Exclusive $b \to s\,\ell^+\ell^-$ decays

– First attempts to fly –

Christoph Bobeth

TU Munich – Excellence Cluster Universe

BEACH 2012 – Wichita

I) Introduction to $b \to s\,\ell^+\ell^-$

A) Motivation

B) Effective theory of $b \to s + (\gamma,\, \ell^+\ell^-)$ decays

C) Theoretical approach to exclusive decays

II) Optimised observables in $B \to K^*(\to K\pi)\ell^+\ell^-$

A) Form factor relations

B) @ Large Recoil

C) @ Low Recoil

III) First "global" fits of $b \to s\,\gamma$ and $b \to s\,\ell^+\ell^-$ data

– Introduction –

Motivation

Effective theory

Exclusive decays

Flavour changes in SM – only via $W^{\pm}$ exchange

$U_i = \{u, c, t\}$:

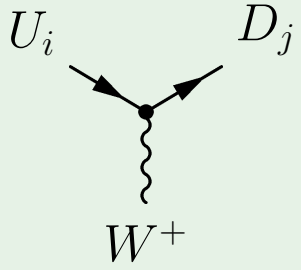
$Q_U = +2/3$

$D_j = \{d, s, b\}$:

$Q_D = -1/3$

$$\mathcal{L}_{\mathrm{CC}} = \frac{g_2}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t}) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma^{\mu} P_L \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^{+}_{\mu}$$

$\sim$ Cabibbo-Kobayashi-Maskawa (CKM) matrix



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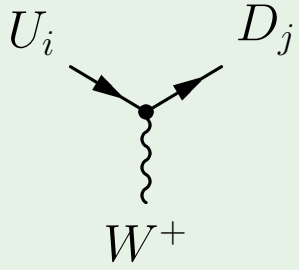

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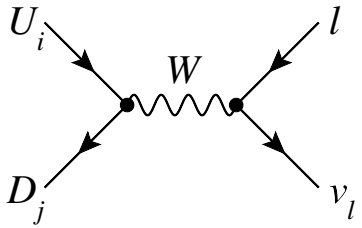

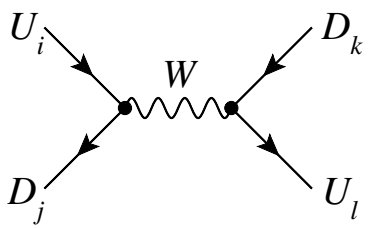

$H \to \ell\nu_\ell$

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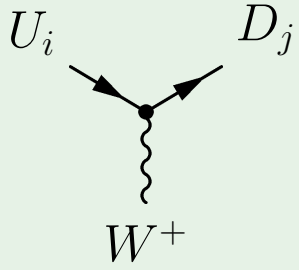

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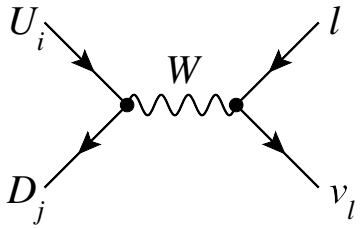

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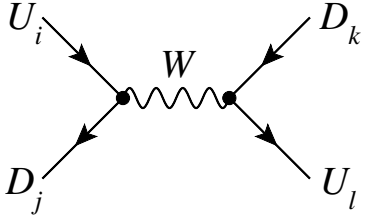

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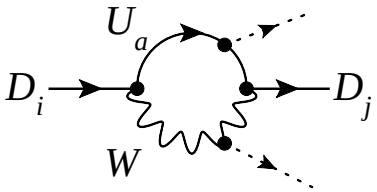

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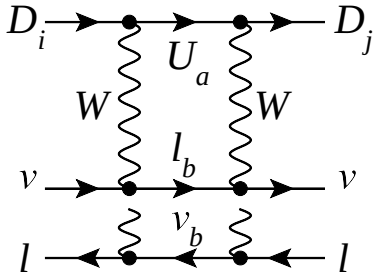

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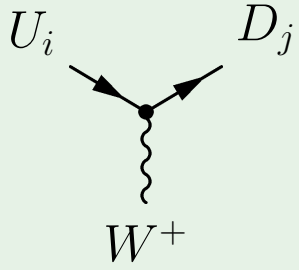

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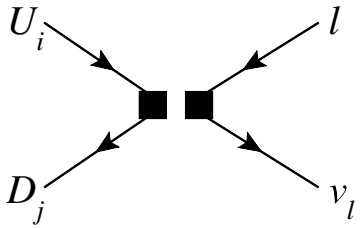

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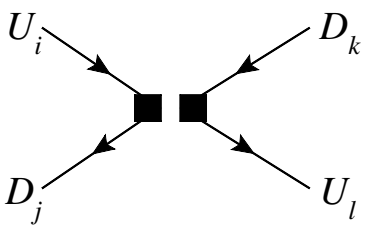

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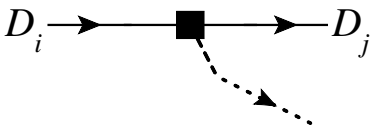

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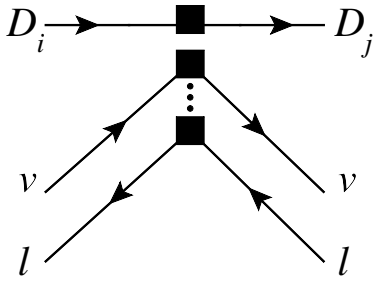

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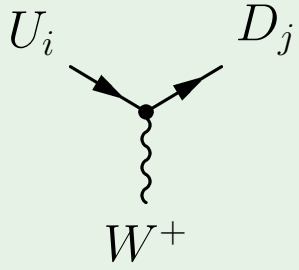


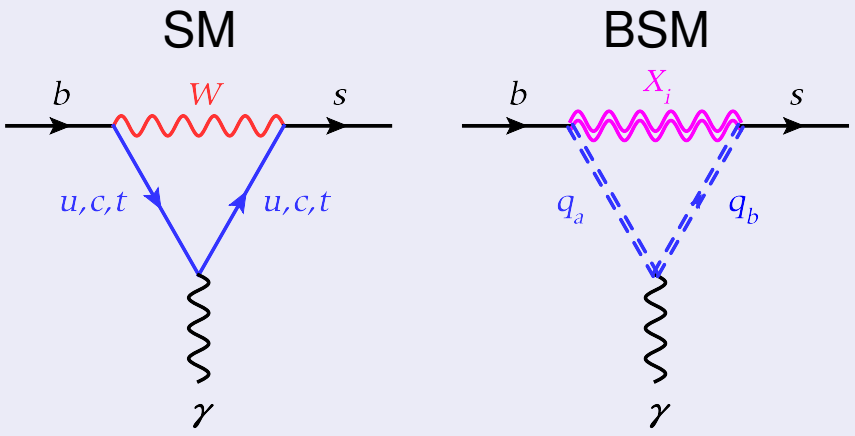
In the SM: FCNC-decays w.r.t. tree-decays are . . .

quantum fluctuations = loop-suppressed

$\Rightarrow$ no suppression of contributions beyond SM (BSM) wrt SM itself

$\Rightarrow$ indirect search for BSM signals

BUT requires high precision, experimentally and theoretically !!!



Experimental data: $b \to s\,\ell^+\ell^-$ – number of events

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- CP-averaged results
- vetoed q^2 region around J/ψ and ψ' resonances
- † unknown mixture of B^0 and $B^\pm$

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Outlook / Prospects

Belle reprocessed all data 711 fb^{-1} $\to$ final analysis ?

CDF recorded about 9.6 fb^{-1} $\to$ final analysis presented at ICHEP 2012

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$\to$ by the end of 2017 about $5 - 7$ fb^{-1}

ATLAS / CMS pursue also analysis of $B \to K^*\mu\bar{\mu}$ and $B \to K\mu\bar{\mu}$

Belle II / SuperB expects about (10-15) K events $B \to K^*\ell\bar{\ell}$ ($\gtrsim$ 2020) [A.J.Bevan arXiv:1110.3901]

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More details on data from

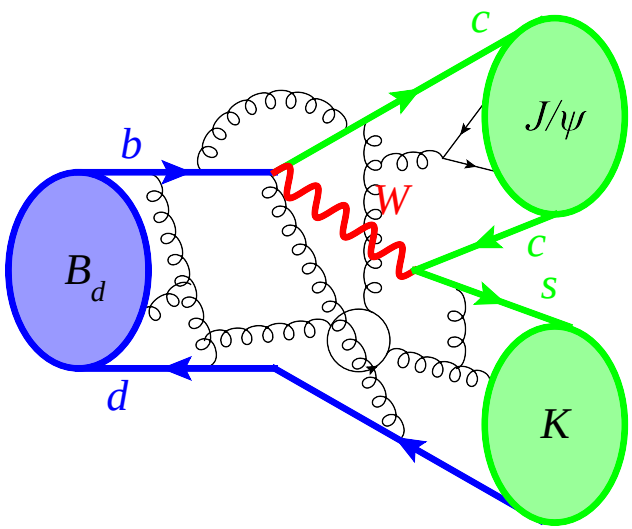
BaBar Justin Albert on Tuesday
CDF Robert F. Harr on Tuesday
LHCb David Hutchcroft on Thursday

B-Hadron decays are a Multi-scale problem . . .

. . . Typical interaction (IA) scales

electroweak IA	$\gg$	hadron in restframe, external momenta	$\gg$	QCD-bound state effects
$M_W \approx 80$ GeV $M_Z \approx 91$ GeV $m_t \approx 172$ GeV		$p_{\text{ext}} \sim M_B \approx 5$ GeV		$\Lambda_{\text{QCD}} \approx 0.5$ GeV

electroweak IA is “short-distance = local” compared to QCD IA



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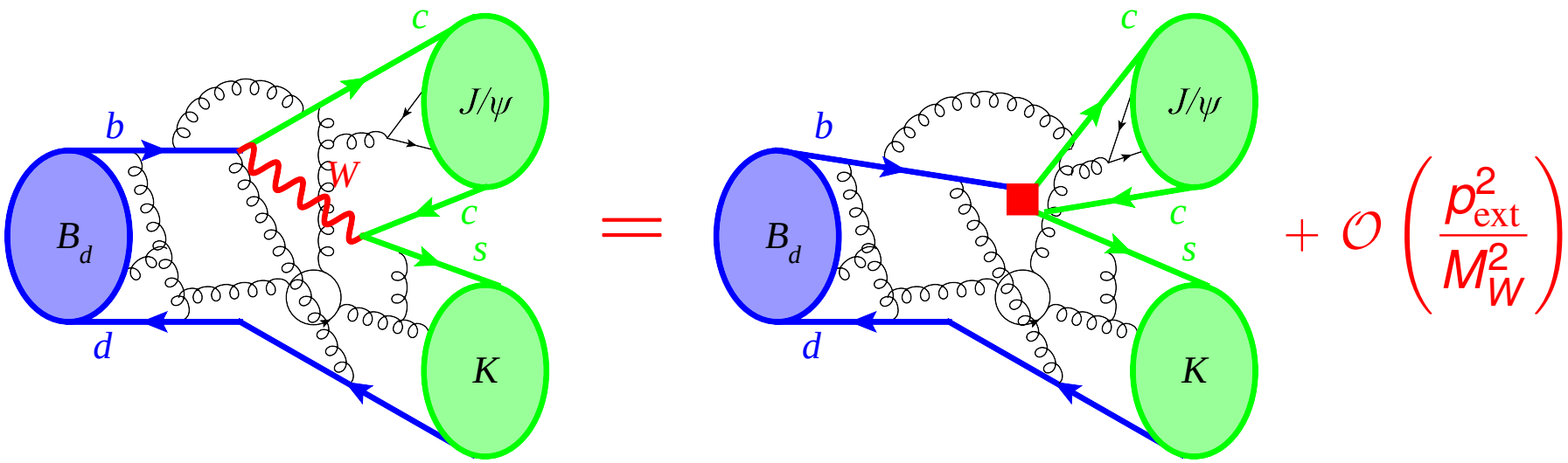
$\Rightarrow$ Effective theory (EFT) of electroweak IA = separation of scales



