
Spares



Pion energy vs Lepton energy (CM)

Fit to the Dalitz Plot Density

Projection on the π^0 energy in the $K^{\pm}$ rest frame:



Data-MC differences mostly below the 1% level

Remaining differences taken into account by systematics.

Introduction: K_{e4} amplitude

$K^\pm \to \pi^+\pi^- e^\pm \nu$ amplitude
is a product of weak leptonic current and (V-A) hadronic current:

$$\frac{G_w}{\sqrt{2}} V_{us}^* \bar{u}_\nu \gamma_\lambda (1-\gamma_5) v_e \, \langle \pi^+\pi^- | V^\lambda - A^\lambda | \mathrm{K}^+ \rangle, \quad \text{where}$$

$$\langle \pi^+\pi^- | A^\lambda | \mathrm{K}^+ \rangle = \frac{-i}{m_K} \big(F (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})^\lambda + G (\mathbf{p}_{\pi^+} - \mathbf{p}_{\pi^-})^\lambda + R (\mathbf{p}_e + \mathbf{p}_\nu)^\lambda \big)$$

R enters in the decay rate multiplied by lepton mass squared => this term is negligible for K_{e4}

and

$$\langle \pi^+\pi^- | V^\lambda | \mathrm{K}^+ \rangle = \frac{-H}{m_K^3} \epsilon^{\lambda\mu\rho\sigma} (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-} + \mathbf{p}_e + \mathbf{p}_\nu)_\mu \times (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})_\rho (\mathbf{p}_{\pi^+} - \mathbf{p}_{\pi^-})_\sigma .$$

In the above expressions, $\mathbf{p}$ is the four-momentum of each particle, F, G, R are three axial-vector and H one vector complex form factors with the convention $\epsilon^{0123} = 1$.

F,G,R,H form factors (FF) depend on decay Lorentz invariants, so their parameterisation (or some tabulation) is needed to describe data.

Chiral Perturbative Theory and Ke4 decays

At energy <~ 1 GeV an effective theory ChPT is used to describe the physical observables in terms of external momenta and light quark masses (Weinberg 1979).

Isospin symmetry ($m_u=m_d$, $\alpha_{QED}=0$) translates into relation between decay modes:
Rates (l - lepton): $\Gamma(K_{l4}(+-)) = ½\,\Gamma(K_{l4}(0+)) + 2\,\Gamma(K_{l4}(00))$;

Taking into account the lifetimes of K^+ ($1.238\ 10^{-8}$ s) and K_L ($5.116\ 10^{-8}$ s):
Branching ratios: $Br(K_{l4}(+-)) = 0.121\ Br(K_{l4}(0+)) + 2\ Br(K_{l4}(00))$
experiments (PDG):

$K_{e4}(+-)$ ($K^\pm \to \pi^+\pi^-\nu e^\pm$): $Br = (4.09 \pm 0.10)\ 10^{-5}$; events: 418000
$K_{e4}(00)$ ($K^\pm \to \pi^0\pi^0\nu e^\pm$): $Br = (2.2 \pm 0.40)\ 10^{-5}$; events: 37
$K_{e4}(0+)$ ($K_L \to \pi^\pm\pi^0\nu e^\pm$): $Br = (5.20 \pm 0.10)\ 10^{-5}$; events: 6131

Experimental precision improvement is needed.
Predictions using Form Factor calculations by ChPT at O(p2,p4,p6)
(Bijnens, Colangelo, Gasser, Nucl. Phys. B427 1994):

Using S118 value as input $K_{e4}(+-)$: $(3160 \pm 140)\ s^{-1}$ Br: $(3.91 \pm 0.17)\ 10^{-5}$
prediction $K_{e4}(00)$: $(1625 \pm 90)\ s^{-1}$ Br: $(2.01 \pm 0.11)\ 10^{-5}$
$K_{e4}(0-)$: $(917 \pm 170)\ s^{-1}$ Br: $(4.69 \pm 0.87)\ 10^{-5}$

Improved measurement will provide tests of ChPT predictions

Why/How measure $\pi\pi$ scattering lengths?

