



## Bottomonium with an effective molecular potential

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### Abstract

A molecular potential is used for the first time as an effective potential for the overall interaction in bottomonium. The rotational contributions of P states are calculated. Known experimental energy levels of bottomonium are fitted and values of predicted energy levels are calculated. The radii of some states are calculated.

**Keywords:** bottomonium; bottomonium spectroscopy; heavy mesons; hadron spectroscopy.

### 1. Introduction

The static potential plays an important role in the description of heavy quarks. With respect to this subject the pioneering work of Eichten *et al.* [1] was followed by many other works. After more than 30 years some important features of the static interaction are not yet completely understood. That is one of the reasons why a large number of approximate methods and effective potentials have been proposed for the description of the overall interaction in quarkonia [1]–[31]. This is a large number but is not exhaustive at all of the enormous effort developed in the field.

When dealing with heavy mesons we can use the non-relativistic quantum mechanics formalism with constituent masses for the quarks and anti-quarks. In this work we use a molecular potential for describing low energy states of bottomonium. This description allows the obtainment of the contribution of the rotational energy of P states, values of theoretically predicted energy levels, and calculation of the radii of some levels. It is, thus, a unique and useful calculation that contributes to the understanding of bottomonium states. None of the above listed papers has used a molecular potential in the description of quarkonia states.

As it is widely known confinement is not well understood and there are models that do not consider it inside hadrons. For example, the original MIT bag model treats confinement only at the wall by making the vector current null at it [32]. In the case of the chiral bag model [33] confinement is treated by means of the continuity of the axial vector current at the wall. We do not need to worry about confinement complications because we only deal with low energy levels.

All the experimental data used below for the energies of bottomonium states were taken from Beringer *et al.* (PDG) [34]. All energy values below are in *MeV* unless noted otherwise.

### 2. The molecular potential

Two widely used molecular potentials are the Kratzer potential [35, 36, 37]

$$V(r) = -2D \left( \frac{a}{r} - \frac{1}{2} \frac{a^2}{r^2} \right) \quad (1)$$

and the Morse potential [37]

$$V(r) = D \left( e^{-2\alpha x} - 2e^{-\alpha x} \right) \quad (2)$$

in which  $-D$  is the minimum of the well,  $a$  is the distance where  $V = -D$ , and  $x = (r - a)/a$ . In the Kratzer potential the 1<sup>st</sup> term is a QCD-like term and

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its 2<sup>nd</sup> term takes care of the well-known repulsion of the strong force for small distances [38]. In the Morse potential the leading term of the expansion for  $x \ll 1$  is of the form  $-C/r$ , and for  $r < a$  the Morse potential increases faster than Kratzer's, presenting, thus, a very strong repulsion for very short distances. For  $|x| < 1$  both potentials can be expanded about the minimum up to order 3 in  $x$  and produce the expression

$$V(x) = -D + \frac{1}{2}ka^2x^2 - \lambda ka^3x^3 \quad (3)$$

where  $\lambda = \frac{\alpha}{2a}$  for the Morse potential.

The solution of Schrödinger equation for both potentials yields the expression [37, 39]

$$\begin{aligned} E_{\nu L} = & \hbar\omega\left(\nu + \frac{1}{2}\right) - A\left(\nu + \frac{1}{2}\right)^2 \\ & + B_L L(L+1) - D_L L^2(L+1)^2 \\ & - C_{\nu L}\left(\nu + \frac{1}{2}\right)L(L+1) + \dots \end{aligned} \quad (4)$$

for the vibrational and rotational levels above the minimum of the potential ( $-D$ ) where  $\nu, L = 0, 1, 2, 3, \dots$ . In this equation the first term describes harmonic vibrations, the second term takes into account the anharmonicity of the potential, the third term describes rotations with constant moment of inertia, the fourth term represents the centrifugal distortion and the fifth term represents the coupling between vibration and rotation. This expression can also be written as [39]

