BOTTOMONIUM WITH AN EFFECTIVE KRATZER MOLECULAR POTENTIAL

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OUTINE OF TALK

1) Brief comments on the works in the area;
2) Why the Kratzer molecular potential as an effective potential for the overall interaction in bottomonium in the description of its states below threshold?
3) Fitting of such states;
4) Calculation of the rotational contribution of P states;
5) Estimation of the sizes of 1P and 2P states;
6) Estimation of the coupling constant.

SOME IMPORTANT REMARKS

A) The static potential plays an important role in the description of heavy quarks.

B) Pioneering work of Eichten et al.[1] was followed by many other works.

C) After more than 30 years some important features of the static interaction in QCD are not yet completely understood.

D) That is one of the reasons why a large number of approximated methods and effective potentials have been proposed for the description of the overall interaction in quarkonia[1-31] . This is a large number but is not exhaustive at all of the enormous effort developed in the field. All the data used below for the energies of the $b\bar{b}$ states were taken from Nakamura et al.(PDG)[32].

The Kratzer molecular potential

The vibration-rotation spectra of diatomic molecules can be excellently described by the Kratzer potential[33-35]

$$V(r) = -2D\left(\frac{a}{r} - \frac{1}{2}\frac{a^2}{r^2}\right) \tag{1}$$

in which $-D$ is the minimum of the well and a is the distance where $V = -D$. We could have chosen Morse molecular potential, but Kratzer´s is more suitable for our purpose due to its explicit functional form: a) its 1st term is an attractive QCD-like term, and b) its 2nd term takes care of the well-known repulsion of the strong force for small distances (on this see, for example, S. Aoki[30] (R. Jastrow[41])). We do not have to worry about a confining term because we are only dealing with states below threshold and the Coulombian-like term increases with the distance.

Solution of the Schrödinger Equation for V(r)

Using non-relativistic quantum mechanics we obtain the energy levels[33]

$$E_{nL} = C + h\upsilon\left(v+\frac{1}{2}\right) - A\left(v+\frac{1}{2}\right)^2 + E_R L(L+1)$$
$$-VR\left(v+\frac{1}{2}\right)\left(L+\frac{1}{2}\right)^2 + \ldots \quad (2)$$

where v, L are the integers $0,1,2,3,\ldots$ and $C, h\upsilon, A, A_{VR}$ are constants, and $E_R = \hbar^2 / 2\Theta$ is a constant within the same L, in which Θ is the moment of inertia. The first term is a constant related to the depth of the potential, the second term describes harmonic vibrations, the third term takes into account the anharmonicity of the potential, the fourth term describes rotations with constant moment of inertia and the fifth term represents the first correction to the coupling between vibrations and rotations. In this work we disregard the fifth term. In 2007 Setare and Karimi (42) have shown that SU(2) is the dynamical group associated with the bound region of the spectrum.

The spin-spin interaction

The spin-spin interaction is given by

$$\Delta E_{\vec{s}_1 \cdot \vec{s}_2} = A_S \frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2} \tag{3}$$

in which A_S is a constant, m_1 and m_2 are the constituent masses of the corresponding quarks and

$$\vec{s}_1 \cdot \vec{s}_2 = \begin{cases} \frac{1}{4}, & S = 1 \\ -\frac{3}{4}, & S = 0 \end{cases} \tag{4}$$

where S is the total spin. For the $b\bar{b}$ system $\Delta E_{\vec{s}_1 \cdot \vec{s}_2}$ is in the range of 50 MeV and is, thus, very significant.

Fitting bottomonium levels to the Kratzer potential

1) We deal only with the levels below threshold, that is, up to the $3S$ levels.

2) We take out the spin-spin interaction energy out from the hyperfine doublets $[\eta_b(1S), \Upsilon(1S)]$, $[\eta_b(2S), \Upsilon(2S)]$, $[\eta_b(3S), \Upsilon(3S)]$ and obtain the 3 degenerate levels Υ_0, Υ_1, and Υ_2 with respective energies 9442.3 MeV, 10014.2 MeV, and 10348.9 MeV.

3) Fitting these 3 levels (L=0) to the potential we obtain the constants $C_{\bar{b}b} = 9067.4\text{MeV}$, $h\upsilon_{b\bar{b}} = 809.1\text{MeV}$, and $A_{b\bar{b}} = 118.6\text{MeV}$.

