

Excited charmonium spectroscopy from lattice QCD

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BEACH 2012, Wichita, Kansas, 27th July 2012

Hadron Spectrum Collaboration



Outline

- Introduction
- Spectroscopy in lattice QCD
- Excited charmonium spectrum
- Scattering and resonances
- Summary and outlook

JHEP 07 (2012) 126

Meson Spectroscopy

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BESIII



LHC

KLOE2



CLAS12



...

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BESIII



LHC

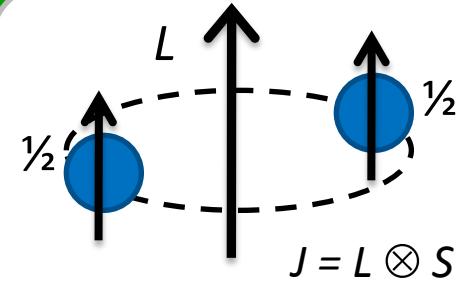


KLOE2

CLAS12

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Quark-antiquark pair: $n^{-2S+1}L_J$



Meson Spectroscopy

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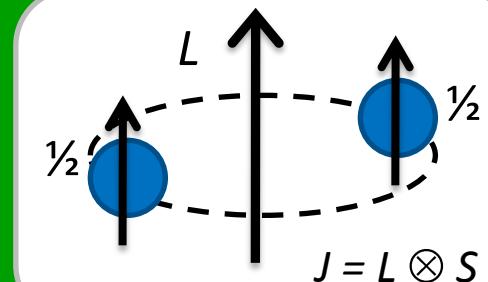
...

Quark-antiquark pair: $n^{2S+1}L_J$

Parity: $P = (-1)^{(L+1)}$

Charge Conj Sym: $C = (-1)^{(L+S)}$

$J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}, 2^{--}, 2^{++}, 2^{-+}, \dots$



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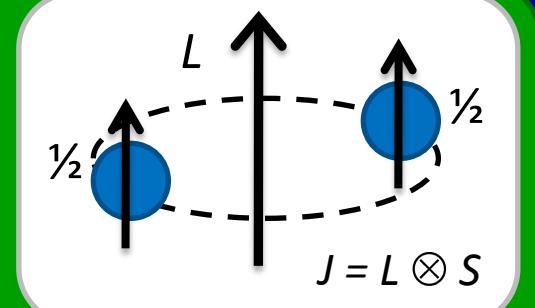
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Exotics ($J^{PC} = 1^{-+}, 2^{+-}, \dots$)? – can't just be a $q\bar{q}$ pair

Probe low energy d.o.f. of QCD

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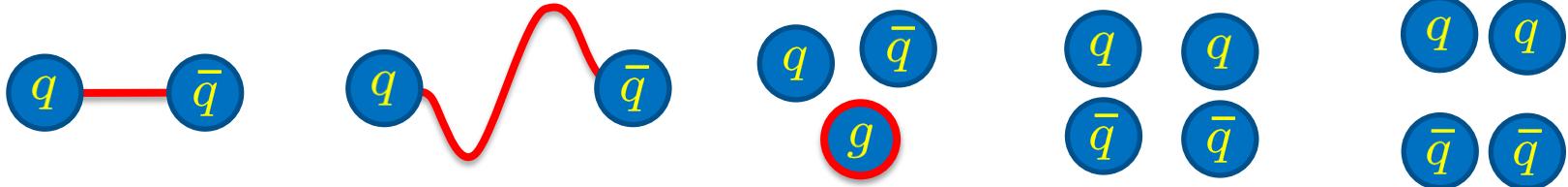
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cc - 1



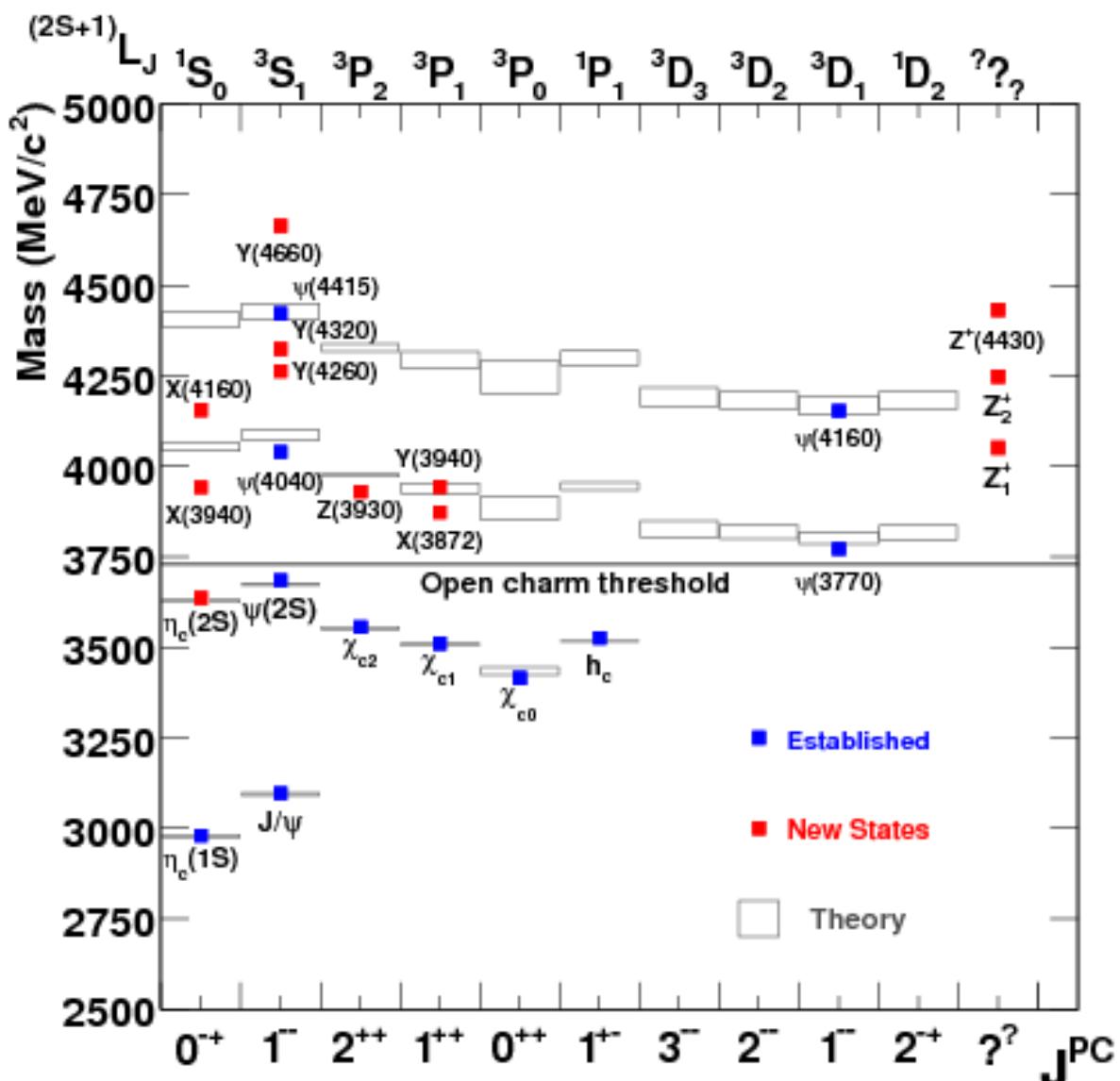
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e.g. hybrids, multi-mesons

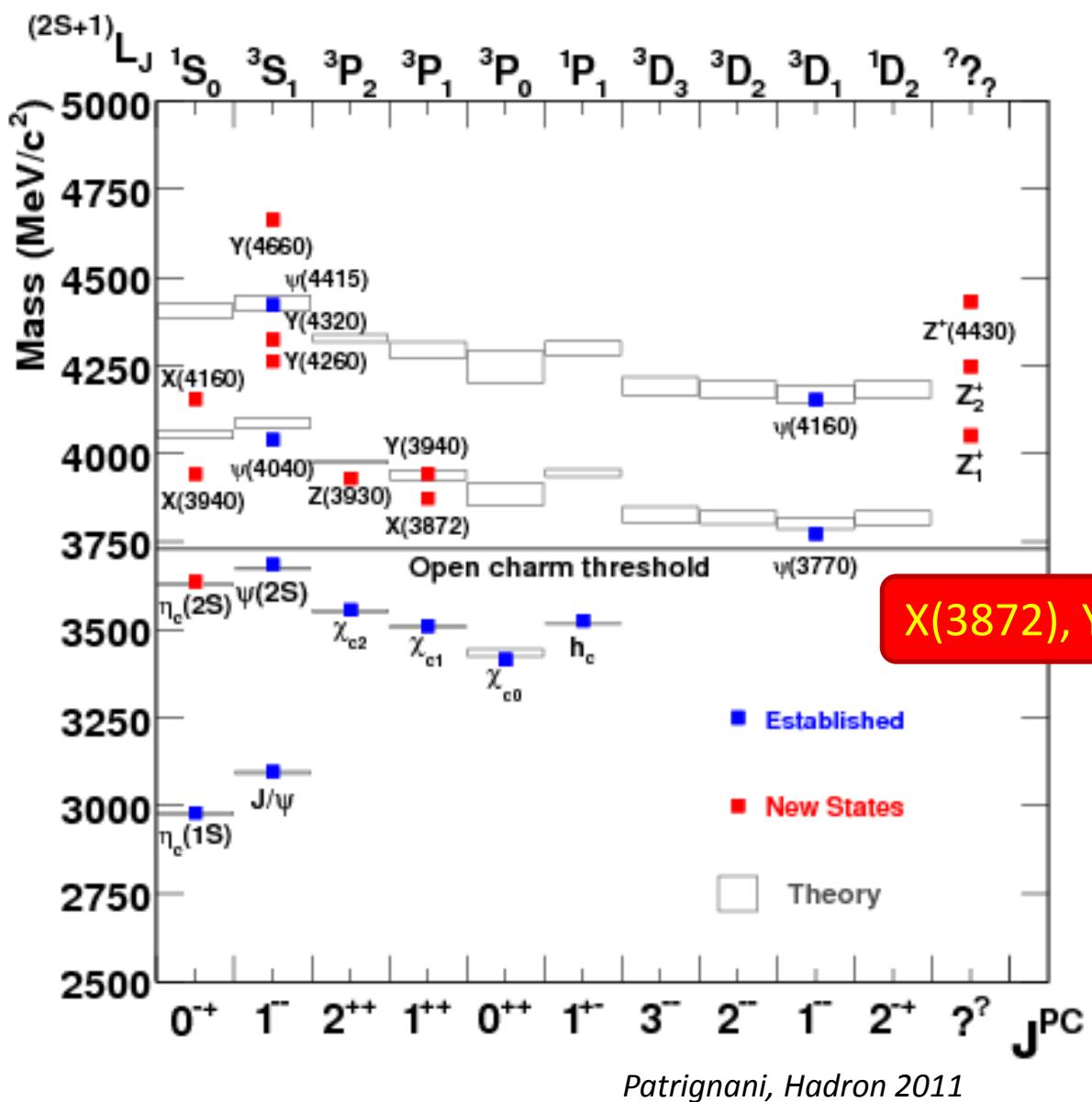
Charmonium Spectroscopy

copy



Patrignani, Hadron 2011

copy



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Spectroscopy on the lattice

Calculate energies and matrix elements (“overlaps”, Z 's)
from correlation functions of meson interpolating fields

$$Z_i^{(n)} \equiv < 0 | \mathcal{O}_i | n >$$

$$C_{ij}(t) = < 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 >$$

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definite $J^P C$

Here up to 3 derivs and $\mathbf{p} = 0$

‘Distillation’ PR D80, 054506

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$$C_{ij}(t) \xrightarrow[t \rightarrow \infty]{} \frac{Z_i^{(0)} Z_j^{(0)*}}{2E_0} e^{-E_0 t}$$

Variational Method

Large basis of operators → matrix of correlators

Generalised eigenvector problem:

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

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Var. method uses orthog of eigenvectors; don't just rely on separating energies

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Dynamical (unquenched) calculation [$N_f = 2+1$]

Anisotropic – finer in temporal dir ($a_s/a_t \approx 3.5$), $a_s \approx 0.12$ fm

Two volumes: 16^3 , 24^3 ($L_s \approx 1.9, 2.9$ fm)

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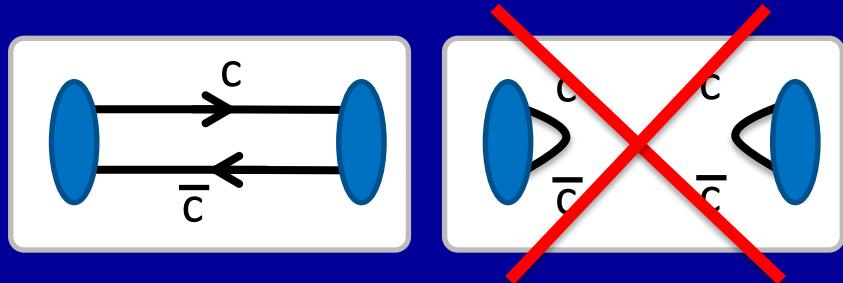
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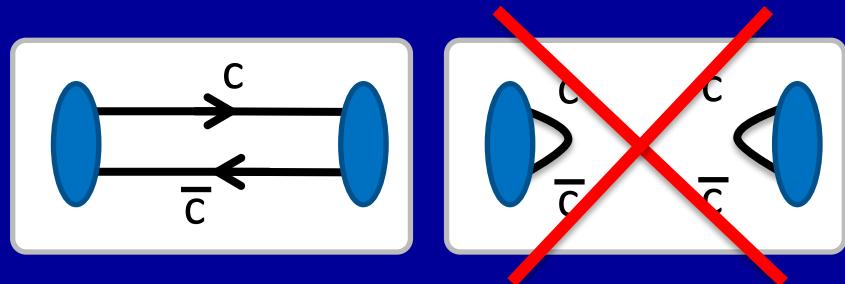
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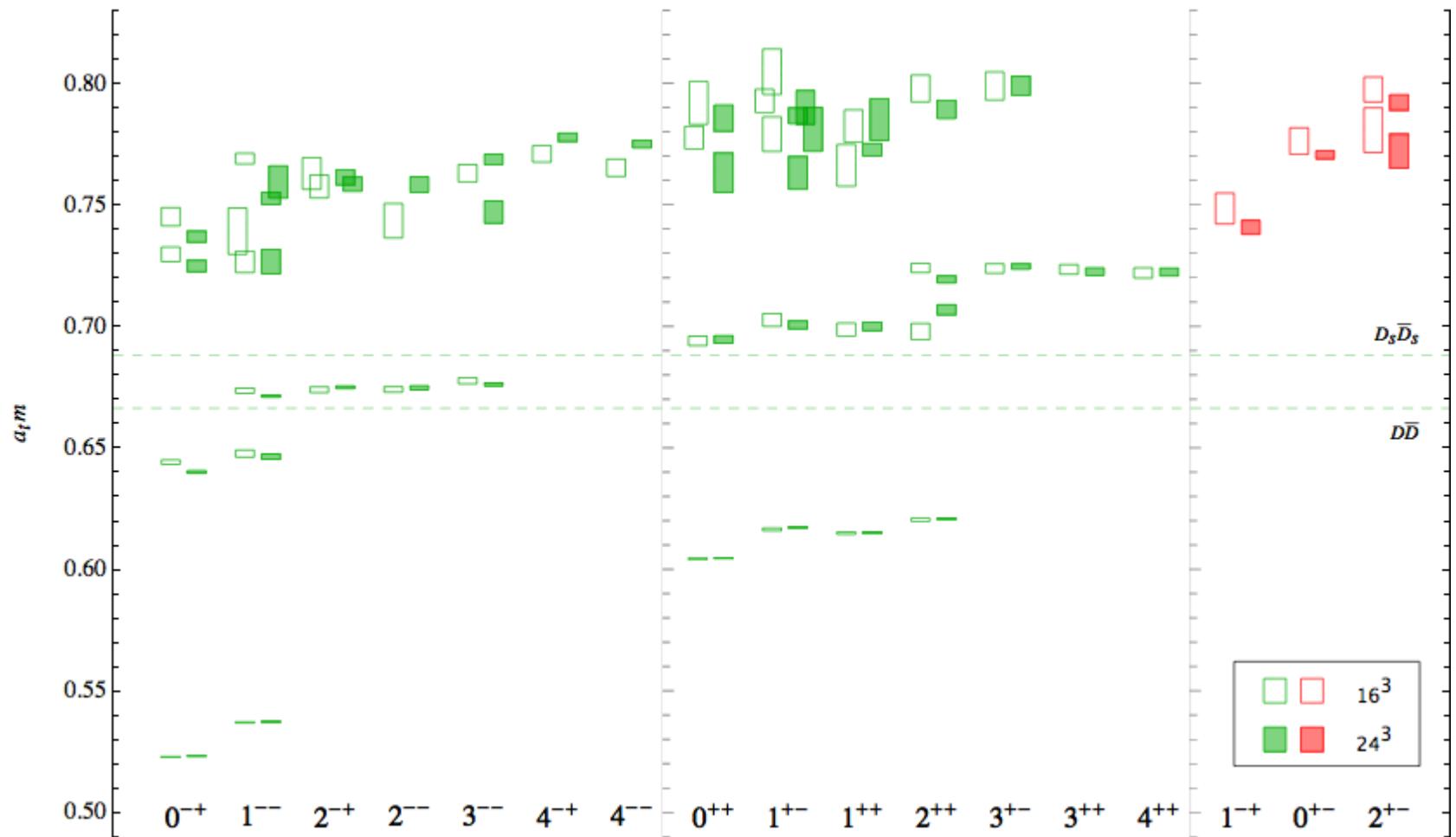
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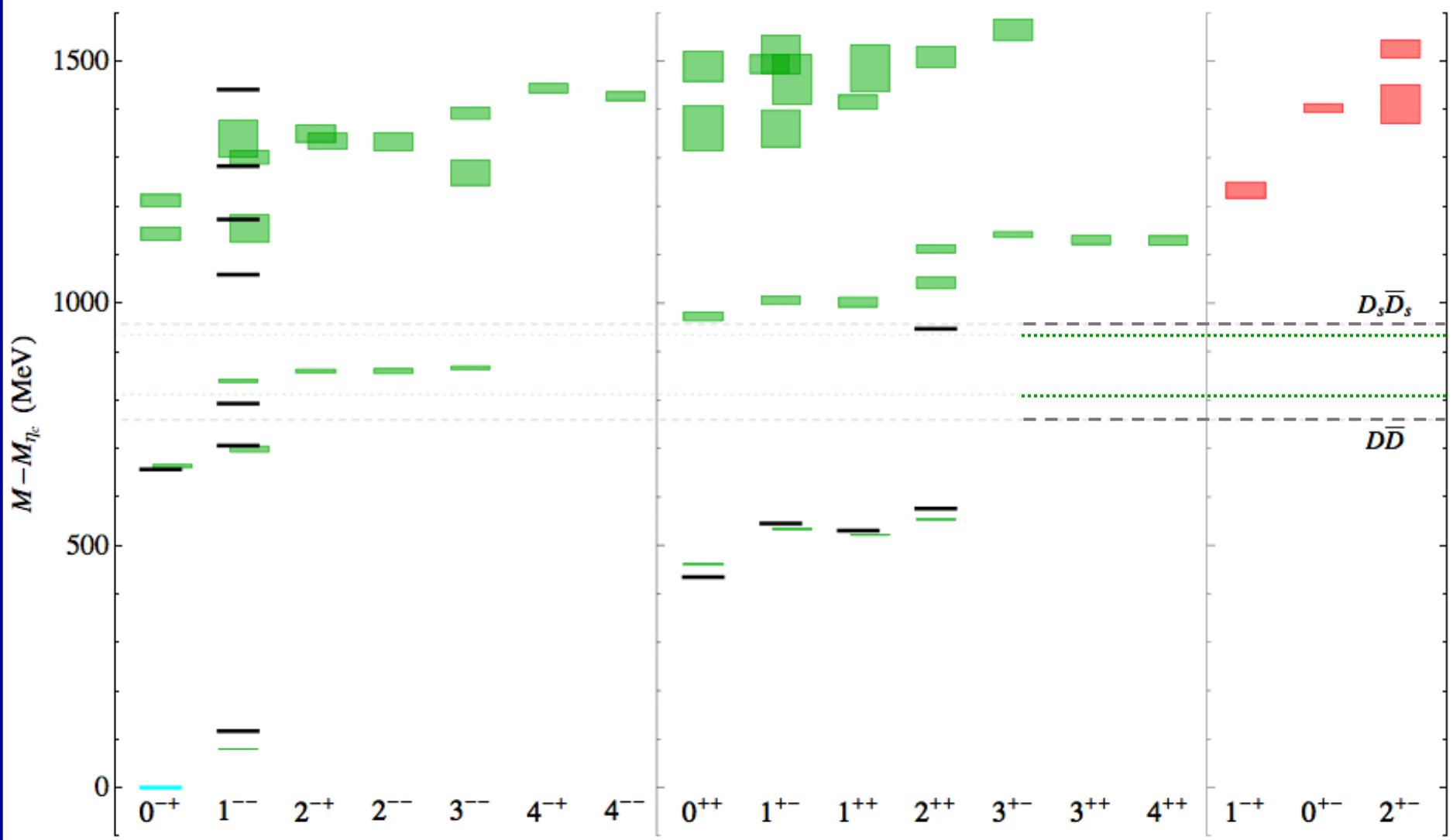


