

# Rare and CP violating Kaon decays: a probe of TeV scale physics

BEACH 2012

Wichita

23.7.2012

Martin Gorbahn

TUM-IAS &

Excellence Cluster 'Universe'



# This Talk



Wichita: Cowtown

Past:  
Why are rare Kaon decays  
so rare?

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Precision theory prediction  
for  $K_L \rightarrow \pi \bar{u} u$  and  $\epsilon_K$



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Wichita: Cowtown



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Present:

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for  $K_L \rightarrow \pi \bar{\nu} \nu$  and  $\epsilon_K$

Future:

What can we learn about  
New Physics from Kaons?



# Why are Kaon Decays so rare?

Before the charm quark: why are the two Branching ratios

$$\text{Br}(\text{K}_\text{L} \rightarrow \mu^+ \mu^-) \simeq 6.84(11) \cdot 10^{-9} \quad \text{Br}(\text{K}_\text{L} \rightarrow \gamma\gamma) \simeq 5.47(4) \cdot 10^{-4}$$

so different in size?

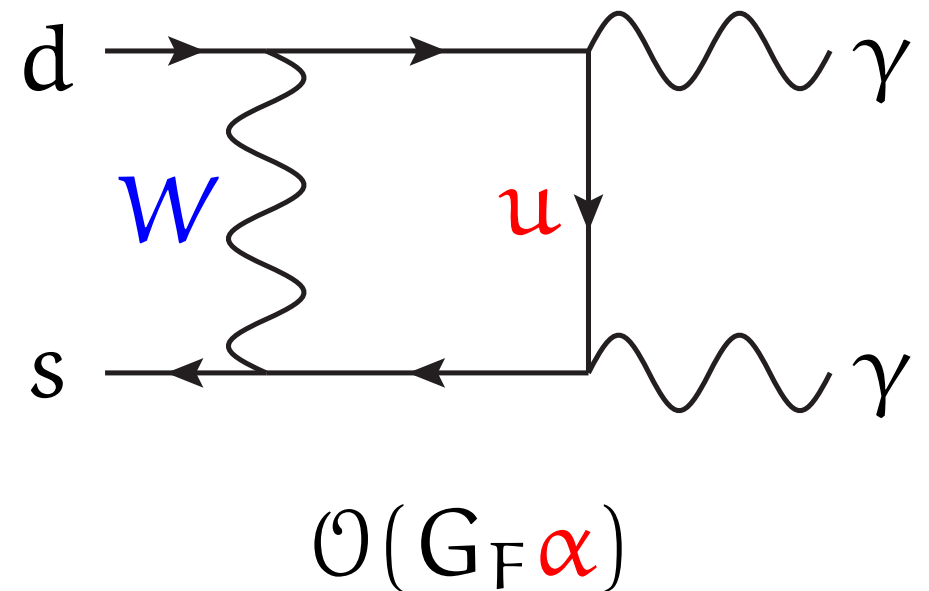
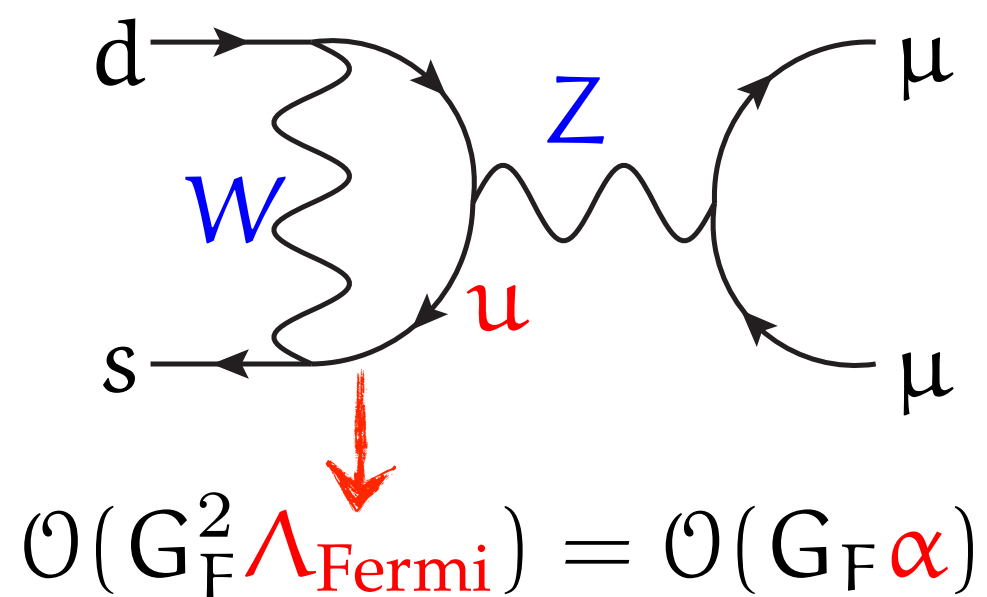
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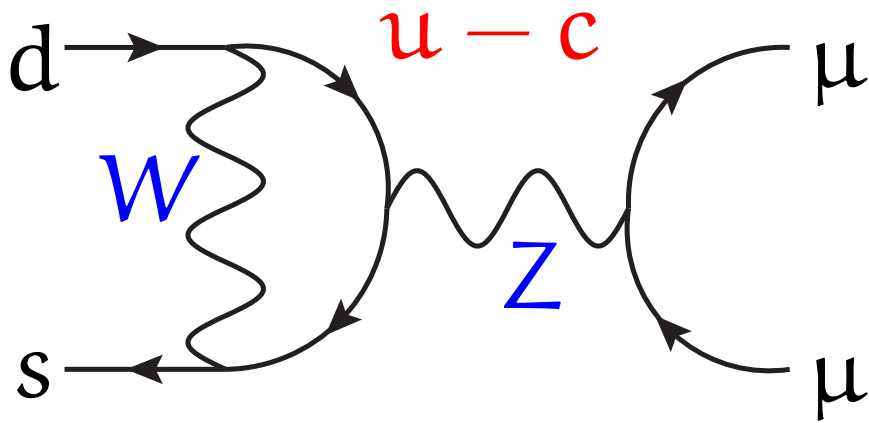
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$K_L \rightarrow \mu^+ \mu^-$  : The 2  $\mu$ s are in J=0 state  $\rightarrow$  no 1  $\gamma$  coupling



# The GIM Mechanism

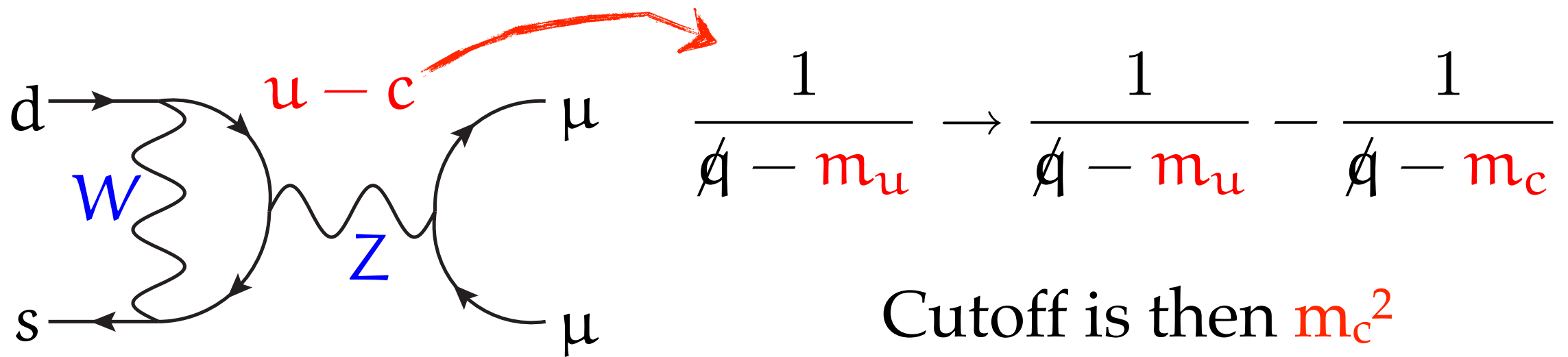
GIM: charm quark to suppress neutral currents





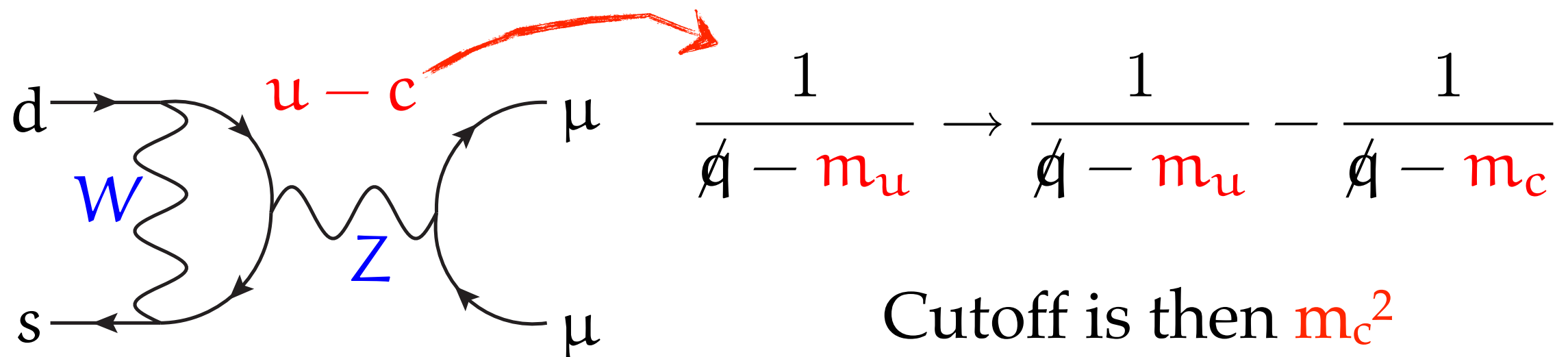
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Quadratic GIM explains the smallness of  $\text{Br}(K_L \rightarrow \mu^+ \mu^-)$

$\frac{m_c^2}{M_W^2}$  dependence: predict charm quark

# Charm Quark Mass

Quadratic GIM suppresses light quark contribution



Sensitive to short distances (**SD**)

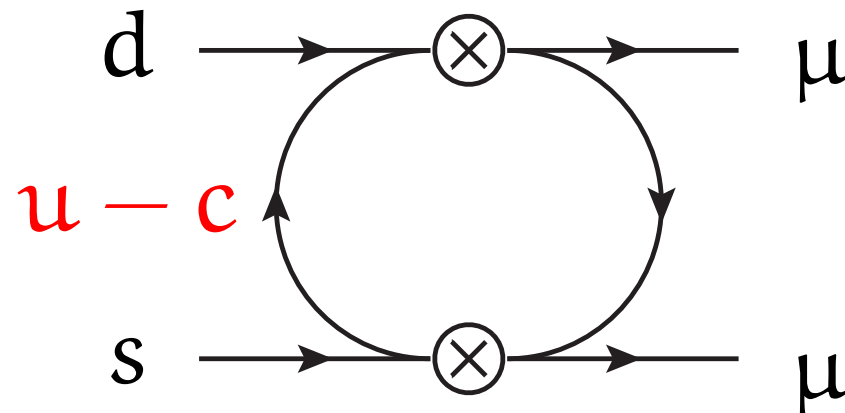
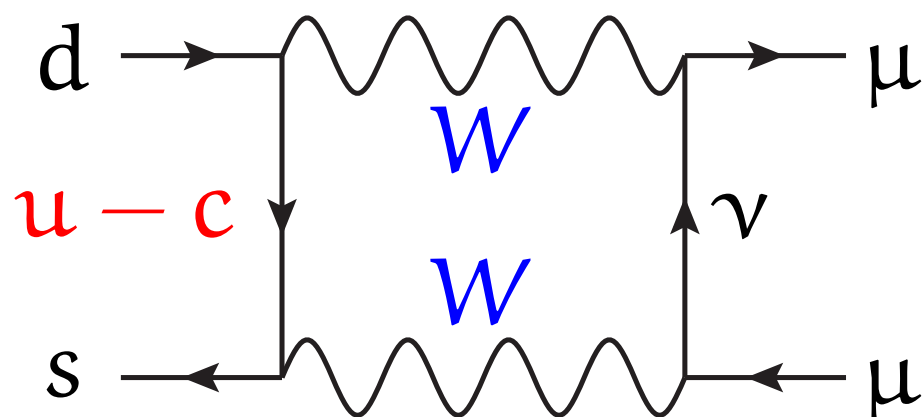
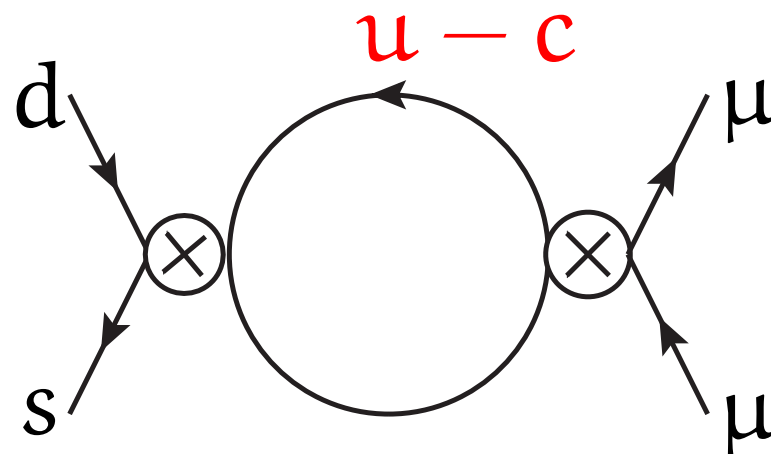
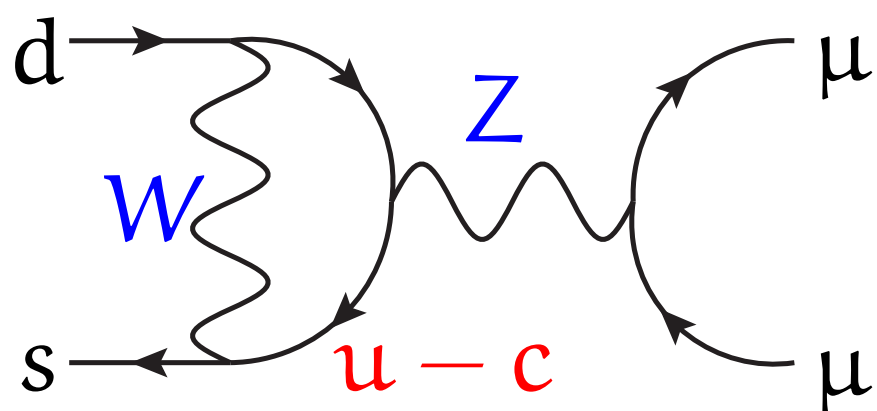


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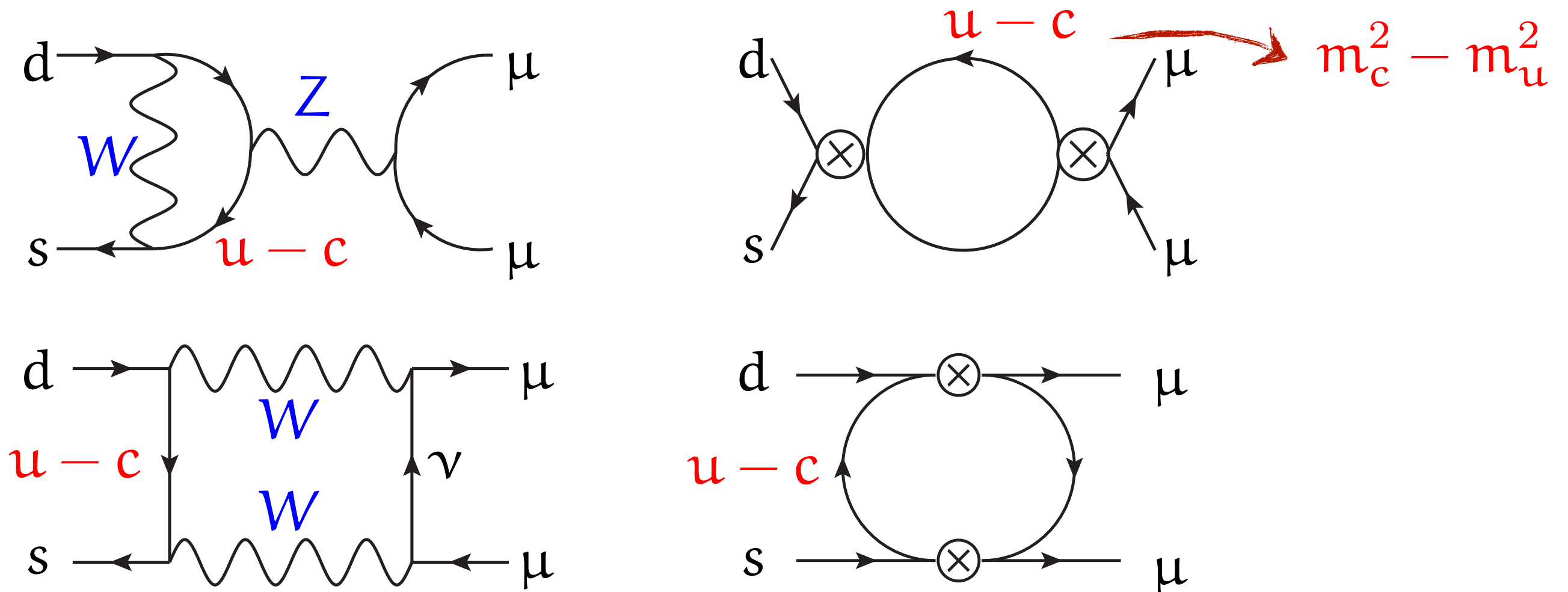