4) Now we take into account the centrifugal term of the potential and consider the $\chi_b(P)$ states. Taking into account the spin-orbit contribution

$\Delta E_{SL} = \Delta\left[J(J+1) - L(L+1) - S(S+1)\right]$ we obtain $E_{Rb\bar{b}1} = 459.3\text{MeV}$ for the states $\chi_{b0}(1^3P_0)$, $\chi_{b1}(1^3P_0)$, and $\chi_{b2}(1^3P_0)$; and $E_{Rb\bar{b}2} = 123.6\text{MeV}$ for the states $\chi_{b0}(2^3P_0)$, $\chi_{b1}(2^3P_0)$ and $\chi_{b2}(2^3P_0)$.

5) Checking the consistency of the calculations

As the third term in Eq. (3) cannot be larger than the second term, we should always have $v < \frac{1}{2}\left(\frac{h\upsilon_{b\bar{b}}}{A_{b\bar{b}}} - 1\right) \approx 2.9$, and thus $v = 0,1,2$;

6) Calculation of the parameters of the potential

Relating the constants above to the parameters of the potential *a* and *D* we find

D=10.500MeV

a=0.48fm

7) Estimation of the coupling constants

Using the harmonic constant and the calculated constants we obtain the coupling constants of about

a) 10.3 for the hadronic coupling constant;

b) 36.5GeV/fm^2 for the overall effective coupling (from the harmonic part).

These figures are completely reasonable. Altmeyer et al. (42) report *g=4.2+/-1.8* for pions. The experimental value is about 6.08.

Estimation of the sizes of $1P$ and $2P$ states of bottomonium

Using the above values of $E_{Rb\bar{b}}$, the reduced mass $M_{b\bar{b}} = 2500\text{MeV}$, and defining the sizes of the mesons by

$$R_{b\bar{b}} = \left(\sqrt{\left\langle \frac{1}{r^2} \right\rangle} \right)^{-1} = \sqrt{\frac{\hbar^2}{2M_{b\bar{b}} E_{Rb\bar{b}}}}$$

Table 1. Estimation of radii for P states of bottomonium below threshold

Meson	*Radius* (*fm*)
$\chi_{0b}(1P)$	0.257
$\chi_{1b}(1P)$	0.257
$\chi_{2b}(1P)$	0.257
$\chi_{0b}(2P)$	0.350
$\chi_{1b}(2P)$	0.350
$\chi_{2b}(2P)$	0.350

These values are much smaller than those found in Ref (31)

But these values are quite reasonable as is shown below:

having in mind that for π^- and K^- Povh and Hüfner[36] have reported radii of about 0.66 fm and 0.61fm, respectively. The above values are much smaller than those found in Ref. (31).

R. Tarrach (37) reports 0.68 fm for charged *pions*.

C.-W. Hwang (39) reports 0.66 fm for charged pions, 0.59 fm for charged *kaons*, and 0.43 fm for D(+).

The experimental results for charged *pions* (39) are 0.66 fm and for charged *kaons* are 0.58 fm (40).
Also, for π^- and K^- Povh and Hüfner[36] have reported radii of about 0.66 fm and 0.61fm, respectively

Summary of results

1) The levels of bottomonium below threshold can be fitted to a Kratzer molecular potential which is a completely new approach;
2) This effective potential can have a broader use in interactions involving bottomonium;
3) We calculate the rotational contributions for L=1 and L=2 levels;
4) We make an estimation of the sizes of P states;
5) A very reasonable estimation of the hadronic coupling constant is achieved.

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THANK YOU!

DETAILS OF CALCULATIONS

4. Fitting bottomonium levels to the Kratzer potential

We deal only with the levels below threshold, that is, up to the $3S$ levels. The energy difference of the hyperfine doublets $\eta_b(1S)$ and $\Upsilon(1S)$ due to the spin-spin interaction is $9460.3-9390.9=69.4$MeV. Removing the splitting we obtain a degenerate level Υ_0 with an energy of $9460.3-(1/4)69.4=9390.9+(3/4)69.4\approx 9442.3$MeV.

Since $2S$ and $3S$ states are also spin-spin interaction hyperfine doublets (see, for example, Bai-Qing, L. and Kuang-Ta, C.[35]). Using for the energies of $\eta_b(2S)$ and $\eta_b(3S)$ the predicted values from Ref. 35, 9987.0MeV and 10330MeV, respectively, we take the spin-spin energy off and obtain the degenerate levels Υ_1 and Υ_2 with respective energies $10023.3-(1/4)36.3\approx 10014.2$MeV, and $10355.2-(1/4)25.2$=10348.9MeV.