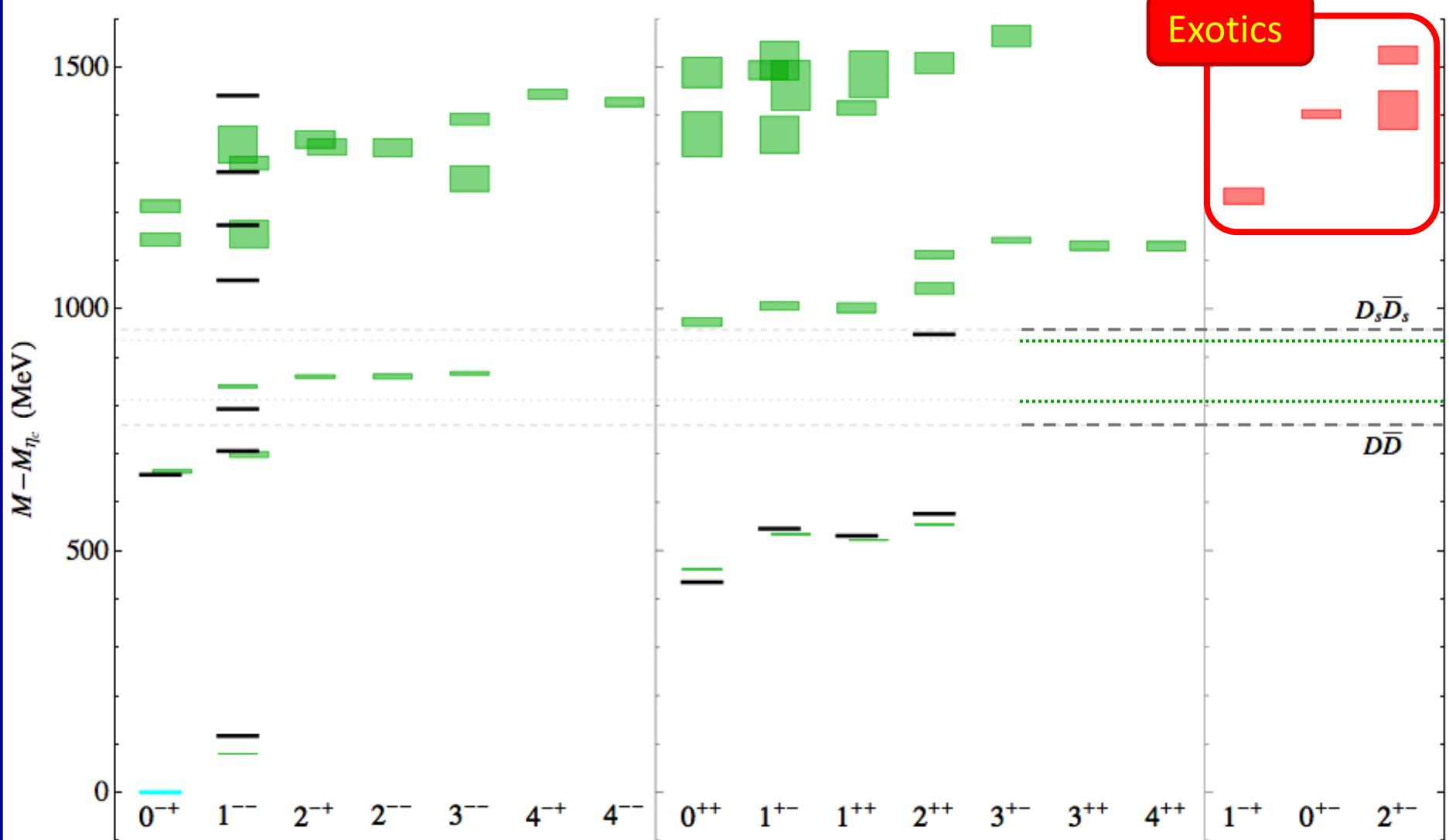
Hadron Spectrum Collaboration – lattice details: PR D78 054501, PR D79 034502

JHEP 07 (2012) 126 – Liuming Liu, Graham Moir, Mike Peardon, Sinéad Ryan, CT, Pol Vilaseca;
Jo Dudek, Robert Edwards, Bálint Joó, David Richards