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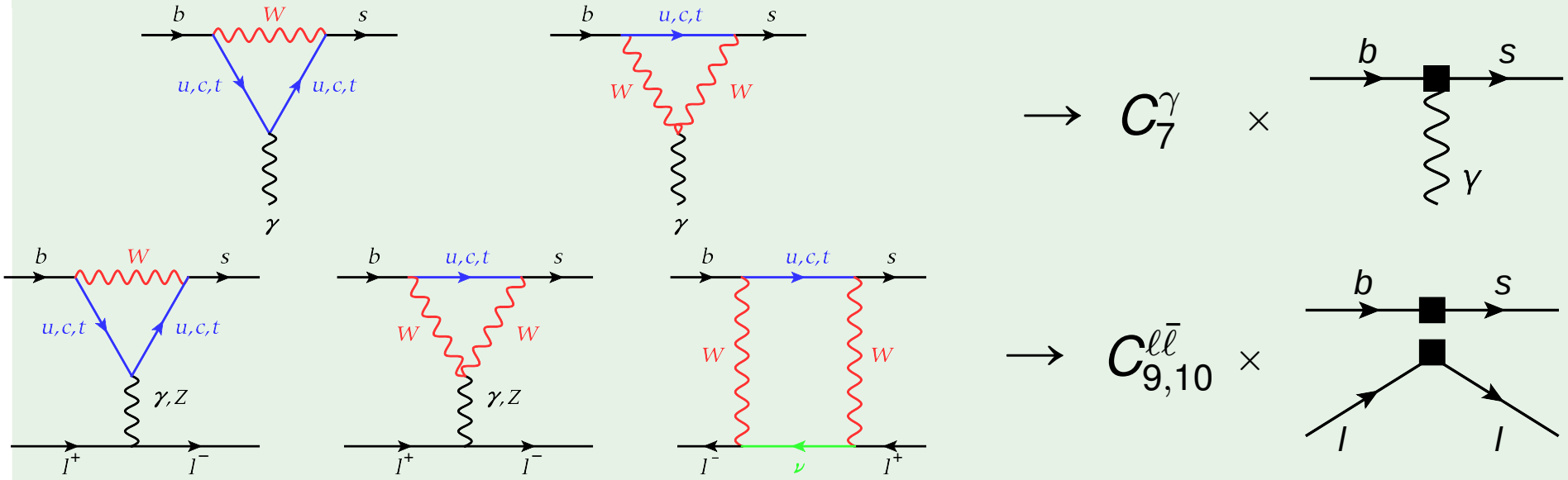

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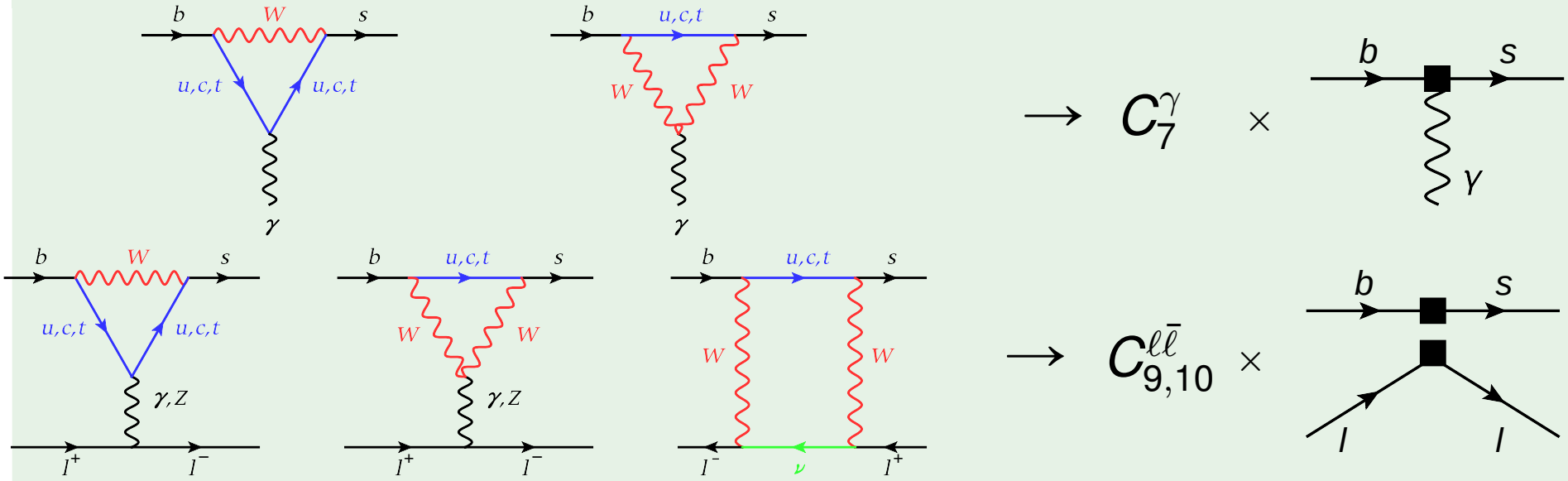

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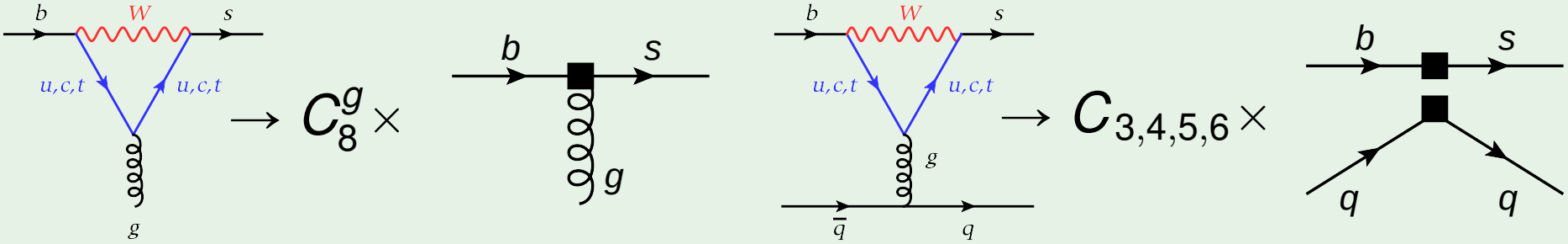
$b \to s + gluon$ and $b \to s + \bar{q}q$



$$\mathcal{O}_8^g = m_b[\bar{s}\,\sigma^{\mu\nu} P_R T^a\, b] G_{\mu\nu}^a \qquad \mathcal{O}_{3,4} = [\bar{s}\,\gamma^\mu(1,\,T^a) P_L\, b] \textstyle\sum_q [\bar{q}\,\gamma_\mu(1,\,T^a)\,q]$$

Extension of EFT beyond the SM ...

$$\mathcal{L}_{\rm eff}\left(\mu_b\right) = \mathcal{L}_{\rm QED\times QCD}\left(u,d,s,c,b,\ e,\mu,\tau,\ ???\right)$$
$$+\frac{4G_F}{\sqrt{2}}V_{\rm CKM}\sum_{\rm SM}(C_i+\Delta C_i)\mathcal{O}_i+\sum_{\rm NP}C_j\mathcal{O}_j(???)$$

- $\Rightarrow$ ΔC_i ... NP contributions to SM C_i
- $\Rightarrow$ $\sum_{\rm NP} C_j\mathcal{O}_j$ ... NP operators (e.g. $C'_{7,9,10}$, $C^{(\prime)}_{S,P}$, ...)
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model-dep. 1) decoupling of new heavy particles @ NP scale: $\mu_{NP} \gtrsim M_W$

2) RG-running to lower scale $\mu_b \sim m_b$ (potentially tower of EFT's)

C_i are correlated, depend on fundamental parameters

model-indep. extending SM EFT-Lagrangian $\rightarrow$ new C_j

C_j are UN-correlated free parameters

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$$+\frac{4G_F}{\sqrt{2}}V_{\rm CKM}\sum_{\rm SM}(C_i+\Delta C_i)\mathcal{O}_i+\sum_{\rm NP}C_j\mathcal{O}_j(???)$$

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$\Rightarrow$ $\sum_{\rm NP} C_j\mathcal{O}_j$ ... NP operators (e.g. $C'_{7,9,10}$, $C^{(')}_{S,P}$, ...)

$\Rightarrow$??? ... additional light degrees of freedom ($\Leftarrow$ not pursued in the following)

model-dep. 1) decoupling of new heavy particles @ NP scale: $\mu_{NP} \gtrsim M_W$

2) RG-running to lower scale $\mu_b \sim m_b$ (potentially tower of EFT's)

C_i are correlated, depend on fundamental parameters

model-indep. extending SM EFT-Lagrangian $\rightarrow$ new C_j

C_j are UN-correlated free parameters

Towards Observables

$\Rightarrow$ EFT universal starting point for calculation of observables

!!! Non-perturbative input required for hadronic matrix elements

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- Heavy Quark Expansion: only few universal non-perturbative parameters from $B \to X_{u,c}\,\ell\bar{\nu}_\ell$ and $B \to X_s\gamma$ photon spectrum
- only @ e^+e^- – B-factories

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Exclusive decays

- $B_{s,d} \to \ell\bar{\ell}$: decay constants $f_{B_{s,d}}$ from QCD-Lattice calculation
- $B \to \{K,\, K^*\} + \ell\bar{\ell}$: ($q^2$ = dilepton invariant mass)
  - @ low-q^2: QCD factorisation (QCDF)
  - @ high-q^2: local OPE of 4-quark contributions
- $B \to \{K,\, K^*\} + \nu\bar{\nu}$

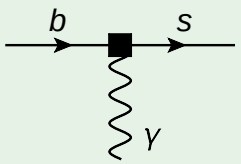
$\Rightarrow$ $B \to \{K,\, K^*\}$ form factors from: Light-Cone Sum Rules LCSR (@ low-q^2)
or QCD-Lattice (@ high-q^2)

... other decays, $\Delta F = 2$, ...

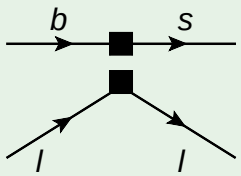
Exclusive $B \to \{K,\, K^*\} + \ell^+\ell^-$

Hadronic amplitude $B \to K^*(\to K\pi)\,\ell^+\ell^-$ neglecting 4-quark operators

$$\mathcal{M} = \langle K\pi |\; C_7 \times$$



$$+ \; C_{9,10} \times$$



$$|B\rangle$$

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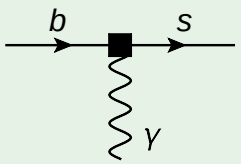

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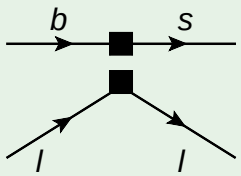

$$|B\rangle$$

$\mathcal{M}$ may expressed in terms of transversity amplitudes ($m_\ell = 0$)

. . . using narrow width approximation & intermediate K^* on-shell

$\Rightarrow$ "just" requires $B \to K^*$ form factors V, $A_{1,2}$, $T_{1,2,3}$ in K^*-transversity amp's:

$$A_\perp^{L,R} \sim \sqrt{2\,\lambda}\left[(C_9 \mp C_{10})\frac{V}{M_B + M_{K*}} + \frac{2\,m_b}{q^2}C_7\,T_1\right],$$

$$A_\parallel^{L,R} \sim -\sqrt{2}\,(M_B^2 - M_{K*}^2)\left[(C_9 \mp C_{10})\frac{A_1}{M_B - M_{K*}} + \frac{2\,m_b}{q^2}C_7\,T_2\right],$$

$$A_0^{L,R} \sim -\frac{1}{2\,M_{K*}\sqrt{q^2}}\left\{(C_9 \mp C_{10})\Big[\ldots A_1 + \ldots A_2\Big] + 2\,m_b C_7\Big[\ldots T_2 + \ldots T_3\Big]\right\}$$

Exclusive $B \to \{K,\, K^*\} + \ell^+\ell^-$

Hadronic amplitude $B \to K^*(\to K\pi)\,\bar{\ell}\ell$ — including 4-quark operators

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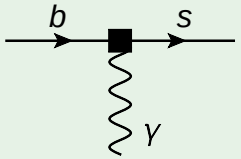
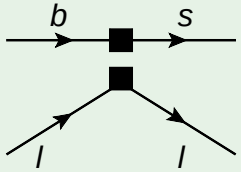
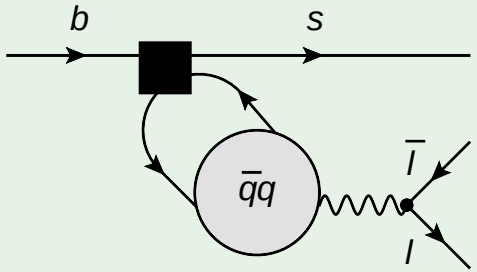


$$+\; C_{9,10} \times$$



$$+ \sum_i C_i \times$$



$$|B\rangle$$

. . . but 4-Quark operators have to be included

- current-current $b \to s + (u\bar{u},\, c\bar{c})$
- QCD-penguin operators $b \to s + q\bar{q}$ $(q = u, d, s, c)$

$\Rightarrow$ large peaking background around $q^2 = (M_{J/\psi})^2,\, (M_{\psi'})^2$:

$B \to K^{(*)}(q\bar{q}) \to K^{(*)}\ell^+\ell^-$

Exclusive $B \to \{K,\, K^*\} + \ell^+\ell^-$

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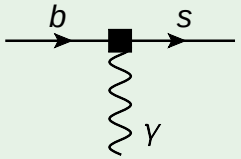

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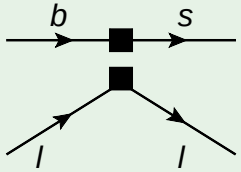