$$\begin{aligned} E_{\nu L} = & \hbar\omega\left(\nu + \frac{1}{2}\right) - \frac{(\hbar\omega)^2}{4D}\left(\nu + \frac{1}{2}\right)^2 \\ & + B_L L(L+1) - D_L L^2(L+1)^2 \\ & - C_{\nu L}\left(\nu + \frac{1}{2}\right)L(L+1) + \dots \end{aligned} \quad (5)$$

where  $\omega, D, \alpha, a$  and  $m$  (the reduced mass) are related by the expression

$$\omega^2 = \frac{2\alpha^2 D}{ma^2} \quad (6)$$

for the Morse potential. The constant  $B_L$  is given by  $B_L = \hbar^2/2ma^2$  where  $m$  is the reduced mass of the constituent quark and antiquark, that is,  $mc^2 = \frac{1}{2}M_b c^2$ , where  $M_b$  is the mass of the  $b$  quark. Details on the connections between Morse and Kratzer potentials can be found in references [40] and [41].

### 3. The fitting

The first two levels,  $\eta_b(1S)$  and  $\Upsilon(1S)$  are a hyperfine doublet because of the spin-spin interaction, but since

Table 1: The levels considered in the fitting of bottomonium.

$(\nu, L)$	Particle	Mass ( $MeV/c^2$ )
(0, 0)	$\Upsilon(1S)$	$9460.30 \pm 0.26$
(0, 1)	$\chi_b(1P)$	$9901.6 \pm 4.5$
(1, 0)	$\Upsilon(2S)$	$10023.26 \pm 0.31$
(0, 2)	$\Upsilon(1D)$	$10161.1 \pm 10.0$
(1, 1)	$\chi_b(2P)$	$10261.4 \pm 4.2$

our Hamiltonian does not depend on spin, we can thus use the  $S$  states  $\Upsilon(1S)$  and  $\Upsilon(2S)$  for the fitting. In the case of  $\chi_b(1P)$  and  $\chi_b(2P)$  states we should take out the spin-orbit interaction contribution which is given by

$$\Delta E_{SL} = \Delta [J(J+1) - L(L+1) - S(S+1)] \quad (7)$$

where  $J = |\vec{J}| = |\vec{L} + \vec{S}|$  in which  $S = 1$  for  $\chi_b(P)$  states. Applying Eq. (7) to the states  $\chi_{b0}(1P)$ ,  $\chi_{b1}(1P)$  and  $\chi_{b2}(1P)$  we obtain the average values  $\Delta_1 = 10.1 \pm 0.2$ , and  $E_{1P} = 9901.6 \pm 0.5$  where  $E_{1P}$  is the energy of the degenerate level  $\chi_b(1P)$ . Doing the same for the states  $\chi_{b0}(2P)$ ,  $\chi_{b1}(2P)$  and  $\chi_{b2}(2P)$  we obtain, similarly, the average values  $E_{2P} = 10261.4 \pm 0.6$ ,  $\Delta_2 = 6.9 \pm 0.3$ , where  $E_{2P}$  is the energy of the degenerate level  $\chi_b(2P)$ . The energies of the states  $\Upsilon_1(1D)$  and  $\Upsilon_3(1D)$  have not yet been found experimentally, and thus we took for the degenerate level  $\Upsilon(1D)$  the energy of the level  $\Upsilon_2(1D)$  which is about 10161.1 but with a larger error bar that we estimated in the following way. In the case of the spin-orbit interaction of  $1P$  states the degenerate level  $\chi_b(1P)$  has an energy of  $9901.6 \pm 0.5$  which is just about 8.84 from the middle state  $\chi_{b1}(1P)$  which has the energy  $9892.76 \pm 0.40$ , and in the case of  $2P$  states the degenerate level  $\chi_b(2P)$  has an energy of  $10261.4 \pm 0.6$  which is about 5.94 from  $10255.46 \pm 0.72$  which is the energy of the middle state  $\chi_{b1}(2P)$ . Therefore, we can assume that the degenerate level  $\Upsilon(1D)$  has an energy of about  $10163.7 \pm 10.0$ . Tab. 1 presents a summary of the levels used in the fitting that includes  $E_{1P}$  and  $E_{2P}$  with the corresponding values of  $\nu$  and  $L$ .