Charmonium – volume comparison

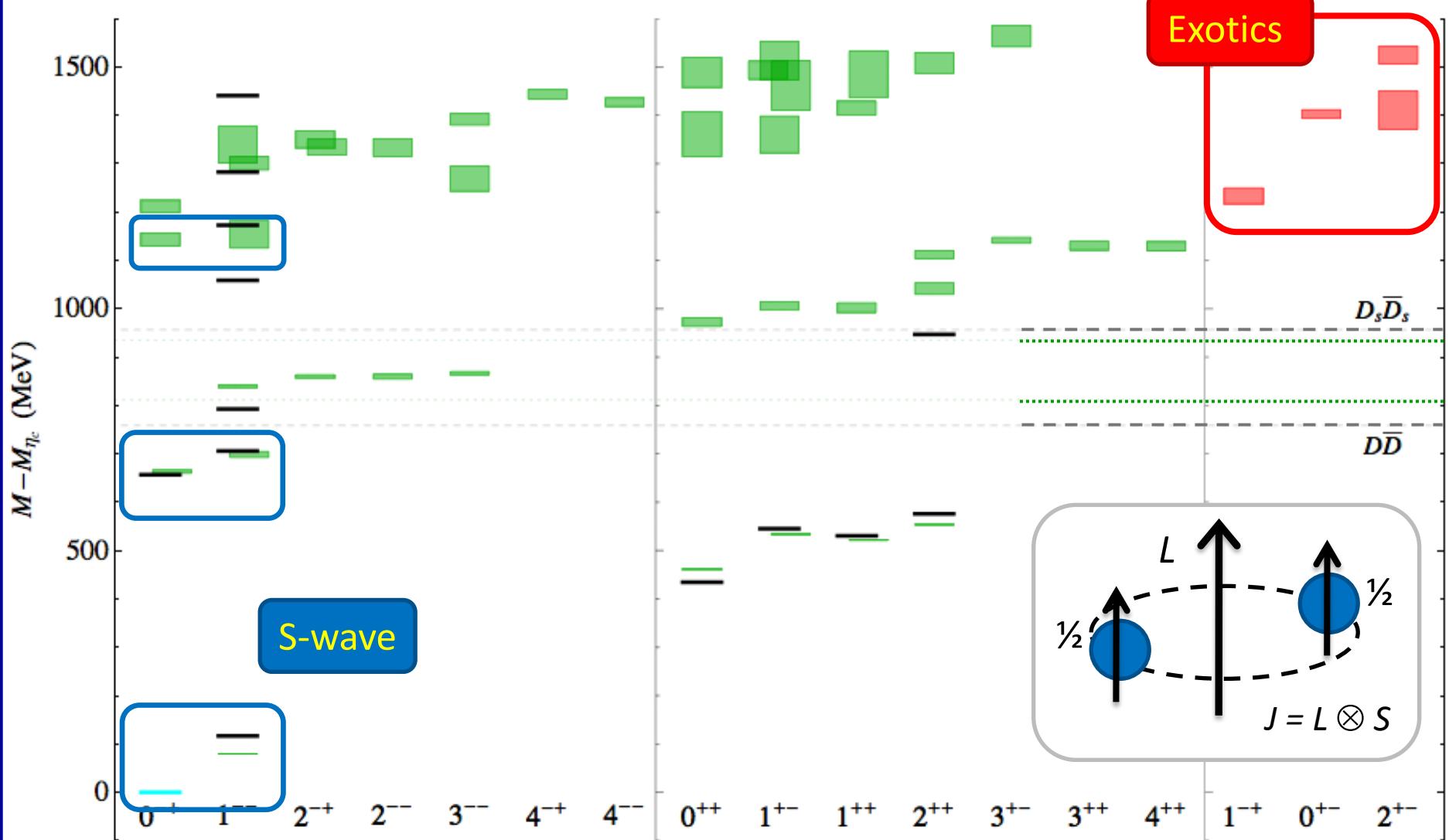






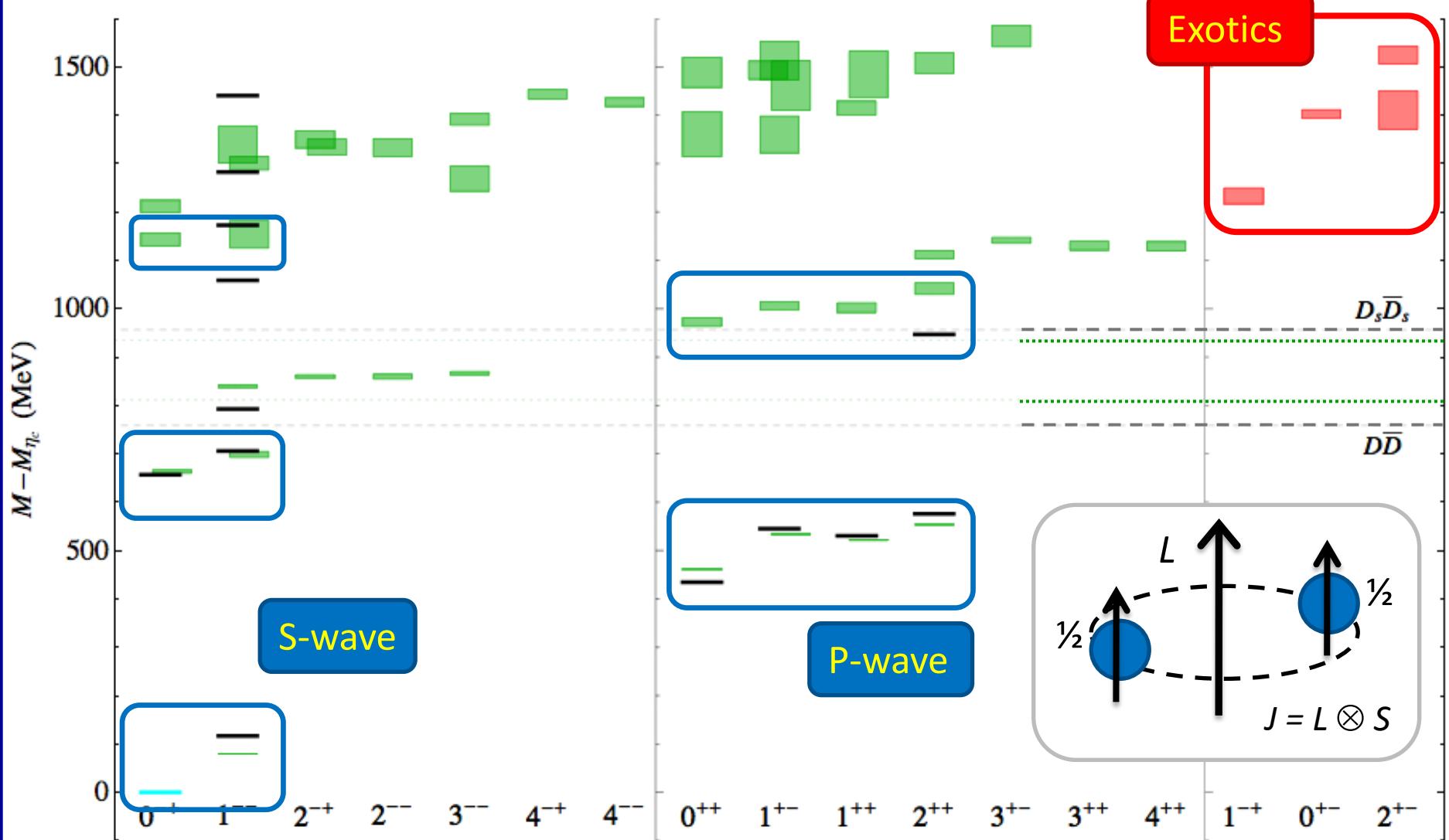
$24^3, M_\pi \approx 400$ MeV

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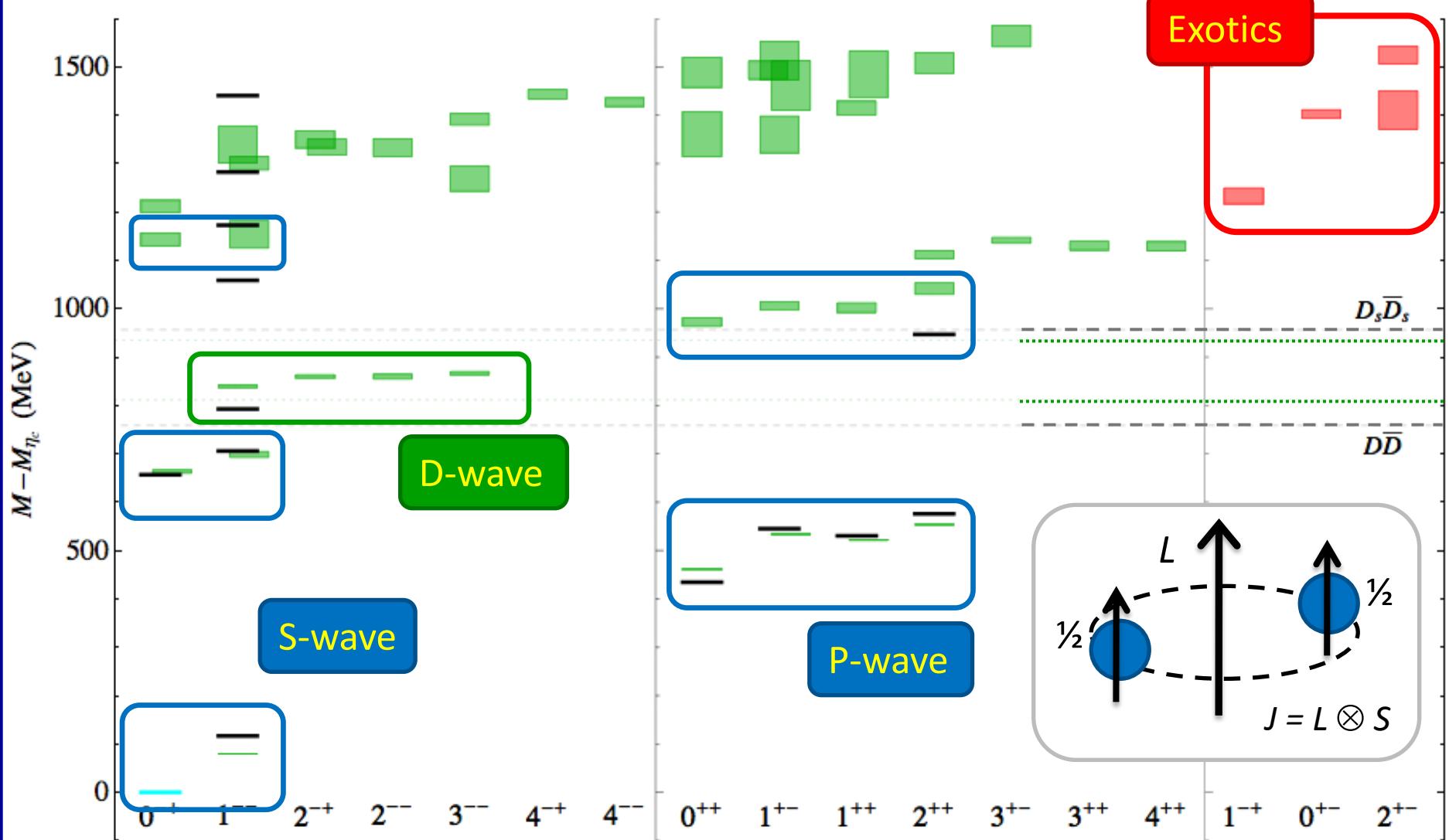
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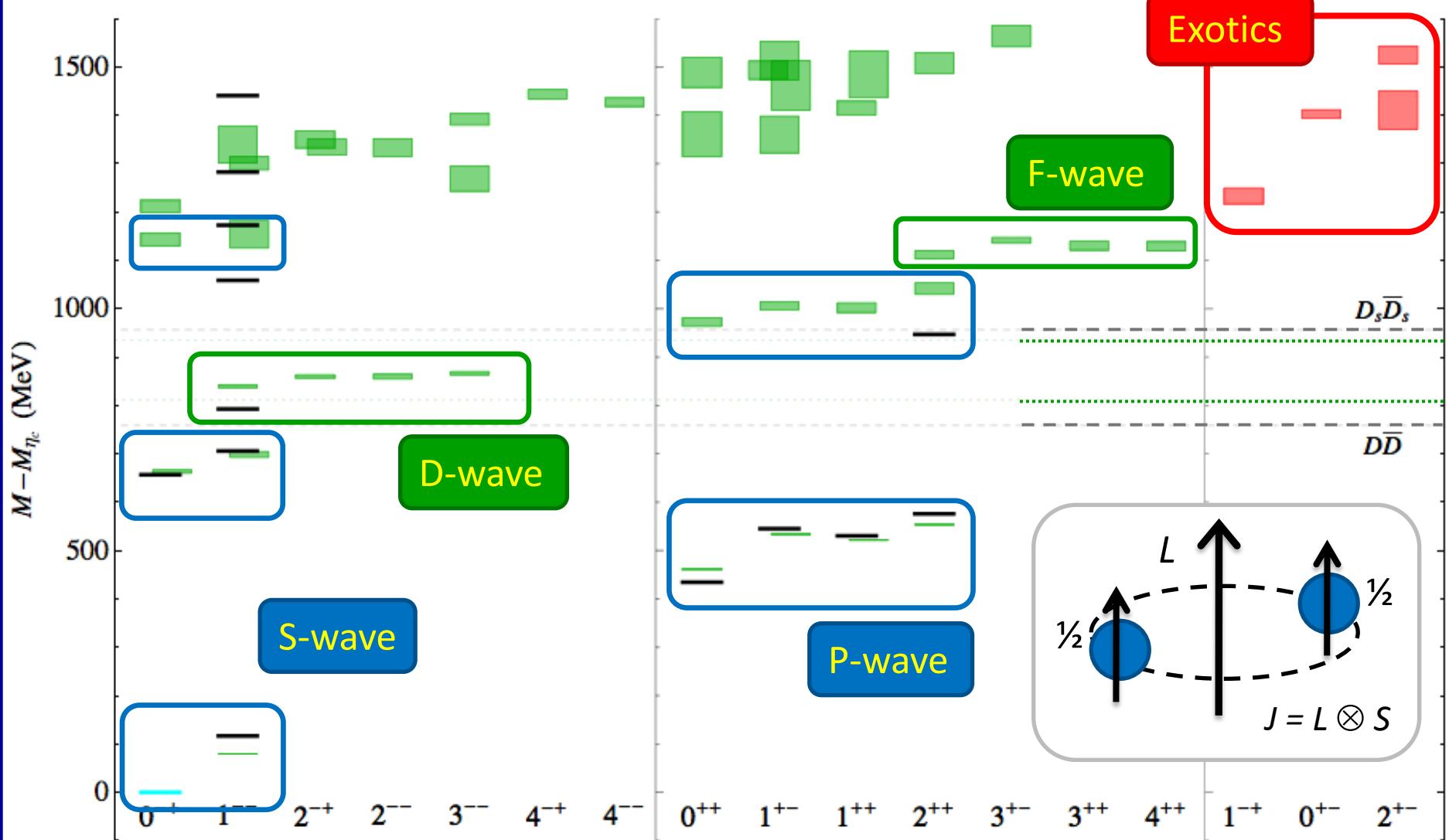
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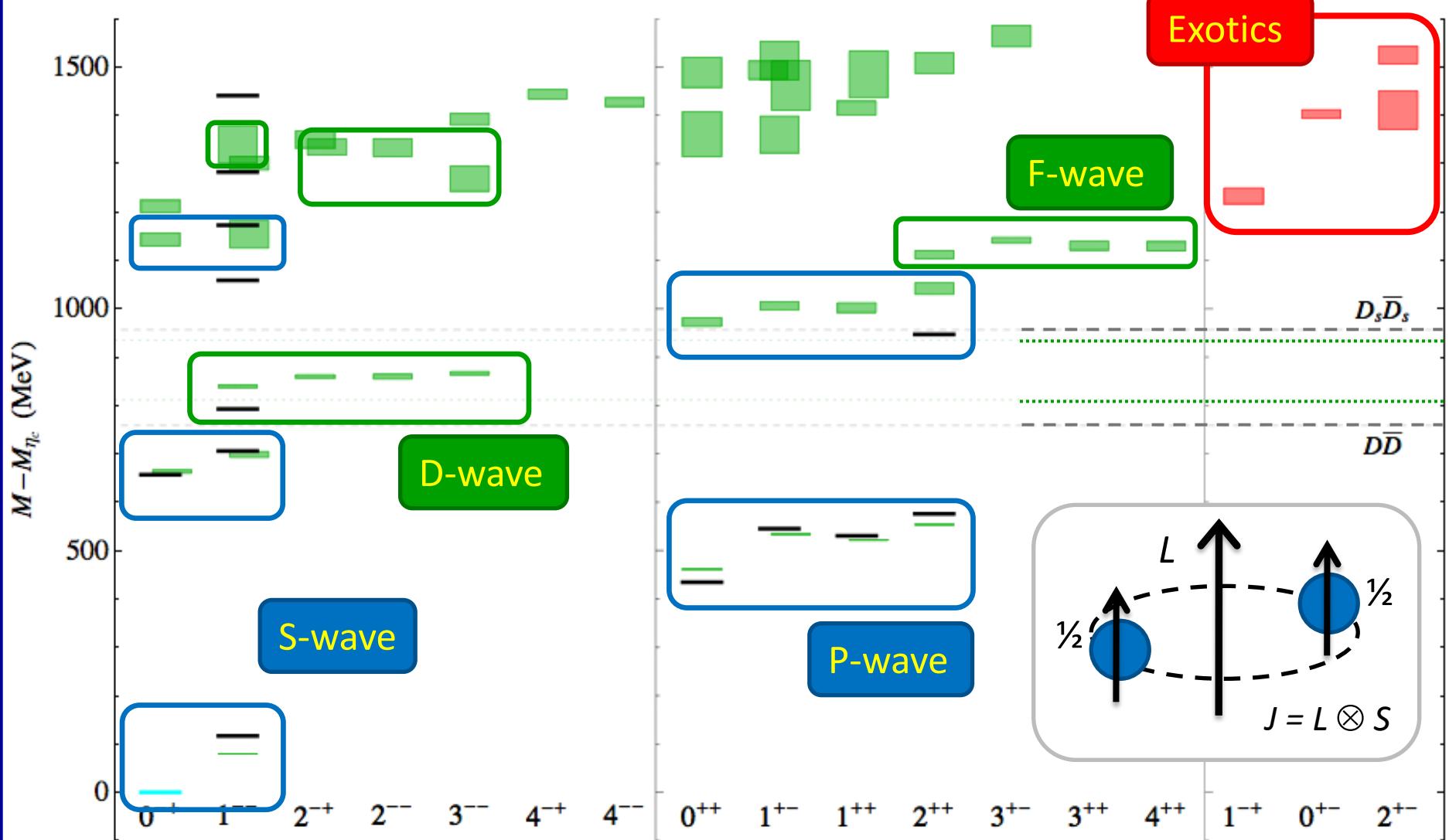
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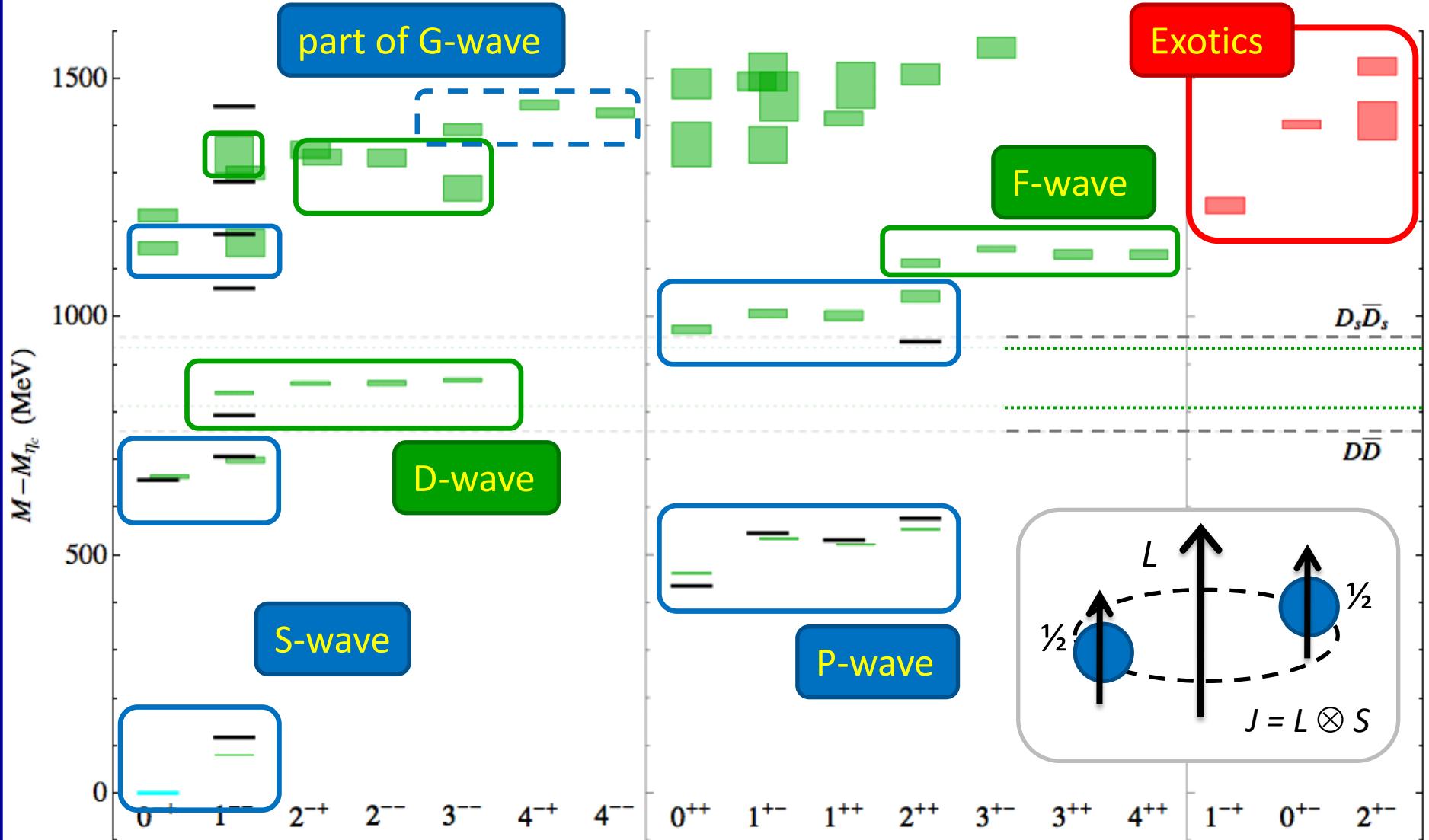
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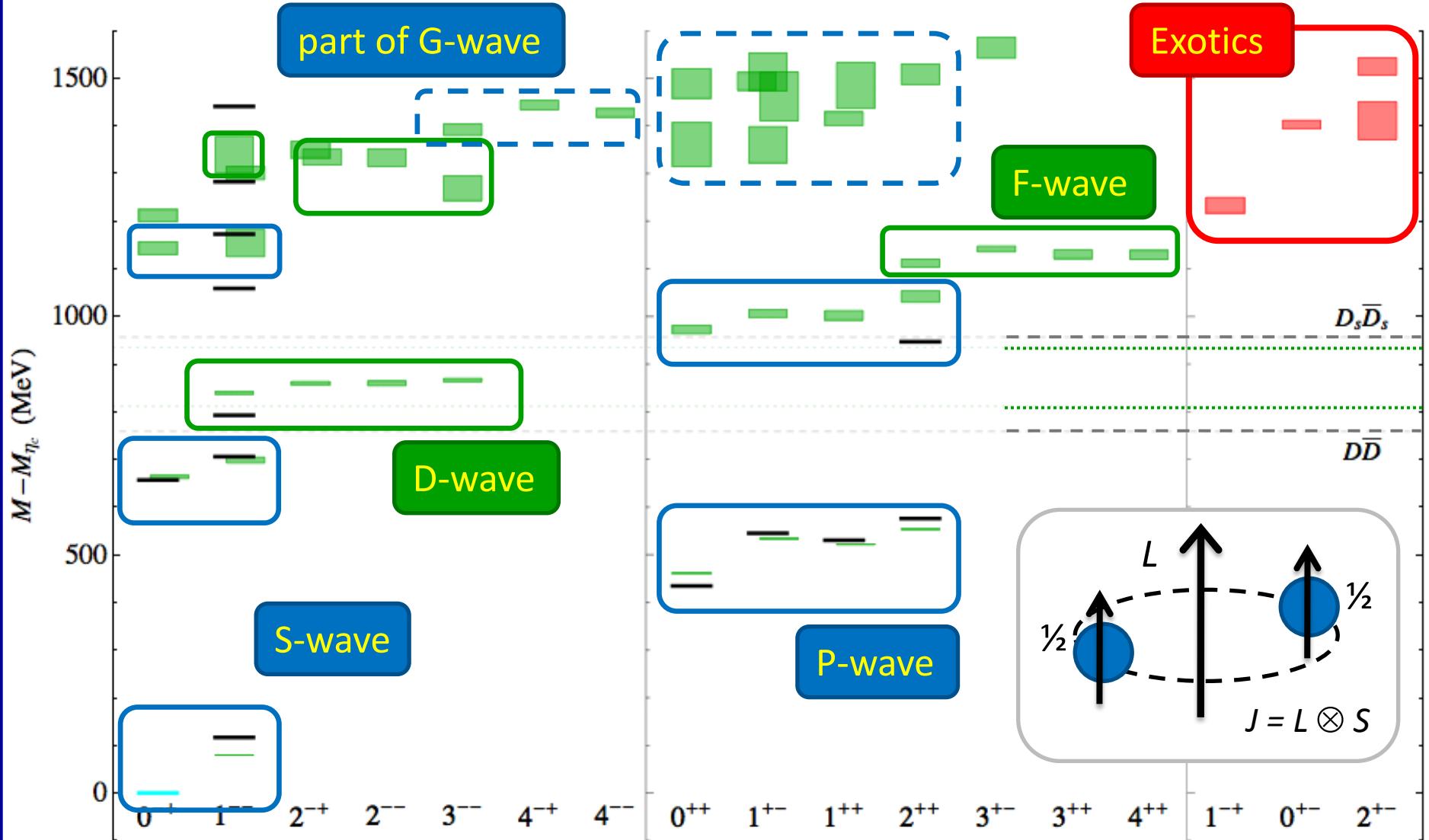
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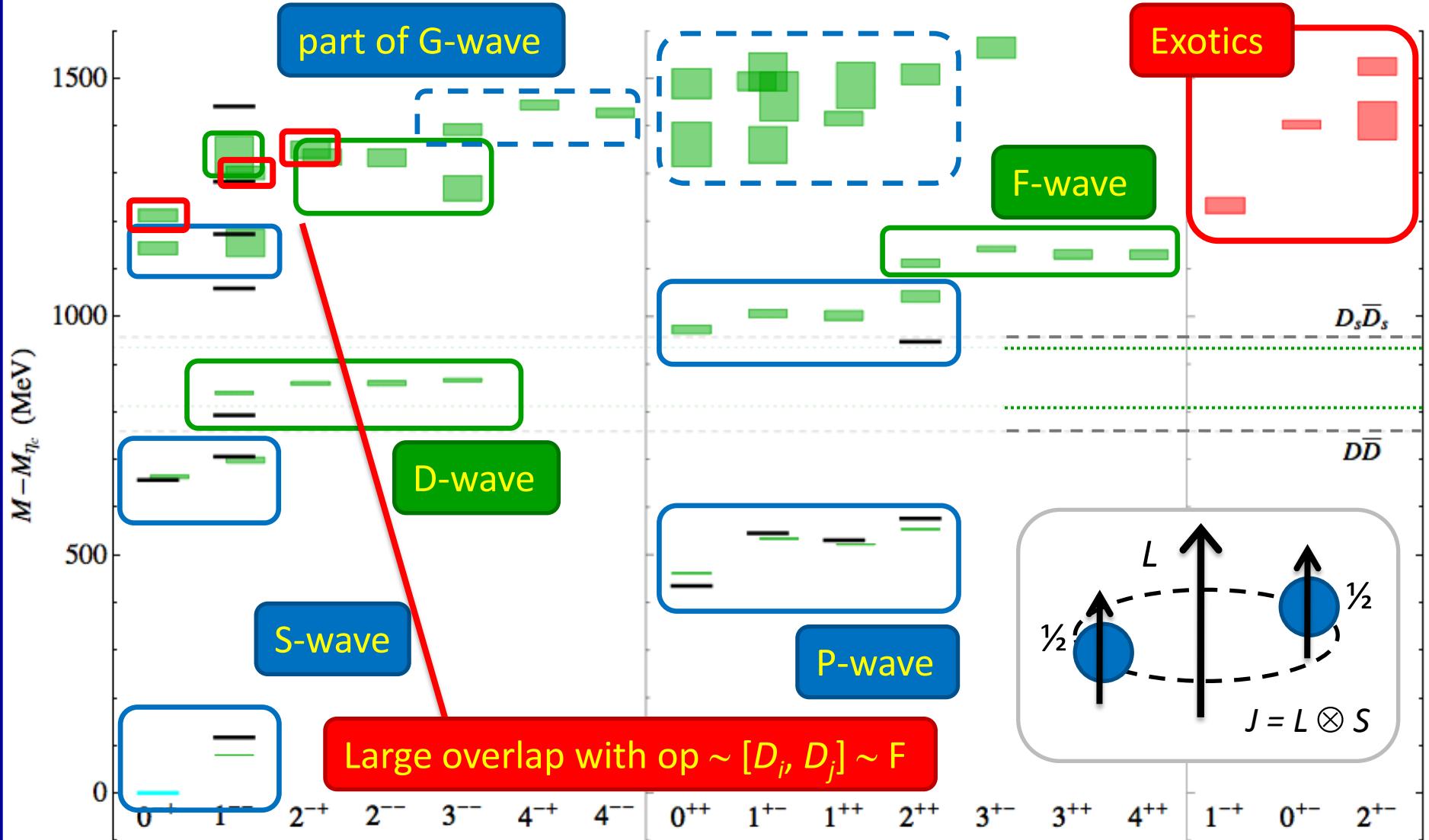
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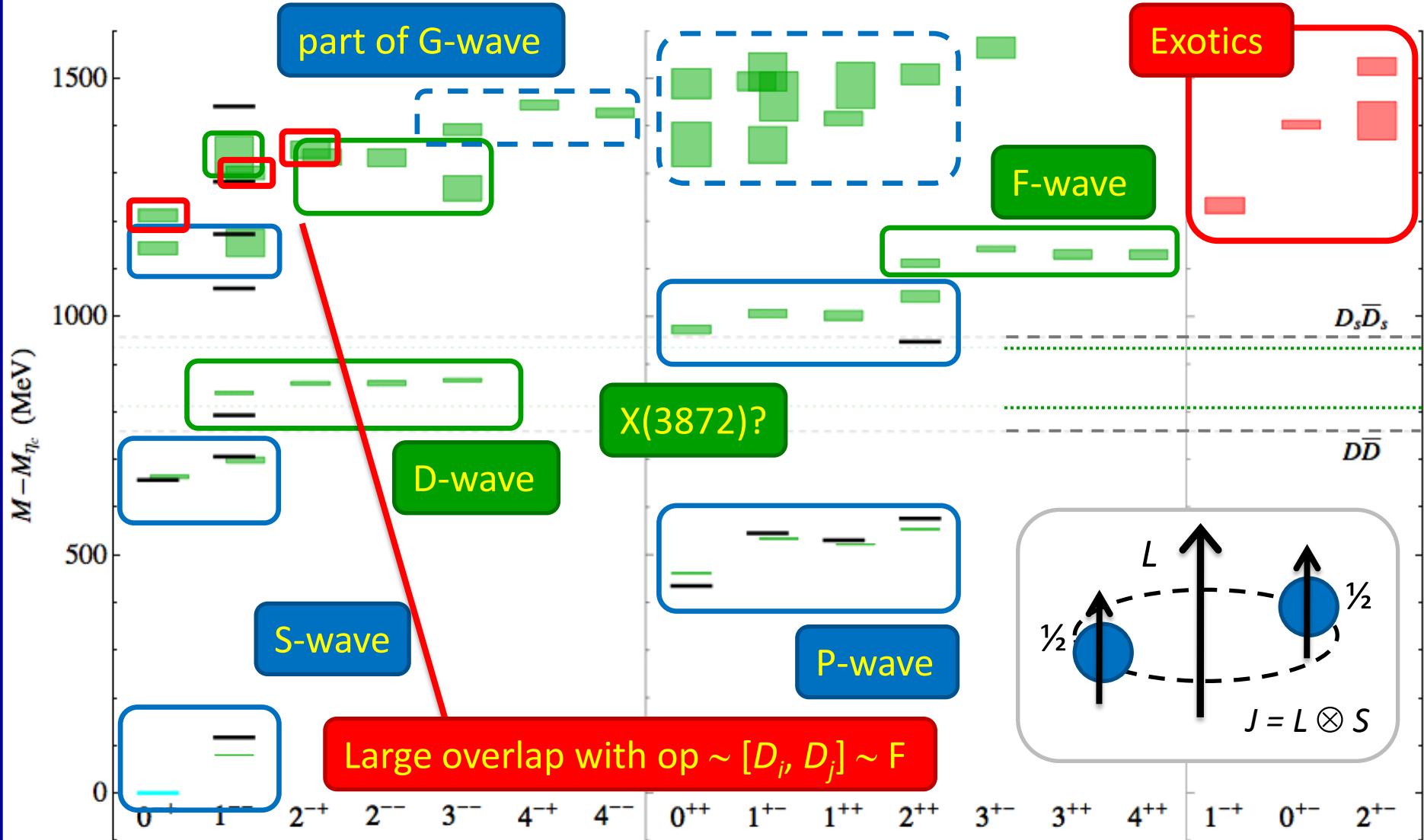
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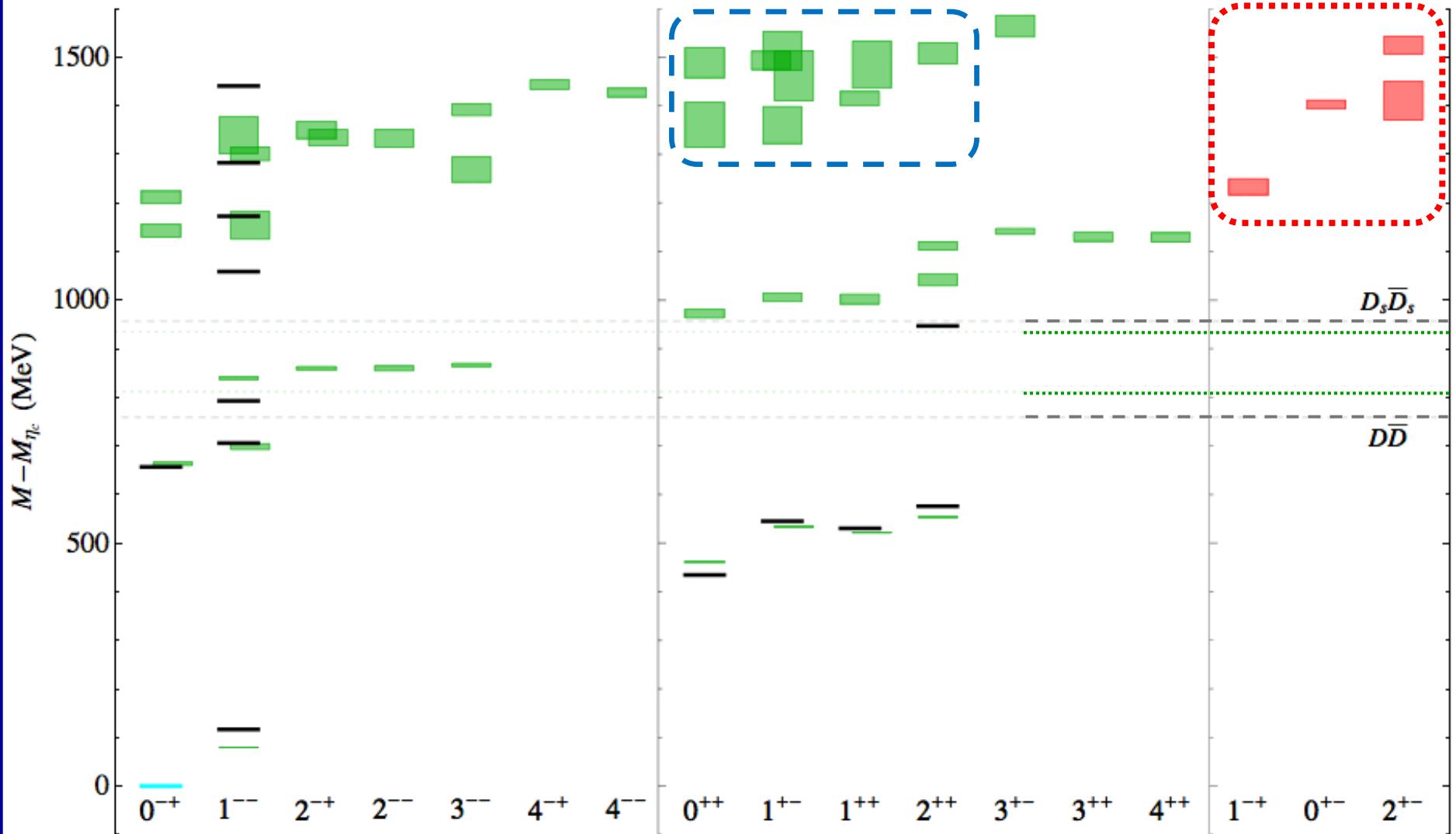
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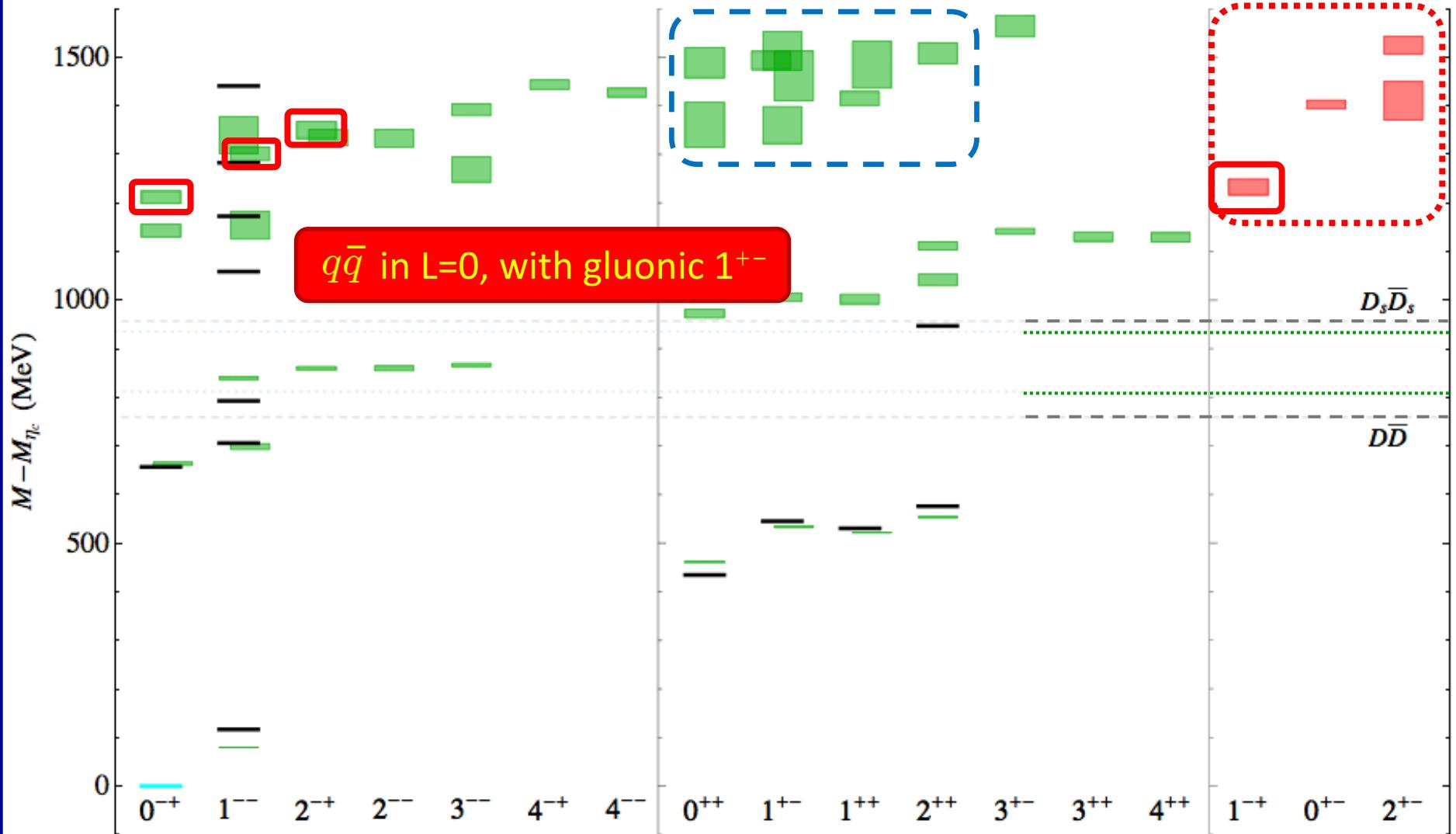
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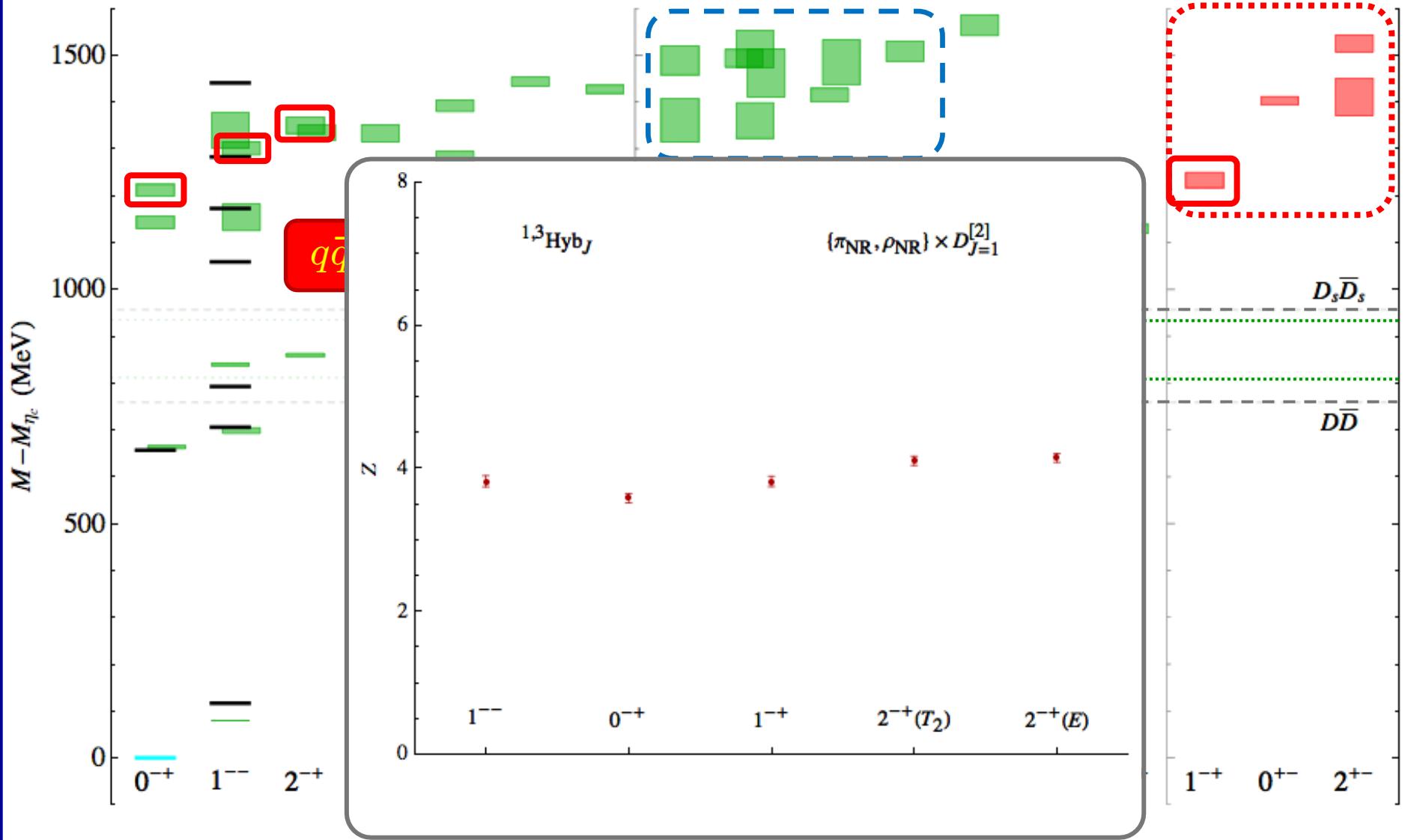
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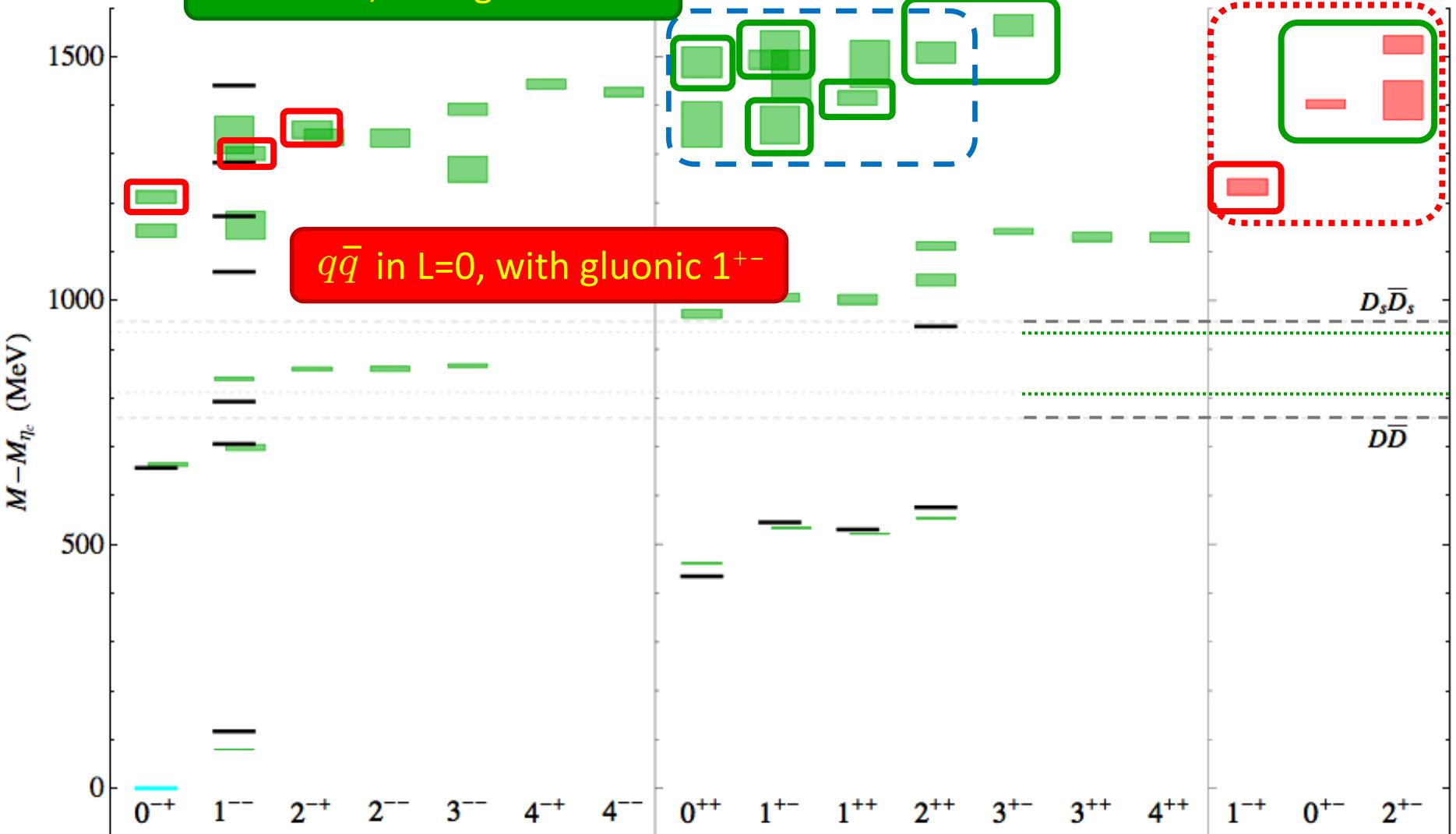
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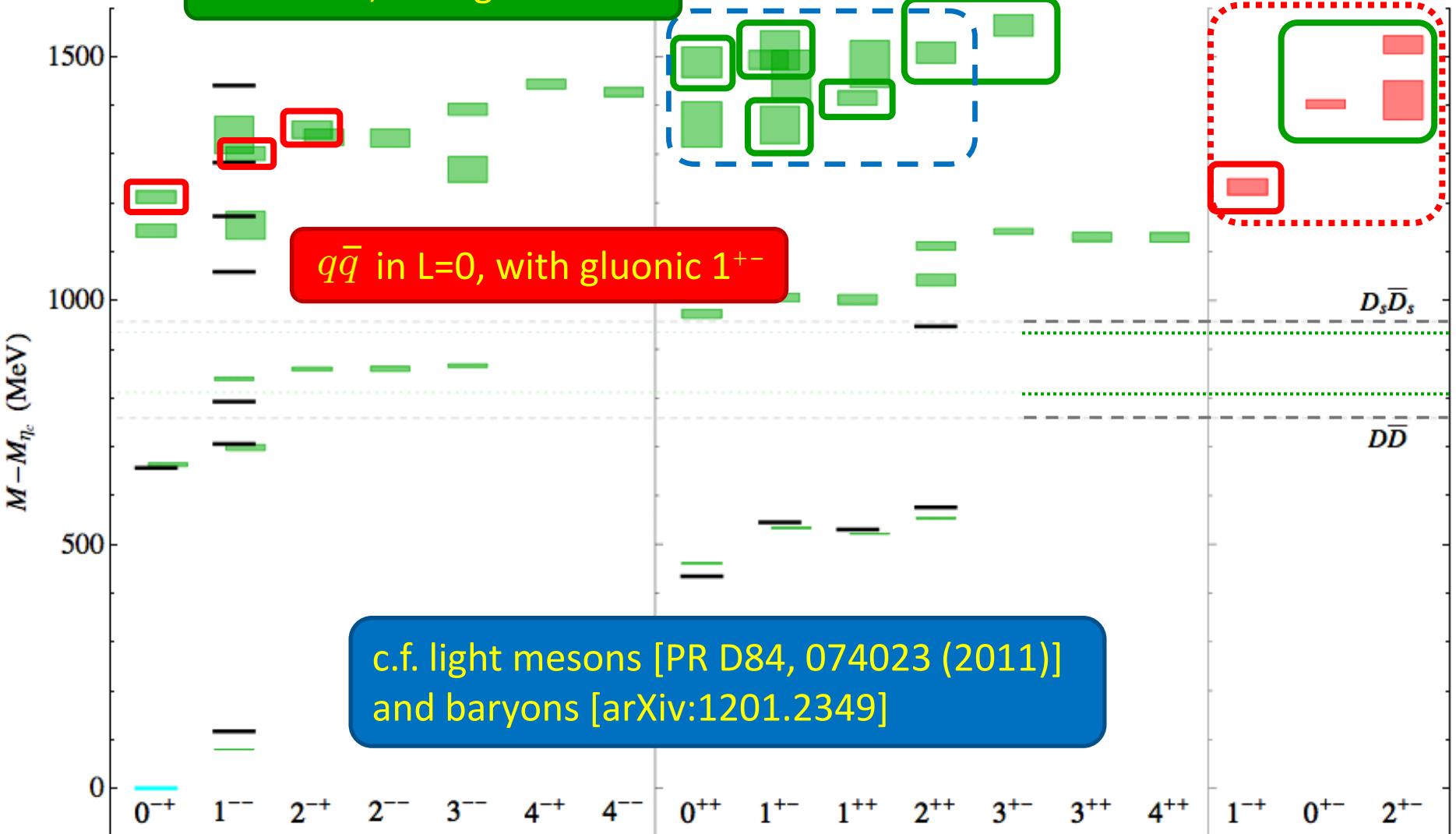
$q\bar{q}$ in L=1, with gluonic 1^{+-}