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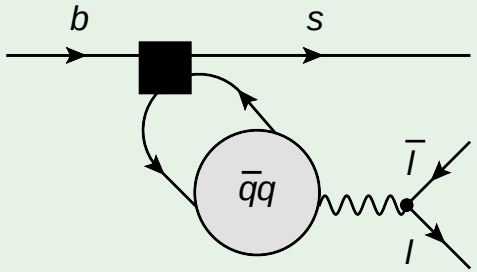

$$|B\rangle$$

q^2 - regions in $b \to s\,\ell^+\ell^-$

$K^{(*)}$-energy in B-rest frame: $E_{K^{(*)}} = (M_B^2 + M_{K^{(*)}}^2 - q^2)/(2\,M_B)$

q^2-region	low-q^2: $q^2 \ll M_B^2$	high-q^2: $q^2 \sim M_B^2$
$K^{(*)}$-recoil	large recoil: $E_{K^{(*)}} \sim M_B/2$	low recoil: $E_{K^{(*)}} \sim M_{K^{(*)}} + \Lambda_{\rm QCD}$
theory method	QCDF, nl OPE: $q^2 \in [1, 6]$ GeV^2	OPE + HQET: $q^2 \geqslant (14 \ldots 15)$ GeV^2

[QCDF: Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]
[non-local OPE: Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]
[local OPE: Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]

Exclusive $B \to \{K,\, K^*\} + \ell^+\ell^-$

Hadronic amplitude $B \to K^*(\to K\pi)\,\bar{\ell}\ell$ — including 4-quark operators

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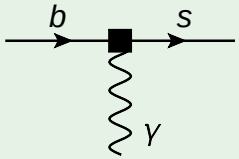

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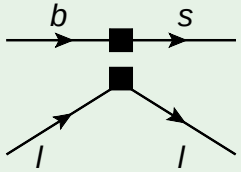

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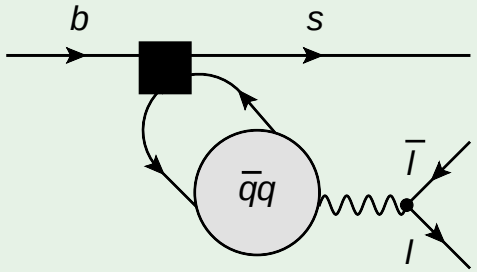
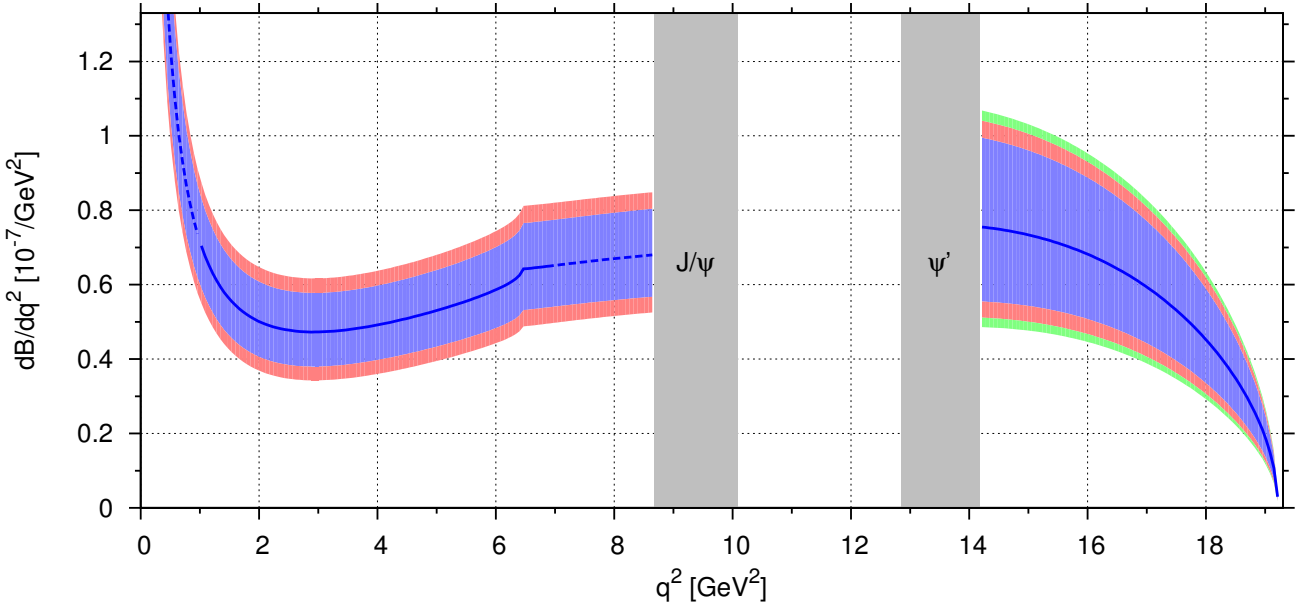


$$|B\rangle$$

$\longrightarrow$ vetoed in experiment

$\mathrm{d}Br[B \to K^*\bar{\ell}\ell]/\mathrm{d}q^2$



Open Issues

- $B \to K$ and $B \to K^*$ form factors at high-q^2 (from Lattice)
  preliminary results without final uncertainty estimate:
  [Liu/Meinel/Hart/Horgan/Müller/Wingate arXiv:0911.2370, 1101.2726]
- better understanding of sub-leading contributions
  1) QCD factorization at low-q^2
  2) OPE at high-q^2 - known up to sub-leading form factors (Lattice?)
  [Grinstein/Pirjol hep-ph/0404250; Beylich/Buchalla/Feldmann arXiv:1101.5118]
- inclusion of $c\bar{c}$-tails at low-q^2 in numerical evaluation
  [Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]
- non-P wave $K\pi$ background to $K\pi$ pairs from K^* at high experimental statistics ???

$\Rightarrow$ Last point addressed recently: [Becirevic/Tayduganov arXiv:1207.4004]

S-wave $K\pi$ pairs from $B \to K_0^* \ell^+ \ell^-$: negligible @ high-$q^2$,

error below 10% for $q^2 \lesssim 1$ GeV^2 and 4 GeV$^2 \lesssim M_{J\psi}^2$,

upto 25% around $q^2 \approx 2$ GeV^2 (depending on observable)

– Optimised Observables –

in $B \to K^*(\to K\pi)\ell^+\ell^-$

@ Large Recoil = low-q^2

@ Low Recoil = high-q^2

$B \to K^* [\to K\pi] + \ell^+\ell^-$:

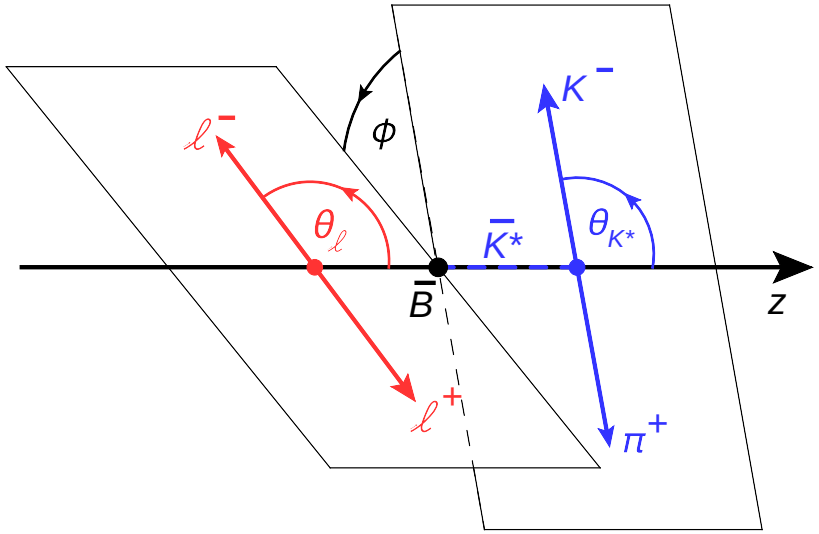
4-body decay with intermediate on-shell K^* (vector)

1) $q^2 = m^2_{\ell\bar{\ell}} = (p_\ell + p_{\bar{\ell}})^2 = (p_B - p_{K*})^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_B, \vec{p}_\ell)$ in $(\ell\bar{\ell})$ – c.m. system

3) $\cos\theta_K$ with $\theta_K \angle(\vec{p}_B, \vec{p}_K)$ in $(K\pi)$ – c.m. system

4) $\phi \angle(\vec{p}_K \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



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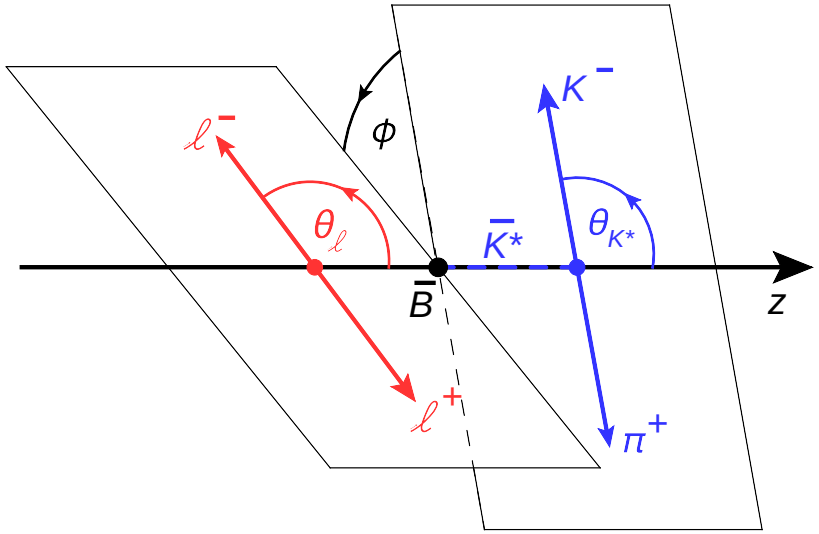

$J_i(q^2)$ = "Angular Observables"

$$\frac{32\pi}{9}\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_\ell$$
$$+ J_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi + J_5 \sin 2\theta_K \sin\theta_\ell \cos\phi$$
$$+ (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_\ell + J_7 \sin 2\theta_K \sin\theta_\ell \sin\phi$$
$$+ J_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi + J_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi$$

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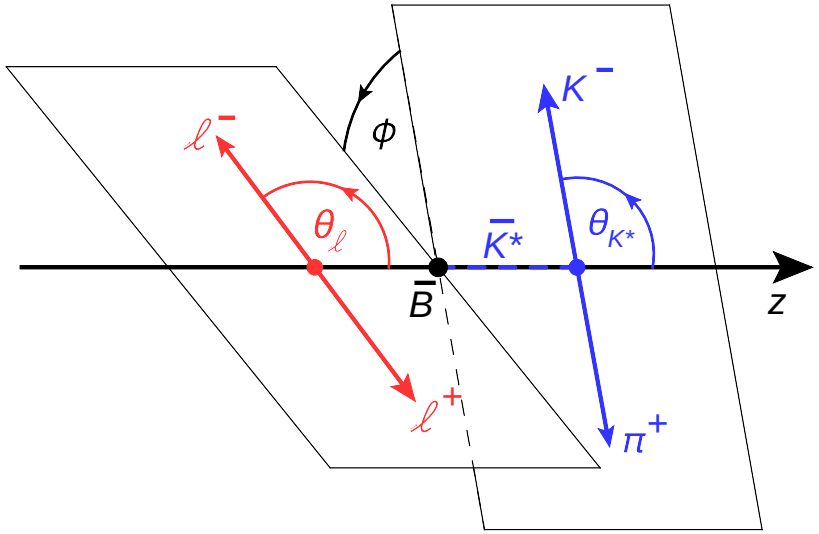

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$\Rightarrow$ "$2 \times (12 + 12) = 48$" if measured separately: A) decay + CP-conj and B) for $\ell = e, \mu$

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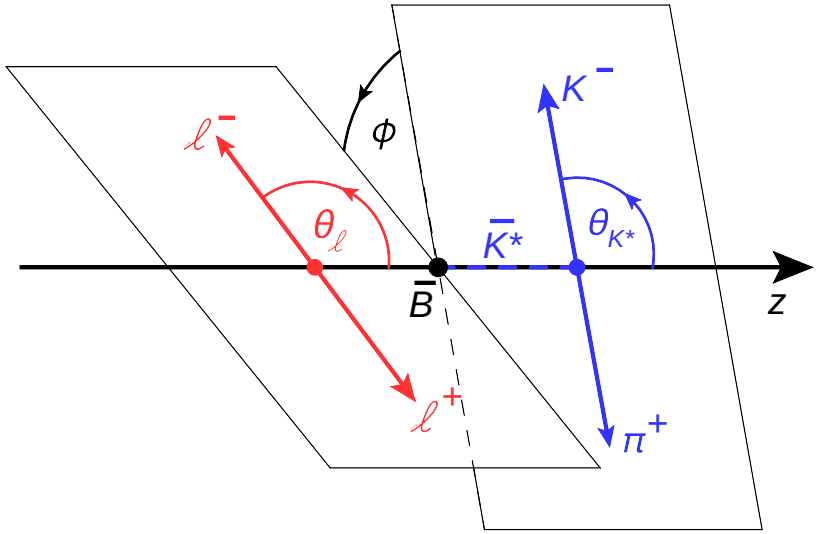