### 4. Results and discussion

Fitting the levels of Tab. 1 to Eq. (4) we obtain the following values for its parameters:  $\hbar\omega = 25039.8 \pm 34.0$ ;  $A = 12238.4 \pm 34.0$ ;  $B_L = 323.4 \pm 4.5$ ;  $D_L = 25.9 \pm 4.5$ ;  $C_{\nu L} = 101.6 \pm 4.5$ .

From Eqs. (4) and (5) we obtain that  $A = (\hbar\omega)^2/4D$ , and thus  $D = 12807.9 \pm 100.5$ . This means that there is no bottomonium state with an energy above  $12807.9 \pm 100.5$ . Using the values of  $D$  and  $\hbar\omega$  above in Eq. (6) we obtain  $a/\alpha = (2.60 \pm 0.01) \times 10^{-2} fm$ , and

Table 2: Radii of three states of bottomonium calculated with the use of Eq. (8). The numbers in the particle names refer to the quantum number  $n$  of QCD models.

$(\nu, L)$	Particle	Radius (fm)
(0, 0)	$\Upsilon(1S), \eta_b(1S)$	$0.179 \pm 0.003$
(1, 0)	$\Upsilon(2S), \eta_b(2S)$	$0.218 \pm 0.003$
(2, 0)	$\Upsilon(3S)$	$0.256 \pm 0.003$

from the value of  $B_L$  we have that  $a = (0.16 \pm 0.01) \text{ fm}$  which is a very reasonable figure since the Compton wavelength of bottomonium is about  $0.26 \text{ fm}$  if we use a constituent mass of  $4.5 \text{ GeV}/c^2$ . Of course, we obtain a very similar figure if we use the uncertainty principle. Using the above values of  $a$  and  $a/\alpha$  we obtain  $\alpha = 6.15 \pm 0.01$ .

As we showed above, a molecular potential is harmonic about its minimum, and thus we can calculate the value of the constant  $k = m\omega^2$  which can be written as  $k = mc^2(\hbar\omega)^2/(\hbar c)^2$ . Using the above values we obtain  $k \approx 3.68 \times 10^4 \text{ GeV}/\text{fm}^2 \approx 5.9 \times 10^{24} \text{ N/m}$  which is a quite fair number. For a distance of  $0.2 \text{ fm}$  it produces a force  $F \approx 10^9 \text{ N}$ .

As it was shown above Kratzer and Morse potentials, when expanded about their minima, yield Eq. (3). And for such a potential Robinett [42] obtained the following equation for the average value of position for S states

$$\begin{aligned} \langle r \rangle_\nu &= a + \frac{3\alpha\hbar\omega}{2m\omega^2 a} \left( \nu + \frac{1}{2} \right) \\ &= a + \frac{3a\hbar\omega}{4\alpha D} \left( \nu + \frac{1}{2} \right) \end{aligned} \quad (8)$$

where we have taken into account that  $\lambda = \alpha/2a$ . We did not calculate the radii of the states  $\Upsilon(4S)$ ,  $\Upsilon(10860)$  and  $\Upsilon(11020)$  because these are states far from equilibrium. Using the above values for the constants we obtain the results shown in Tab. 2 for the radii of three states of bottomonium.