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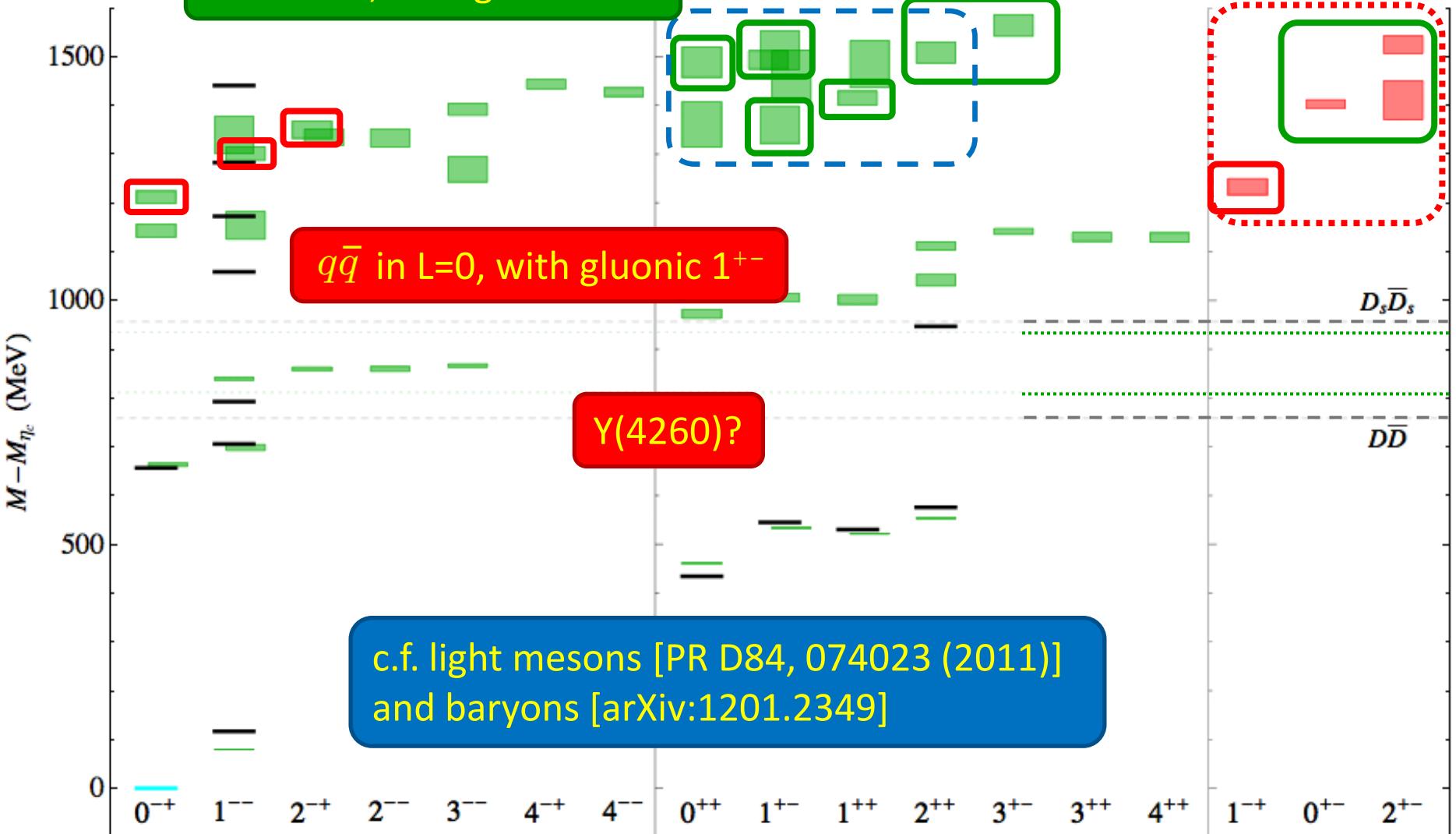