CP-conj. decay $B^0 \to K^{*0}(\to K^+\pi^-)\ell^+\ell^-$: $d^4\bar{\Gamma}$ from $d^4\Gamma$ by replacing

$$\text{CP-even} \quad : \quad J_{1,2,3,4,7} \quad \longrightarrow \quad + \bar{J}_{1,2,3,4,7}[\delta_W \to -\delta_W]$$

$$\text{CP-odd} \quad : \quad J_{5,6,8,9} \quad \longrightarrow \quad - \bar{J}_{5,6,8,9}[\delta_W \to -\delta_W]$$

with weak phases δ_W conjugated

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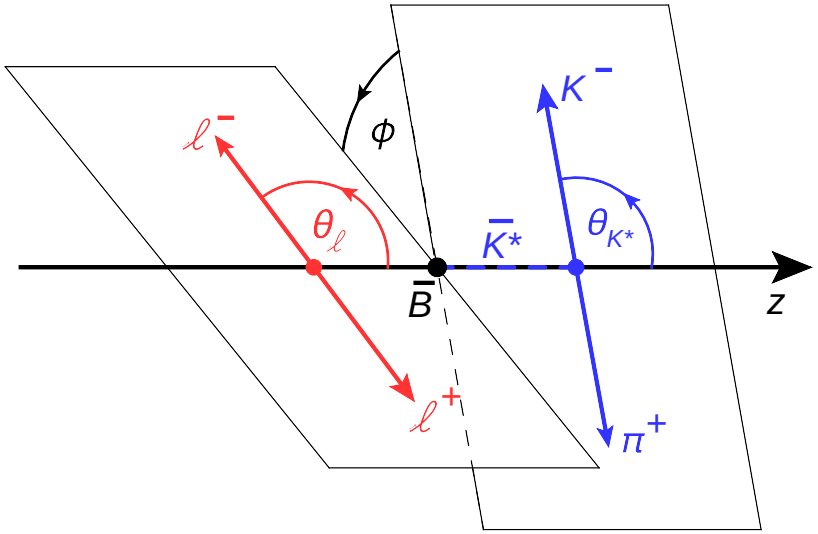

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with weak phases δ_W conjugated

1) CP-odd : $A_{\rm CP} \sim (J_i - \bar{J}_i) \sim d^4(\Gamma + \bar{\Gamma})$ = flavour-untagged B samples

2) (naive) T-odd $J_{7,8,9}$: $A_{\rm CP} \sim \cos\delta_s \sin\delta_W \to$ not suppressed by small strong phases δ_s

[CB/Hiller/Piranishvili arXiv:0805.2525, Altmannshofer et al. arXiv:0811.1214]

Angular observables

$$J_i(q^2) \sim \{\mathrm{Re}, \mathrm{Im}\} \left[A_m^{L,R} \left(A_n^{L,R} \right)^* \right]$$

$$\sim \sum_a (C_a F_a) \sum_b (C_b F_b)^*$$

$A_m^{L,R} \ldots K^*$-transversity amplitudes $m = \perp, \|, 0$

$C_a \ldots$ short-distance coefficients

$F_a \ldots$ form factors

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$A_m^{L,R}$ ... K^*-transversity amplitudes $m = \perp, \|, 0$

C_a ... short-distance coefficients

F_a ... form factors

simplify when using form factor relations:

$m_b \to \infty$ limit: [Isgur/Wise PLB232 (1989) 113, PLB237 (1990) 527]

$$T_1 \approx V, \qquad T_2 \approx A_1, \qquad T_3 \approx A_2 \frac{M_B^2}{q^2}$$

large K^* recoil limit: $E_{K*} \sim M_B$ [Charles et al. hep-ph/9812358, Beneke/Feldmann hep-ph/0008255]

$$\xi_\perp \equiv \frac{M_B}{M_B + M_{K*}} V \approx \frac{M_B + M_{K*}}{2E_{K*}} A_1 \approx T_1 \approx \frac{M_B}{2E_{K*}} T_2$$

$$\xi_\| \equiv \frac{M_B + M_{K*}}{2E_{K*}} A_1 - \frac{M_B - M_{K*}}{M_{K*}} A_2 \approx \frac{M_B}{2E_{K*}} T_2 - T_3$$

Sub-leading corrections to TransAmp's

$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil

$$A_i^{L,R} \sim C^{L,R} \times f_i \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \parallel, 0$)

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K*}} V, \quad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K*})\,A_1, \quad f_0 = \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K*}\,(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

Sub-leading corrections to TransAmp's

$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil

FF symmetry breaking

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda)$$

$$C^{L,R} = (C_9 \mp C_{10}) + \kappa \frac{2m_b^2}{q^2} C_7,$$

1 SD-coefficient $C^{L,R}$ and 3 FF's f_i ($i = \perp, \|, 0$)

$$C_7^{\rm SM} \approx -0.3,\ C_9^{\rm SM} \approx 4.2,\ C_{10}^{\rm SM} \approx -4.2$$

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("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

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$\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$

Low hadronic recoil

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Large hadronic recoil

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2 SD-coefficients $C_{\perp,\parallel}^{L,R}$ and 2 FF's $\xi_{\perp,\parallel}$

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$\Rightarrow$ limited, end-point-divergences at $\mathcal{O}(\lambda)$

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“Transversity” Observables @ Large Recoil . . .

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. . . in order to have reduced form factor $\xi_{\perp,\parallel}$ uncertainty

$$A_T^{(2)} = \frac{|A_\perp^L|^2 + |A_\perp^R|^2 - |A_\parallel^L|^2 - |A_\parallel^R|^2}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2},$$

$$A_T^{(3)} = \frac{|A_0^L A_\parallel^{L*} + A_0^{R*} A_\parallel^R|}{\sqrt{\left(|A_0^L|^2 + |A_0^R|^2\right)\left(|A_\perp^L|^2 + |A_\perp^R|^2\right)}},$$

$$A_T^{(4)} = \frac{|A_0^L A_\perp^{L*} - A_0^{R*} A_\perp^R|}{|A_0^{L*} A_\parallel^L + A_0^R A_\parallel^{R*}|},$$

$$A_T^{(5)} = \frac{|A_\parallel^L A_\perp^{R*} + A_\perp^L A_\parallel^{R*}|}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2},$$

$$A_T^{(\mathrm{re})} = \frac{2\,\mathrm{Re}\left[A_\parallel^L A_\perp^{L*} - A_\parallel^R A_\perp^{R*}\right]}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2},$$

$$A_T^{(\mathrm{im})} = \frac{2\,\mathrm{Im}\left[A_\parallel^L A_\perp^{L*} + A_\parallel^R A_\perp^{R*}\right]}{|A_\perp^L|^2 + |A_\perp^R|^2 + |A_\parallel^L|^2 + |A_\parallel^R|^2}$$

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece 0807.2589 + 1005.0571, Becirevic/Schneider 1106.3283]

. . . and extended operator basis in [Matias/Mescia/Ramon/Virto arXiv:1202.4266]

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$$A_T^{(4)} = \sqrt{\frac{J_5^2 + (2\,J_8)^2}{(2\,J_4)^2 + J_7^2}}, \qquad A_T^{(5)} = \frac{\sqrt{16\,J_{1s}^2 - 9\,J_{6s}^2 - 36\,(J_3^2 + J_9^2)}}{8\,J_{1s}},$$

$$A_T^{(\mathrm{re})} = \frac{J_{6s}}{4\,J_{2s}}, \qquad A_T^{(\mathrm{im})} = \frac{J_9}{2\,J_{2s}}$$

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Observables @ Low Recoil . . .

[CB/Hiller/van Dyk arXiv:1006.5013, 1105.0376]

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$$H_T^{(1)} = \frac{\sqrt{2} J_4}{\sqrt{-J_{2c}\left(2J_{2s} - J_3\right)}} = \text{sgn}(f_0)$$

test OPE framework →
duality violating contributions

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"long-distance free"

$$H_T^{(2)} = \frac{J_5}{\sqrt{-2J_{2c}\left(2J_{2s} + J_3\right)}} \qquad H_T^{(3)} = \frac{J_{6s}/2}{\sqrt{(2J_{2s})^2 - (J_3)^2}}$$

. . . and "long-distance free" CP-asymmetries $a_{\text{CP}}^{(1,2,3)}$

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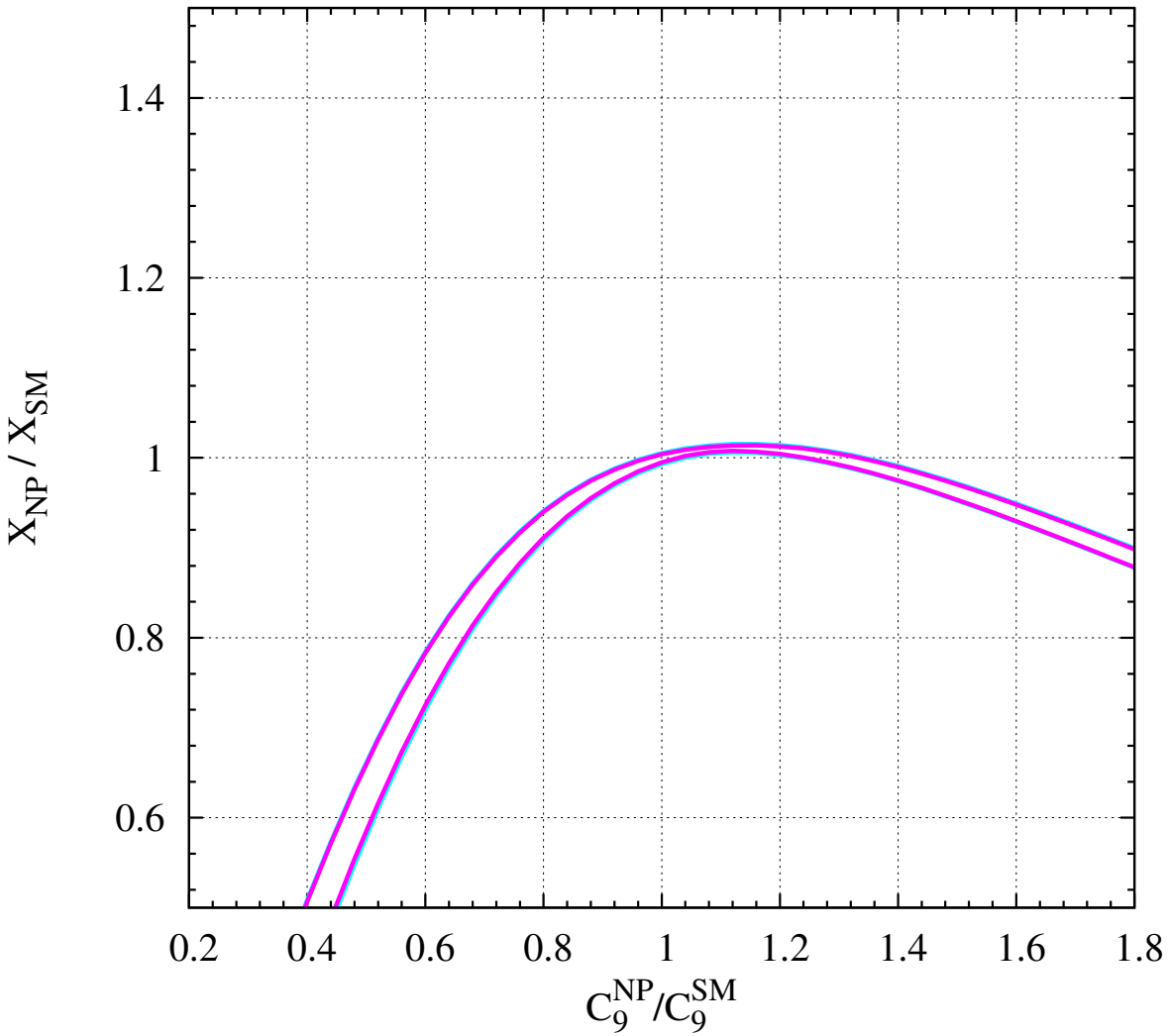
"short-distance free" → measure form factors $f_{0,\|,\perp}$ (SM-operator basis only)

$$\frac{f_0}{f_\|} = \frac{\sqrt{2} J_5}{J_{6s}} = \frac{-J_{2c}}{\sqrt{2} J_4} = \frac{\sqrt{2} J_4}{2J_{2s} - J_3} = \sqrt{\frac{-J_{2c}}{2J_{2s} - J_3}} = \frac{\sqrt{2} J_8}{-J_9},$$

$$\frac{f_\perp}{f_\|} = \sqrt{\frac{2J_{2s} + J_3}{2J_{2s} - J_3}} = \frac{\sqrt{-J_{2c}\,(2J_{2s} + J_3)}}{\sqrt{2} J_4}, \qquad \frac{f_0}{f_\perp} = \sqrt{\frac{-J_{2c}}{2J_{2s} + J_3}}$$

Sensitivity of $H_T^{(2,3)}$ – example: real C_9



- varying C_9 around SM value C_9^{SM}
- $X_{[q^2_{min}, q^2_{max}]} = \int_{q^2_{min}}^{q^2_{max}} X(q^2) dq^2$
- observables X_{NP} normalised to SM-prediction X_{SM}
- bands = theory uncertainties