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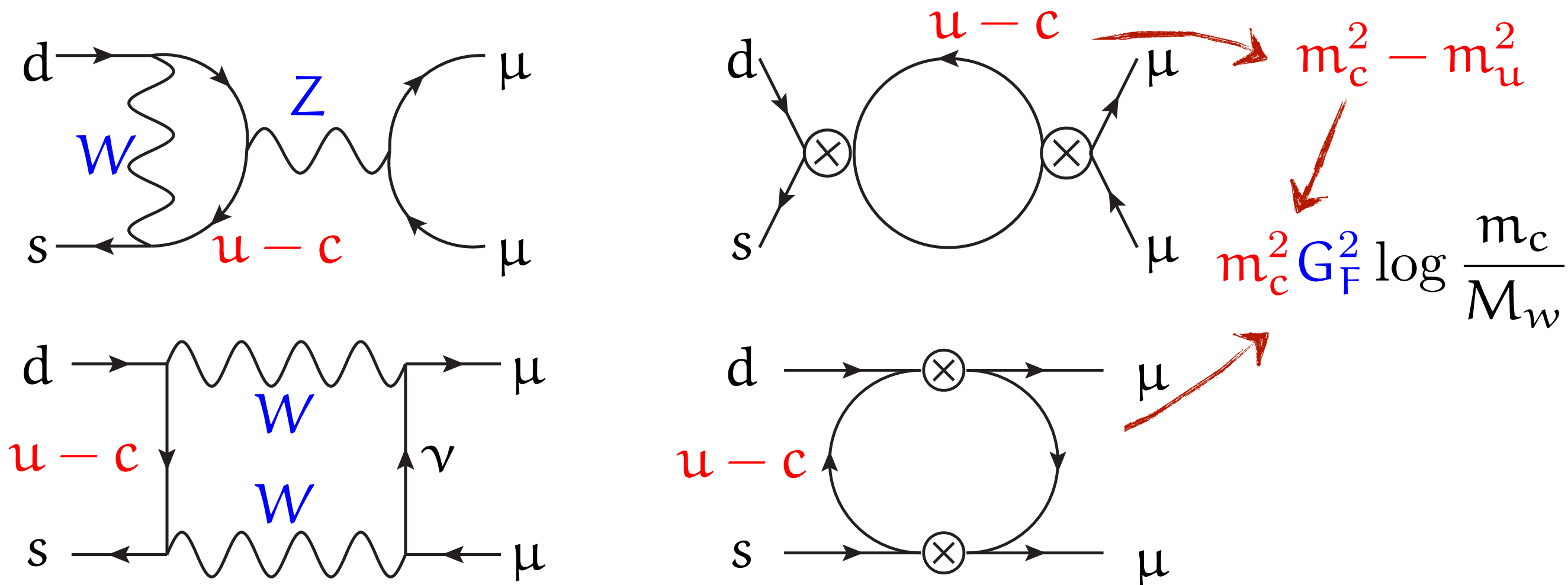


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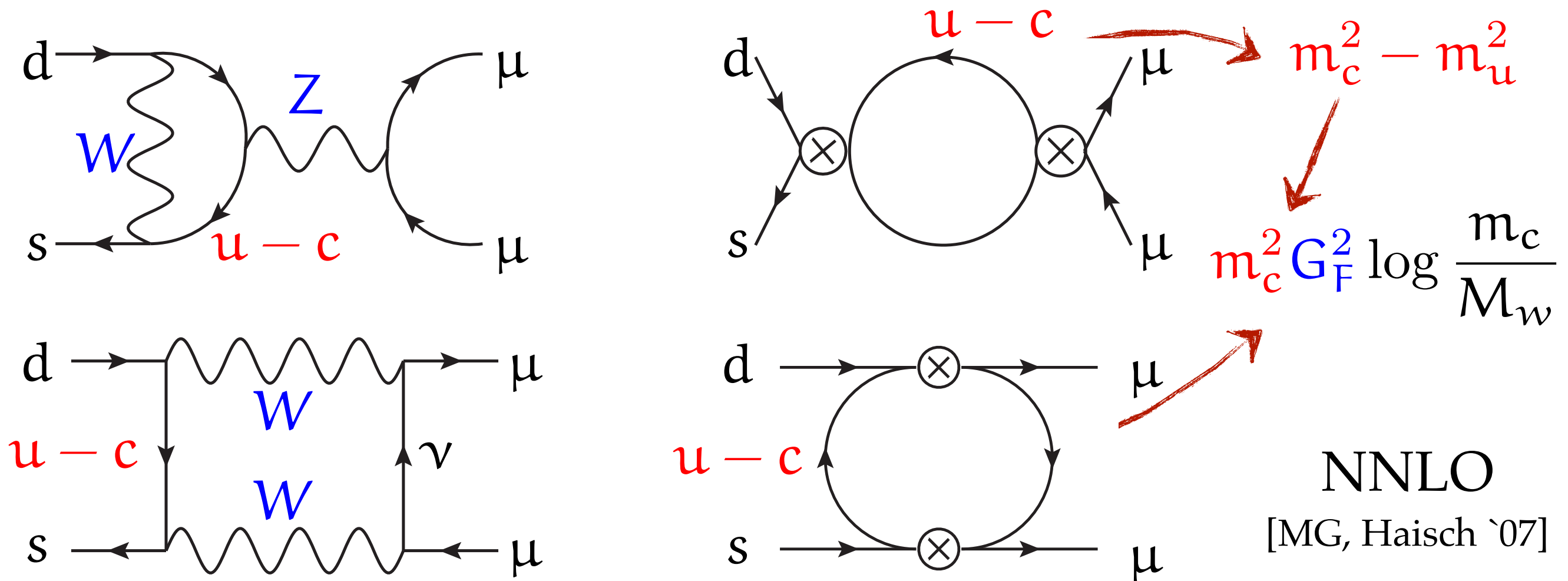


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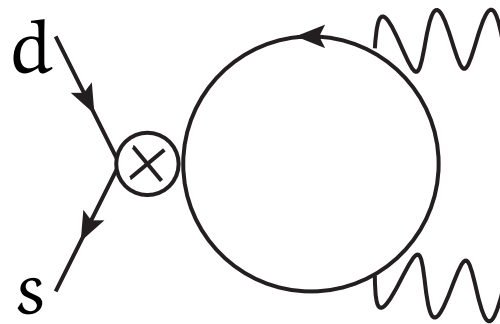
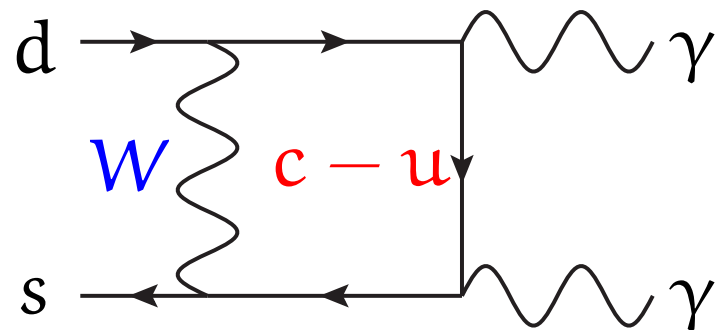
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# Contributions to $K_L \rightarrow \mu^+ \mu^-$

No quadratic suppression for  $K_L \rightarrow \gamma\gamma$

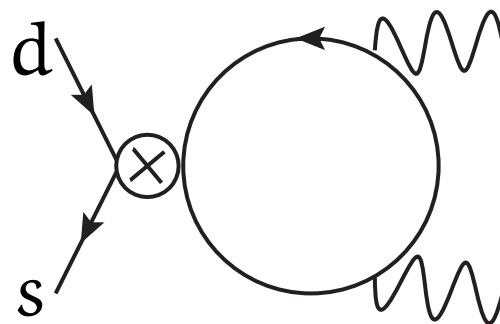
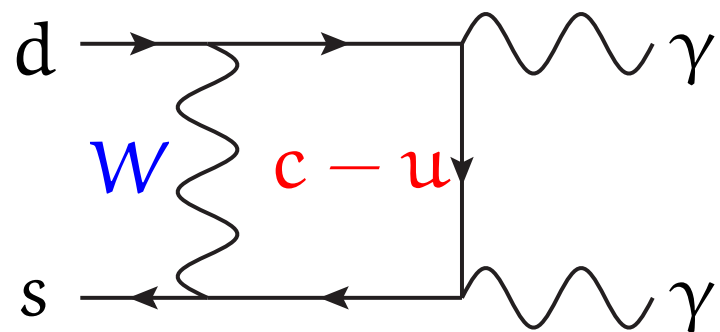


$$G_F \log \frac{\Lambda_{QCD}}{m_c}$$

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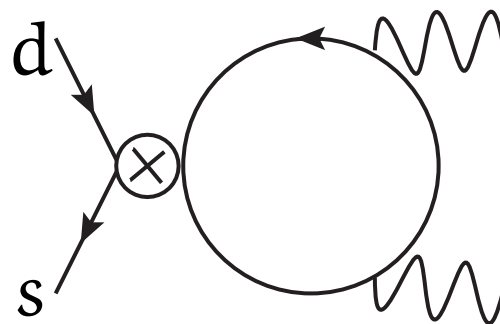
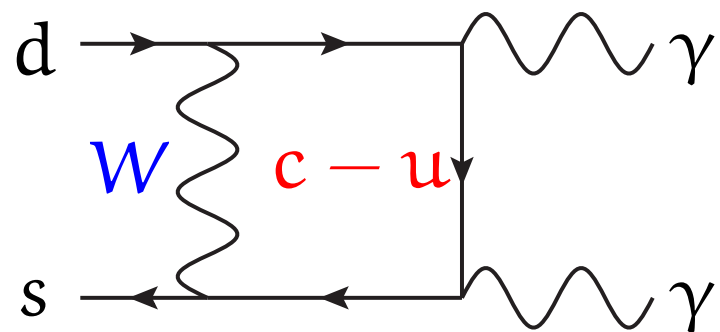
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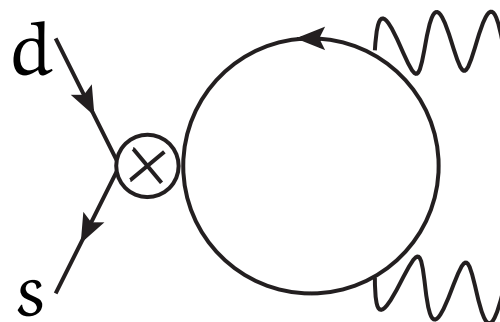
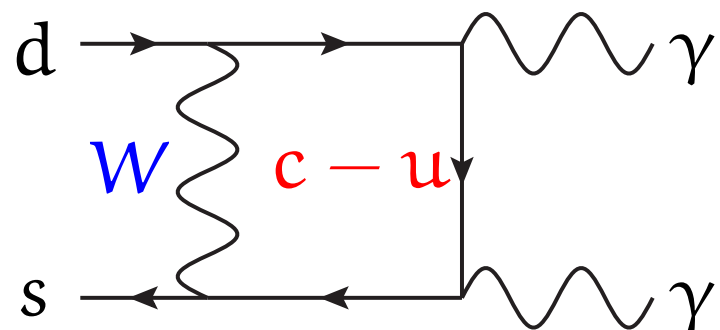
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including top

$$\text{Br}(K_L \rightarrow \mu^+ \mu^-) \simeq ((-0.95 \pm ???)^2 + 6.7) \cdot 10^{-9}$$

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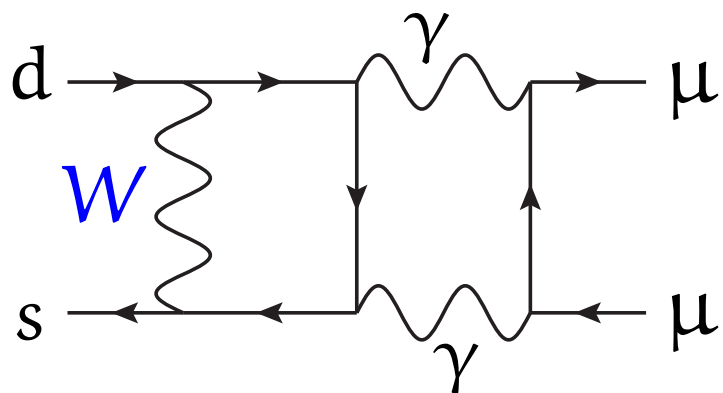


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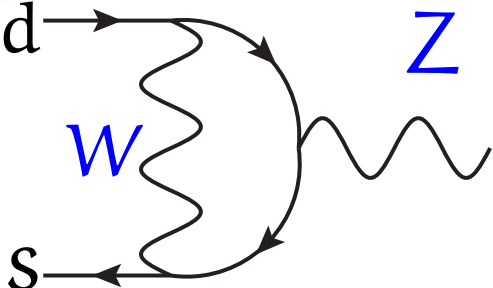
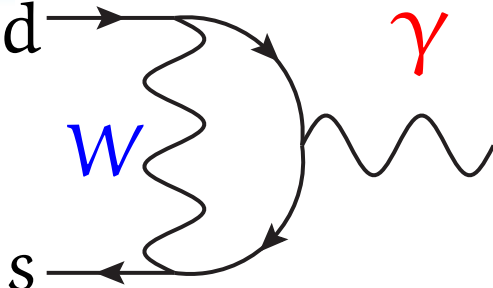
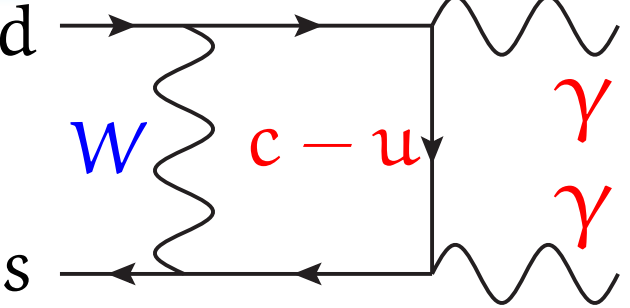


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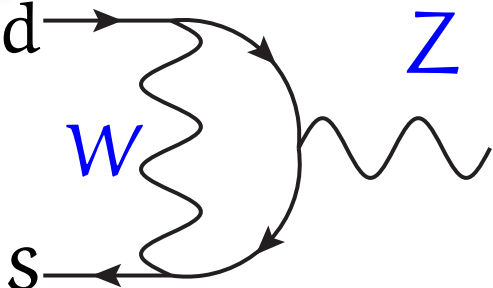
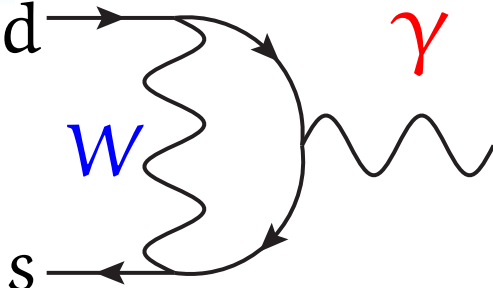
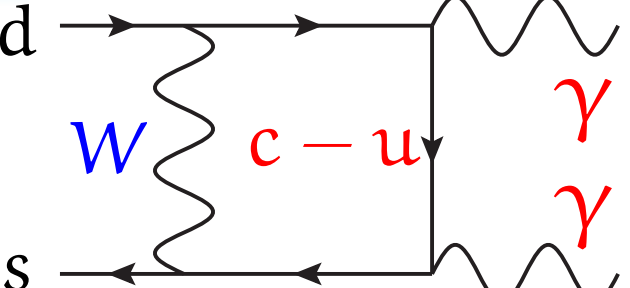
Dispersive

Absorptive

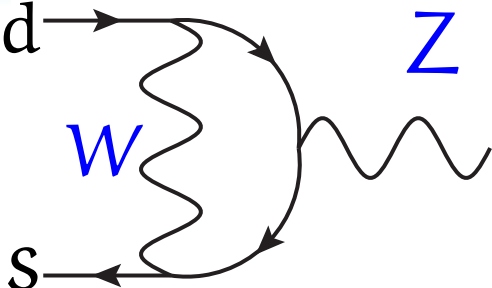
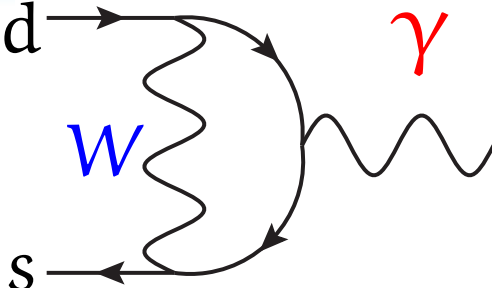
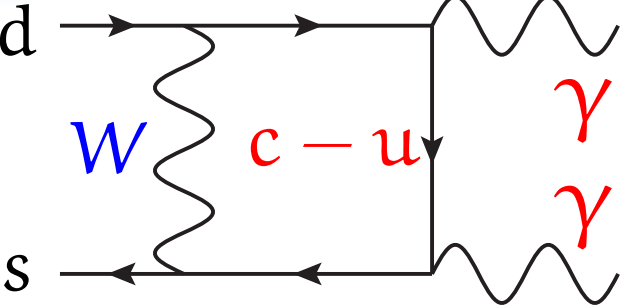
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$K_L \rightarrow \mu^+ \mu^-$	SD	—	$\alpha_e$ LD