We can also calculate the energies of the states  $\eta_b(2S)$ ,  $h_b(1P)$  and  $h_b(2P)$ . Since the Hamiltonian does not depend on spin, we can say that the difference in energy between the states with  $\nu = 1, L = 0$  and the states with  $\nu = 0, L = 0$  should be the same for states with  $S = 0$  or  $S = 1$ , and thus the energy of the state  $\eta_b(2S)$  should be about  $9391.0 \pm 2.8 + [10023.26 \pm 0.31 - (9460.30 \pm 0.26)] = 9954.0 \pm 3.4$ . We can calculate the energy of the state  $h_b(1P)$  in a similar way. The difference in energy between the degenerate level  $\chi_b(1P)$  and the state  $\Upsilon(1S)$  should be approximately the same between  $h_b(1P)$  and  $\eta_b(1S)$ , that is, the energy of  $h_b(1P)$  is about  $9390.9 \pm 2.8 + [9901.60 \pm 0.5 - (9460.30 \pm 0.26)] =$

Table 3: The experimental and calculated levels of bottomonium. The calculated values are in bold face. The number in each parenthesis of the particle name refers to the value of the quantum number  $n$  of QCD models which also appears in the 3<sup>rd</sup> column.

$(\nu, L)$	$n^{2S+1}L_J$	Particle	Mass (MeV/c <sup>2</sup> )
(5, 0)	$6^3S_1$	$\Upsilon(11020)$	$11019 \pm 8$
(4, 0)	$5^3S_1$	$\Upsilon(10860)$	$10865 \pm 8$
(3, 0)	$4^3S_1$	$\Upsilon(4S)$	$10579.4 \pm 1.2$
(2, L)	$3^3P_J$	$\chi_b(3P)$	$10530 \pm 5(\text{stat.})$ $\pm 9(\text{syst.})$
(2, 0)	$3^3S_1$	$\Upsilon(3S)$	$10355.2 \pm 0.5$
(1, 1)	$2^3P_2$	$\chi_{b2}(2P)$	$10268.65 \pm 0.55$
(1, 1)	$2^3P_1$	$\chi_{b1}(2P)$	$10255.46 \pm 0.55$
(1, 1)	$2^3P_0$	$\chi_{b0}(2P)$	$10232.5 \pm 0.6$
(1, 1)	$2^1P_1$	$h_b(2P)$	<b><math>10192.0 \pm 0.8</math></b>
(0, 2)	$1^3D_3$	$\Upsilon_3(1D)$	
(0, 2)	$1^3D_2$	$\Upsilon_2(1D)$	$10163.7 \pm 1.4$
(0, 2)	$1^3D_1$	$\Upsilon_1(1D)$	
(0, 2)	$1^1D_2$	$\eta_b(1D)$	
(1, 0)	$2^3S_1$	$\Upsilon(2S)$	$10023.26 \pm 0.31$
(1, 0)	$2^1S_0$	$\eta_b(2S)$	<b><math>9953.86 \pm 0.57</math></b>
(0, 1)	$1^3P_2$	$\chi_{b2}(1P)$	$9912.21 \pm 0.40$
(0, 1)	$1^3P_1$	$\chi_{b1}(1P)$	$9892.76 \pm 0.40$
(0, 1)	$1^3P_0$	$\chi_{b0}(1P)$	$9859.44 \pm 0.52$
(0, 1)	$1^1P_1$	$h_b(1P)$	<b><math>9832.3 \pm 3.2</math></b>
(0, 0)	$1^3S_1$	$\Upsilon(1S)$	$9460.30 \pm 0.26$
(0, 0)	$1^1S_0$	$\eta_b(1S)$	$9391.0 \pm 2.8$

$9832.2 \pm 3.6$ . Similarly, the energy of  $h_b(2P)$  is about  $9953.86 \pm 3.37 + [10261.4 \pm 0.6 - (1023.26 \pm 0.31)] = 10192.0 \pm 4.3$ . We did not include in the fitting the levels above  $\chi_b(2P)$  because it is well known that molecular potentials do not describe well very high levels in energy because for such levels the rigid rotator approximation cannot be used. Tab. 3 presents the experimental values [34] of the energies of states of bottomonium and those that were calculated above.

## 5. Conclusion

It is shown that the levels of bottomonium up to  $\chi_b(2P)$  states can be described by a molecular potential. The fitting made possible the calculation of the parameters of the molecular potential, and prediction of the energies and radii of some states. The above results add important information for the understanding of bottomonium.

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