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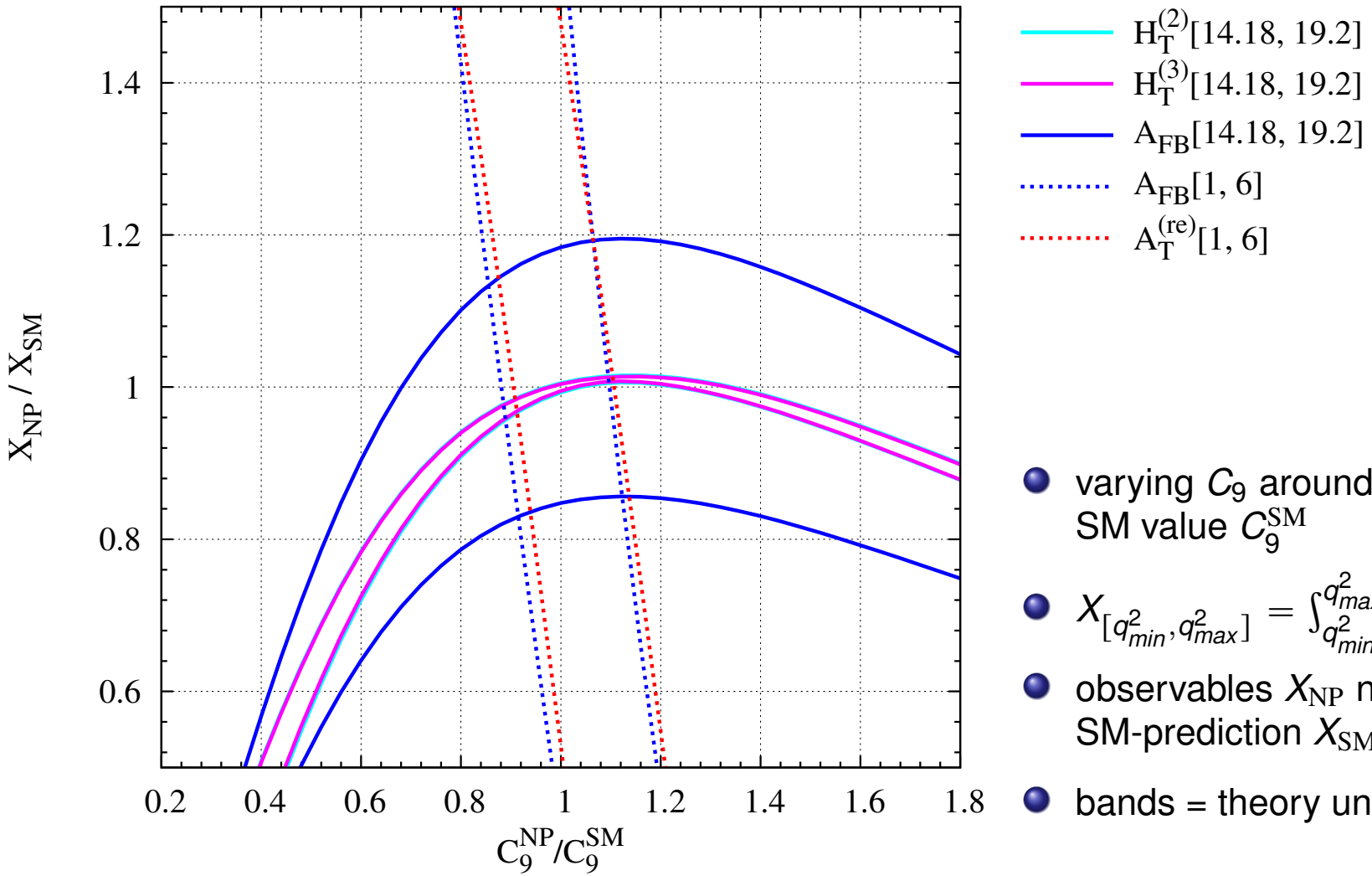

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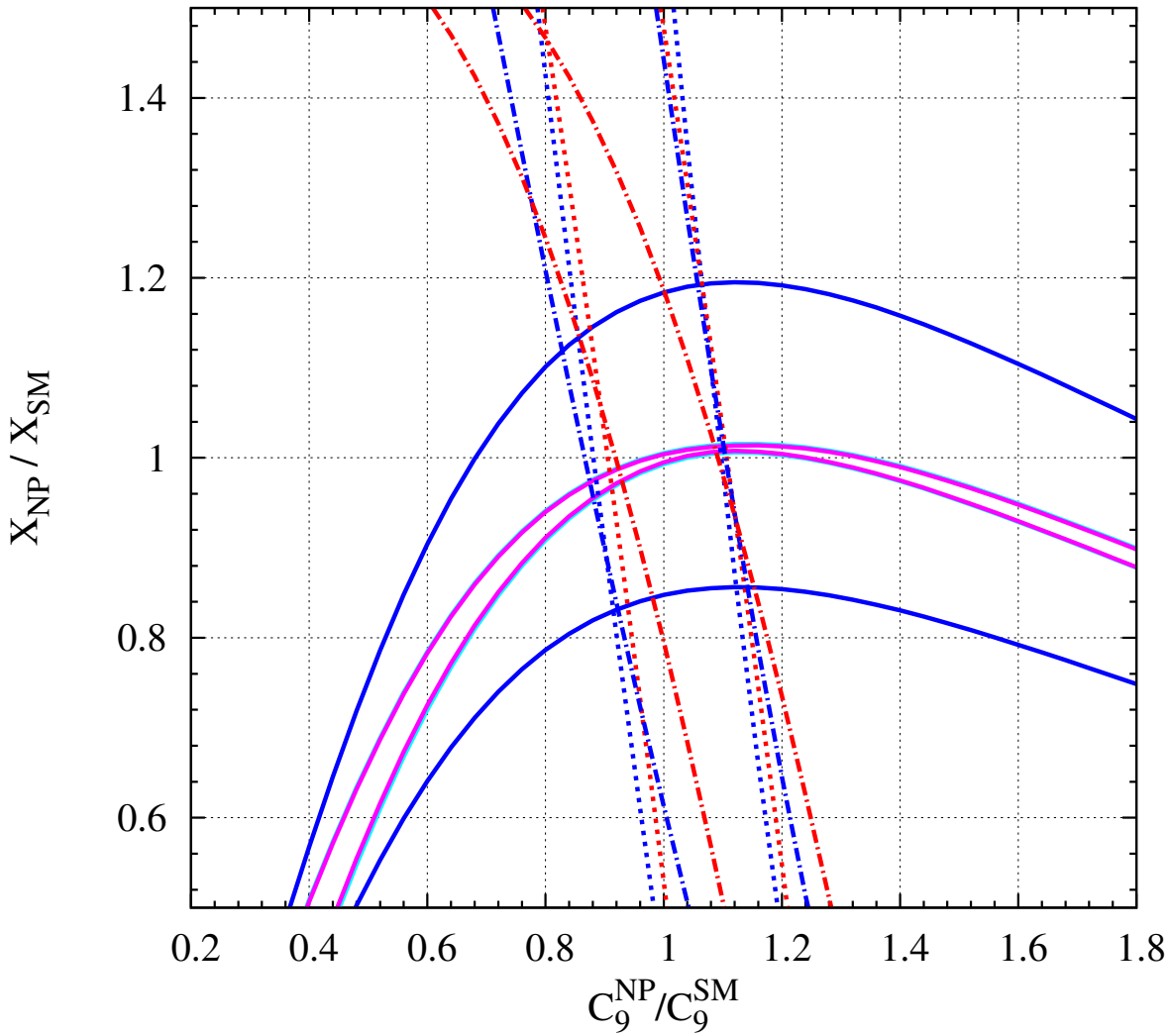

$H_T^{(2)}[14.18, 19.2]$
$H_T^{(3)}[14.18, 19.2]$
$A_{FB}[14.18, 19.2]$
$A_{FB}[1, 6]$
$A_T^{(re)}[1, 6]$
$A_{FB}[2, 4.3]$
$A_T^{(re)}[2, 4.3]$

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Towards a global analysis of rare $\Delta B = 1$ decays

– Model-independent –

"Global Fit" = combination of $b \to s + (\gamma,\ \ell^+\ell^-)$ observables

Parameters of interest

$$\vec{\theta} = (C_i)$$

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Nuisance parameters

$\vec{\nu}$

1) process-specific

FF's, decay const's,
LCDA pmr's,
sub-leading Λ/m_b,
renorm. scales: $\mu_{b,0}$

2) general

quark masses, CKM, ...

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$$O(\vec{\theta}, \vec{\nu})$$

depend usually on sub-set of $\vec{\theta}$ and $\vec{\nu}$

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$$\text{pdf}(O = o)$$

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Fit strategies: 1) Put theory uncertainties in likelihood:

- sample $\vec{\theta}$-space (grid, Markov Chain, importance sampling...)
- theory uncertainties of O_i at each $(\vec{\theta})_i$: vary $\vec{\nu}$ within some ranges $\Rightarrow \sigma_{\text{th}}(O[(\vec{\theta})_i])$
- use Frequentist or Bayesian method $\Rightarrow$ 68 & 95 % (CL or probability) regions of $\vec{\theta}$

$$\chi^2 = \sum \frac{(O_{\text{ex}} - O_{\text{th}})^2}{\sigma_{\text{ex}}^2 + \sigma_{\text{th}}^2}$$

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Fit strategies: 2) Fit also nuisance parameters:

- sample $(\vec{\theta} \times \vec{\nu})$-space (grid, Markov Chain, importance sampling...)
- accounts for theory uncertainties by fitting also $(\vec{\nu})_i$
- use Frequentist or Bayesian method $\Rightarrow$ 68 & 95 % (CL or probability) regions of $\vec{\theta}$ and $\vec{\nu}$

Strategy 1)

$\Rightarrow$ Model-independent analysis with different sets of operators

$\Rightarrow$ Using inclusive and exclusive $b \to s + (\gamma,\ \ell^+\ell^-)$ data

- Descotes-Genon/Ghosh/Matias/Ramon arXiv:1104.3342
- CB/Hiller/van Dyk arXiv:1105.0376
- Altmannshofer/Paradisi/Straub arXiv:1111.1257
- CB/Hiller/van Dyk/Wacker arXiv:1111.2558

- Becirevic/Kosnik/Mescia/Schneider arXiv:1205.5811
- Altmannshofer/Straub arXiv:1206.0273
- Becirevic/Kou/Le Yaounac/Tayduganov arXiv:1206.1502
- Hurth/Mahmoudi arXiv:1207.0688
- Descotes-Genon/Matias/Ramon/Virto arXiv:1207.2753

...

Strategy 2) Beaujean/CB/van Dyk/Wacker arXiv:1205.1838

- theory uncertainties = nuisance parameters $\Rightarrow$ include them in the fit and profit from "short-distance" free observables @ low recoil = 'fitting form factors"
- use Bayes theorem = Bayesian inference
- based on Population MC (PMC) [Cappé et al. arXiv:0710.4242; Kilbinger et al. arXiv:0912.1614, 1101.0950]
  1) to avoid problems of Markov chains in presence of multi-modal posterior
  2) allows for parallelized evaluation of likelihood
- Flavour tool "EOS": observables for http://project.het.physik.tu-dortmund.de/eos/

$$B \to K^*\gamma, \quad B \to K\,\ell^+\ell^-, \quad B \to K^*\ell^+\ell^-, \quad B_s \to \mu^+\mu^-$$

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Bayes Theorem – update knowledge given some data D and a model M

$$P\left(\vec{\theta},\vec{\nu}\,|\,D,M\right) = \frac{P\left(D,M\,|\,\vec{\theta},\vec{\nu}\right)\,P\left(M\,|\,\vec{\theta},\vec{\nu}\right)}{Z}$$

- $P\left(M\,|\,\vec{\theta},\vec{\nu}\right)$: probability of pmr's $(\vec{\theta},\vec{\nu})$ in model M (prior = the "subjective" part)
- $P\left(D,M\,|\,\vec{\theta},\vec{\nu}\right)$: likelihood of the data D in model M given the pmr's $(\vec{\theta},\vec{\nu})$
- Normalisation factor: Z = evidence

$$Z = \int \mathrm{d}\vec{\theta}\,\mathrm{d}\vec{\nu}\,P\left(D,M\,|\,\vec{\theta},\vec{\nu}\right)\,P\left(M\,|\,\vec{\theta},\vec{\nu}\right)$$

$\Rightarrow$ allows model comparison among M_1, M_2 ...