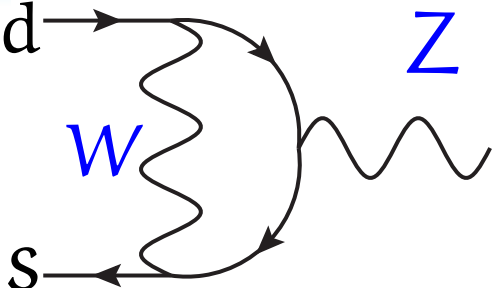
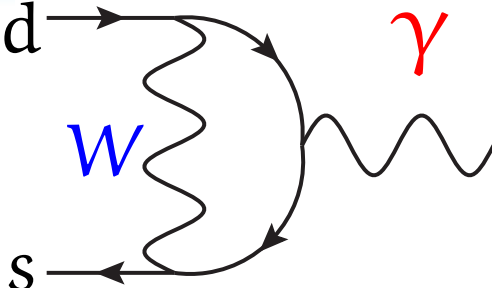
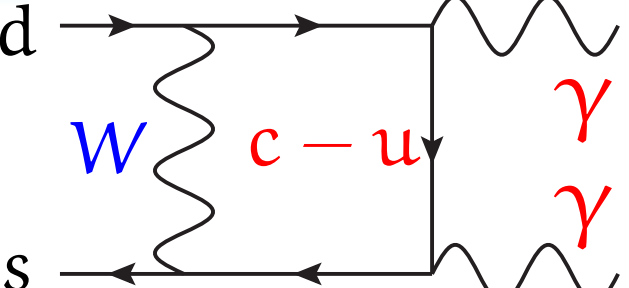
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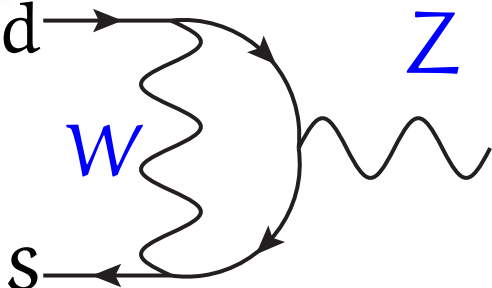
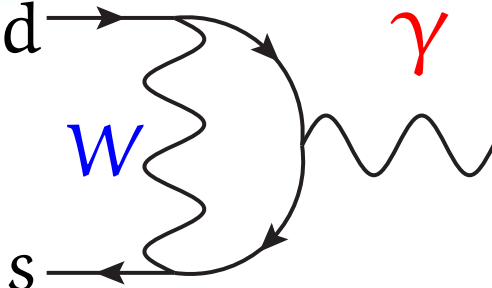
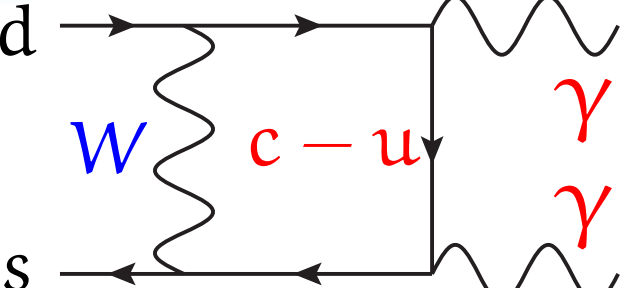
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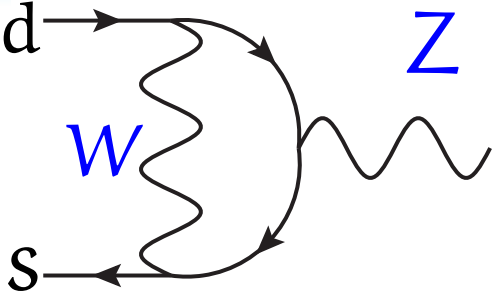
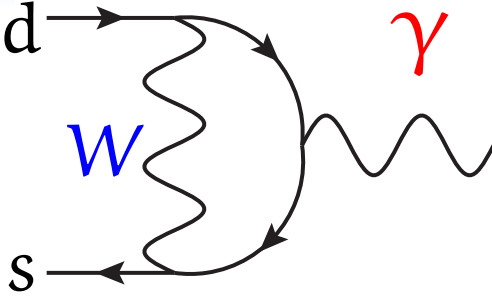
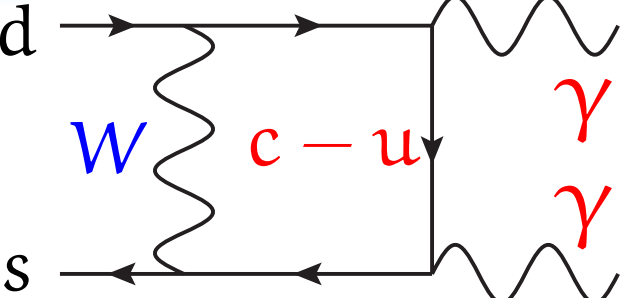
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$K_L \rightarrow \pi l^+ l^-$	SD	SD + $\epsilon_K$ LD	$\alpha_e$ LD

CP violating



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$K_L \rightarrow \pi l^+ l^-$	SD	SD + $\epsilon_K$ LD	$\alpha_e$ LD
CP violating	NLO QCD [Buchalla et. al. '95]	$K_L \rightarrow K_S$ & $K_S \rightarrow \pi^0 l^+ l^-$ 7 [Mescia et. al. '06]	Estimate from $K_L \rightarrow \pi^0 \gamma \gamma$ [Isidori et. al. '04]

# Top quark

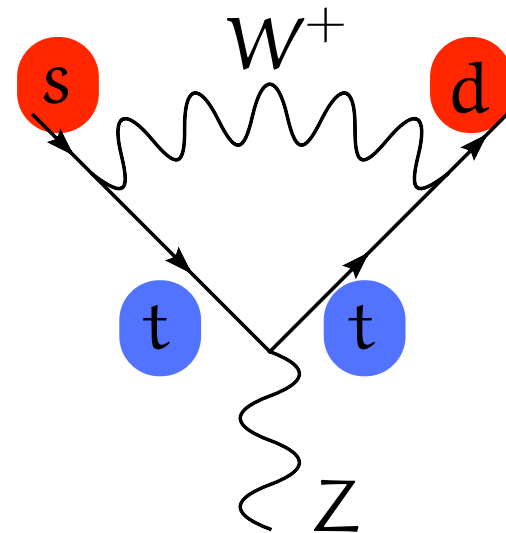
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 $\rightarrow$  top-quark dominates

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$$\frac{m_c^2}{M_W^2} \log \left( \frac{m_c^2}{M_W^2} \right) \lambda = 0.3 \cdot \frac{m_t^2}{M_W^2} \lambda^5$$

$$V_{ij} = \mathcal{O} \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda = \mathcal{O}(0.2)$$



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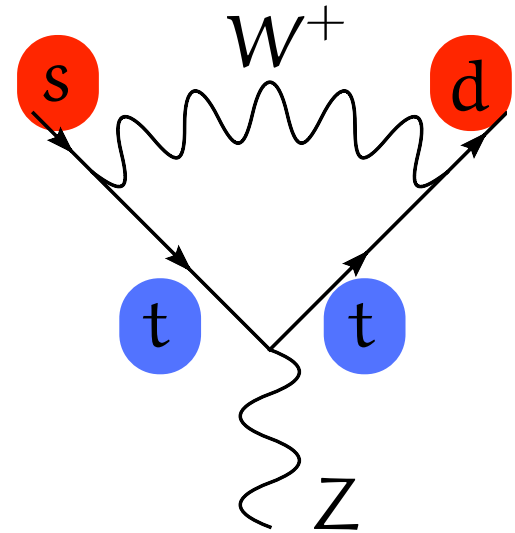
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FCNCs which are dominated by top-quark loops:

$$b \rightarrow s :$$

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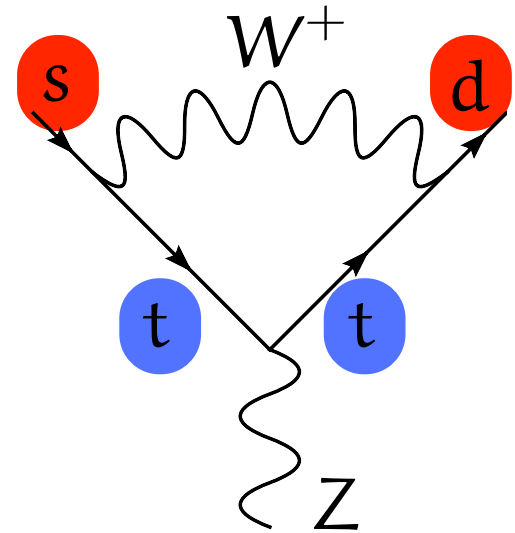
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Kaons test new physics up to 100 TeV

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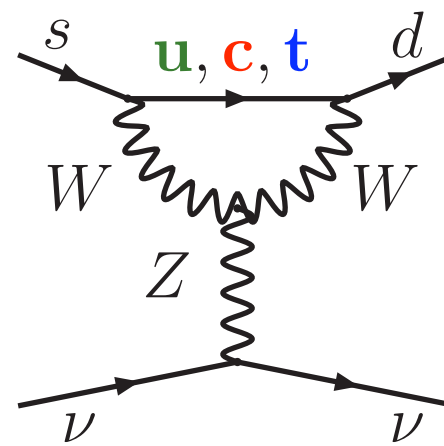
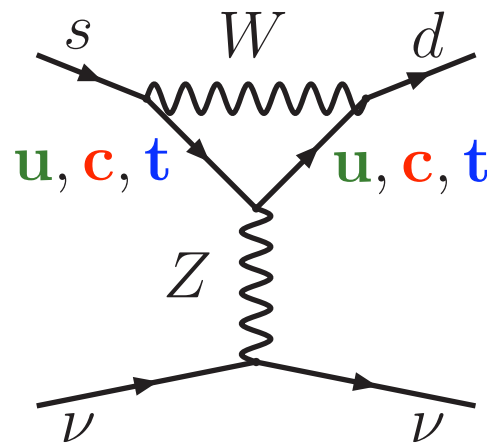
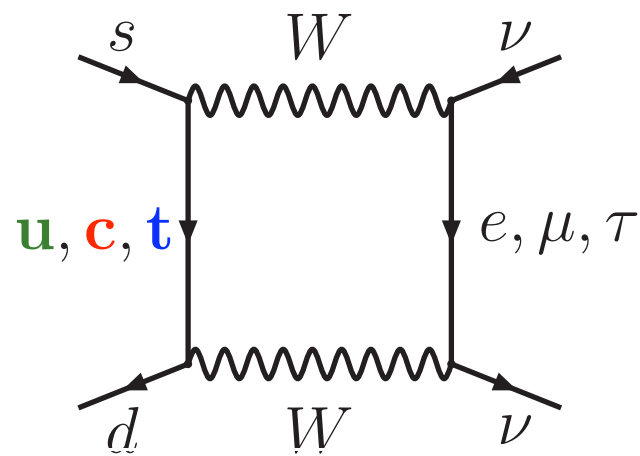
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Hadronic decays: CP violation in mixing  $\varepsilon_K$

Light quark contributions suppressed by quadratic GIM and small 2nd generation complex phase

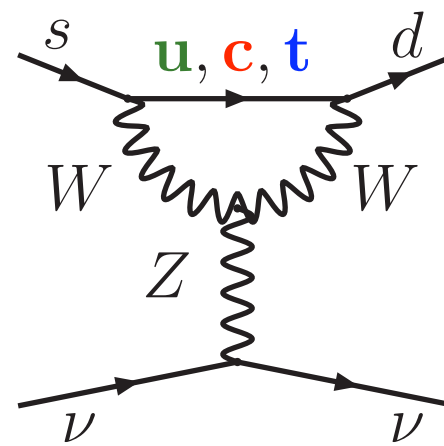
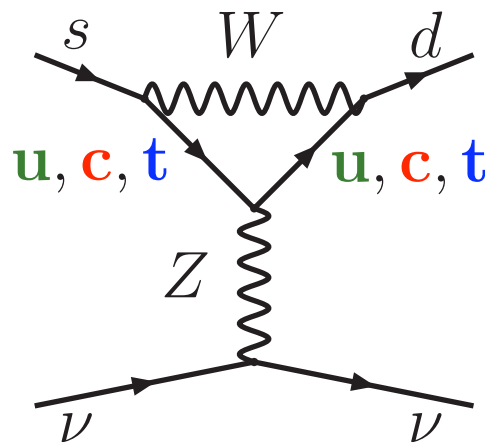
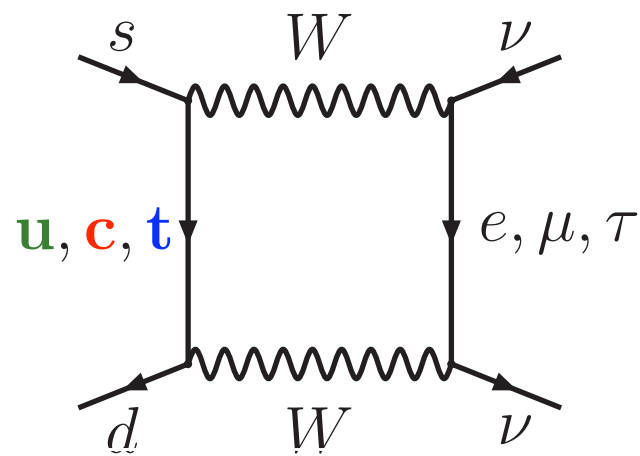
# $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ at $M_W$



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

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Quadratic GIM:

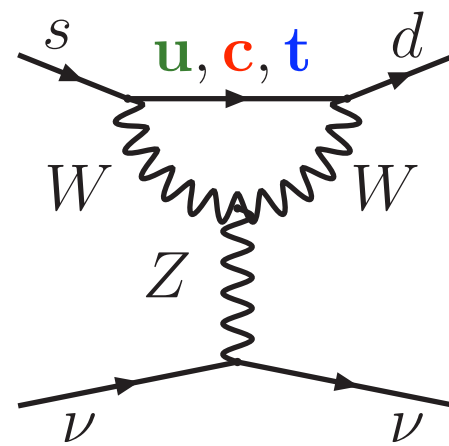
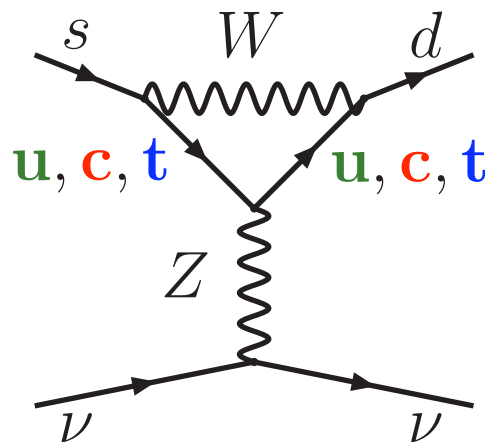
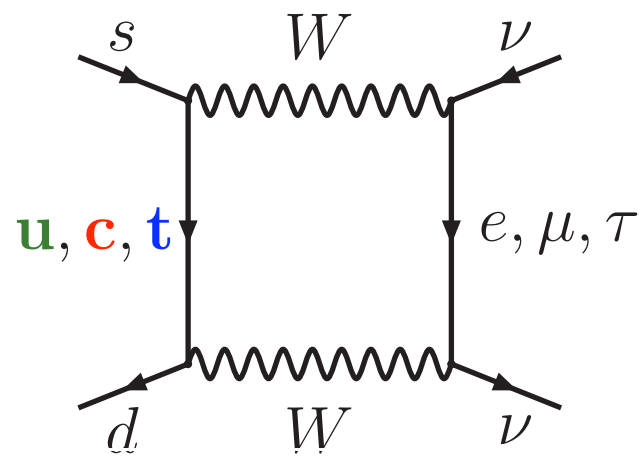
$$\lambda^5 \frac{m_t^2}{M_W^2}$$

Matching (NLO +EW):

[Misiak, Urban; Buras, Buchalla;  
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$$Q_\nu = (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma^\mu \nu_L)$$