We apply Eq. (3) to S states to avoid the fourth term, initially, because for these states $L=0$. Disregarding the fifth term and applying Eq. (3) to the three levels Υ_0, Υ_1, and Υ_2, we have

$$C_{b\bar{b}} + h\upsilon_{b\bar{b}}\left(0+\frac{1}{2}\right) - A_{b\bar{b}}\left(0+\frac{1}{2}\right)^2 = 9442.3 \qquad (5)$$

$$C_{b\bar{b}} + h\upsilon_{b\bar{b}}\left(1+\frac{1}{2}\right) - A_{b\bar{b}}\left(1+\frac{1}{2}\right)^2 = 10014.2 \qquad (6)$$

$$C_{b\bar{b}} + h\upsilon_{b\bar{b}}\left(2+\frac{1}{2}\right) - A_{b\bar{b}}\left(2+\frac{1}{2}\right)^2 = 10348.9 \qquad (7)$$

from which we obtain $C_{\bar{b}b} = 9067.4\text{MeV}$, $h\upsilon_{b\bar{b}} = 809.1\text{MeV}$, and $A_{b\bar{b}} = 118.6\text{MeV}$.

Let us now take into account the centrifugal term in Eq. (3). For this purpose we work with the $\chi_b(P)$ states and take into account the spin-orbit interaction term

$$\Delta E_{SL} = \Delta\left[J(J+1) - L(L+1) - S(S+1)\right]. \qquad (8)$$

Applying Eq. (9) to $\chi_{b0}(1^3P_0)$, $\chi_{b1}(1^3P_0)$ and $\chi_{b2}(1^3P_0)$ we obtain

$$E_{1P} + \Delta_1\left[0 - 1\times 2 - 1\times 2\right] = 9859.4 \qquad (9)$$

$$E_{1P} + \Delta_1\left[1\times 2 - 1\times 2 - 1\times 2\right] = 9892.8 \qquad (10)$$

$$E_{1P} + \Delta_1\left[2\times 3 - 1\times 2 - 1\times 2\right] = 9912.2 \qquad (11)$$

where E_{1P} is the energy of the degenerate level $1P$. We obtain the average values $\Delta_1 = 10.1\text{MeV}$, $E_{1P} = 9901.6\text{MeV}$. Thus, we have $E_{Rb\bar{b}1}1(1+1) = 9901.6 - 9442.3 = 459.3\text{MeV}$, and so, $E_{Rb\bar{b}1} = 229.7\text{MeV}$ for $n = 0$. And applying Eq. (9) to

$\chi_{b0}(2^3P_0)$, $\chi_{b1}(2^3P_0)$ and $\chi_{b2}(2^3P_0)$ we obtain, similarly, the averages $E_{2P} = 10261.4\text{MeV}$, $\Delta_2 = 6.9\text{MeV}$, where E_{2P} is the energy of the degenerate level $2P$. Hence, we have $E_{Rb\bar{b}\,2}1(1+1) = 10261.4 - 10014.2 = 247.2\text{MeV}$, and then $E_{Rb\bar{b}\,2} = 123.6\text{MeV}$ for $n = 1$.

Let us now check the consistence of our calculation. As the third term in Eq. (3) cannot be larger than the second term, we should always have

$$n < \frac{1}{2}\left(\frac{h\upsilon_{b\bar{b}}}{A_{b\bar{b}}} - 1\right) \tag{12}$$

where n is the number of states below threshold. Inserting into this equation the figures obtained above we have $n < 2.91$ which is quite consistent.

5 The sizes of the states of bottomonium below threshold

Using the above values of $E_{Rb\bar{b}}$, the reduced mass $M_{b\bar{b}} = 2500\text{MeV}$, and defining the sizes of the mesons by

$$R_{b\bar{b}} = \left(\sqrt{\left\langle \frac{1}{r^2} \right\rangle} \right)^{-1} = \sqrt{\frac{\hbar^2}{2M_{b\bar{b}} E_{Rb\bar{b}}}} \qquad (13)$$

and remembering that $R_{b\bar{b}}$ is the same within the same L, we obtain the following results

Table 1. Calculated radii for P states of bottomonium below threshold

$Meson$	$Radius\ (fm)$
$\chi_{0b}(1P)$	0.257
$\chi_{1b}(1P)$	0.257
$\chi_{2b}(1P)$	0.257
$\chi_{0b}(2P)$	0.350
$\chi_{1b}(2P)$	0.350
$\chi_{2b}(2P)$	0.350

These values are quite reasonable having in mind that for π^- and κ^- Povh and Hüfner[36] have reported radii of about 0.66 fm and 0.61fm , respectively. The above values are much smaller than those found in Ref. (35).