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Priors

1) flat priors for Wilson coefficients
2) gaussian (symmetric) / LogGamma (asymmetric) priors for

- CKM and quark-mass input
- form factor results from LCSR at low-q^2, only extrapolation to high-q^2 [Ball/Zwicky hep-ph/0412079, Khodjamirian et al. arXiv:1006.4945]
- parametrization of lacking sub-leading contributions @ low- and high-q^2

$\Rightarrow$ about $\mathcal{O}(30)$ nuisance parameters

$\Rightarrow$ test prior dependence

Parameters of interest $C_i(4.2\,\mathrm{GeV})$ - 2D marginalised posterior

→ individual constraints at 95 % CR from

$B \to K^* \gamma$ and

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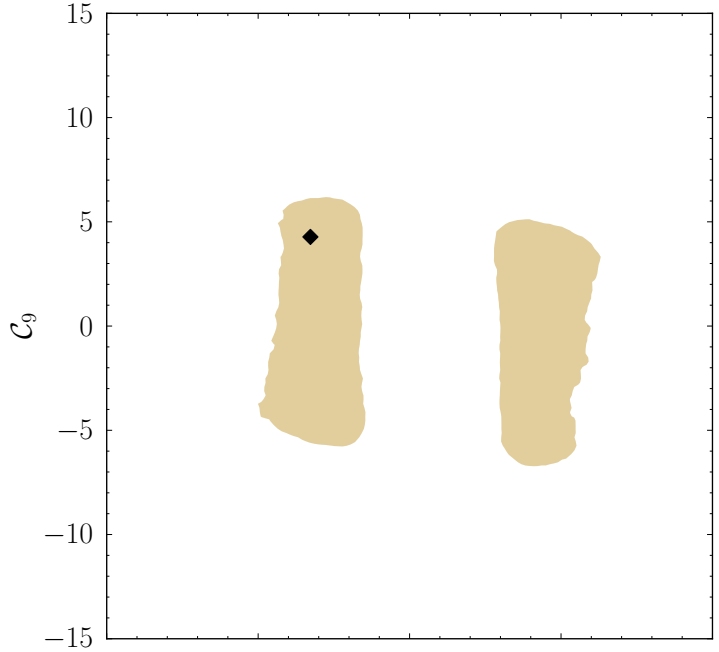
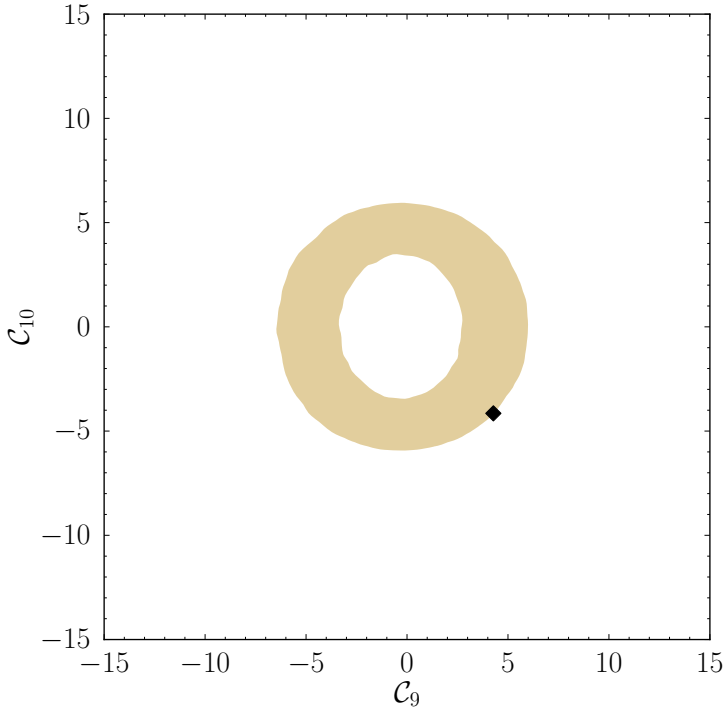
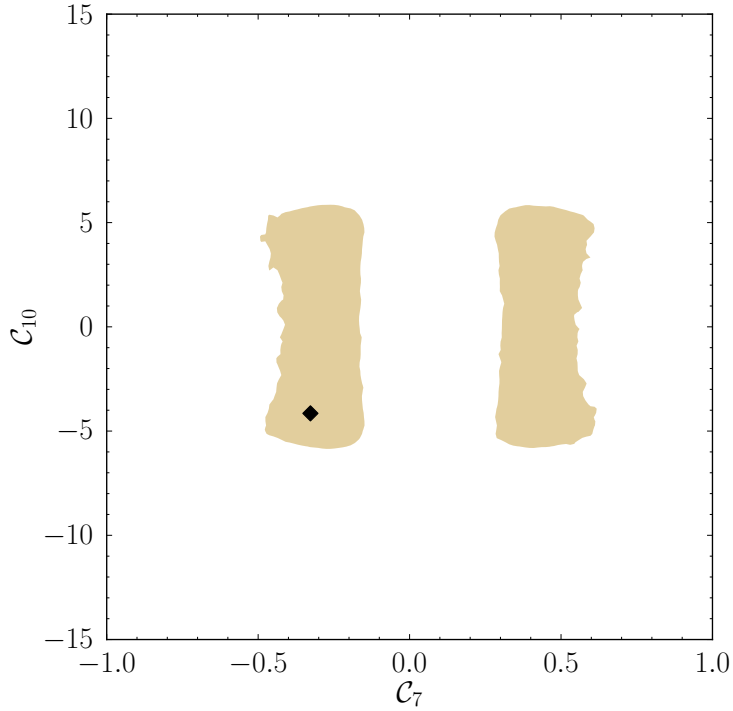




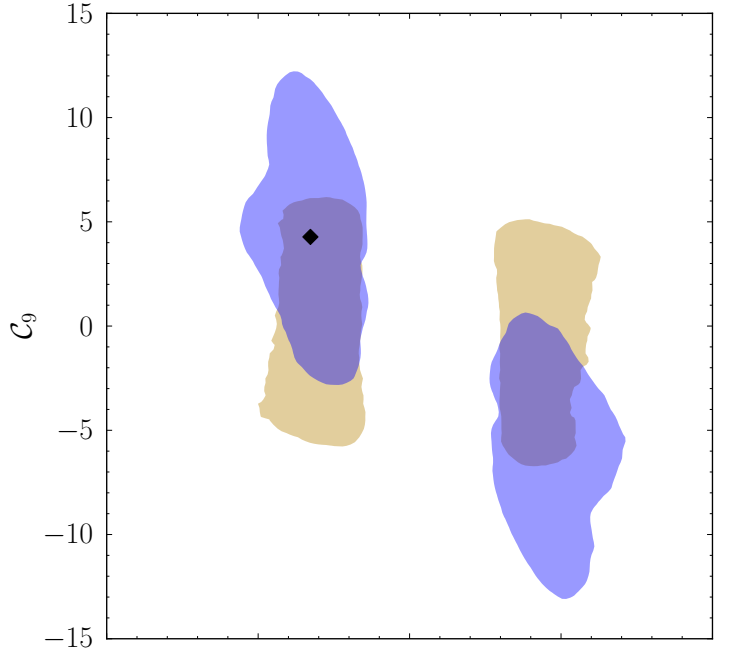


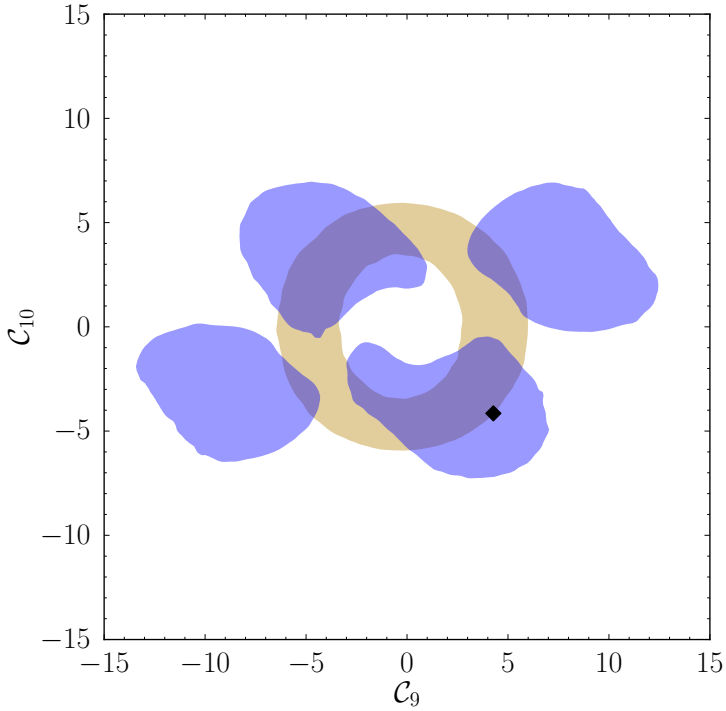
Parameters of interest $C_i(4.2\,\text{GeV})$ - 2D marginalised posterior

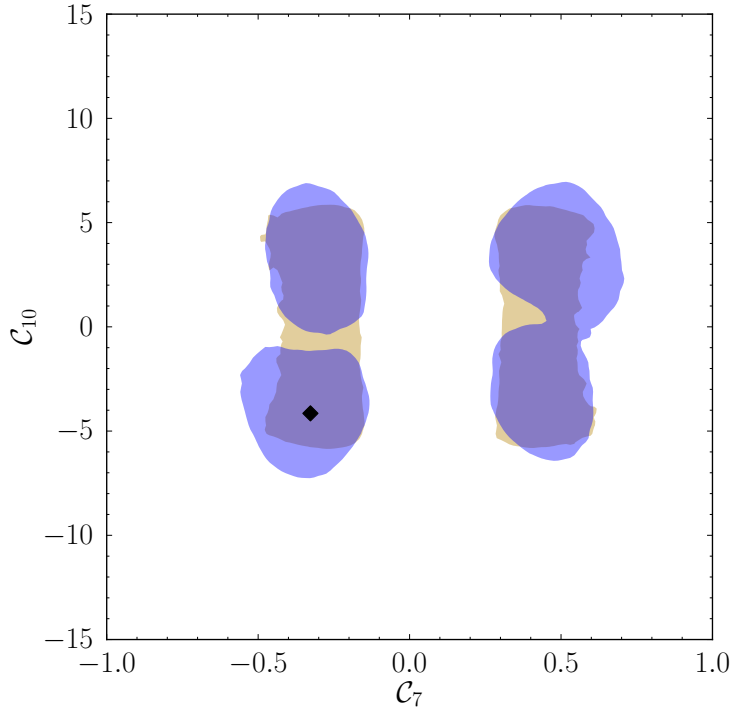
→ individual constraints at 95 % CR from

$B \to K^*\gamma$ and lo+hi-q^2 $B \to K\bar{\ell}\ell$

lo-q^2 $B \to K^*\bar{\ell}\ell$







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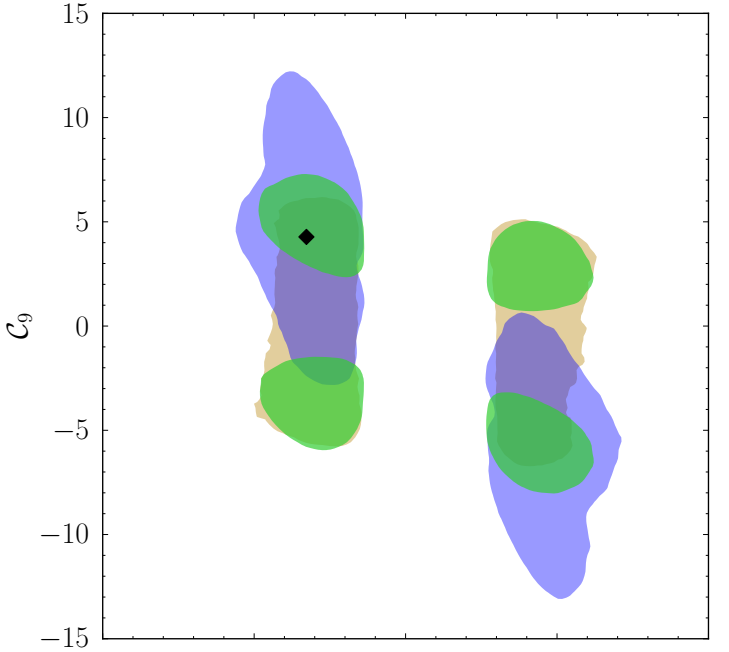
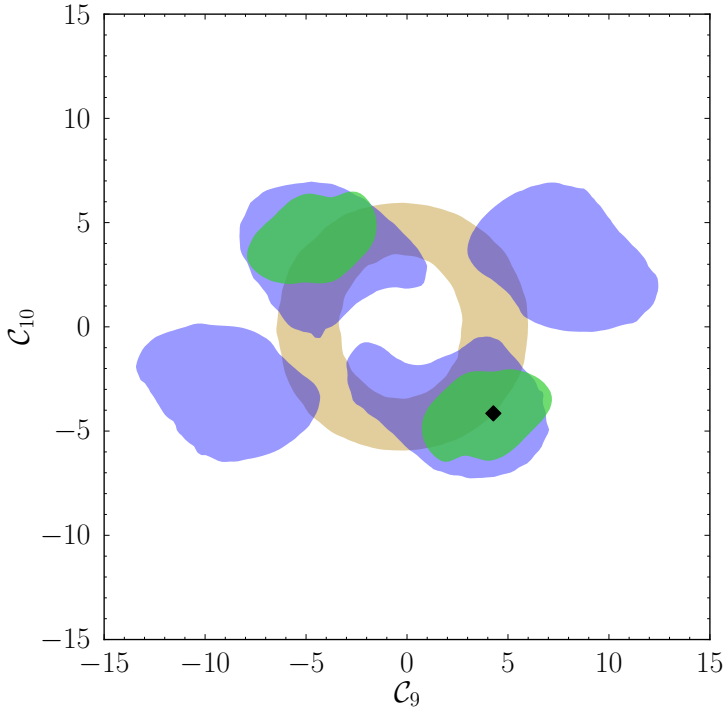
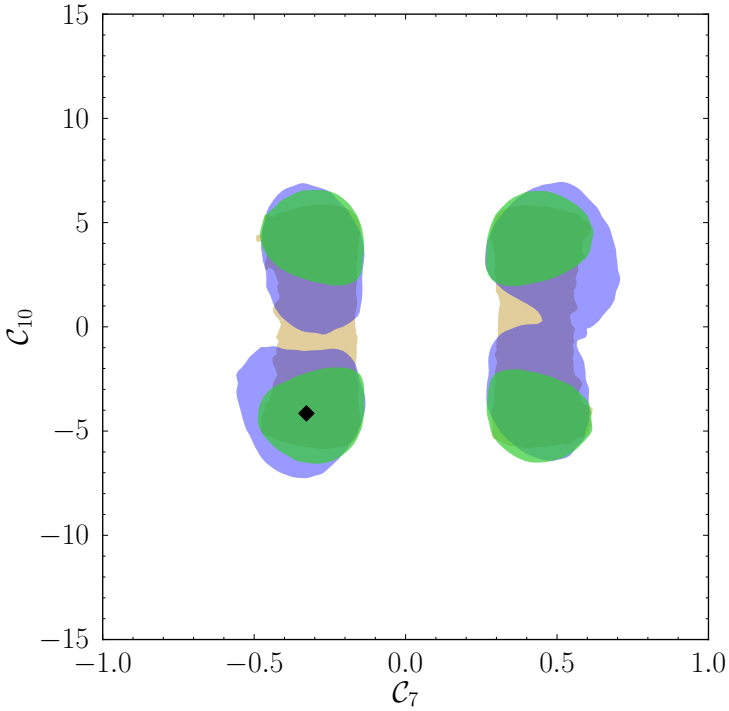