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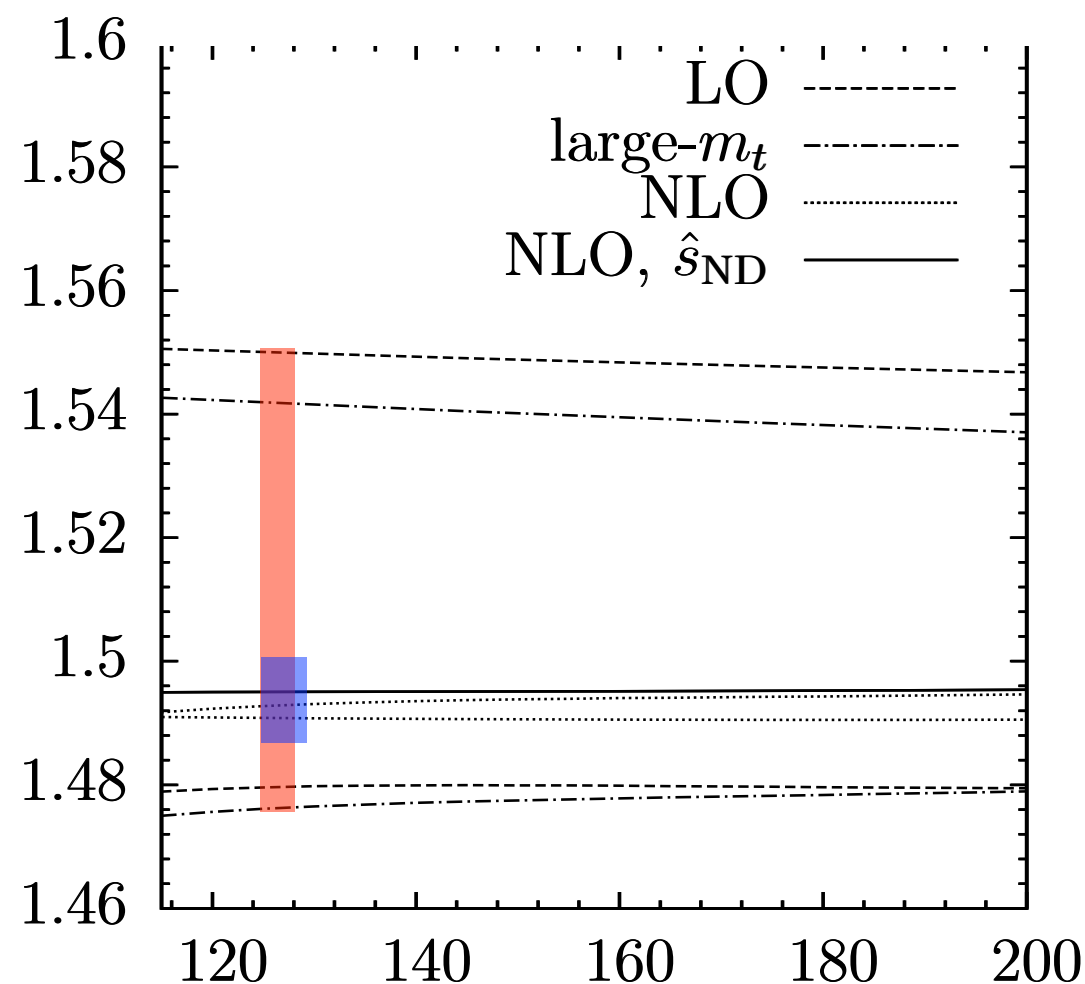
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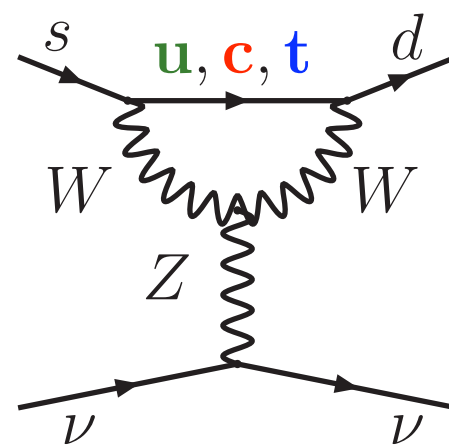
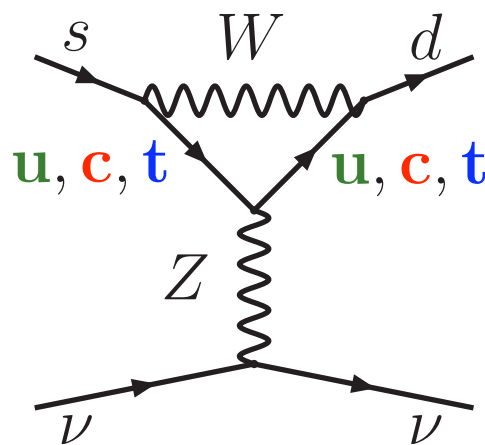
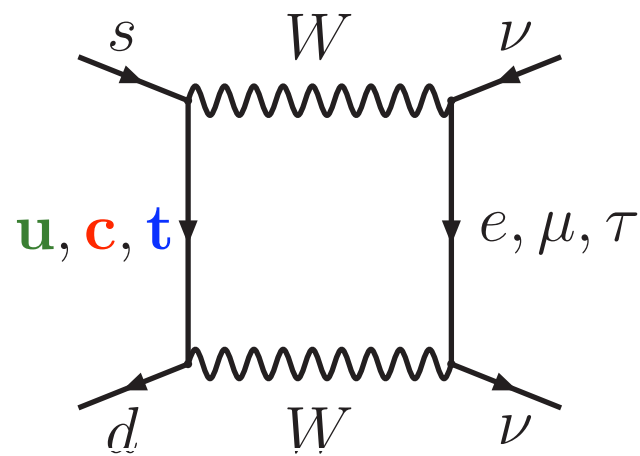
$$Q_\nu = (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma^\mu \nu_L)$$

After 2011 uncertainty below 1%

$\hat{x}_t$



# $K^+ \rightarrow \pi^+ \bar{u} u$ at $M_W$



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

Quadratic GIM:

$$\lambda^5 \frac{m_t^2}{M_W^2}$$

$$\lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$$

Matching (NLO +EW):

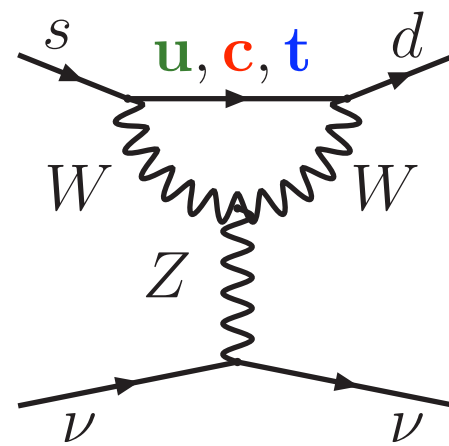
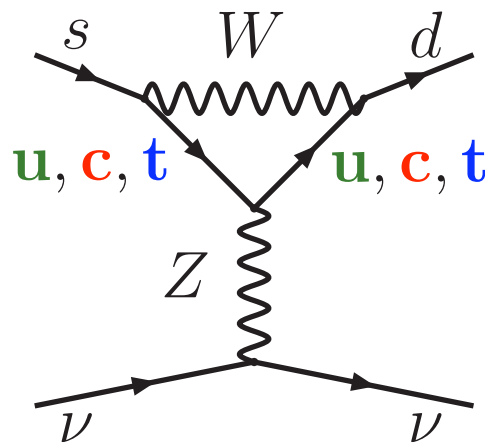
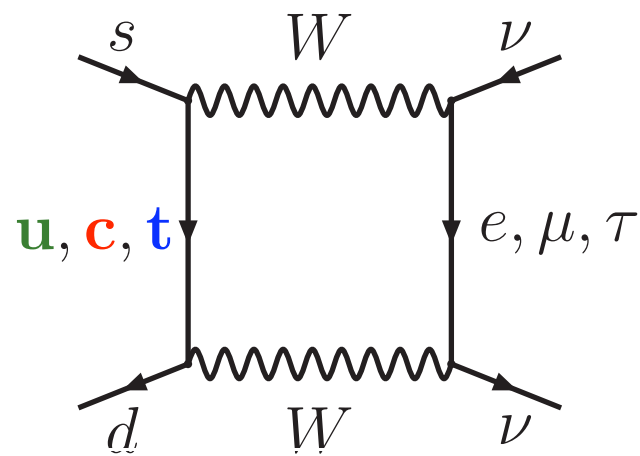
[Misiak, Urban; Buras, Buchalla;  
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$$Q_\nu = (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma^\mu \nu_L)$$

Operator  
Mixing (RGE)



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Operator  
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Matrix element from  $K_{13}$  decays  
(Isospin symmetry:  $K^+ \rightarrow \pi^0 e^+ \nu$ )

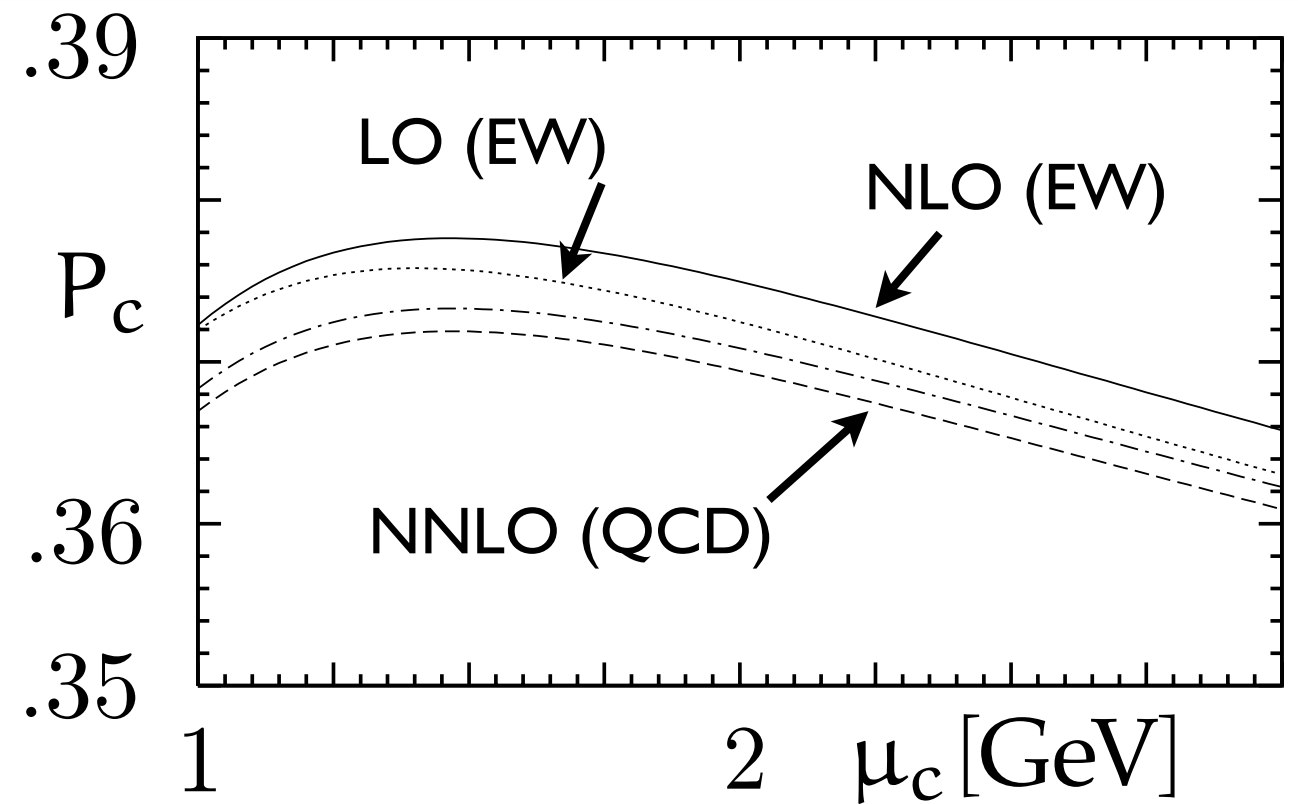
[Mescia, Smith]

# $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ from $M_W$ to $m_c$

$P_c$ : charm quark contribution  
to  $K^+ \rightarrow \pi^+ \bar{\nu} \nu$  (30% to BR)

Series converges very well  
(NNLO:10%  $\rightarrow$  2.5% uncertainty)

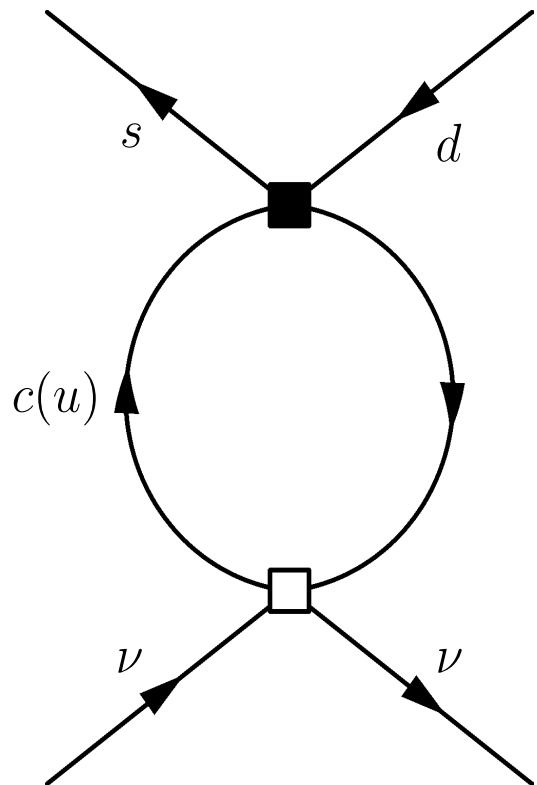
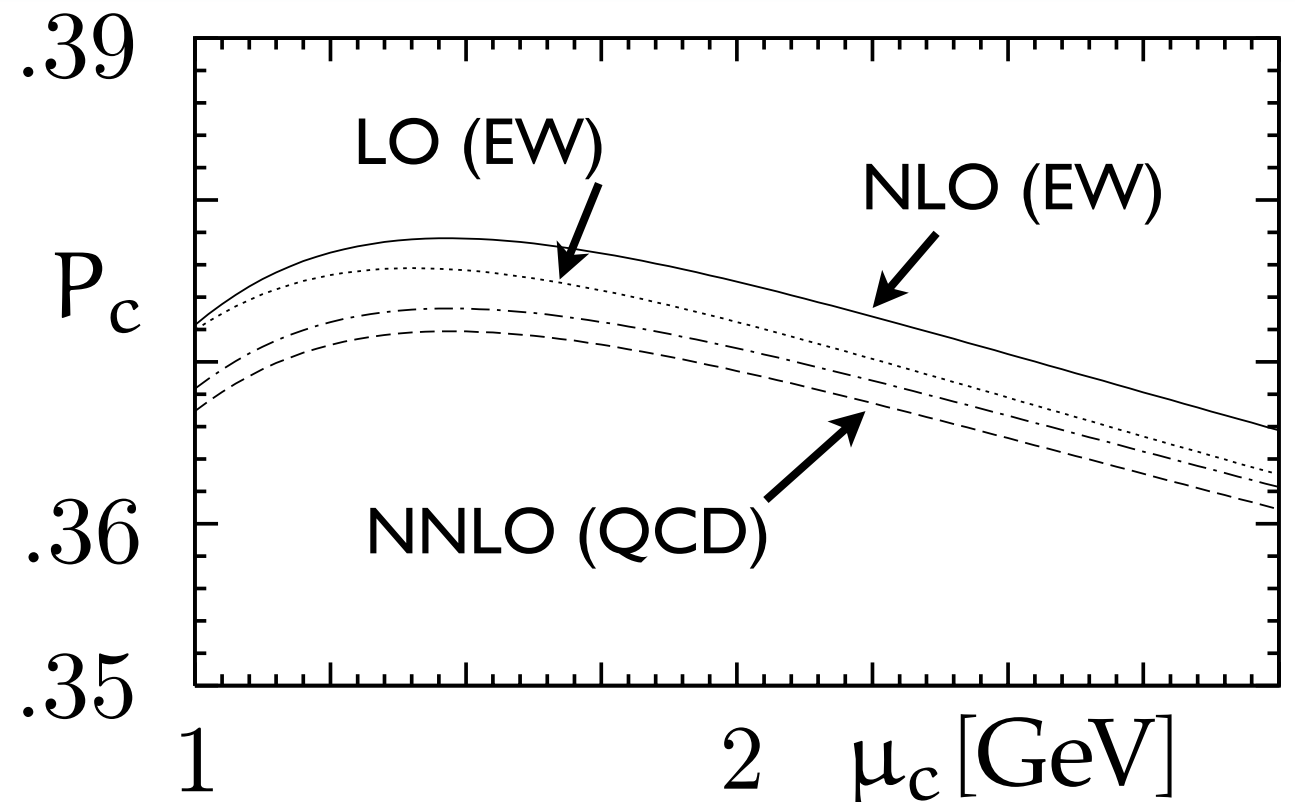
NNLO+EW [Buras, MG, Haisch,  
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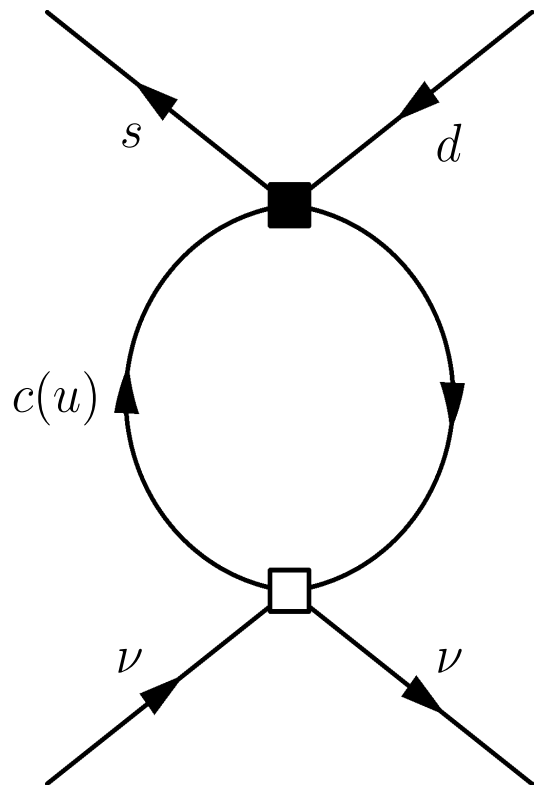
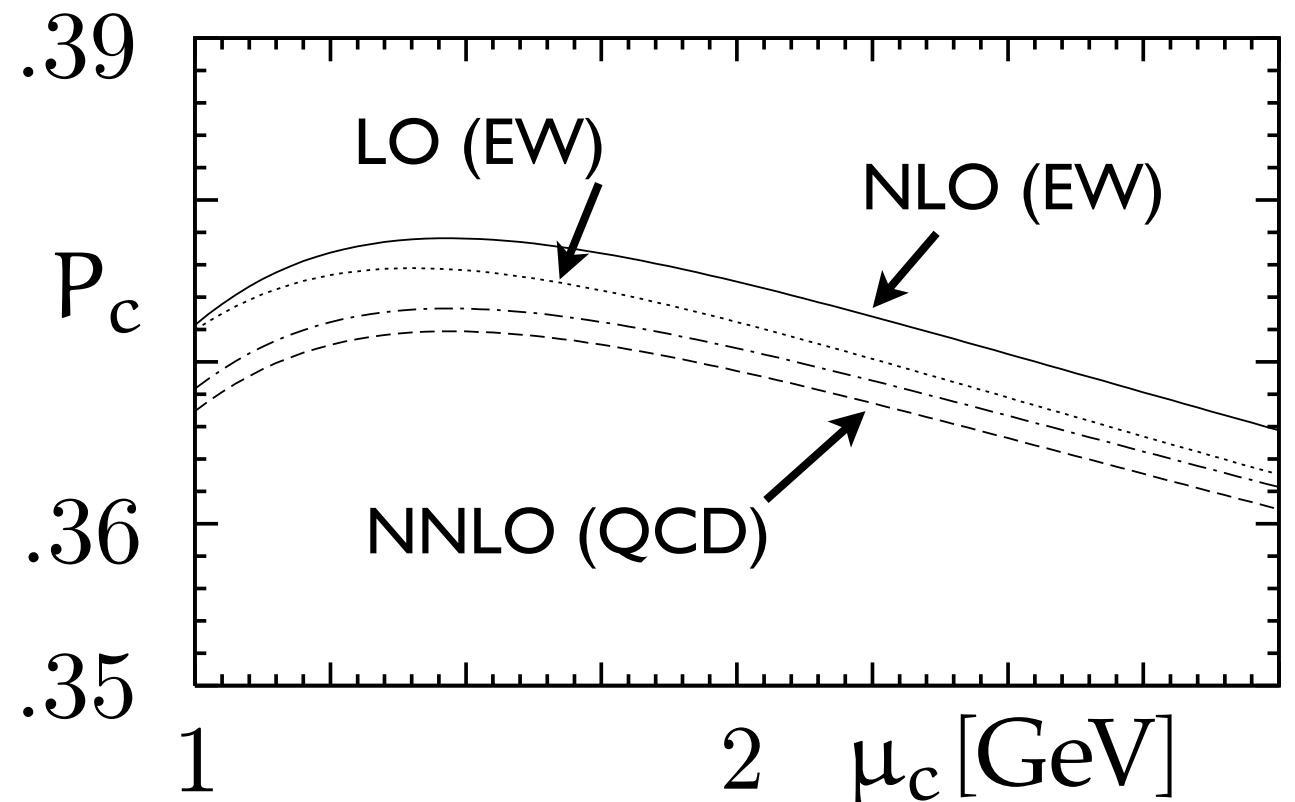