The important free parameter of ChPT is the quark condensate $<q\bar{q}>$, that determines the relative size of mass and momentum terms in the power expansion

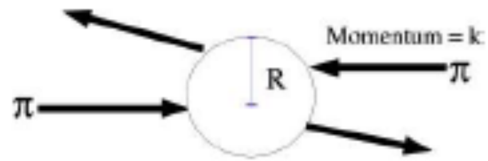
a_0 and a_2 are S-Wave scattering lengths in isospin states I=0,2

At low energy kr << 1 S-wave dominates total cross section

Scattering matrix $S|\pi\pi> = \exp(2i\delta)|\pi\pi>$

may be parametrized with 2 phases

$$\delta_{0,2} = a_{0,2}k + O(k^2)$$



($\pi\pi$) scattering in the center of mass frame

The relation between $<q\bar{q}>$ and the scattering length a_0 and a_2 is known from theory with high precision, so the experimental measurement of a_0 and a_2 provides important constraints for ChPT Lagrangian parameters

Why/How measure $\pi\pi$ scattering lengths?

3 kinds of measurements have been developed:

Pionium atoms: DIRAC CERN/SPS $\pi\pi$ lifetime

K3π modes (cusp) : $BR(K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}) = (1.757 \pm 0.024) \cdot 10^{-2}$

→ NA48/2 CERN/SPS ($16 \cdot 10^{6}$ 2006): $60 \cdot 10^{6}$ (2008)

$BR(K_L \rightarrow \pi^{0}\pi^{0}\pi^{0}) = (19.56 \pm 0.14) \cdot 10^{-2}$

KTeV ($68 \cdot 10^{6}$) and NA48 ($100 \cdot 10^{6}$)

Ke4 decays: $BR(K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu) = (4.09 \pm 0.09) \cdot 10^{-5}$

Very clean environment, known for long but limited statistic:

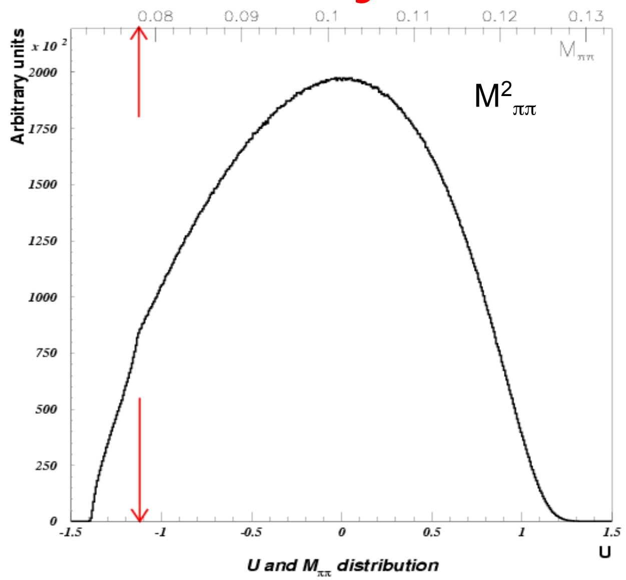
Geneva-Saclay CERN/SPS experiment: $3 \cdot 10^{4}$ (1977)

E685 BNL experiment: $4 \cdot 10^{5}$ (2003)

→ NA48/2 CERN/SPS: $1.15 \cdot 10^{6}$ (2008)