Parameters of interest $C_i(4.2\,\text{GeV})$ - 2D marginalised posterior

→ individual constraints at 95 % CR from

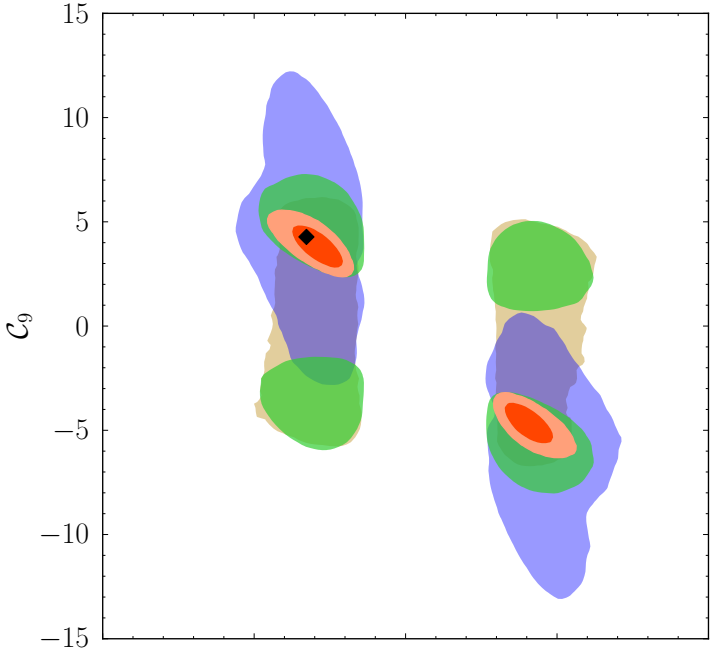
$B \to K^*\gamma$ and lo+hi-q^2 $B \to K\bar{\ell}\ell$

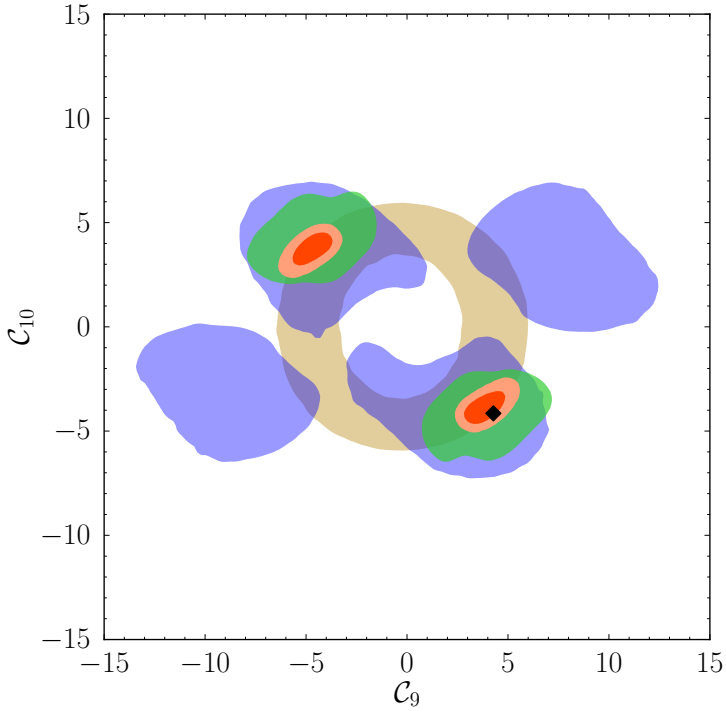
lo-q^2 $B \to K^*\bar{\ell}\ell$

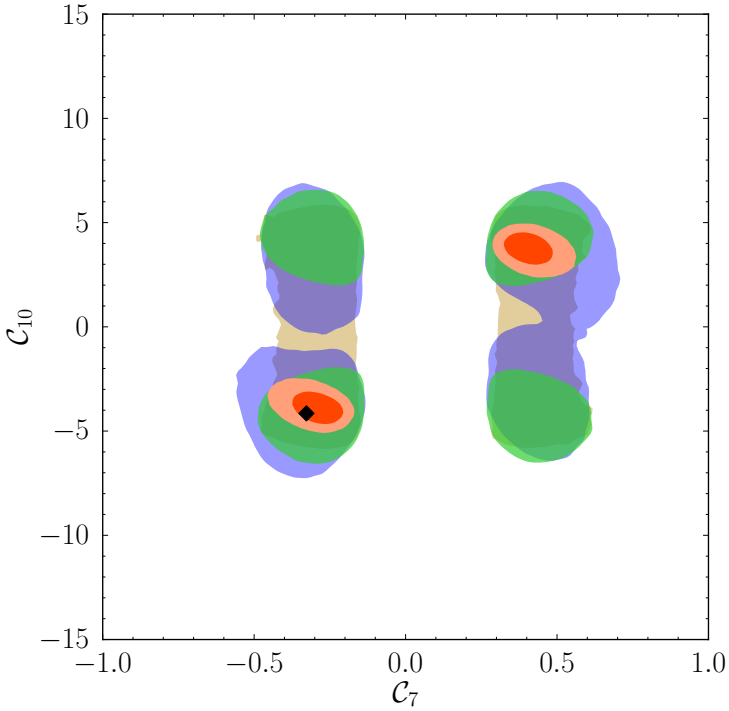
hi-q^2 $B \to K^*\bar{\ell}\ell$

all constraints ($+B_s \to \bar{\mu}\mu$): 68 % (95 %) CR

SM = (◆)

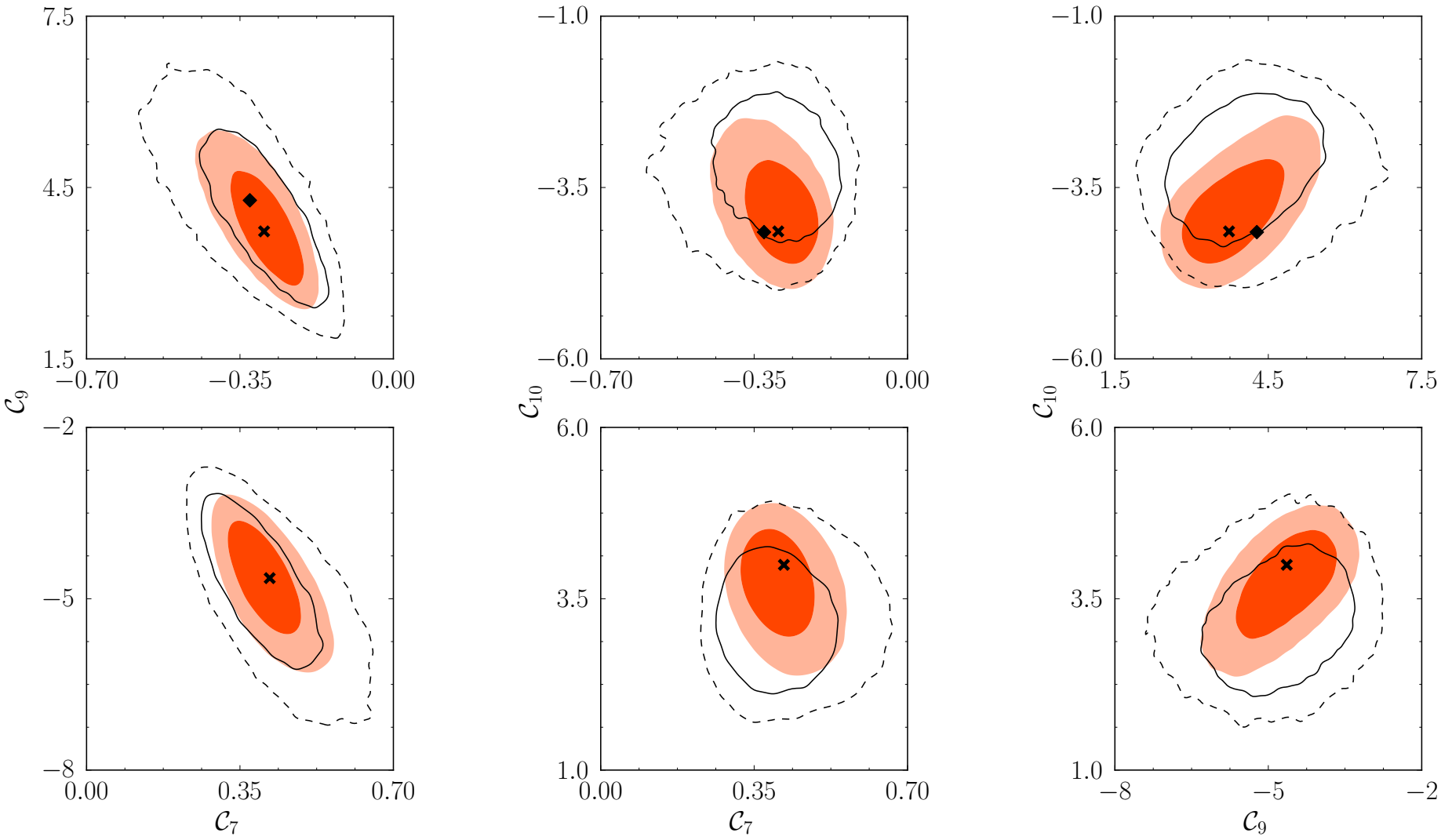






Prior dependence

SM = (◆), best fit point = (×)

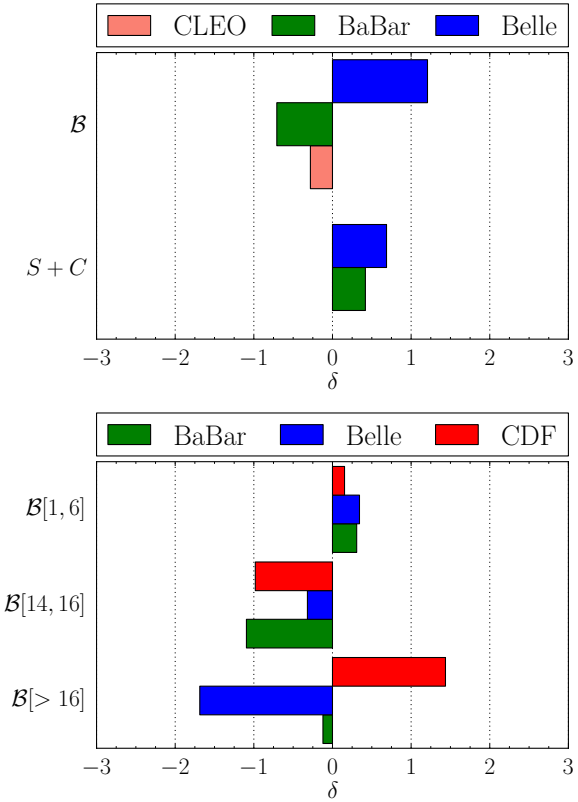


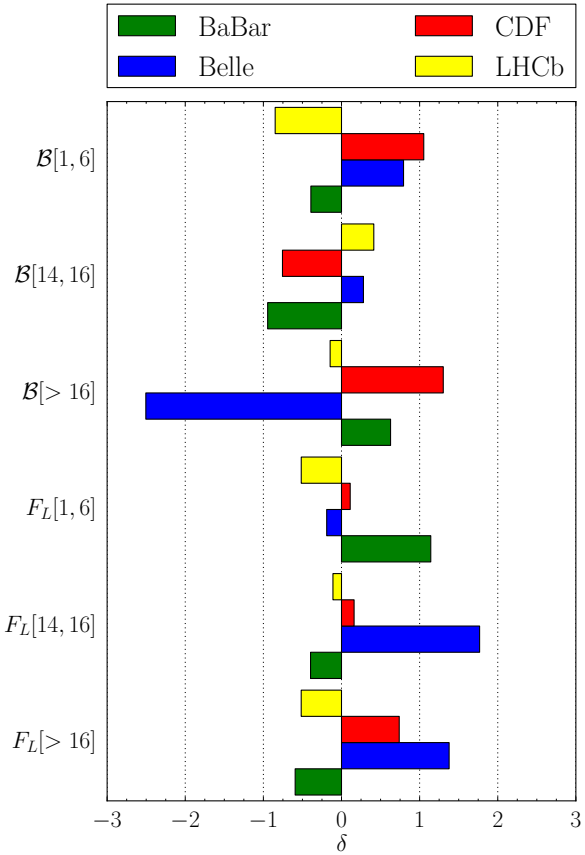
95 % (dashed) and 68 % (solid) credibility regions using 3× larger prior ranges