No GIM below the charm quark mass scale  
higher dimensional operators UV scale dependent  
One loop ChiPT calculation approximately cancels  
this scale dependence  $\delta P_{c,u} = 0.04 \pm 0.02$   
[Isidori, Mescia, Smith '05]

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Could be calculated on the lattice

[Isidori, Martinelli, Turchetti '06]

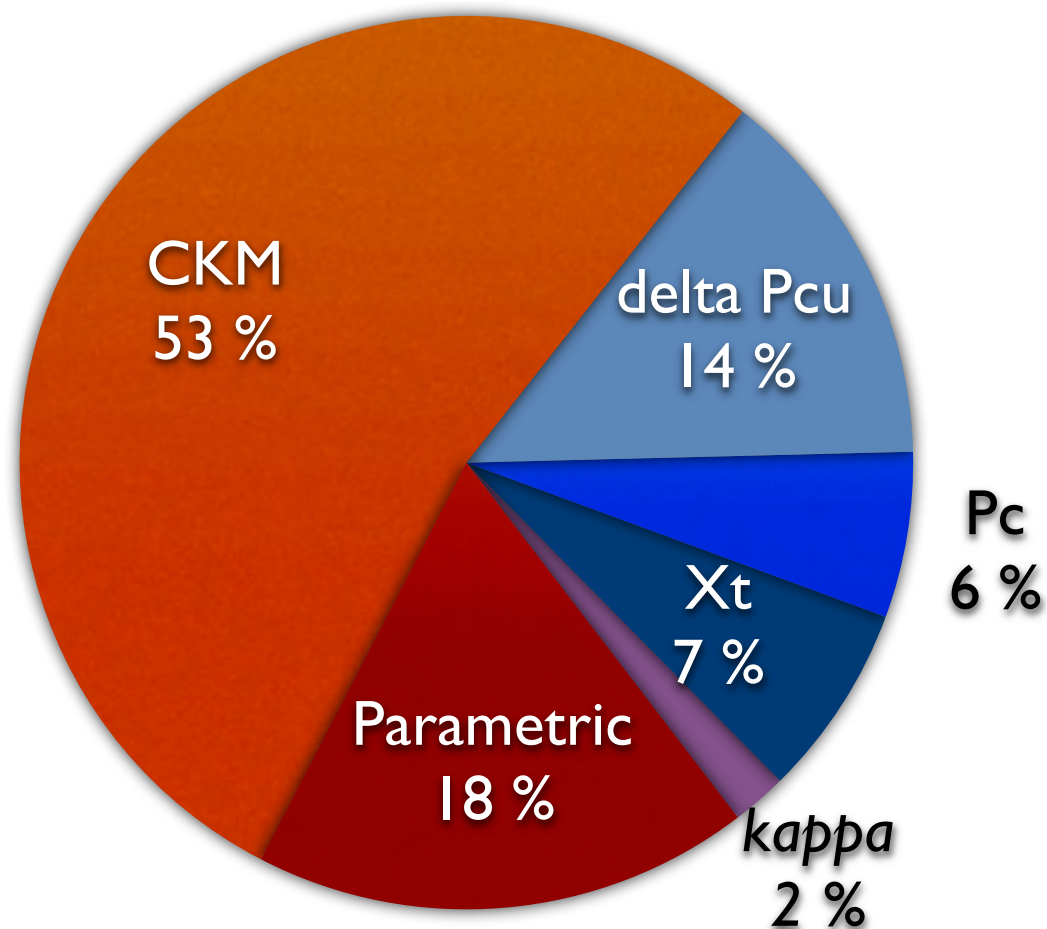
# $K \rightarrow \pi \bar{\nu} \nu$ : Error Budget

$$\text{BR}^{\text{th}}(K^+ \rightarrow \pi^+ \bar{\nu} \nu) = 8.2(3)(7) \cdot 10^{-11}$$

$$\text{BR}^{\text{exp}}(K^+ \rightarrow \pi^+ \bar{\nu} \nu) = 17(11) \cdot 10^{-11}$$

[E787, E949 '08]

NA62 aims at 10% accuracy



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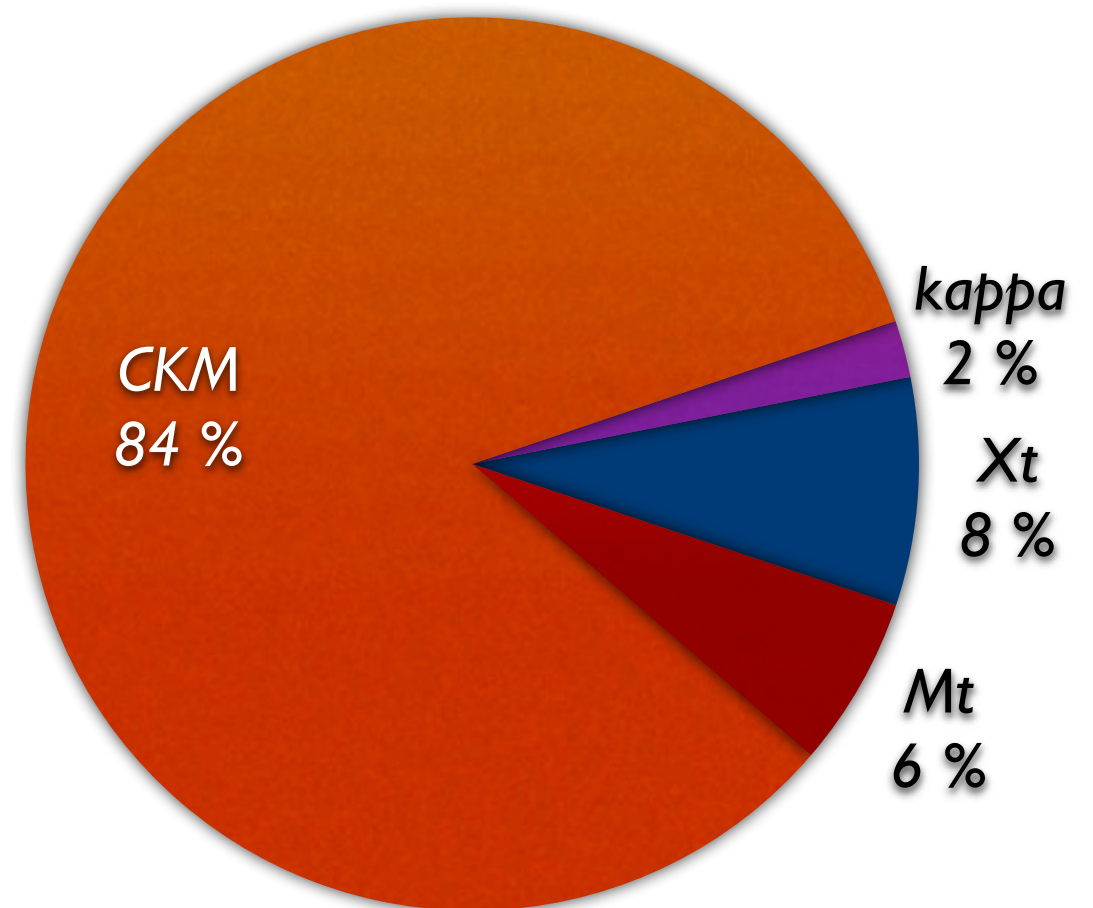
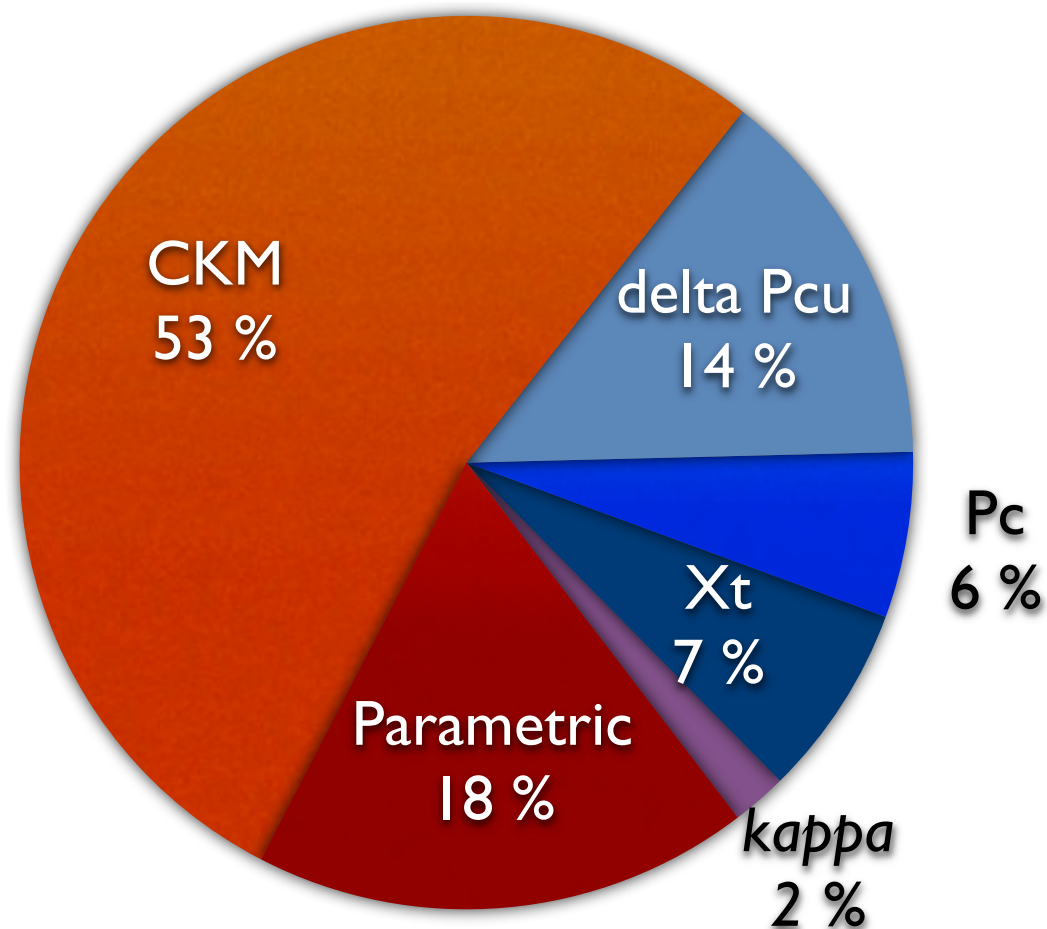
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$$\text{BR}^{\text{exp}}(K^+ \rightarrow \pi^+ \bar{\nu} \nu) < 6.7 \cdot 10^{-8}$$

[E787, E949 '08]

[E391a '08]

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# $\epsilon_K$ : CP violation in Kaon Mixing

$$\epsilon_K \simeq \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle}$$

$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left( \frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

from experiment      small

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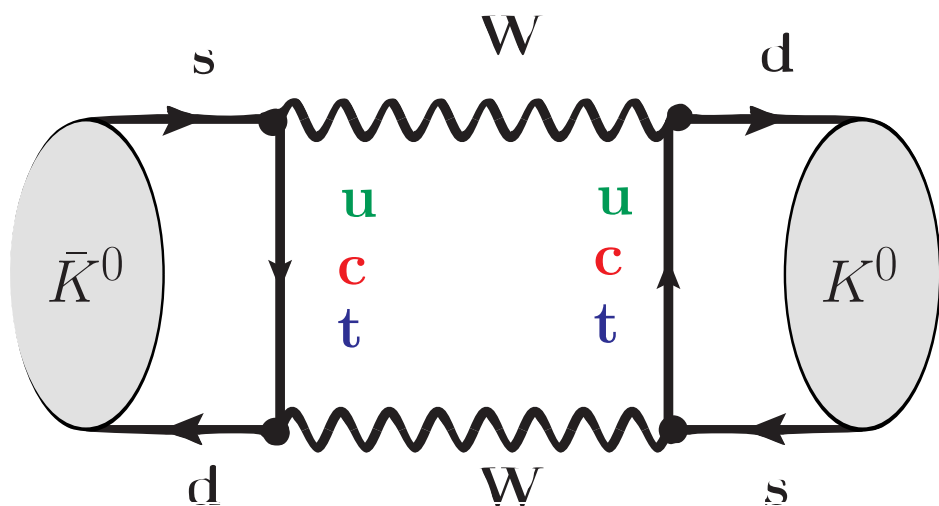
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dispersive part





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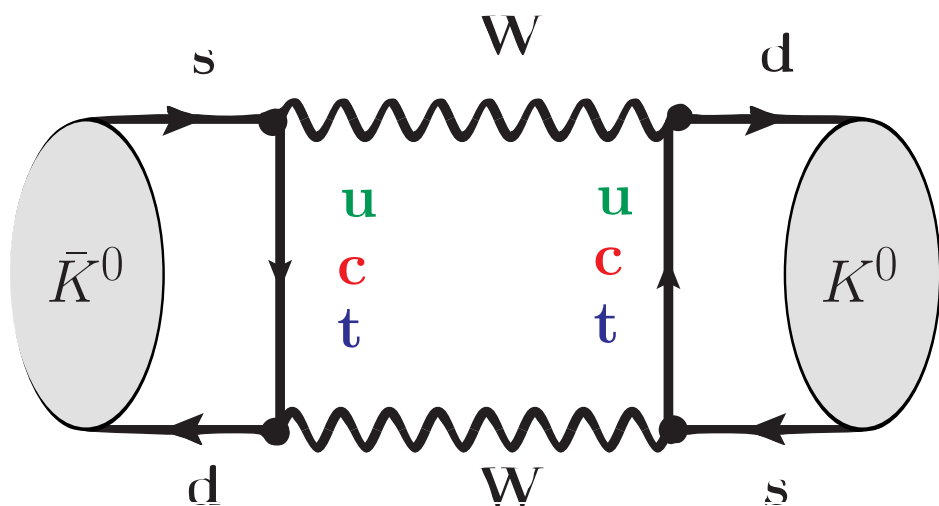
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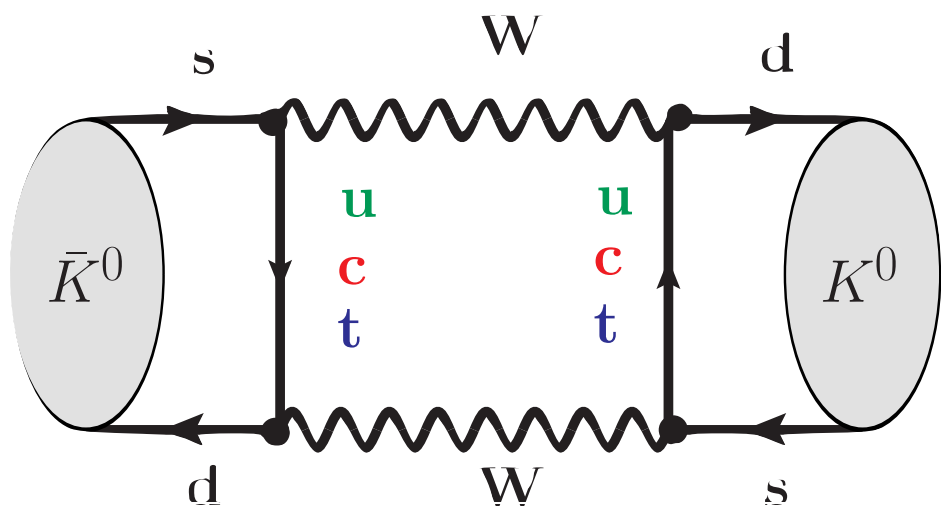
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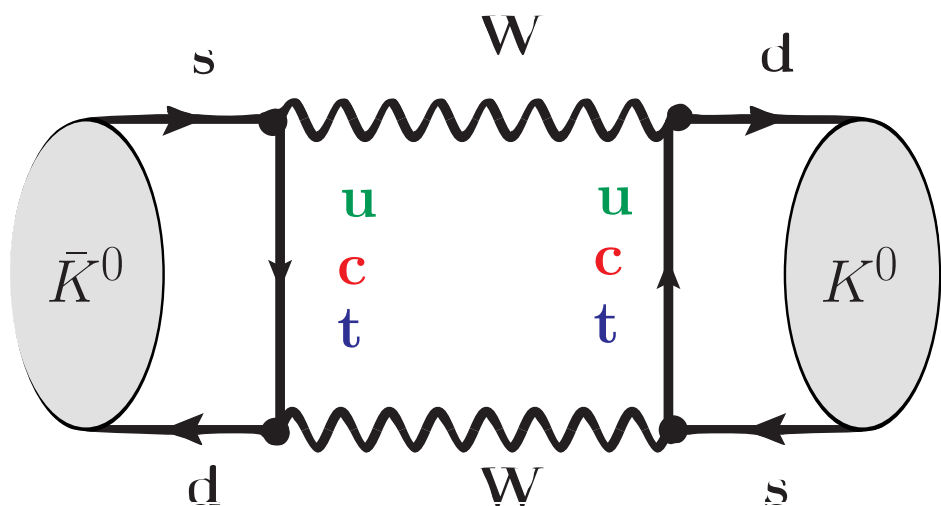
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$\eta_{ct}$ : 3-loop RGE,  
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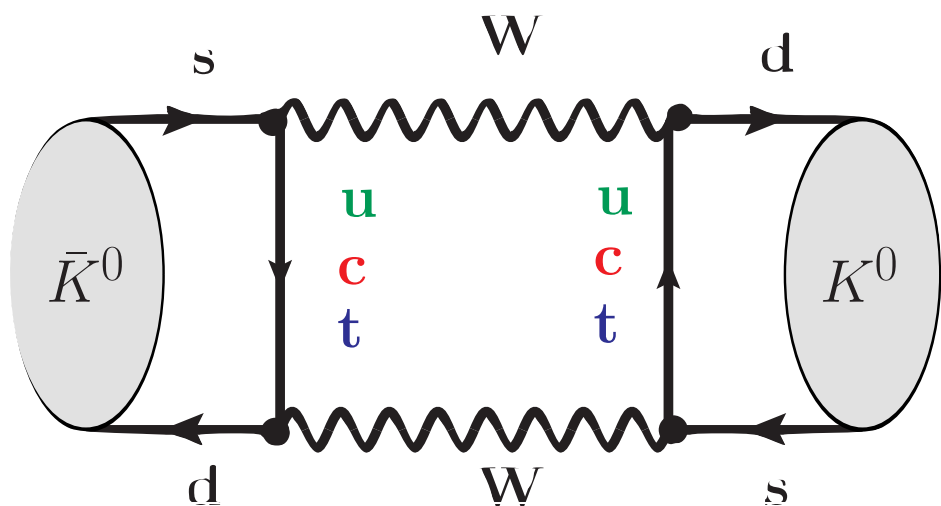
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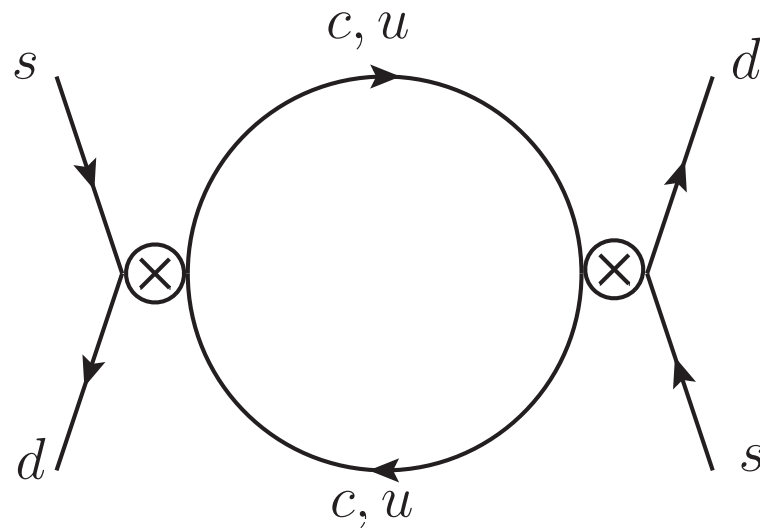
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Local Interaction:

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Lattice:  $\langle K^0 | \tilde{Q} | \bar{K}^0 \rangle$

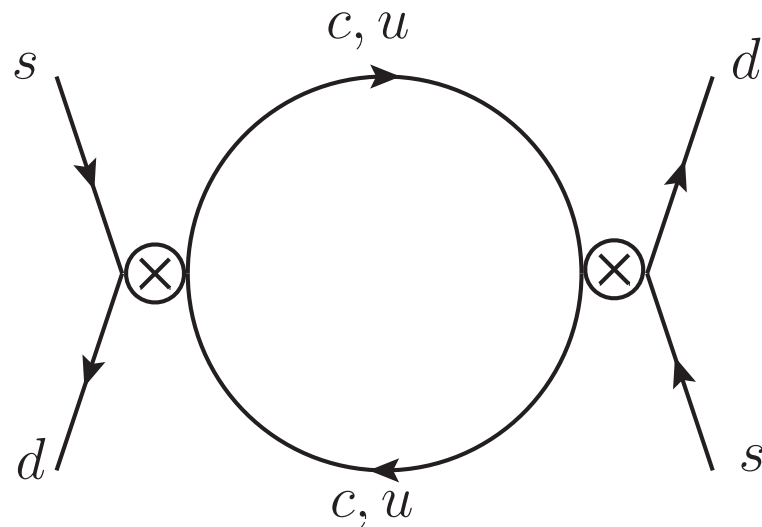
# Long Distance $\epsilon_K$



$$\int d^4x \langle K^0 | H^{|\Delta S|=1}(x) H^{|\Delta S|=1}(0) | \bar{K}^0 \rangle$$

Higher dimensional operator [Cata Peris'04]

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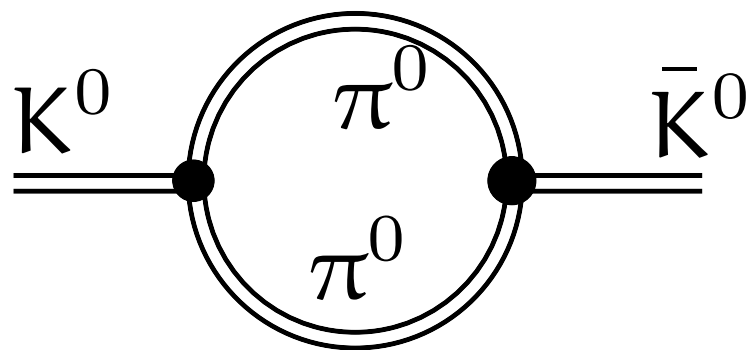
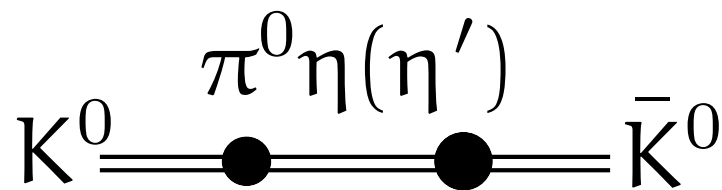
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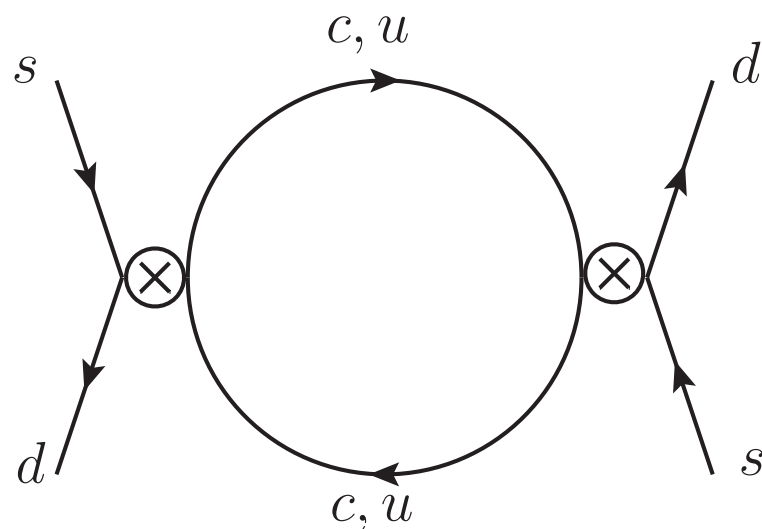
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estimate from  $\ln(m_\pi/m_Q)$  [Buras et.al. '10]

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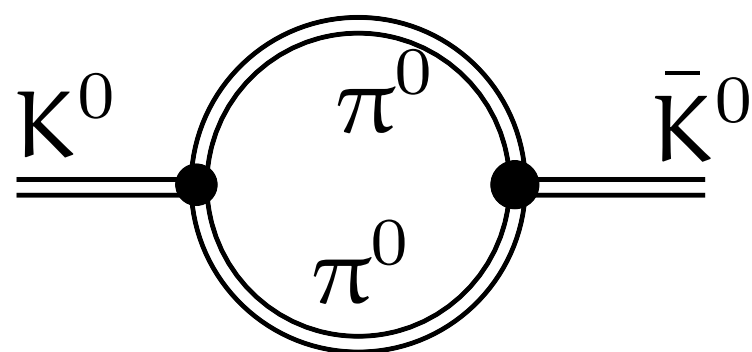
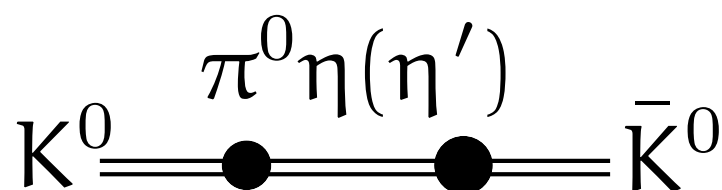
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absorptive part

estimated from  $\epsilon'$

Future: Lattice

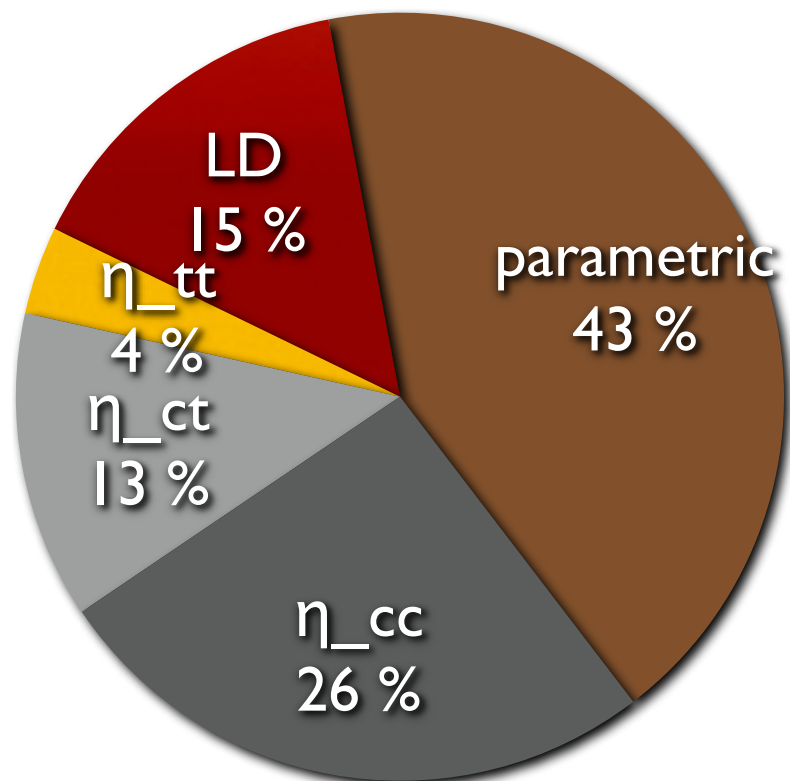
[N. Christ]

$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left( \frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \frac{\text{Im}(A_0)}{\text{Re}(A_0)} \right)$$

# Residual Theory Uncertainty

After Lattice QCD & NNLO progress:  $\eta_{cc}$  dominant uncertainty

$$|\epsilon_K| = 1.81(28) \cdot 10^{-3}$$
$$\stackrel{\text{exp.}}{=} 2.23(1) \cdot 10^{-3}$$



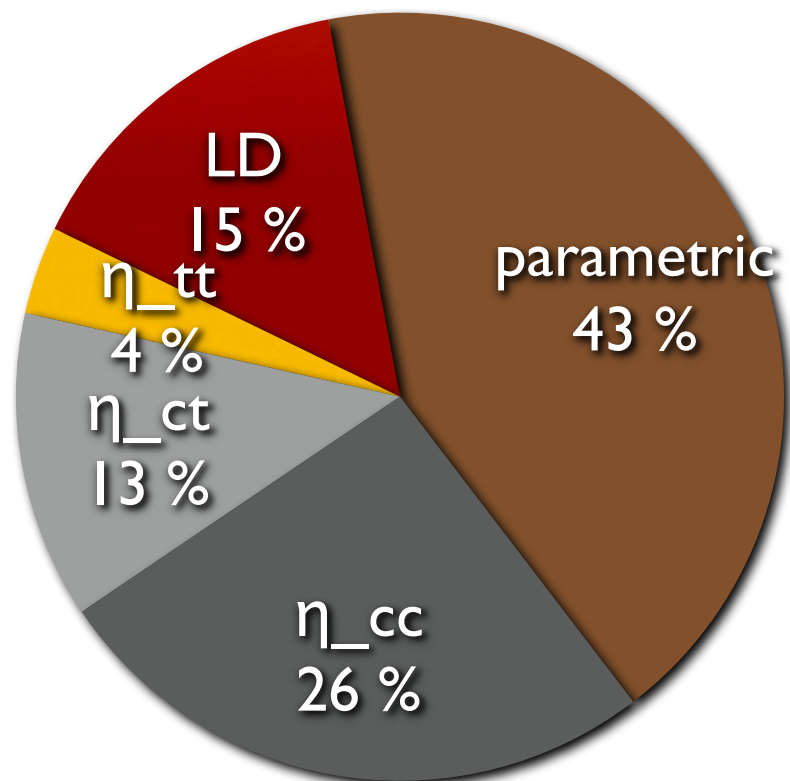


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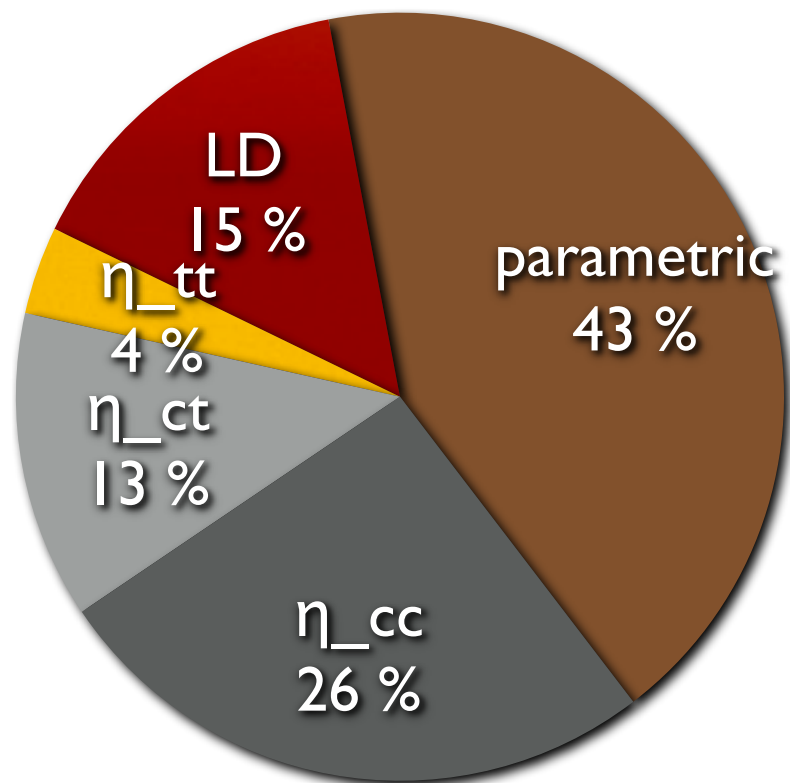
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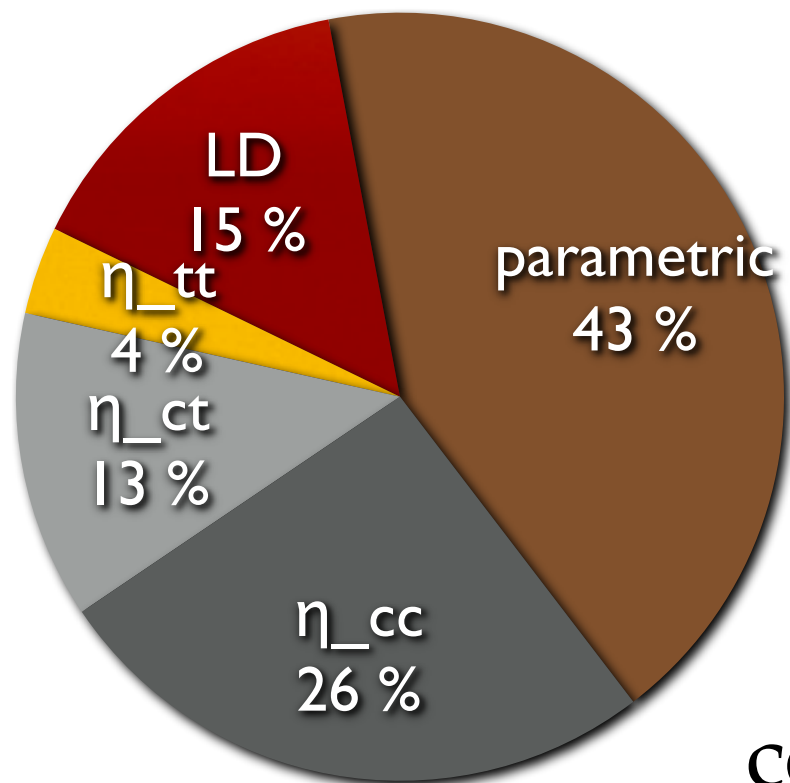
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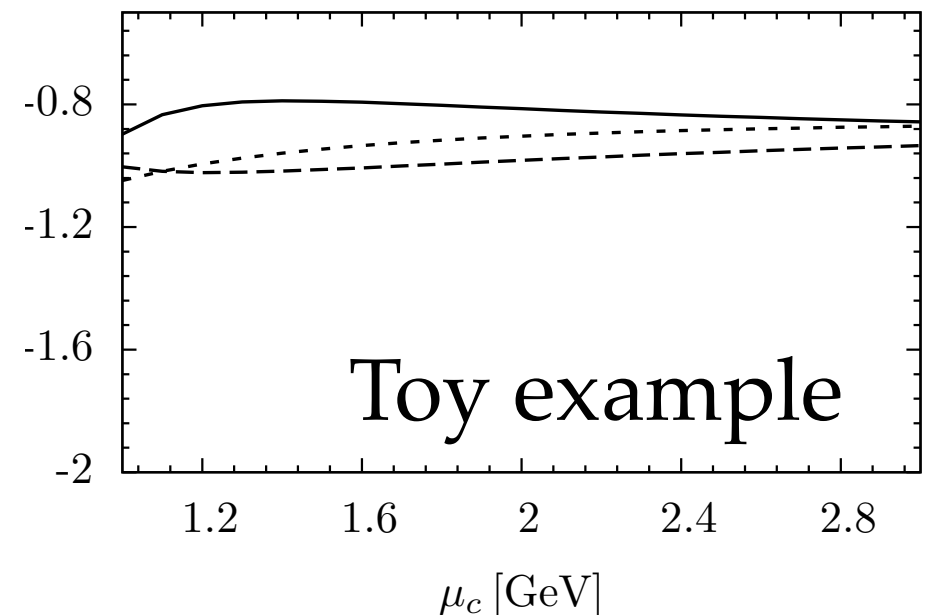
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$\epsilon_K$  & charm possible for next generation Lattice QCD [Christ '11]



$$\frac{\eta_{cc}}{\Lambda_{\text{Lat}}}$$



Requires matching of Lattice and continuum QCD – toy numerics converge well

# 3, Constrain and Interpret NP

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Operator based Approach:

Write down all  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  invariant Operators  
[Buchmüller, Wyler]

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Top-down approach:

Supersymmetry, LHT-Model, RS-Model ...