c.f. light mesons [PR D84, 074023 (2011)]
and baryons [arXiv:1201.2349]

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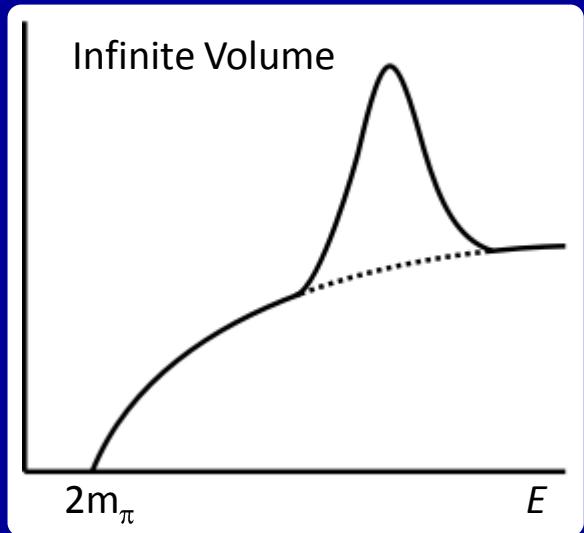


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Scattering in a box

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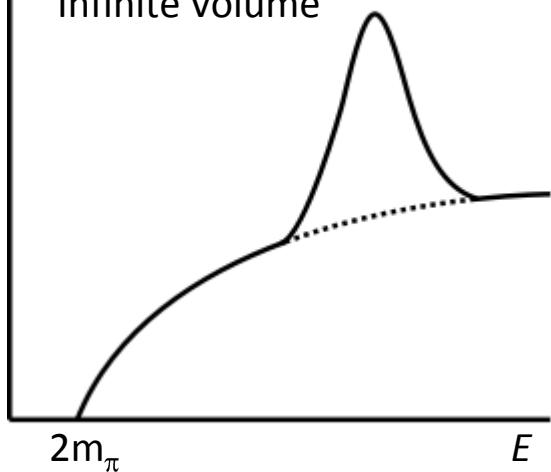
Infinite Volume

Continuous spectrum

$$E_{\pi\pi}(p) = 2\sqrt{m_\pi^2 + \vec{p}^2}$$

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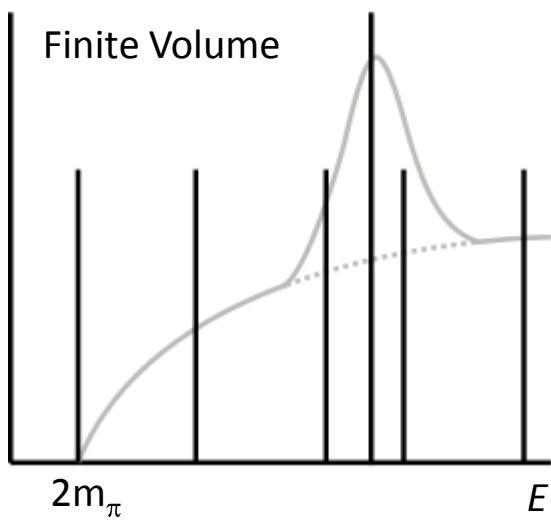


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Finite Volume

Cubic box with periodic boundary conditions

Quantised momenta

$$\vec{p} = \frac{2\pi}{L_s}(n_x, n_y, n_z)$$

→ Discrete spectrum

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Euclidean time: can't directly study dynamical properties like widths

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Lüscher: (elastic) energy shifts in **finite volume** → phase shift

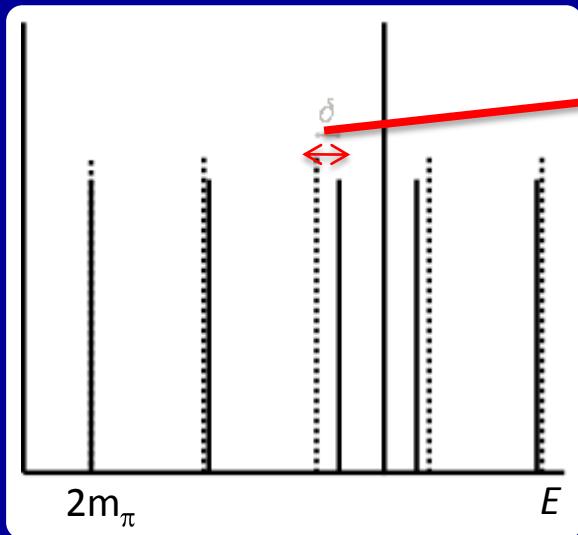
$$f_l(E) = \frac{1}{2ip} (e^{2i\delta_l(E)} - 1) = \frac{1}{p} e^{i\delta_l(E)} \sin \delta_l(E)$$

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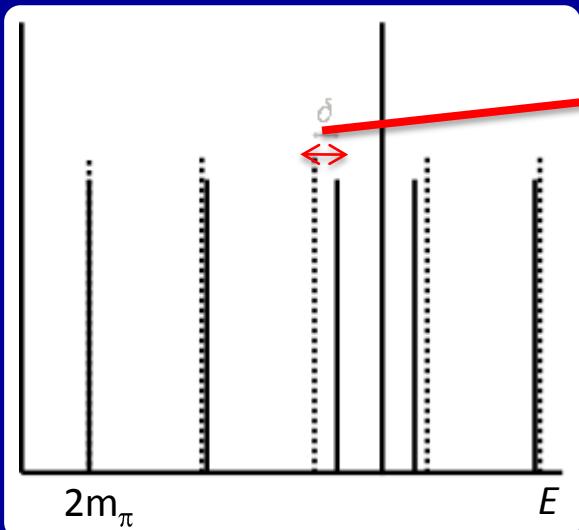
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Extract phase shift at discrete p_{cm}

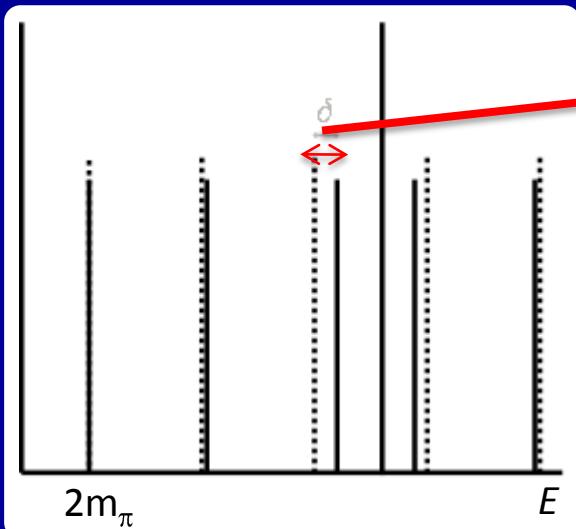
Map out phase shift →
resonance parameters
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ρ , $X(3872)$, $Z^+(4430)$, ...

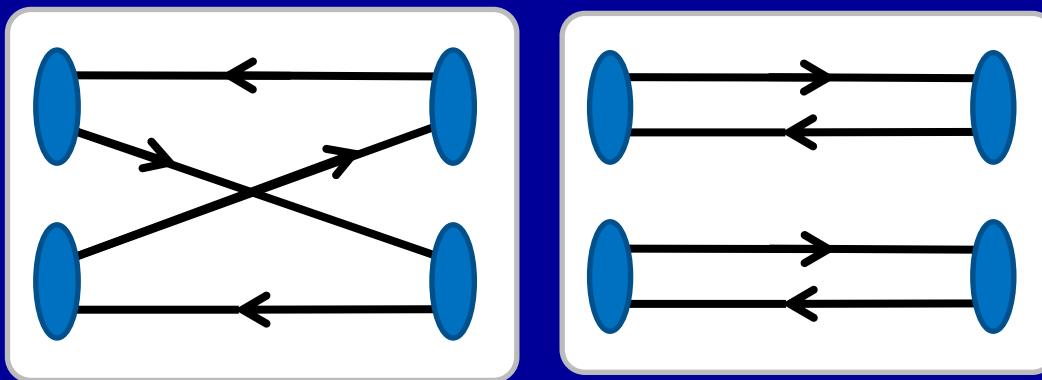
Isospin-2 $\pi\pi$ scattering

Testing new methodology with $\pi\pi$ in isospin-2

arXiv:1203.6041 [to appear in PRD] – Jo Dudek, Robert Edwards, CT

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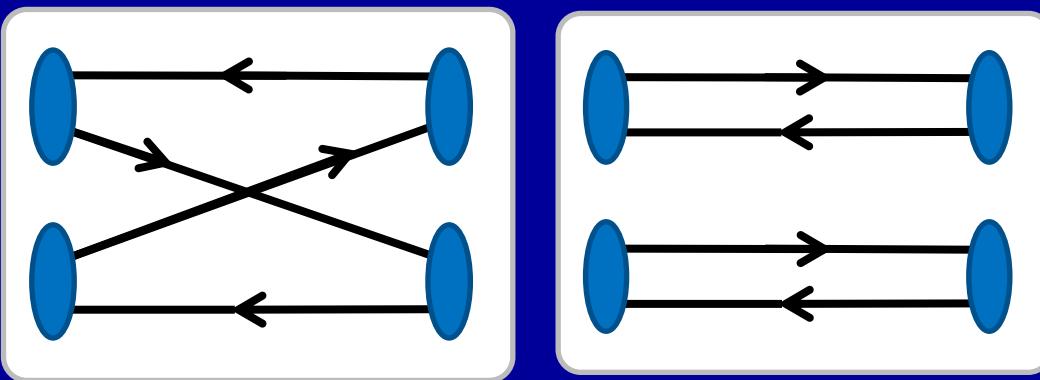
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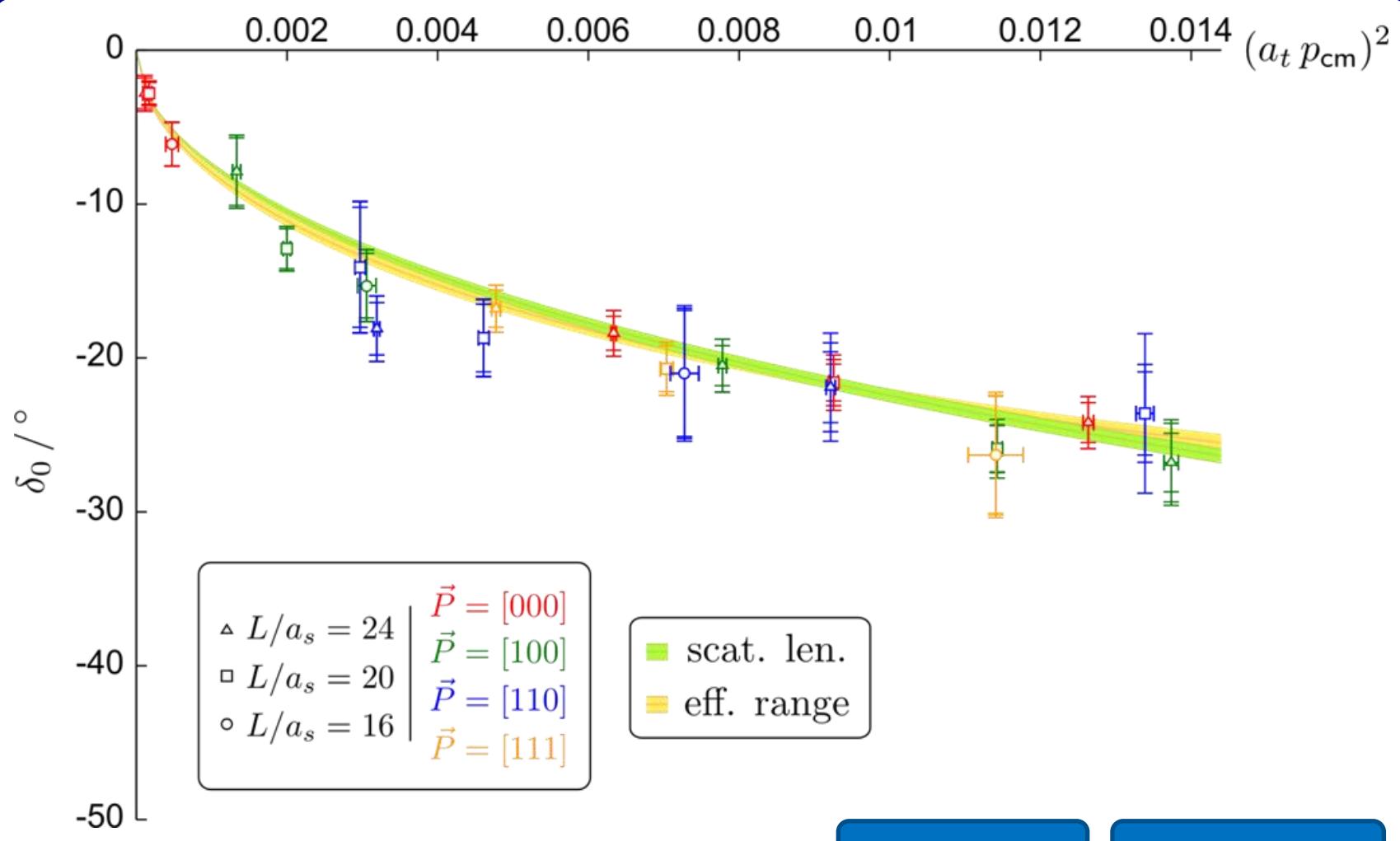
+ similar diagrams

$$\mathcal{O}(\vec{P}) = \sum_{\vec{p}_1, \vec{p}_2} C_\Lambda(\vec{P}, \vec{p}_1, \vec{p}_2) \mathcal{O}_\pi(\vec{p}_1) \mathcal{O}_\pi(\vec{p}_2)$$