"Cusp" effect in $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$ decays

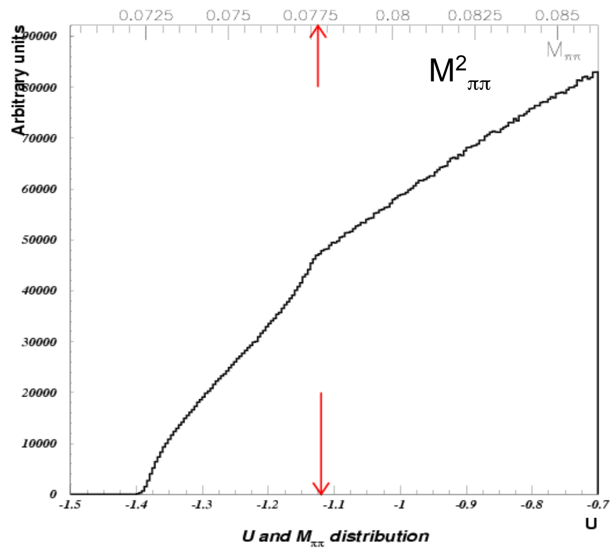
- In $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decay the matrix element is usually given as polynominal expansion as a function of the Dalitz variables U and V

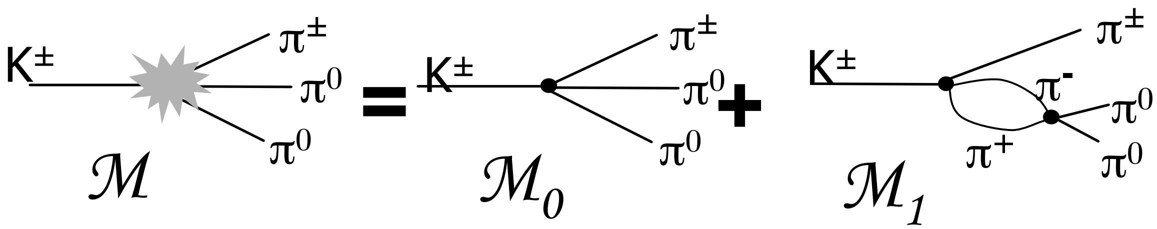


- Thanks to the big statistics collected by NA48/2 and the good energy resolution, for the first time a structure has been observed at the $\pi\pi$ threshold value [Batley et al.,Phys.Lett. B633:173-283,2006]

- Budini and Fonda (1961): "A threshold effect of the kind of a cusp will be observed in the spectrum..." but no data available at the time to test it

- This structure has been more recently interpreted by Cabibbo [Cabibbo Phys. Rev. Lett. 93, 121801 (2004)] as due to the strong $\pi\pi$ rescattering in the $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ final state





Cusp: theoretical approach (CI)

- Phenomenological approach
- The term M0 (no rescattering) is given by the standard PDG expansion
- the first rescattering terms is real below threshold and immaginary above. Interference below threshold.

Kaon rest frame:

$u = 2m_K \cdot (m_K/3 - E_{odd})/m_\pi^2$

$v = 2m_K \cdot (E_1 - E_2)/m_\pi^2$

$$M_0 = A_0(1+g_0 u/2+h_0 u^2/2+k_0 v^2/2)$$

$$M_+ = A_+(1+g_+ u/2+h_+ u^2/2+k_+ v^2/2)$$

$s_{\pi\pi} > (2m_{\pi+})^2$ $\qquad M^2=(M_0)^2+|M_1|^2$

$s_{\pi\pi} < (2m\pi_+)^2$ $\qquad M^2=(M_0)^2+(M_1)^2+2M_0M_1$

Negative interference under threshold

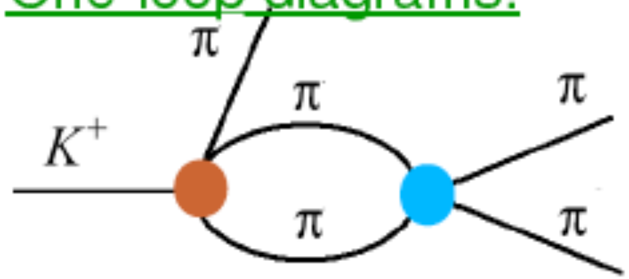
Rescattering amplitude:

$$M_1 = -2/3(a_0 - a_2)m_+ M_+ \sqrt{1-(M_{00}/2m_+)^2}$$

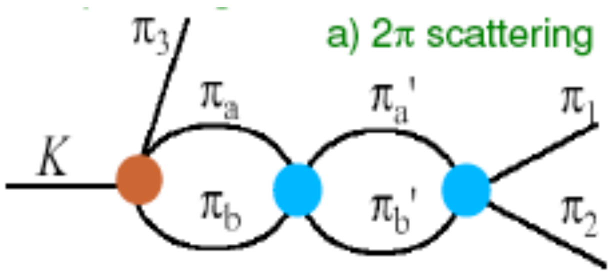
Combination of S-wave $\pi\pi$ scattering lenghts