# Model (in)dependent

Heavy new physics:  $(\bar{D}_L \gamma^\mu S_L)(H^\dagger D_\mu H) \rightarrow \bar{d}_L \gamma^\mu s_L Z^\mu + \bar{u}_L \gamma^\mu c_L Z^\mu$

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study this in a model  
independent way  
and classify models

		Observable								
$O_{\varphi q}^{(1)}$	$(\bar{D}_L \gamma^\mu S_L)(H^\dagger D_\mu H)$	$\checkmark$	$\checkmark$	$\checkmark$	hs	—	—	—	$\checkmark$	
$O_{\varphi q}^{(3)}$	$(\bar{D}_L \gamma^\mu \sigma^i S_L)(H^\dagger D_\mu \sigma^i H)$	$\checkmark$	$\checkmark$	$\checkmark$	hs	hs	$\checkmark$	$\checkmark$	$\checkmark$	
$O_{\varphi d}$	$(\bar{d}_R \gamma^\mu S_R)(H^\dagger D_\mu H)$	$\checkmark$	$\checkmark$	$\checkmark$	hs	—	—	—	$\checkmark$	

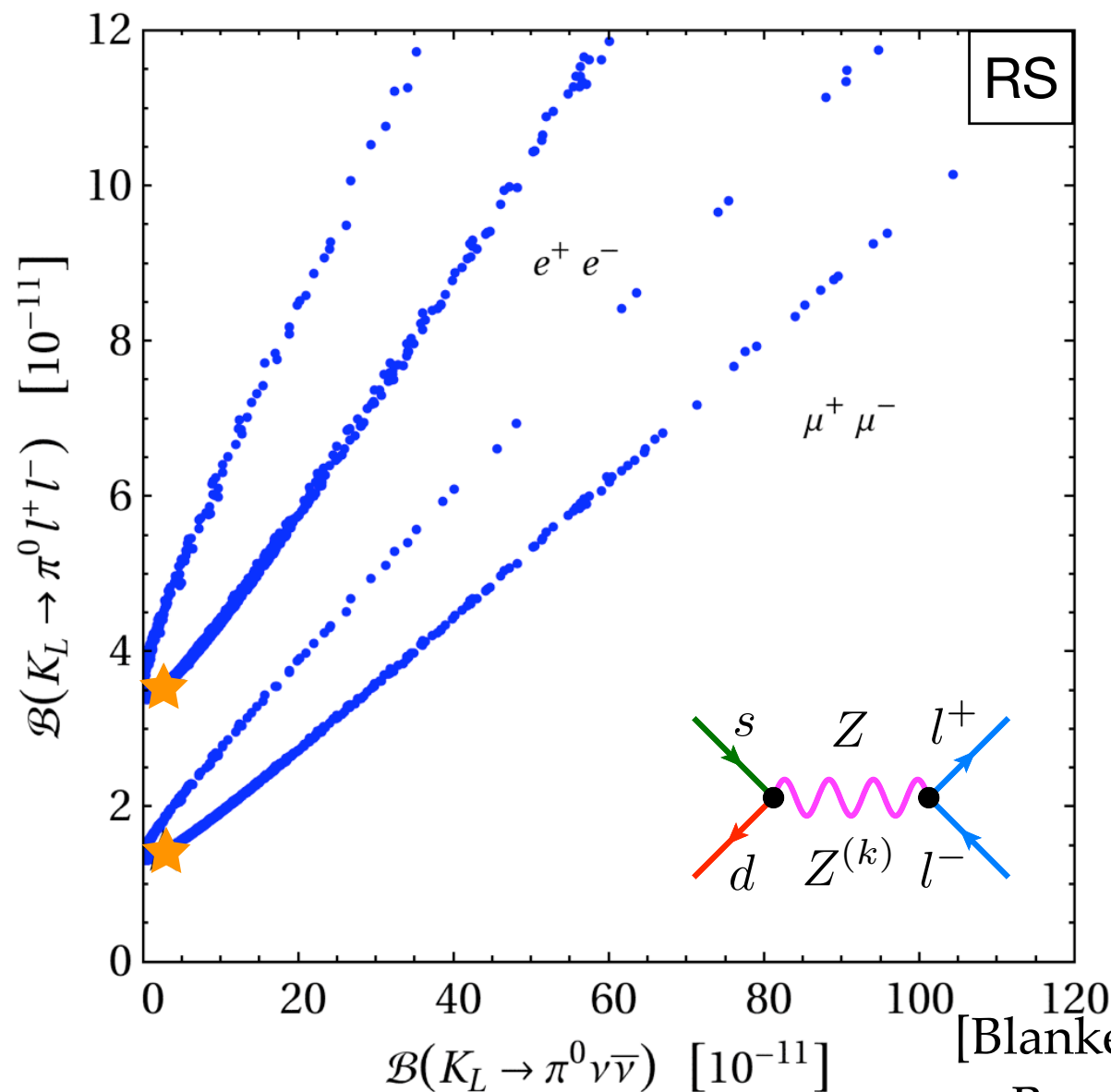
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The axial vector ( $Z_L$ - $Z_R$ ) contribution dominates the  $K_L \rightarrow \pi^0 l^+ l^-$  decay modes in many models of new physics

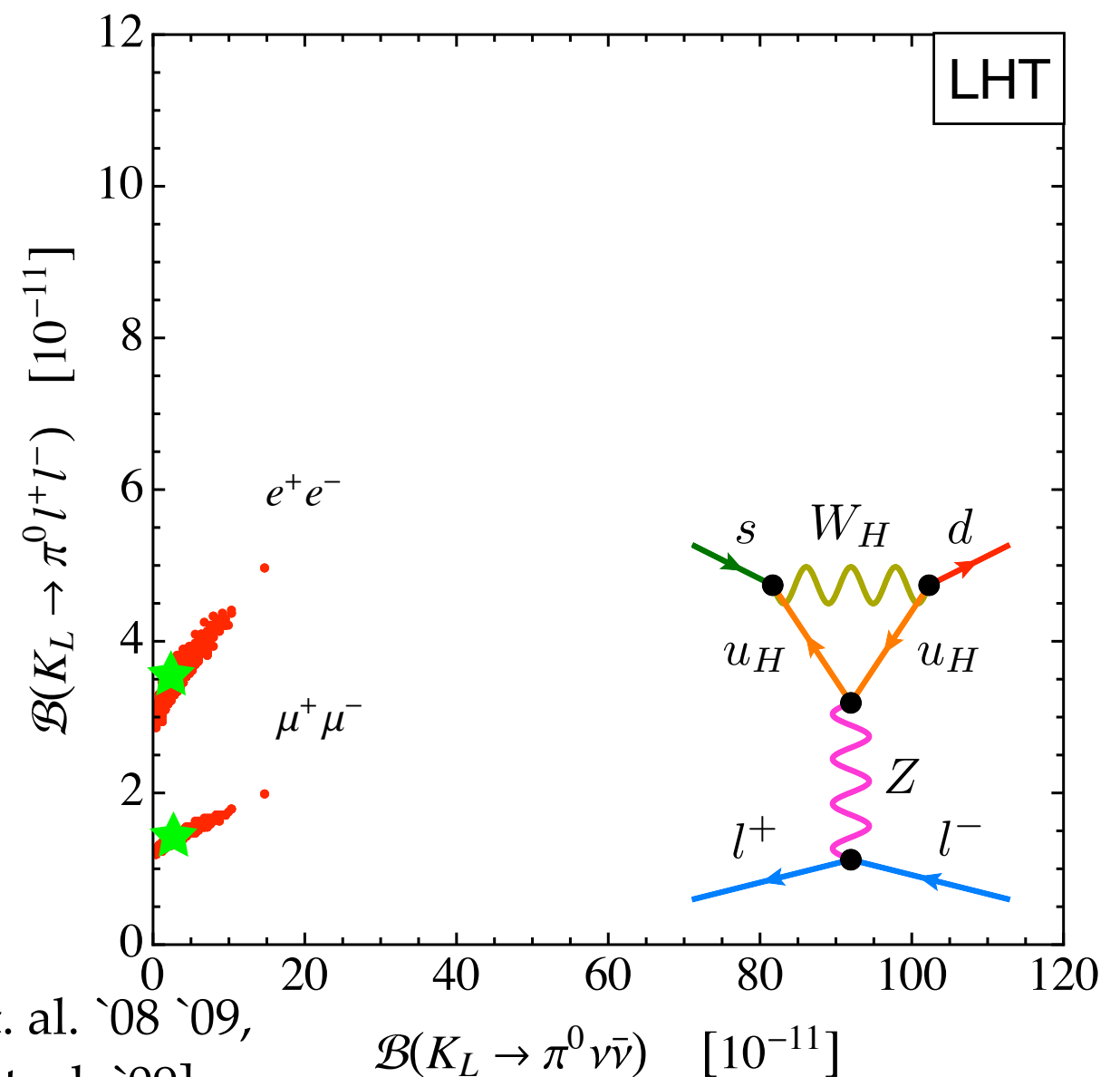
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## Correlations in Randall Sundrum and Little Higgs Models



[Blanke et. al. '08 '09,  
Bauer et. al. '09]



# Correlations with $\varepsilon_K$

If there are only left-handed currents  
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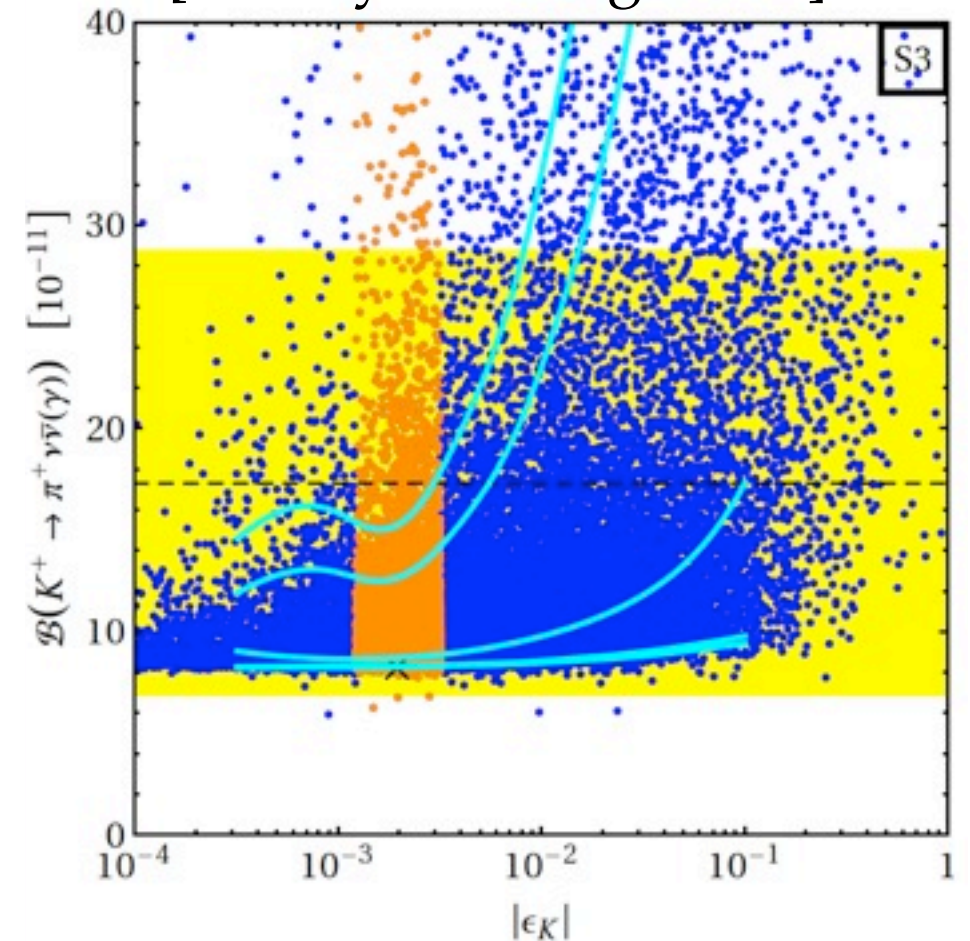
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Can still lead to interesting  
restrictions of the model parameter  
space

RS with common down-type bulk mass  
[Plot by S. Casagrande]



# Correlations with $\varepsilon_K$

Correlations in models with restricted sources of flavour violation  
for example: Gauged  $SU(3)_Q \times SU(3)_U \times SU(3)_D$  [Grinstein et. al '10]

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Using results for arbitrary  
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[Brod, Casagrande, MG in preperation]