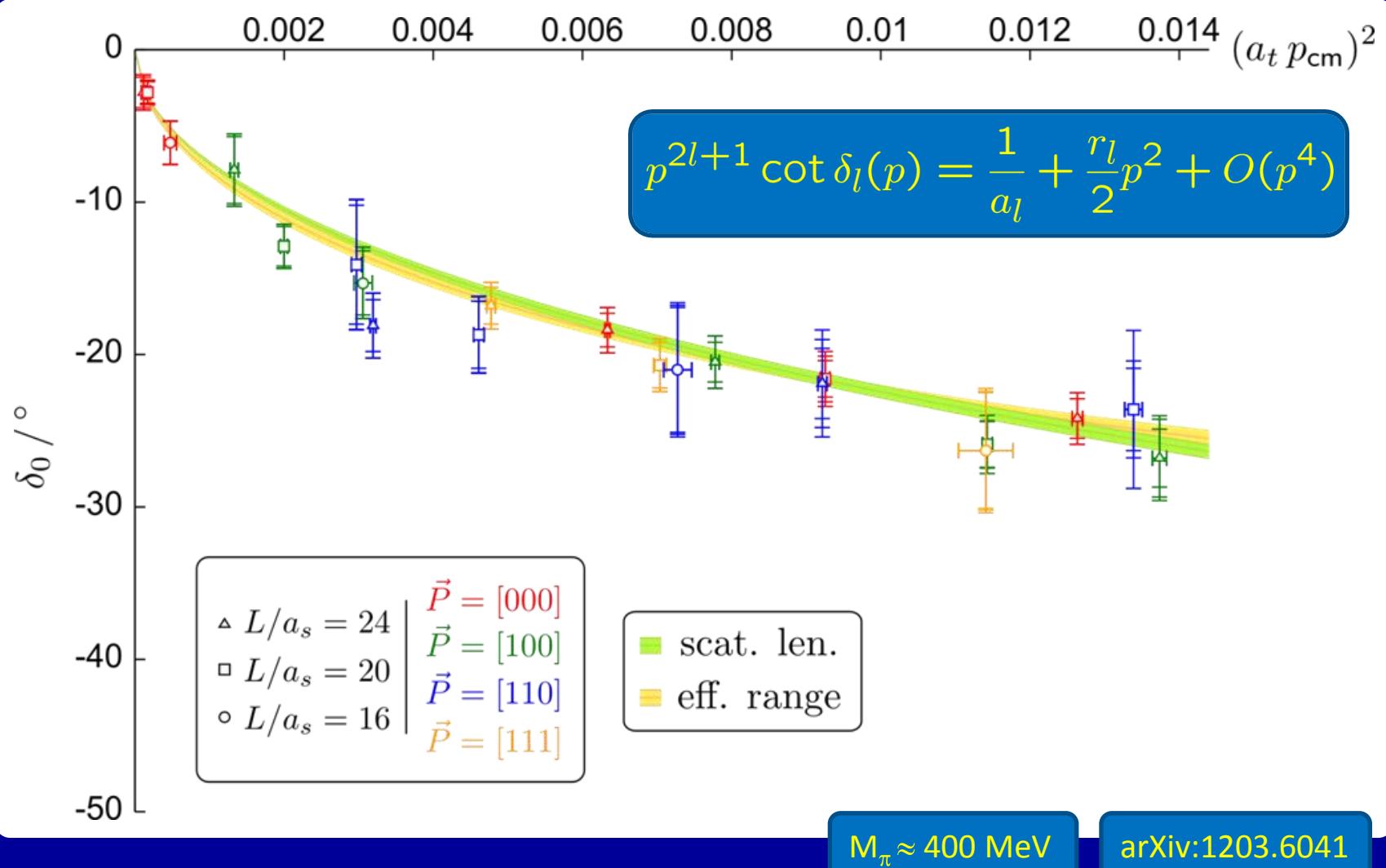
$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

$$\vec{P} = [0, 0, 0], [0, 0, 1], [0, 1, 1], [1, 1, 1]$$

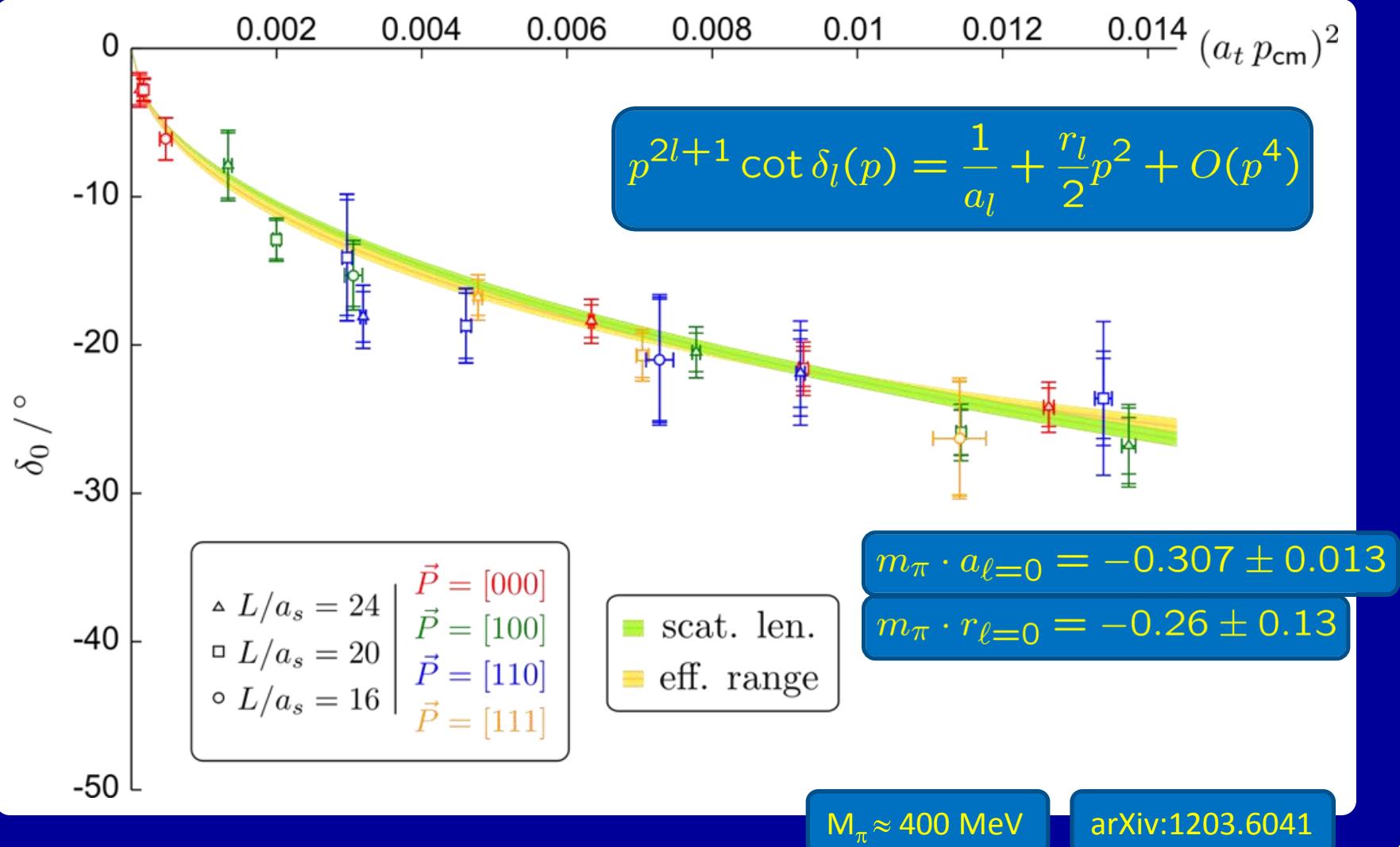
$\pi\pi$ $|l=2$ scattering: $L = 0$



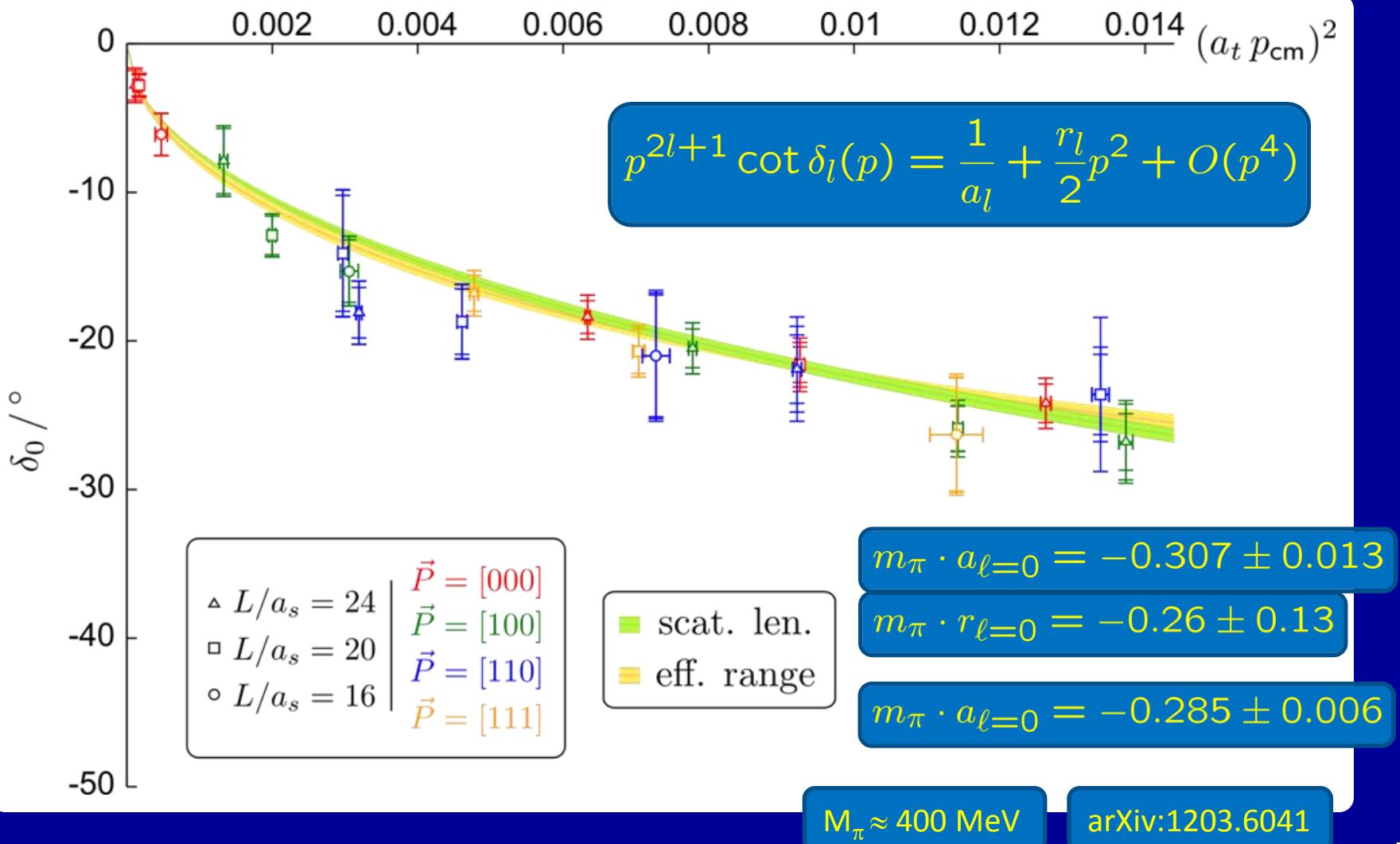
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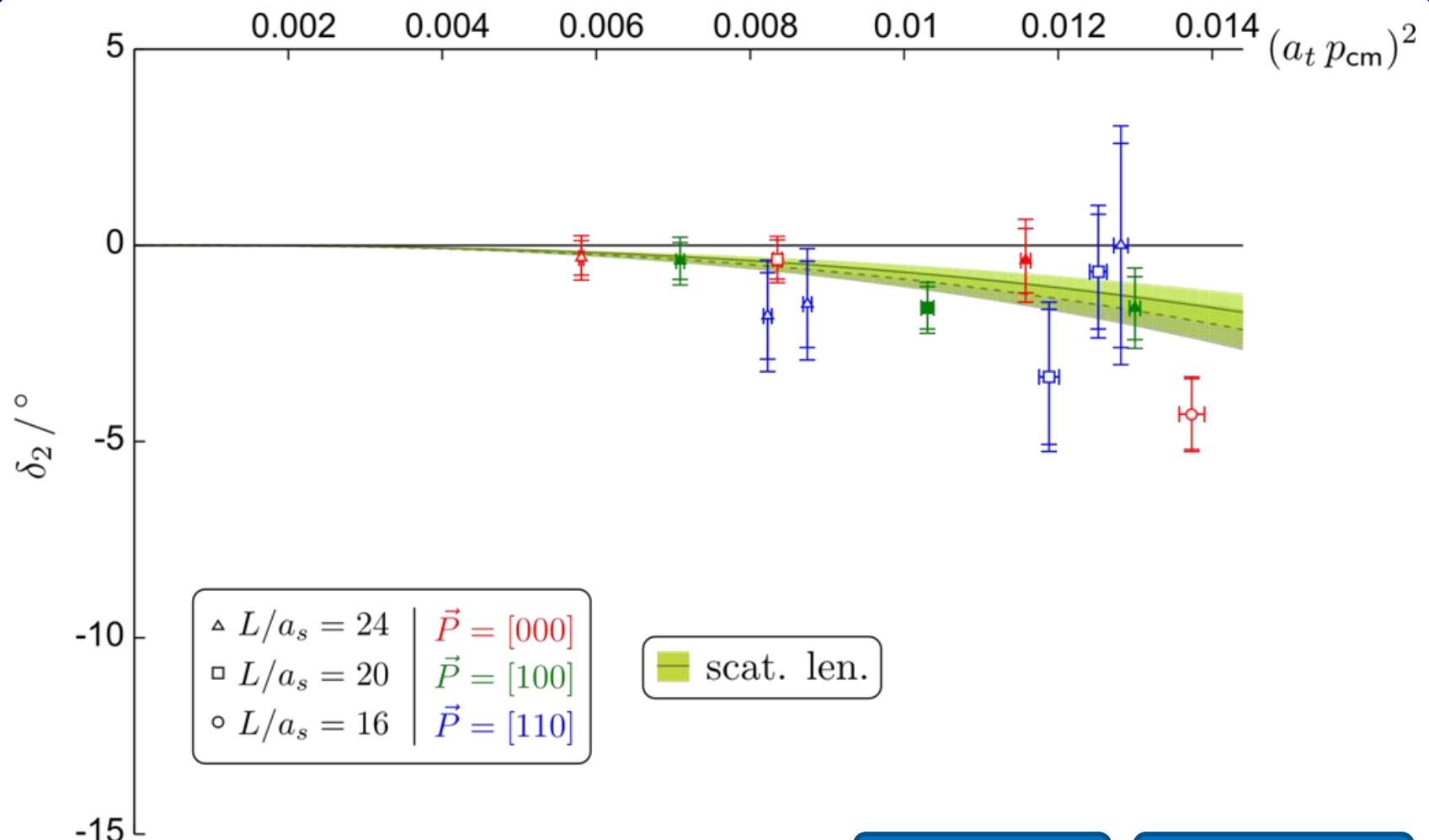
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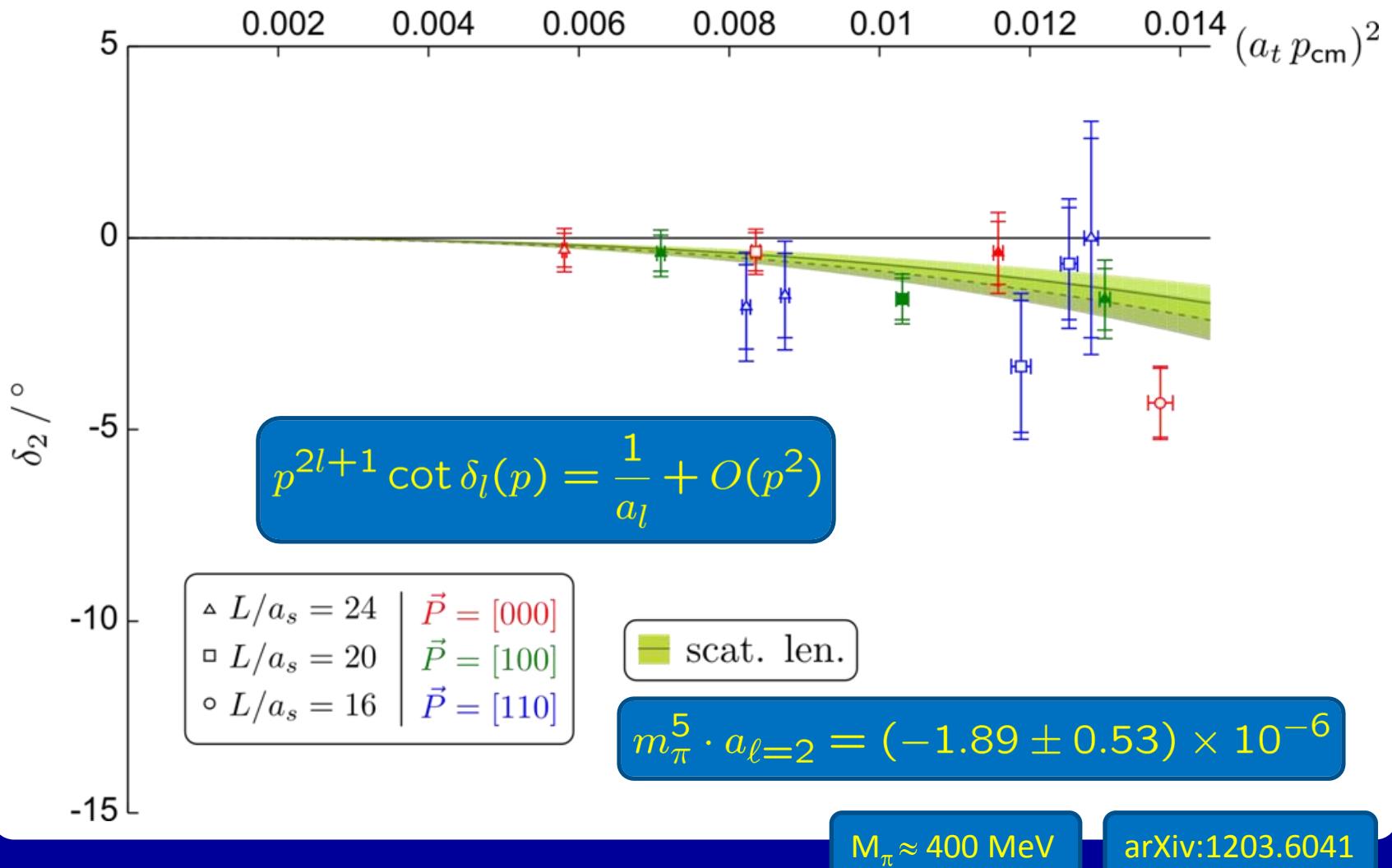
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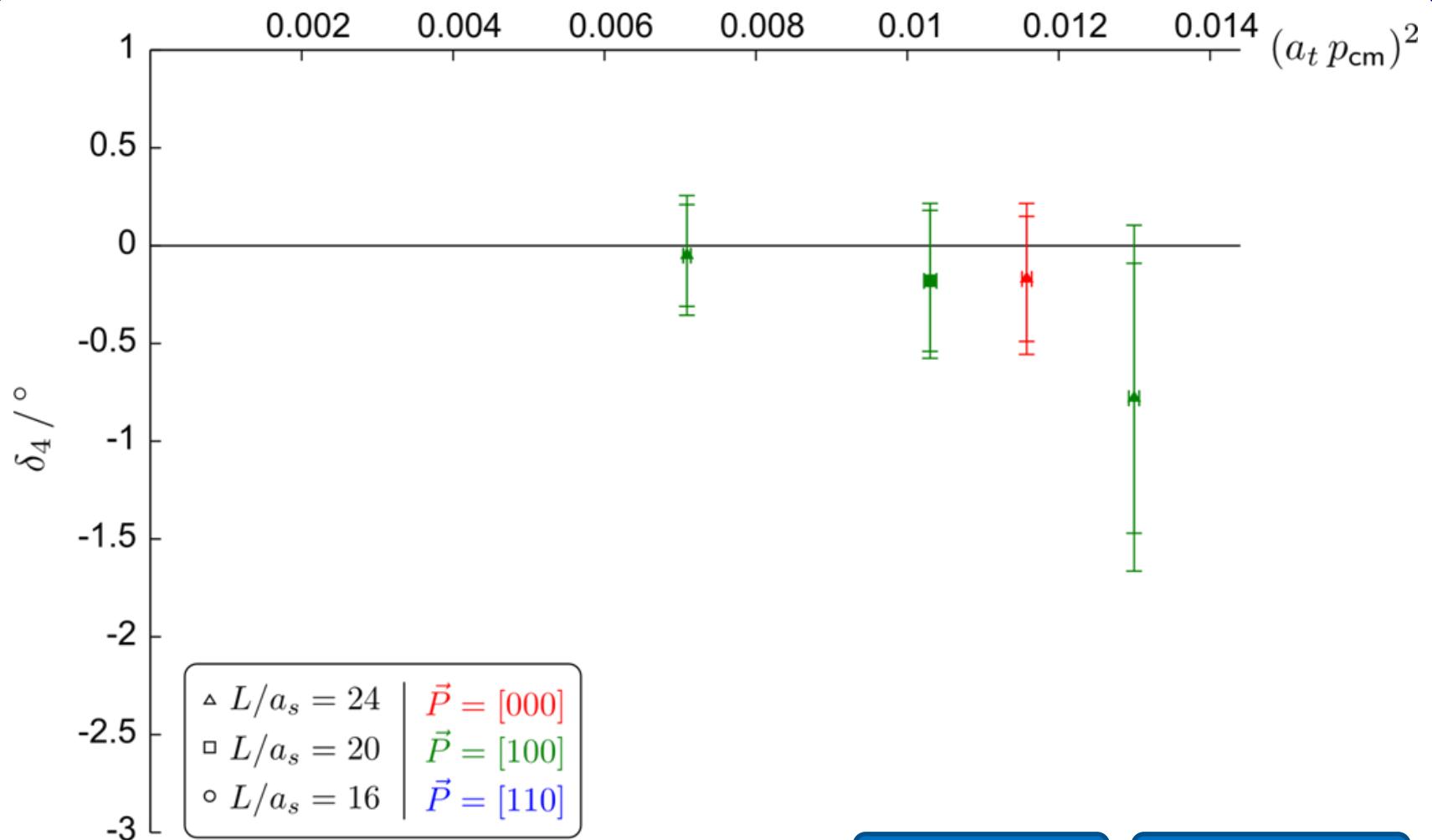
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$\pi\pi$ $|l|=2$ scattering: $L = 2$