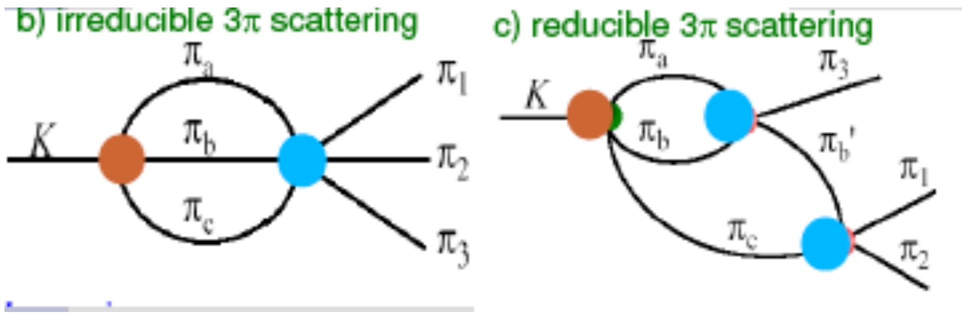
$K^\pm \rightarrow 3\pi^\pm$ amplitude at threshold

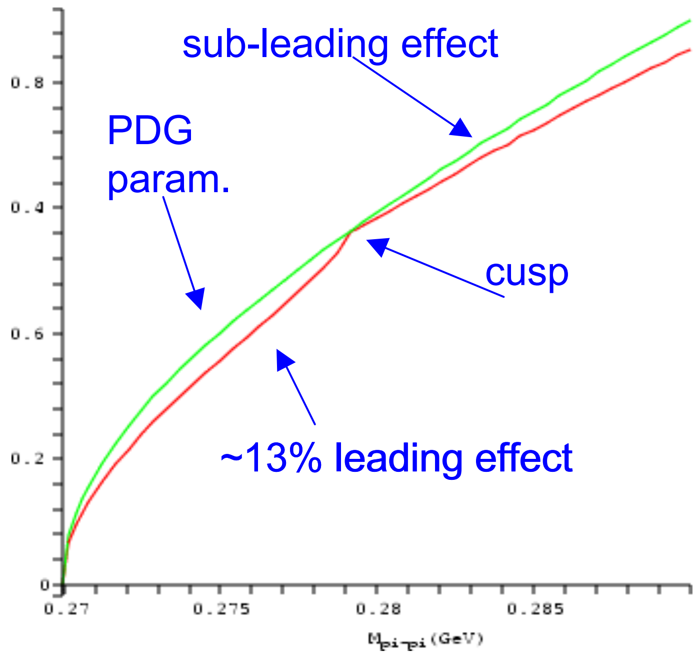
One-loop diagrams:



- Five S-wave scattering lengths (a_x, a_{++}, a_{+-}, a_{+0}, a_{00}) expressed as linear combinations of a_0 and a_2
- Isospin symmetry breaking accounted for following J. Gasser. For example, $a_x = (1+\varepsilon/3)(a_0-a_2)/3$, where $\varepsilon=(m_+^2-m_0^2)/m_+^2=0.065$ is isospin breaking parameter
- Radiative corrections missing; (a_0-a_2) precision ~5%
- V-dependent terms ~ $(k`/2)V^2$ introduced both into "unperturbed" $K^\pm\to\pi^\pm\pi^0\pi^0$ and $K^\pm\to\pi^\pm\pi^+\pi^-$ amplitudes.