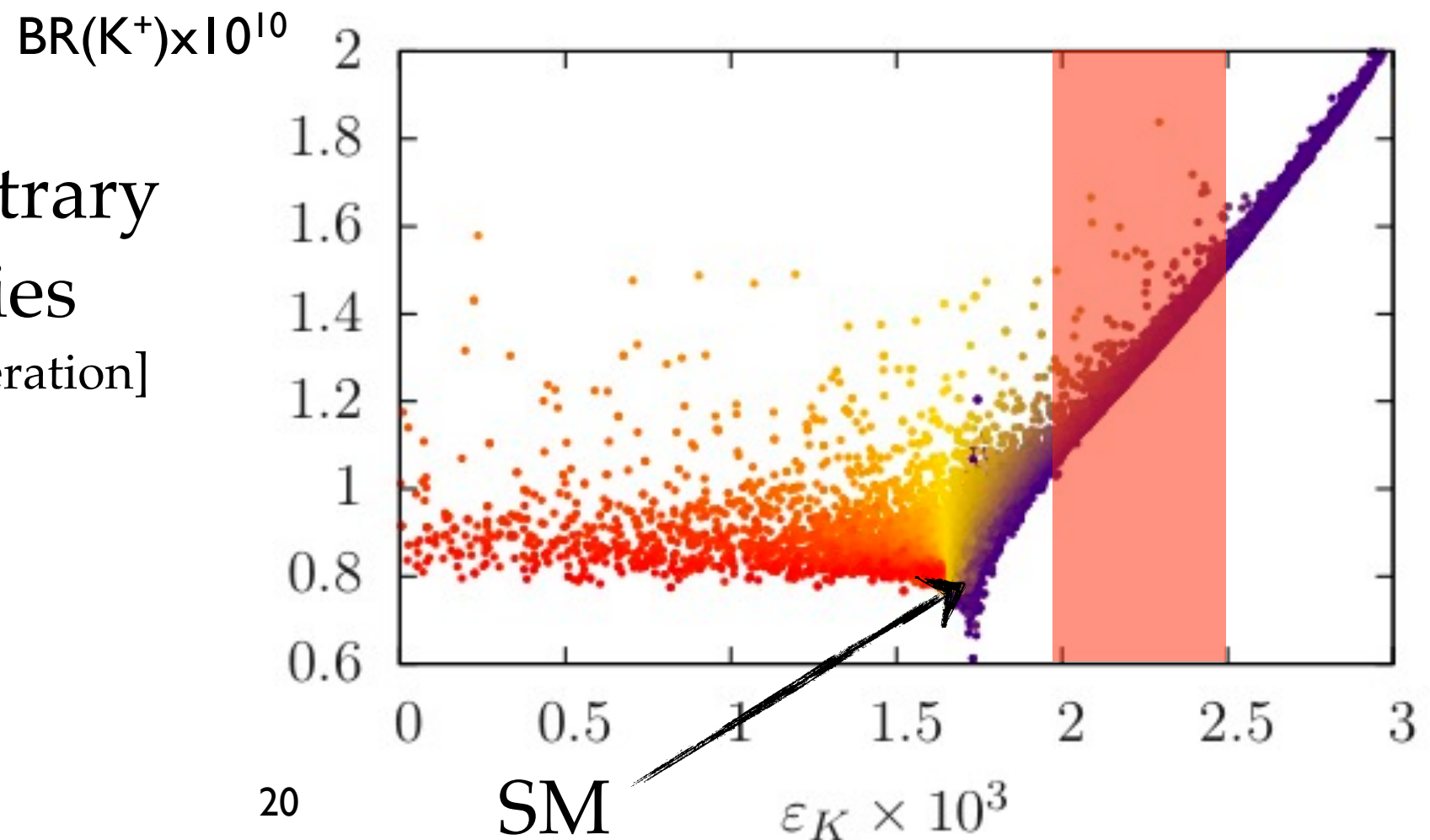
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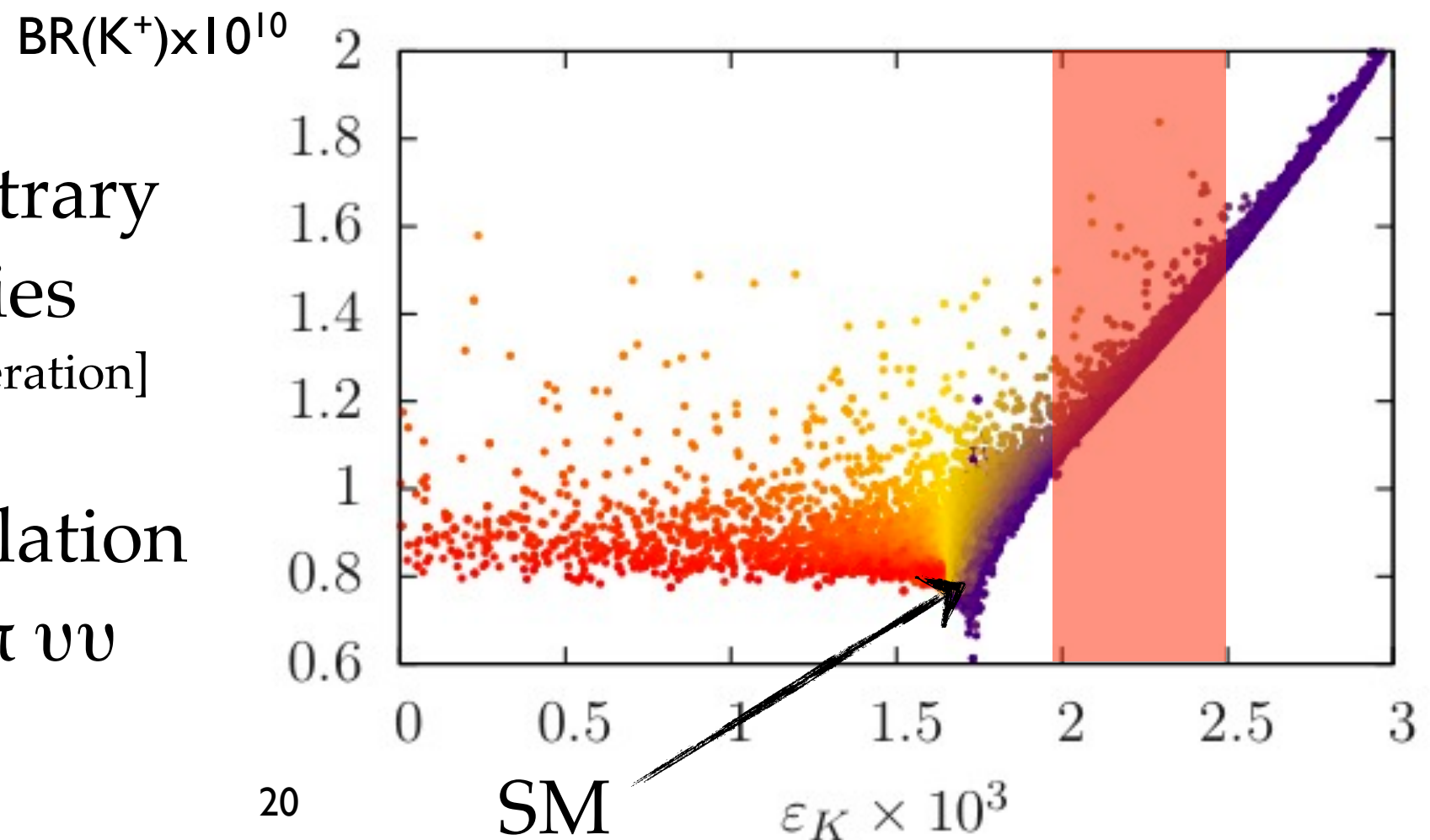
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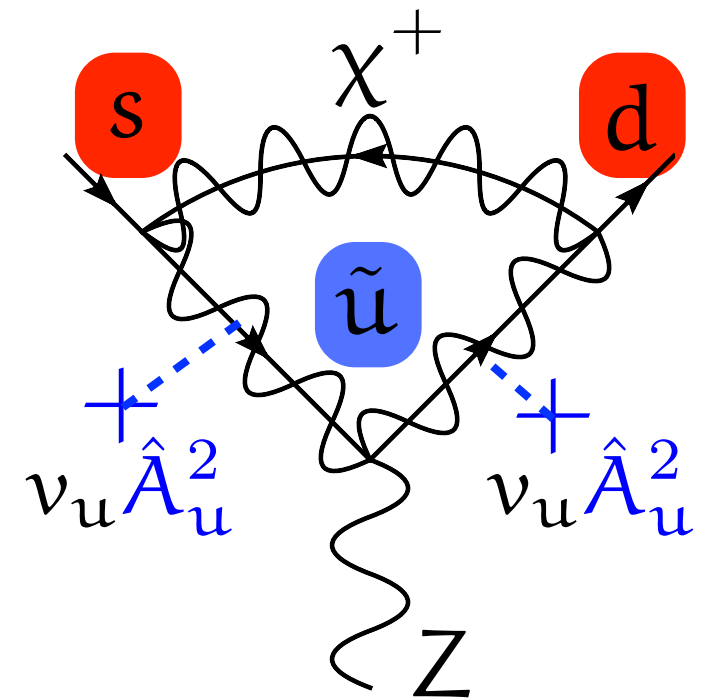
we find a strong correlation  
between  $\epsilon_K$  &  $K \rightarrow \pi \nu \nu$



# No correlations in the MSSM

The MSSM has many sources of flavour violation – Z Penguin sensitive to up-type A-terms [Collangelo, Isidori '98]

$$\hat{\mathcal{M}}_{\tilde{u}}^2 = \begin{pmatrix} \hat{M}_{\tilde{u}_L}^2 & v_u \hat{A}_u^\dagger - v_d \mu \hat{Y}_u^\dagger \\ v_u \hat{A}_u - v_d \mu^* \hat{Y}_u & \hat{M}_{\tilde{u}_R}^2 \end{pmatrix}$$

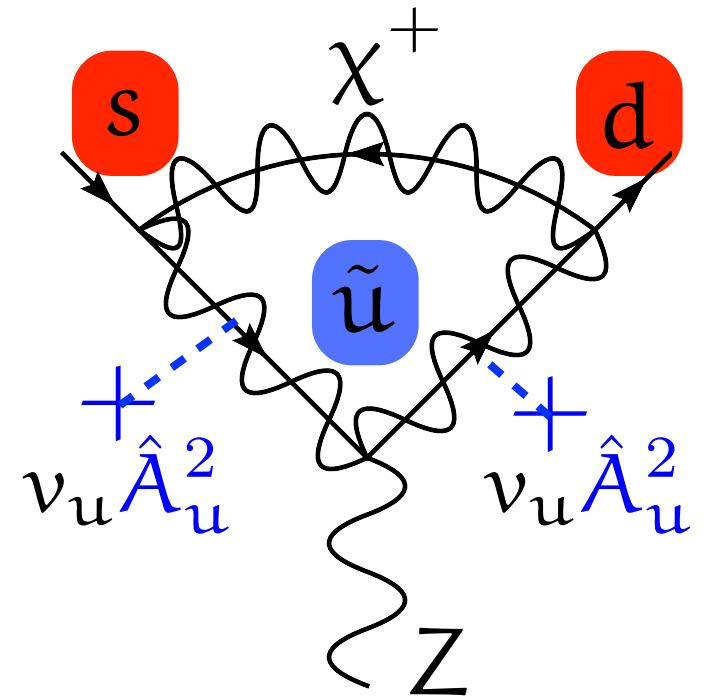




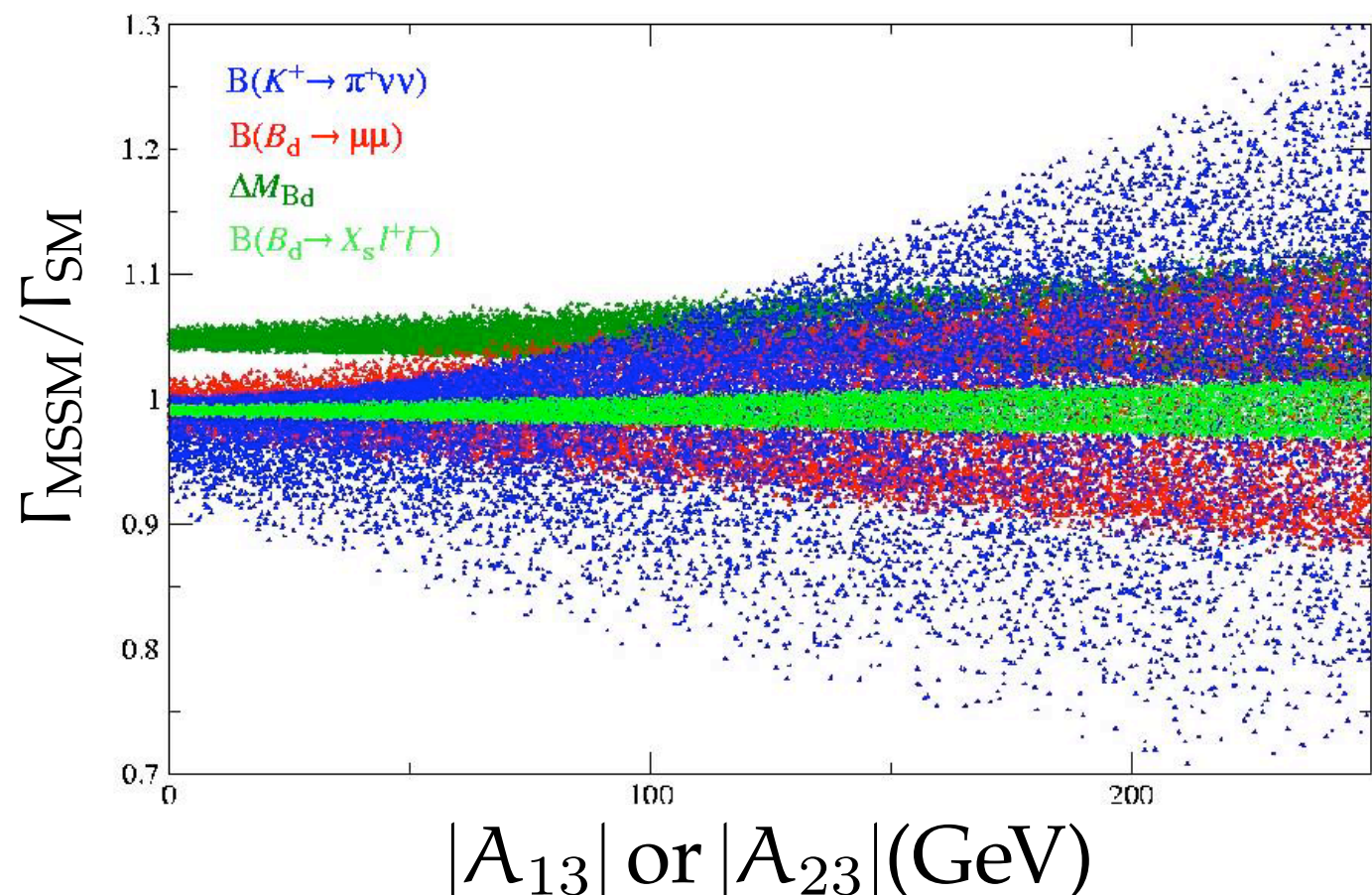
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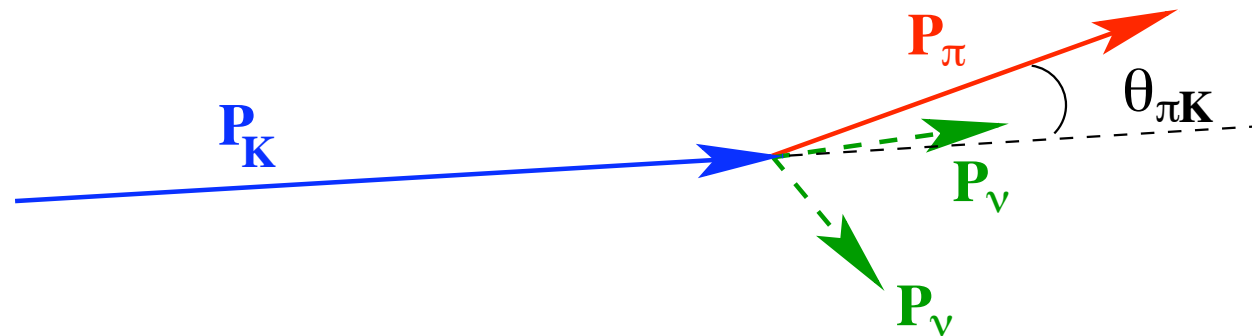


MSSM parameter  
scan shows  
sensitivity to  
 $A_{13}$  &  $A_{23}$   
[Isidori et. al. '06]



# Beyond the Z Penguin

Experiment: Background from frequent  $K^+$ -Decays

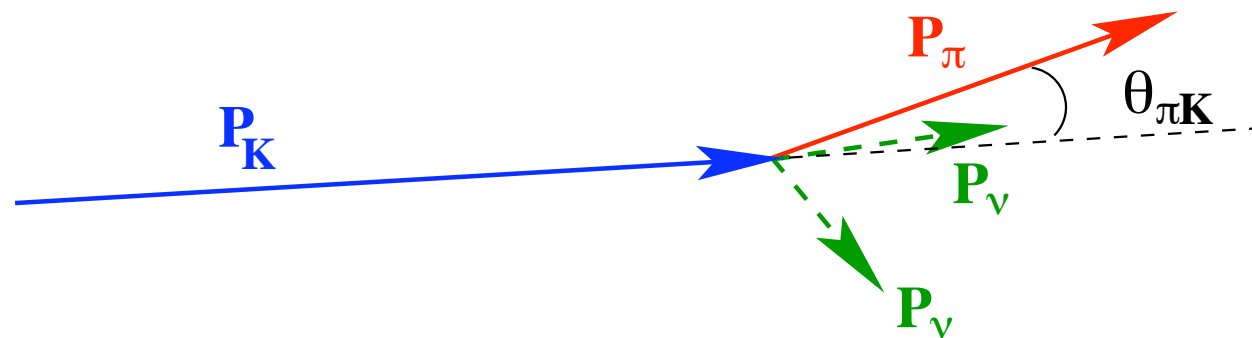


Measure  $p_\pi$  &  $\theta_{\pi K}$

cut on:  $m_{miss}^2 \simeq m_K^2 \left( 1 - \frac{|P_\pi|}{|P_K|} \right) + m_\pi^2 \left( 1 - \frac{|P_K|}{|P_\pi|} \right) - |P_K| |P_\pi| \theta_{\pi K}^2$

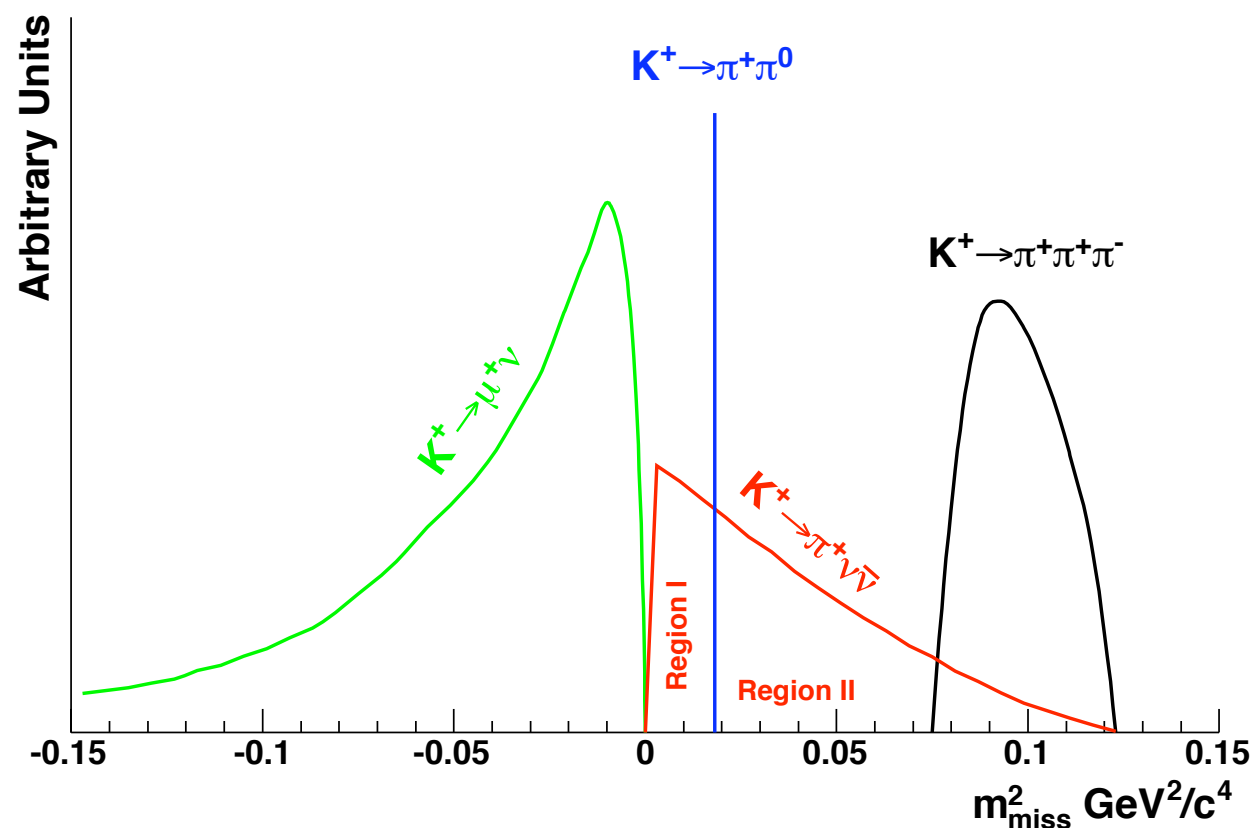
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Couplings to weakly interacting light new particles strongly constrained by  $K^+ \rightarrow \pi^+ + \text{invisible}$  [Kamenik, Smith '11]



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Improvements from theory side possible using Lattice QCD and interplay with perturbative QCD