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Outlook

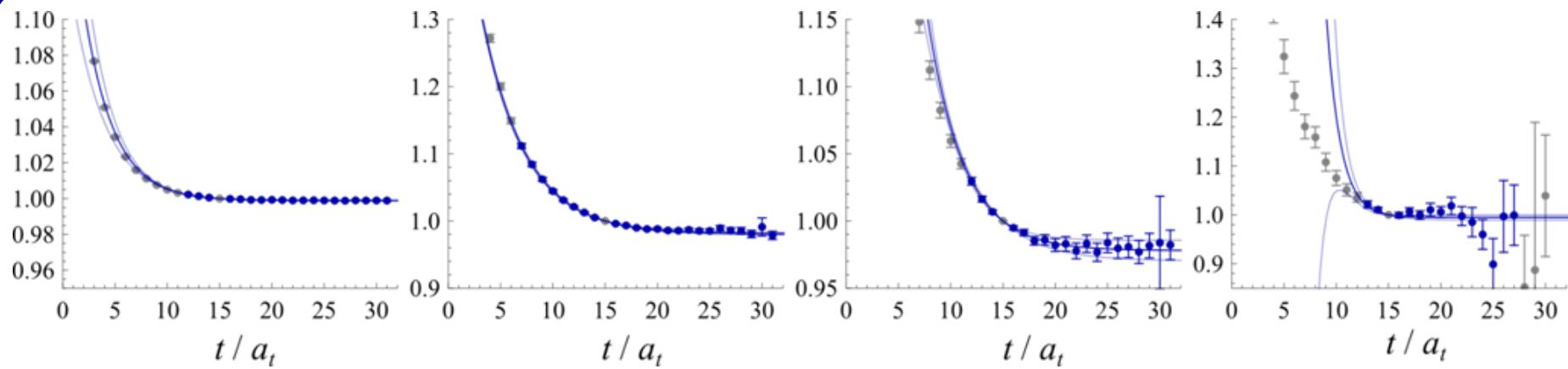
- **Scattering – resonances, decays, ...**
- Disconnected contributions, glueball mixing, etc
- D/D_s mesons, charmed baryons, **rad. transitions**
- Lighter pion masses, larger volumes, ...