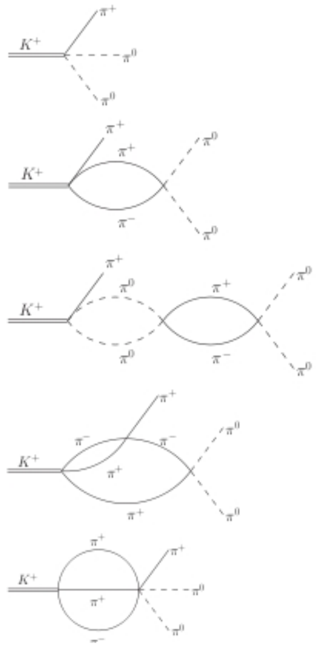




- Other 5 terms rise from two loops calculation (proportional to scattering lengths): effects below and above threshold
- Theoretical error evaluated from the next level expansion (work in progress)

Cusp: theoretical approach (BB)

- Different approach based on effective non-relativistic Lagrangian
- The electromagnetic effects are naturally included in this approach (explicitly omitted in the CI work)
- Different structure of the expansion (different correlation between the terms wrt the Cabibbo-Isidori expansion): kinetic energy and threshold parameter.
- Simultaneus fitting of charged and neutral amplitude to exctract M+ slope parameters (modified with respect to the PDG parametrization)
- Radiative correction, outside the cusp point, included in the BB model



$$
\begin{aligned}
\mathcal{M}^{\text{tree}} &= G_0 + G_1(p_3^0 - M_\pi) + \dots \\
\mathcal{M}^{1-\text{loop}} &= B_1 J_{+-}(s_3) + B_2 J_{00}(s_3) \\
&\quad + [B_3 J_{+0}(s_1) + (s_1 \leftrightarrow s_2)] \\
\mathcal{M}^{2-\text{loop}} &= 2G_0 C_x^2 \underbrace{J_{+-}(s_3) J_{00}(s_3)}_{\text{double loops}} + \dots \\
&\quad + 4H_0 C_x C_{+-} \underbrace{F_+(\dots; s_3)}_{\text{overlapping loops}} \\
&\quad + \mathcal{O}(i\,\epsilon^4) \quad [\not\propto \text{ scatt. lengths}]
\end{aligned}
$$

[Colangelo, Gasser, Kubis, Rusetsky in Phys.Lett.B638:187-194,2006]

[Bissenger, Fuhrer, Gasser, Kubis, Rusetsky in Phys.Lett.B659:576,2008]

[Bissegger, Fuhrer, Gasser, Kubis, Rusetsky in NPH B806:178, 2009]

Cusp: fitting procedure

- Resolution and detector response matrix obtained using accurate Geant3 based simulation

- Both theories can be fitted with the same procedure (fit parameters: g_0, h'_0, a_0-a_2 , a_2 , N)

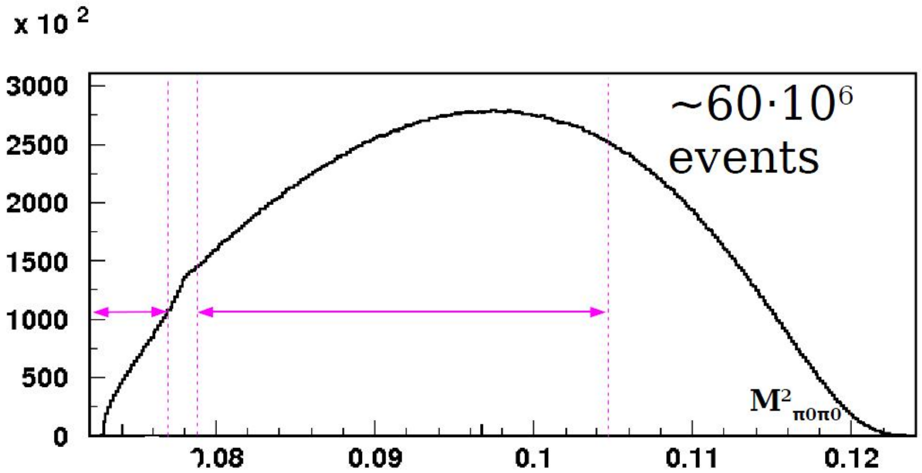
$$M_0 = A_0(1+g_0 u/2+h'_0 u^2/2+k'_0 v^2/2)$$

u,v = Dalitz variables

- The M+ term appearing in the **CI** theory is fixed by the recent measured $K^\pm \to \pi^\pm \pi^+ \pi^-$ slope parameters [Batley et al. Phys.Lett.B649:349-358,2007]