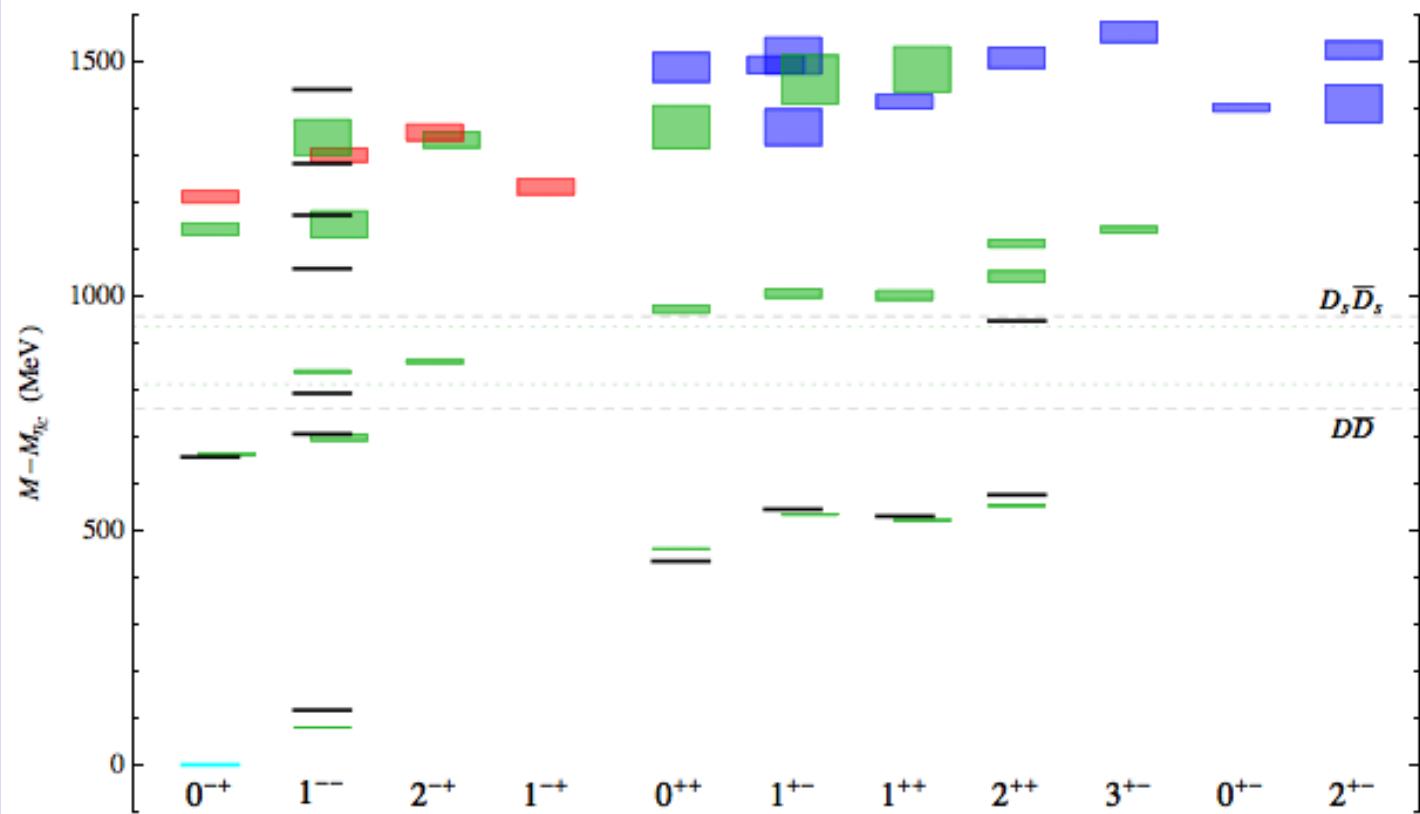
Extra Slides

Charmonium – principal correlators

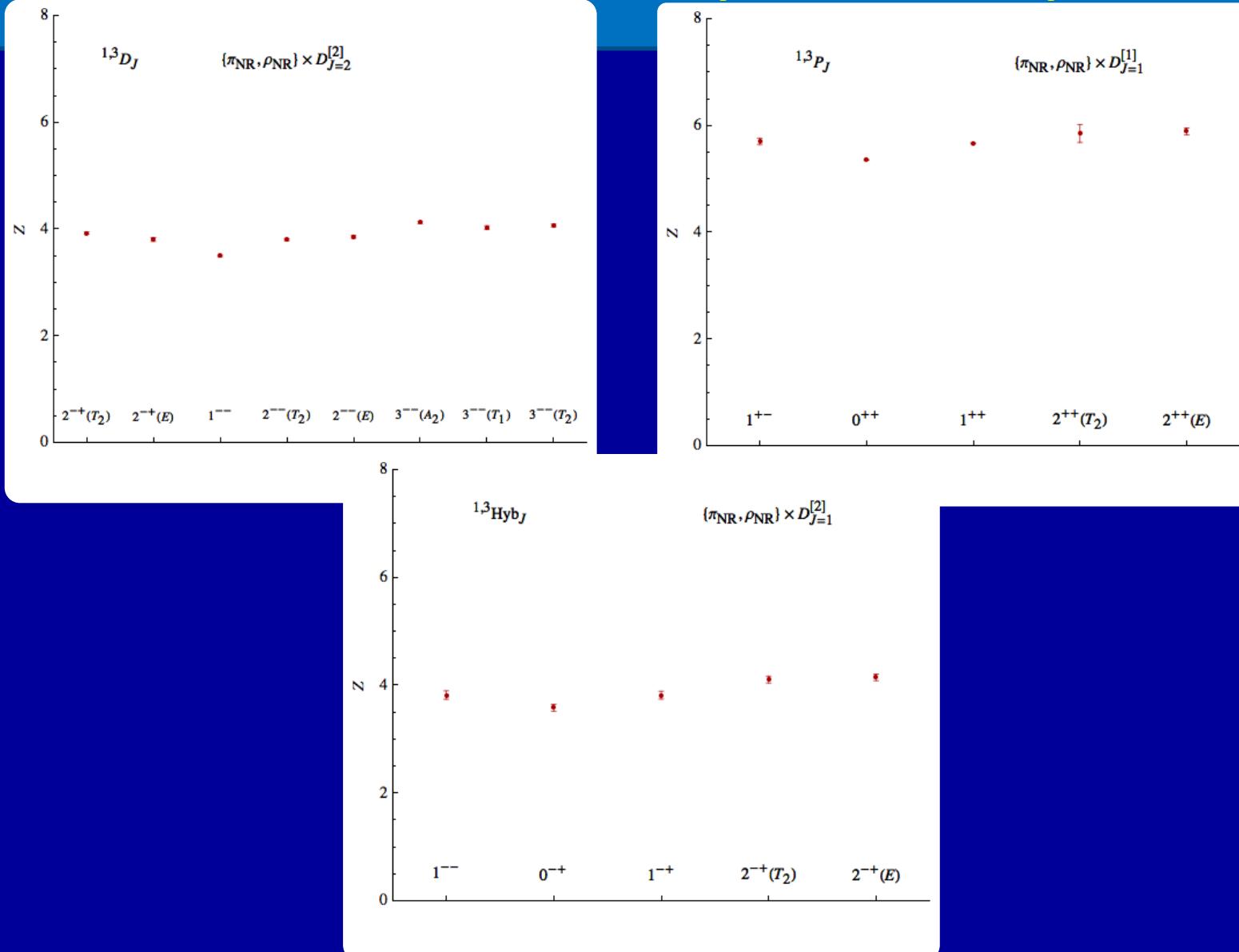
$$\lambda(t) \cdot e^{m(t-t_0)}$$



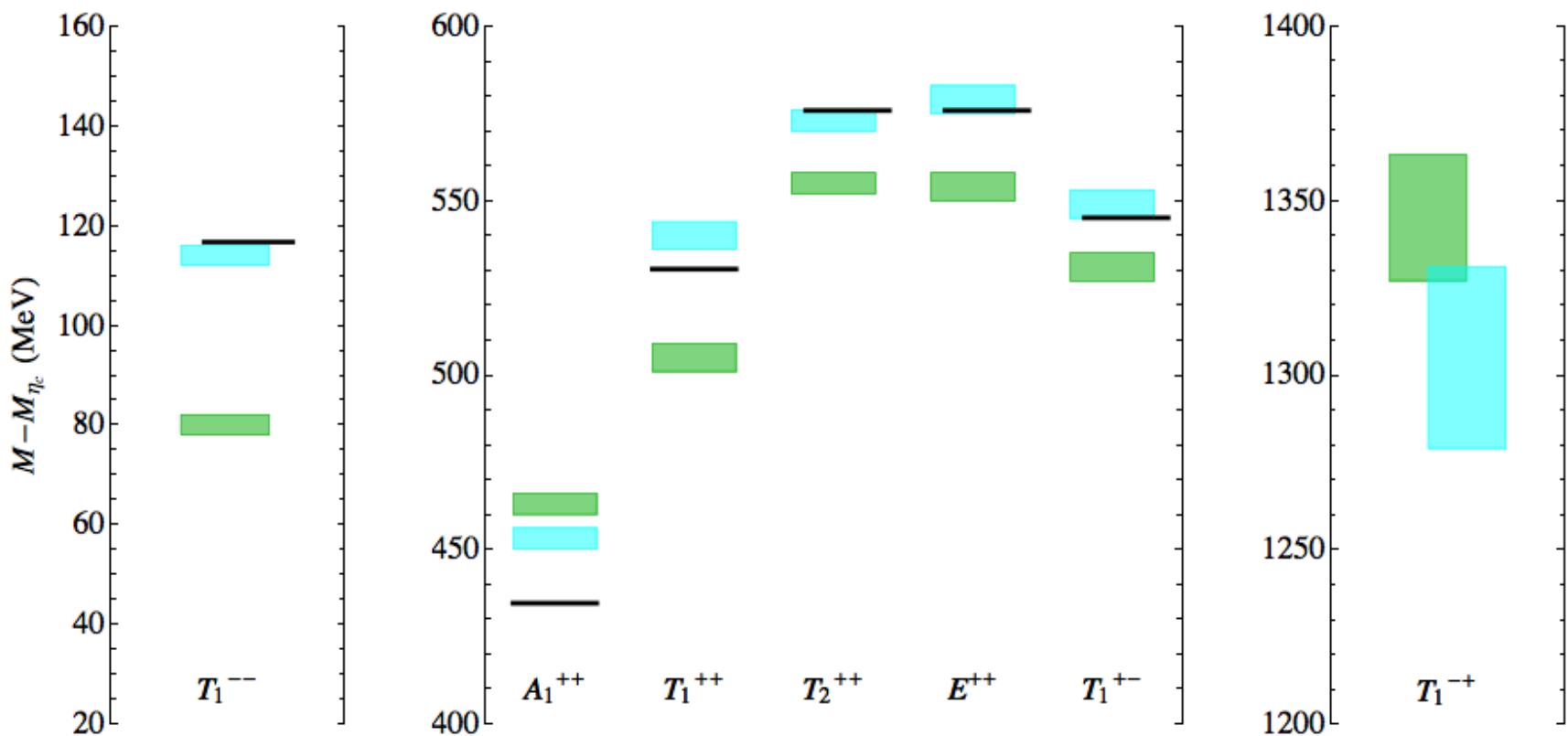
Charmonium – hybrid candidates



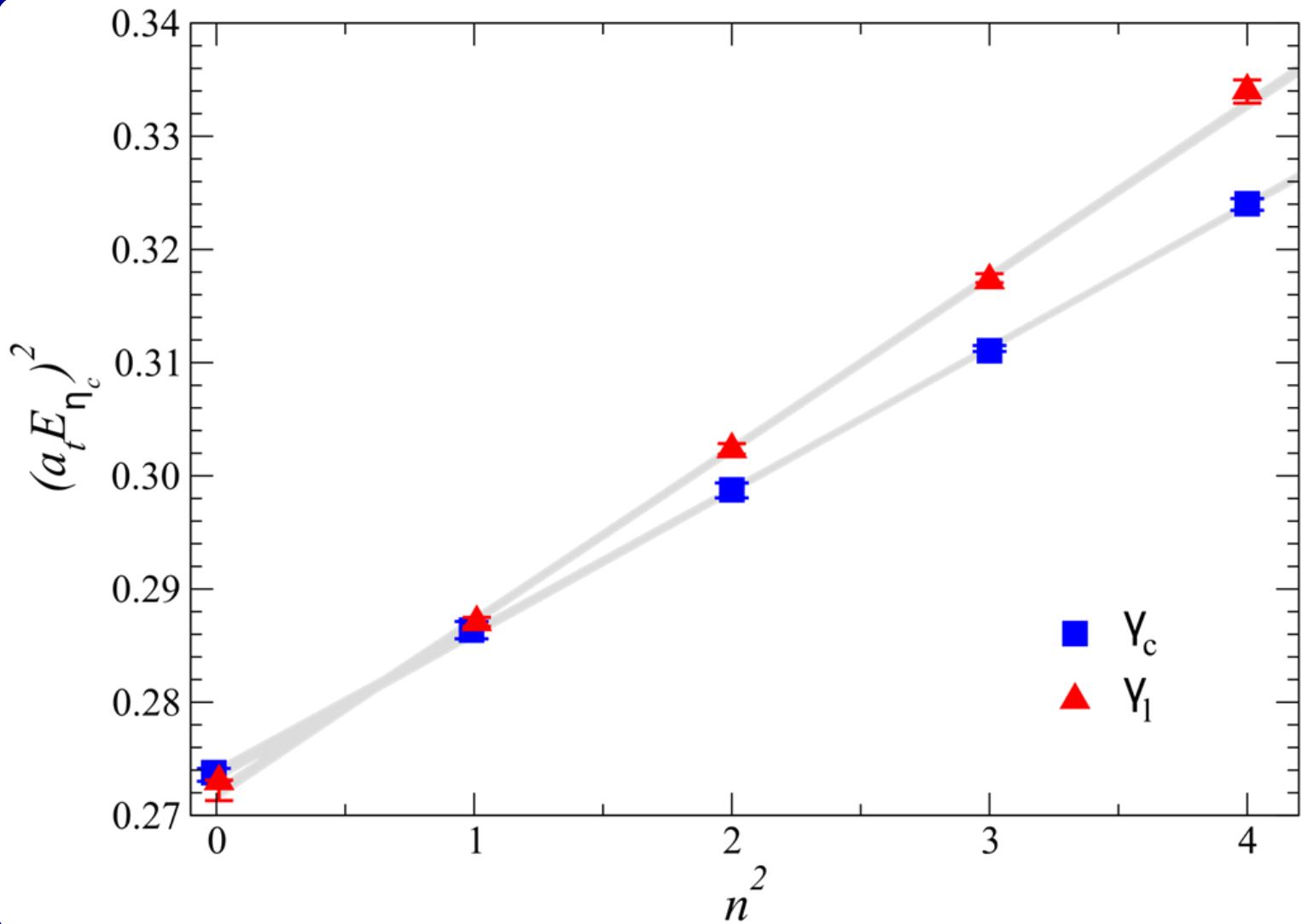
Charmonium – supermultiplets



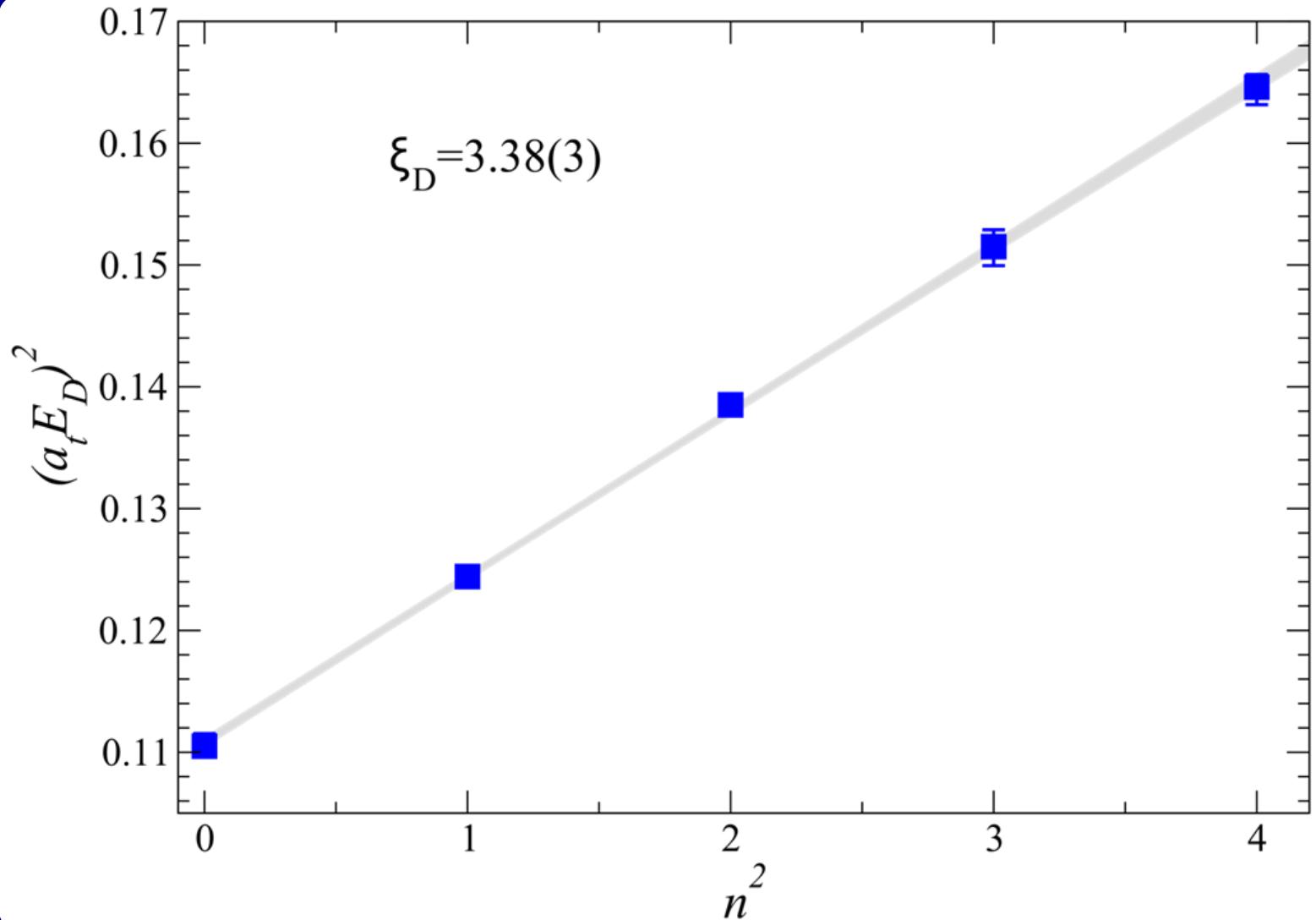
Charmonium – $O(a)$



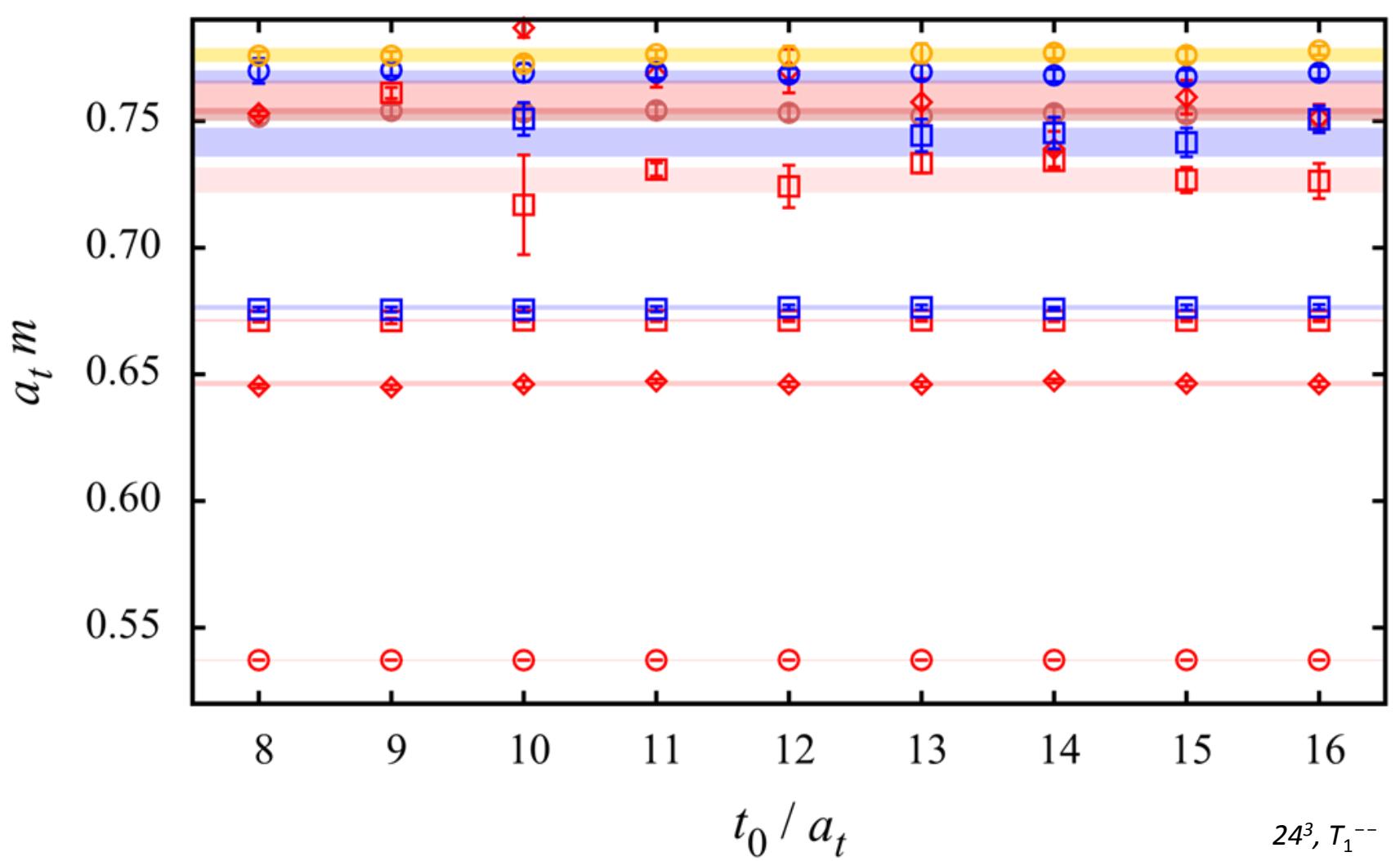
Dispersion relation – η_c



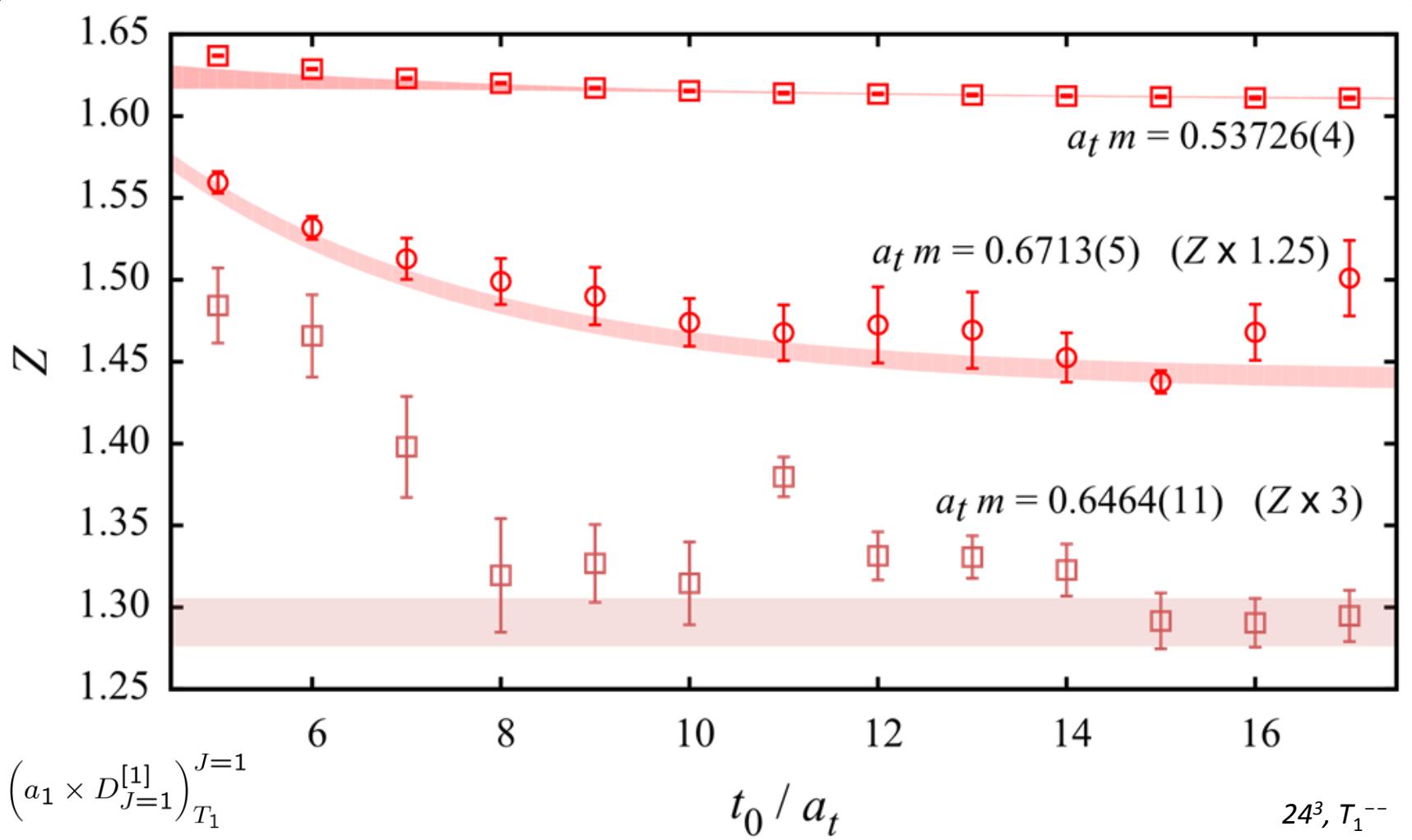
Dispersion relation – D



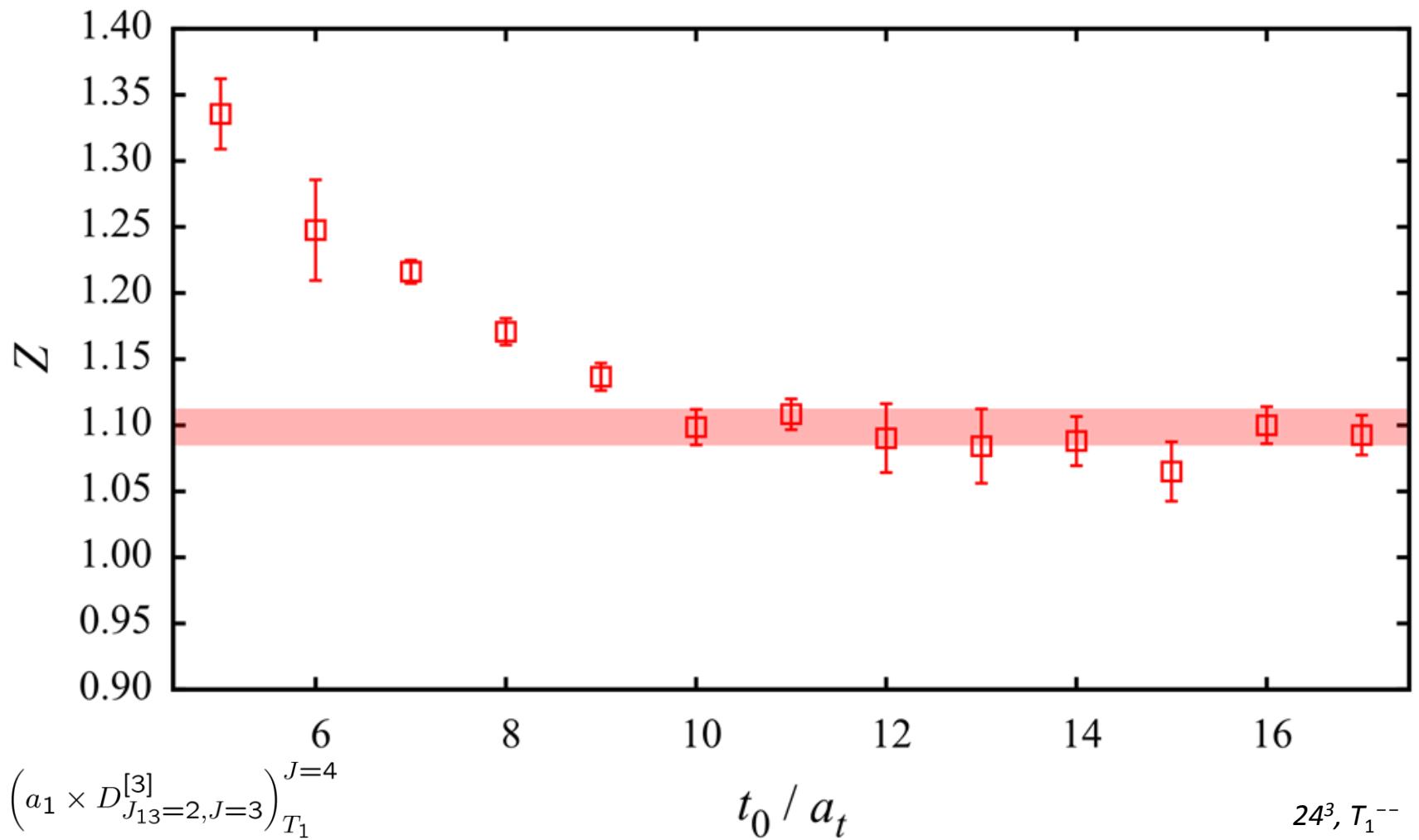
Charmonium systematics – t_0



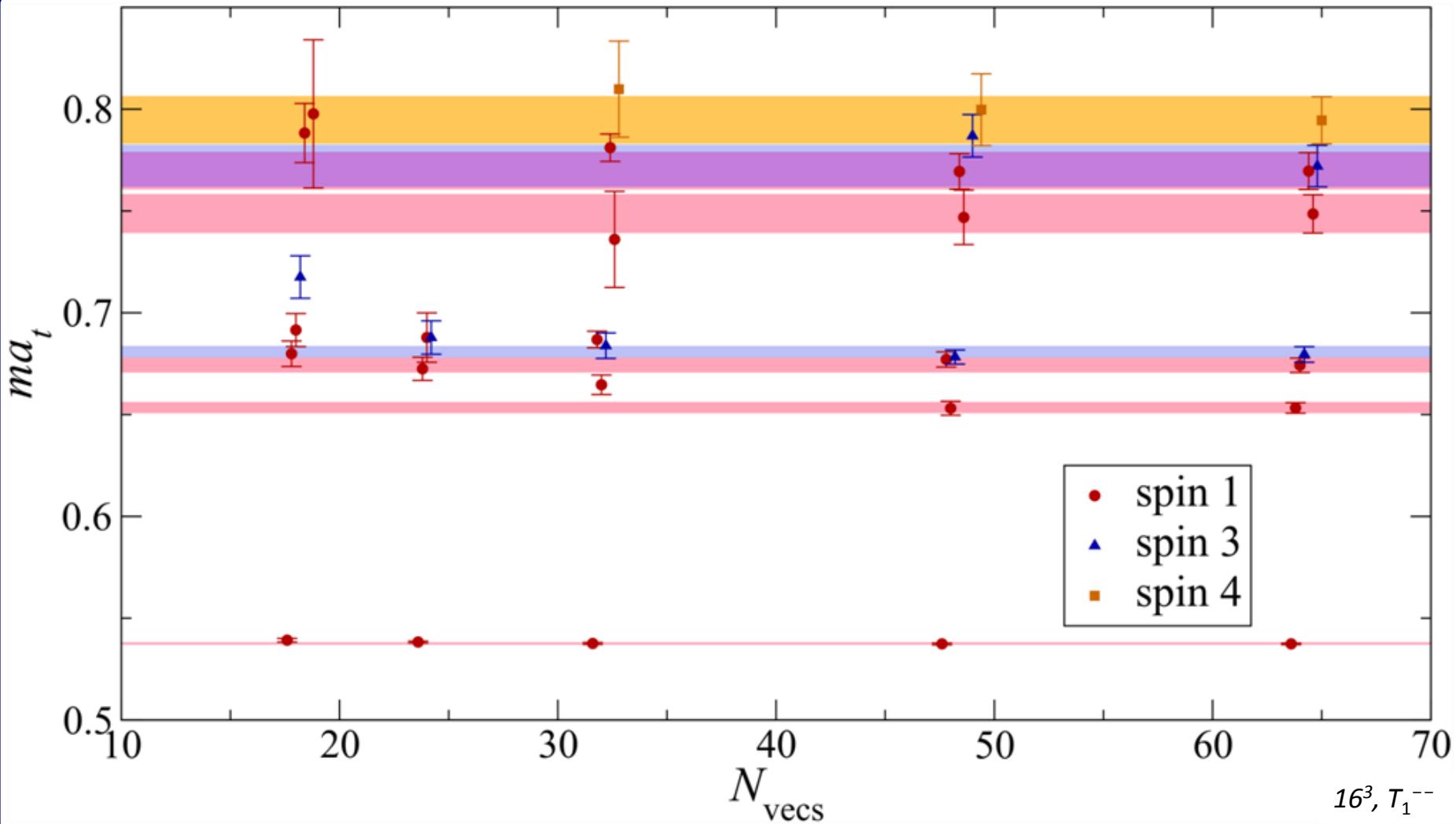
Charmonium systematics – t_0



Charmonium systematics – t_0



Charmonium systematics – N_{vecs}



Charmonium operators

Λ	Λ^{-+}	Λ^{--}	Λ^{++}	Λ^{+-}
A_1	12	6	13	5
A_2	4	6	5	5
T_1	18	26	22	22
T_2	18	18	22	14
E	14	12	17	9

	a_0	π	π_2	b_0	ρ	ρ_2	a_1	b_1
Γ	1	γ_5	$\gamma_0\gamma_5$	γ_0	γ_i	$\gamma_0\gamma_i$	$\gamma_5\gamma_i$	$\gamma_0\gamma_5\gamma_i$