- In the **BB** the M+ term is obtained simultaneously fitting $K^\pm \to \pi^\pm \pi^0 \pi^0$ and $K^\pm \to \pi^\pm \pi^+ \pi^-$ Dalitz plot

- Isospin effects included



Ke4 : Fitting Procedure

The fit parameters are: F_s, F_p, G_p, H_p, and $\delta=\delta_s-\delta_p$

- Define iso-populated boxes in the 5-dimension space of $M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ :

  $10(M_{\pi\pi})\times5(M_{e\nu})\times5(\cos\theta_\pi)\times5(\cos\theta_e)\times12(\phi)=15000$ boxes
- The form factors and phase shift are extracted by minimizing a log-likelihood estimator in 10 independent $M_{\pi\pi}$ bins
- K^+ and K^- samples fitted separately and results combined in each $M_{\pi\pi}$ bin according to their statistical error
- Only relative form factors (F_p/F_s, G_p/F_s, H_p/F_s) are measured (no overall normalization from BR)
- The form factor structure is studied in 10 bins of q^2

Ke4: δ Phase and Scattering Lengths

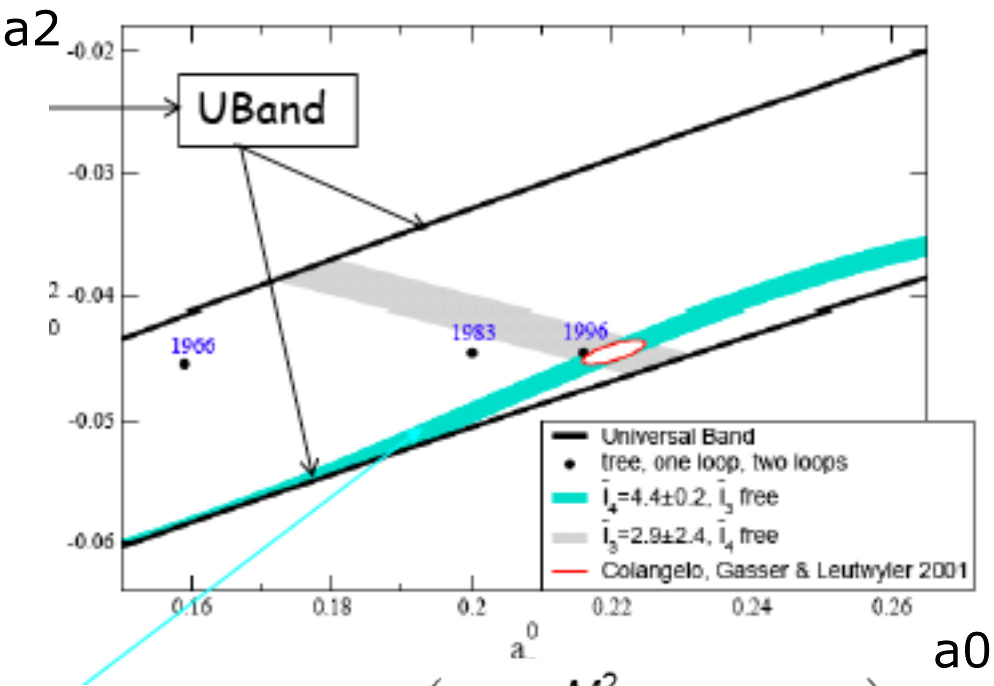
- The extraction of pion scattering lengths from the fitted $\delta=\delta_s-\delta_p$ phase shift needs external theoretical and experimental inputs:
  - The *Roy equations* provide the relation between δ and a0 and a2 near threshold (1) (2) (3)
  - Extrapolating the data from the $M_{\pi\pi}>0.8$ GeV it's possible to fit the result in the threshold region (the uncertainty from the experimental data defines the *Universal Band*)

- Coulomb correction (Gamow factor) and real photons are included in simulation
- Isospin correction prescription given by Gasser [Gasser et al. Eur.Phys.J. C59:777, 2009] results in 11 to 15 mrad in the fitted $M_{\pi\pi}$ range.

See also [M. Knecht, R. Urech, Nucl. Phys. B 519, 329 (1998)]

1)[Ananthanarayan,Colangelo,Gasser,Leutwyler Phys.Rept.353:207-279 (2001)]
2)[Descotes-Genon, Fuchs, Girlanda,Stern Eur.Phys.J.C24:469-483,2002]
3)[Kaminski, Pelaez, Yndurain Phys.Rev.D77 (2008)]

a_0 and a_2 theoretical predictions



$$a_0 m_\pi = \frac{7 M_\pi^2}{32\pi F_\pi^2} = 0.16$$

$$a_2 m_\pi = \frac{-M_\pi^2}{16\pi F_\pi^2} = -0.045$$

S. Weinberg, PRL 17 (1966) 216

$$M_\pi^2 = M^2\left(1 - \frac{M^2}{32\pi^2 F^2}\bar{\ell}_3 + O(p^4)\right)$$

$$M^2 \equiv -\frac{m_u + m_d}{F^2}\langle 0|\bar{q}q|0\rangle$$

$$F_\pi = F\left(1 + \frac{M^2}{16\pi^2 F^2}\bar{\ell}_4 + O(p^4)\right)$$

a0	=	0.220	+- 0.005
a2	=	-0.0444	+- 0.0010
a0-a2	=	0.265	+- 0.004

Colangelo, Gasser, Leutwyler, PRL 86, 5008, (2001)

Analyticity and chiral symmetry predict the relation:

$a_2 = (-0.0444 \pm 0.0008) + 0.236(a_0-0.22) - 0.61(a_0-0.22)^2 - 9.9(a_0-0.22)^3$

Bern – Bonn approach

G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky, PHL B638, 187, (2006)

Model based on a non-relativistic field theory framework
using two expansion parameter

a = generic $\pi\pi$ scattering length at threshold
ε = a formal parameter such that
pion momentum is of order $O(\varepsilon)$
pion kinetic energy is of order $O(\varepsilon^2)$

The present formulation includes terms up to $O(\varepsilon^4, a\varepsilon^3, a^2\varepsilon^2)$
corresponding to 1-loop and 2-loops calculation.
Valid over the full physical region

Bern-Bonn calculation is used to fit simultaneously
m_{00} distribution from $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$ and
m_{++} distribution from $K^{\pm} \rightarrow \pi^{\pm}\pi^+\pi^-$

The Bern-Bonn group calculated radiative correction outside the cusp point

M. Bissegger, A. Fuhrer, J.Gasser, B. Kubis, A. Rusetsky NPH B806, 178 (2009)

<u>So far B.B. approach provides the most complete description of rescattering effect</u>

$K \rightarrow 3\pi$ Dalitz plot definition

- Three body decay can be analized using the Dalitz plot
- The most convenient variables are the Dalitz plot variables u and v
- The Matrix element can be expanded as a function of u and v (just a polynomial expansion)

Dalitz variables:

$u = (s_3 - s_0)/m_\pi^2$

$v = (s_2 - s_1)/m_\pi^2$

$s_i = (p_K - p_i)^2 \quad i = 1,2,3$

$s_0 = (s_1 + s_2 + s_3)/3$

Results: other Dalitz plot parameters

$$M(K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}) = M_0 + M_1$$

Unperturbed amplitude is $M_0 \sim (1+g_0 u/2+h_0 u^2/2+k_0 v^2/2)$

NB: even without M_1 not the same parameters as the PDG ones:

$|M_0|^2_{(PDG)} \sim (1+gu+hu^2+kv^2)$ $[g_0\approx g,\ h_0\approx h-g^2/4,\ k_0\approx k]$

- Technique:
  1. k_0 is extracted from 2-dimensional CI and BB fits
  2. $(a_0\text{-}a_2, g_0, h_0)$; ChPT $a_2(a_0)$; fixed k_0 (its uncertainty -> systematics)

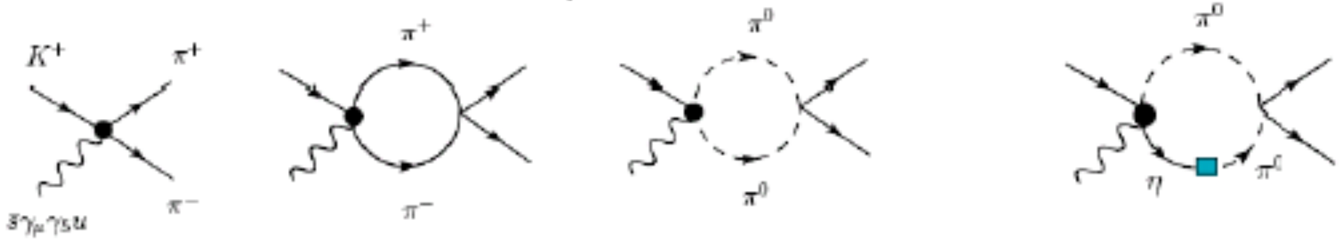
CI parameters:	BB parameters:	uncertainties:
$k_0^{CI} = -0.0095$	$k_0^{BB} = -0.0081$	$\pm 0.0002_{stat.} \pm 0.0005_{syst.}$
$g_0^{CI} = 0.653$	$g_0^{BB} = 0.622$	$\pm 0.001_{stat.} \pm 0.003_{syst.}$
$h_0^{CI} = -0.043$	$h_0^{BB} = -0.052$	$\pm 0.001_{stat.} \pm 0.003_{syst.}$

For the free a_2 the errors are larger.

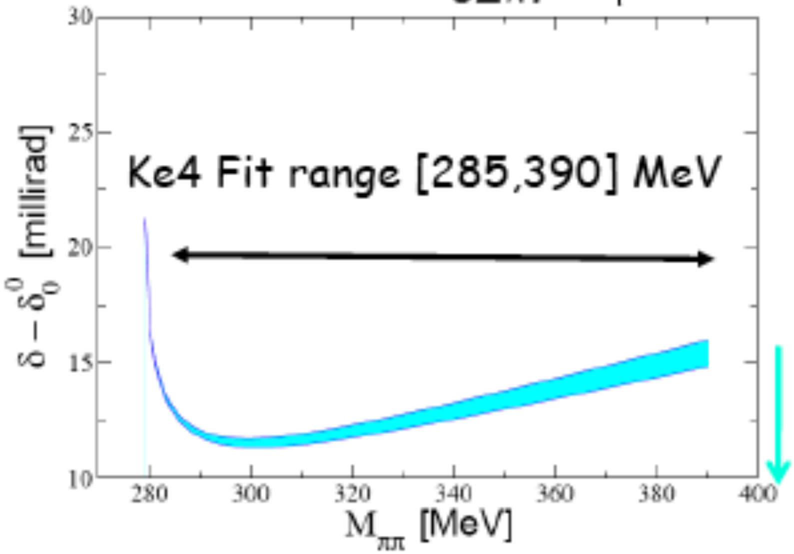
Ke4 charged decays : isospin corrections to δ

CGR EPJ C59 (2009) 777 formulation developed in close contact with NA48,

tree one loop π^0–η mixing



δ_0^0 →
$$\delta = \frac{1}{32\pi F^2}\left\{(4\Delta_\pi + s)\sigma + (s - M_{\pi^0}^2)\left(1+\frac{3}{2R}\right)\sigma_0\right\}$$



$$\Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2,$$

$$\sigma = \sqrt{1 - \frac{4M_\pi^2}{s}},$$

$$R = \frac{m_s - \hat{m}}{m_d - m_u}$$

Correction is ~ 10-15 mrad

Exp. Stat precision is ~ 7-8 mrad