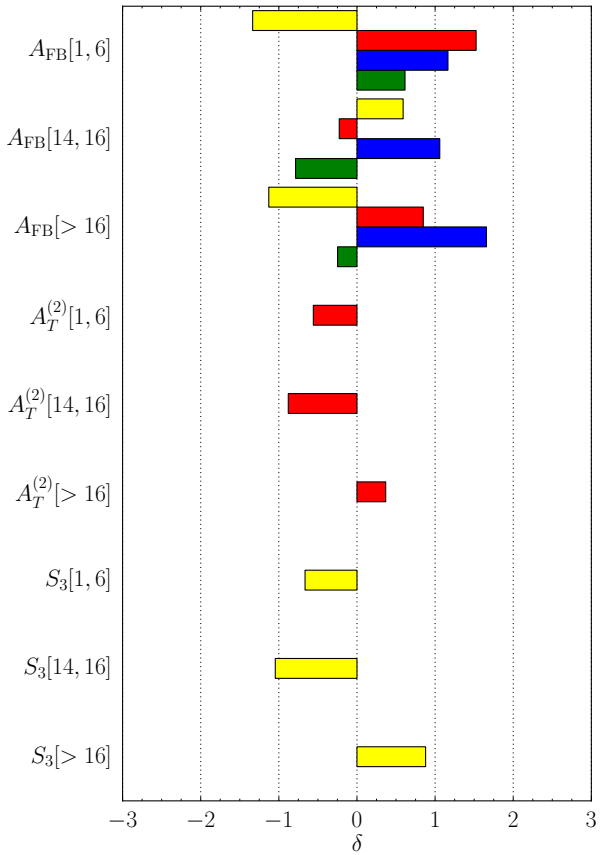
⇒ fit still converges

Pull values of experimental observables

22 observables with 59 measurements





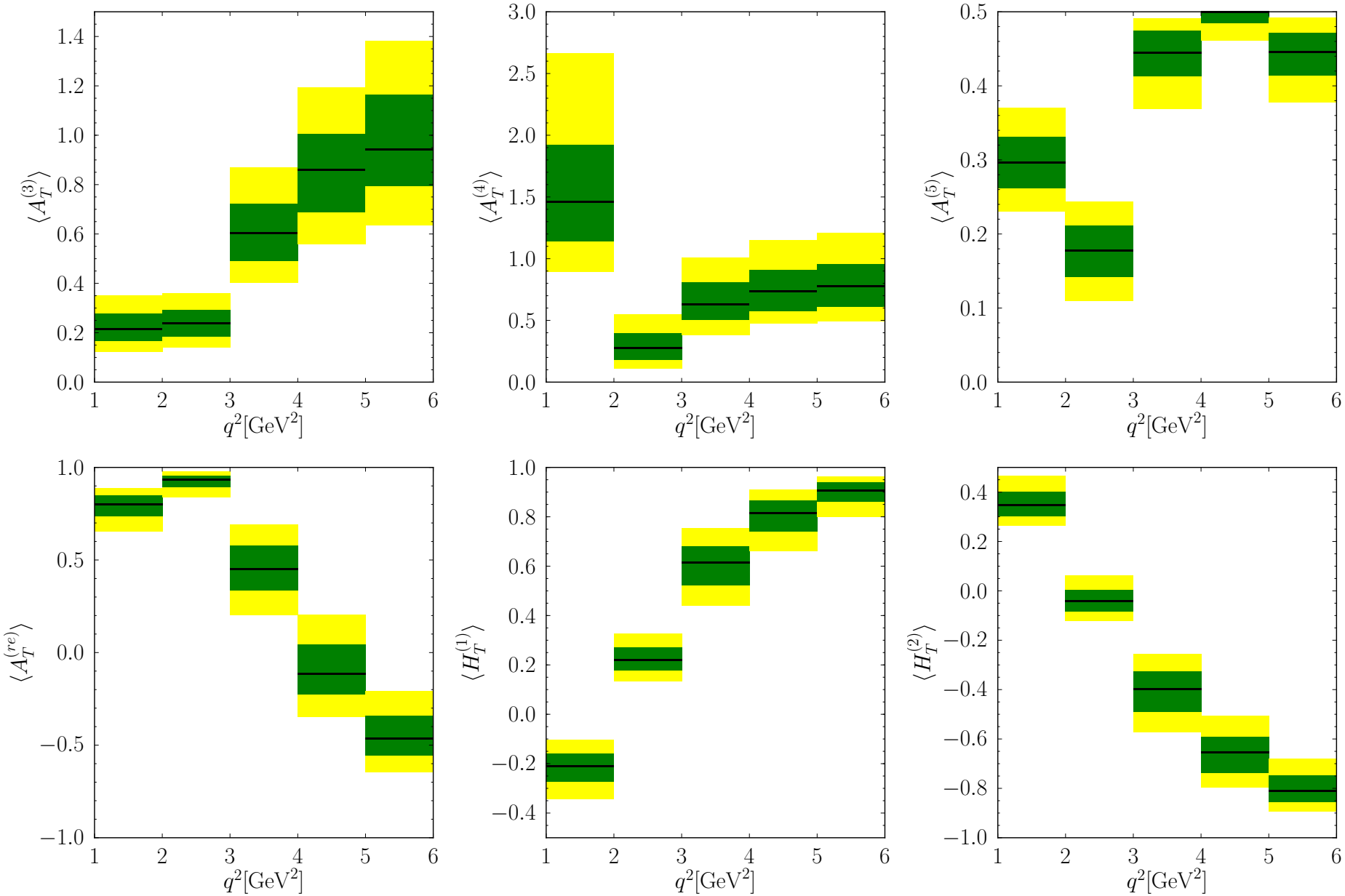


pull definition

$$\delta = \frac{x_{pred}(\vec{\theta},\vec{\nu}) - x}{\sigma}$$

$x_{pred}(\vec{\theta},\vec{\nu})$	theory prediction at best fit point
x	central value of experimental distribution
σ	experimental uncertainty

Prediction of yet unmeasured optimized observables @ low-q^2

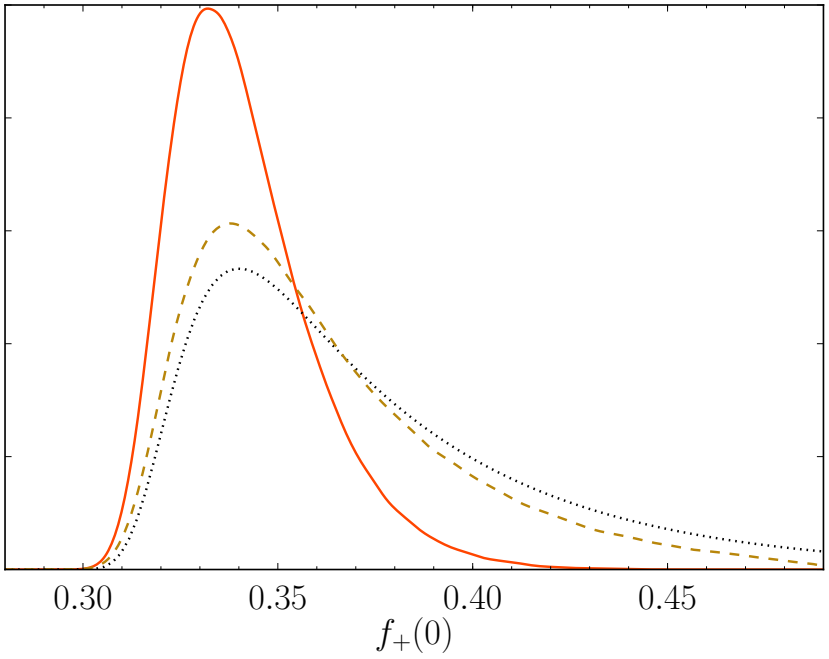


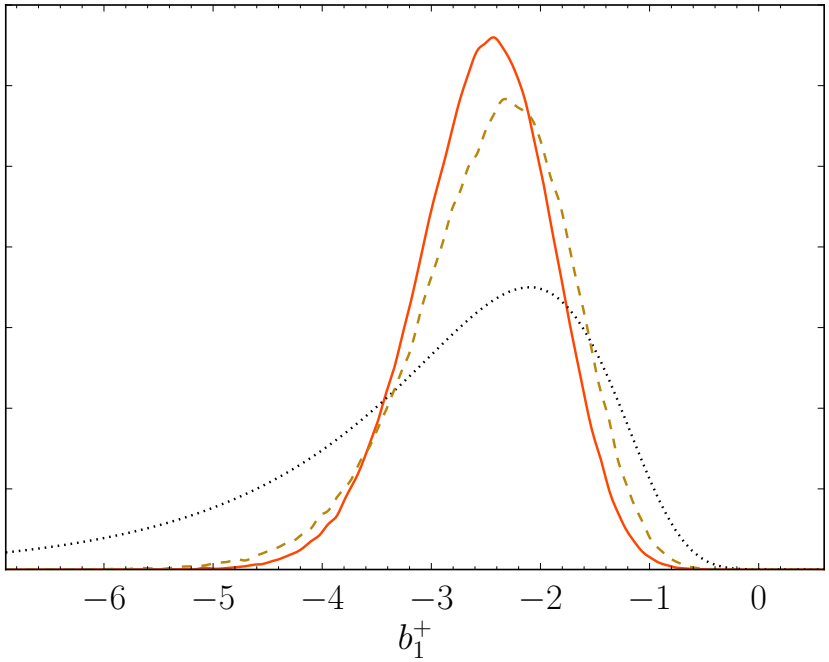
$\Rightarrow$ Measurements outside these predictions would put simple scenario $C_{7,9,10}$ in trouble

Nuisance parameter – example $B \to K$ form factor $f_+(q^2)$

$$f_+(q^2) = \frac{f_+(0)}{1 - q^2/M^2_{\text{res},+}} \left[1 + b_1^+ \left(z(q^2) - z(0) + \frac{1}{2}\left[z(q^2)^2 - z(0)^2\right]\right)\right],$$

$$z(s) = \frac{\sqrt{\tau_+ - s} - \sqrt{\tau_+ - \tau_0}}{\sqrt{\tau_+ - s} - \sqrt{\tau_+ - \tau_0}}, \qquad \tau_0 = \sqrt{\tau_+}\left(\sqrt{\tau_+} - \sqrt{\tau_+ - \tau_-}\right), \qquad \tau_\pm = (M_B \pm M_K)^2$$





⇒ Prior [dotted] from LCSR calculation [Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]

⇒ Posterior of $f_+(0)$ [left] and b_+^1 [right] using

1) $B \to K\ell^+\ell^-$ data only [dashed] vs 2) all data [solid, red]

Summary

- rare $b \to s + (\gamma,\, \ell^+\ell^-)$ are suppressed in the SM → indirect search of New Physics
  - provide strong constraints on generic extensions of flavour sector
- new $b \to s + (\gamma, \ell^+\ell^-)$ data from 2nd generation exp's: LHCb, Belle II and SuperB with high statistics through next decade
- angular observables J_i in exclusive $B \to K^*(\to K\pi)\bar{\ell}\ell$ provide @ low- and high-q^2 combinations with small hadronic uncertainties
- SM test and BSM search require extension of CKM-fit strategy:

global analysis: "combine all data and constrain scenarios"

EOS = new Flavour tool @ TU Dortmund by Danny van Dyk et al.
Download @ http://project.het.physik.tu-dortmund.de/eos/

Upcoming Workshop in September 10 – 11, 2012 @ University of Sussex, Brighton, UK
https://indico.cern.ch/conferenceDisplay.py?ovw=True&confId=198173

– Backup Slides –

Remark on $Br[B_s \to \mu^+\mu^-]$

So far theorists neglected mixing of $B_s \Rightarrow$ predict Br at $t = 0$: $Br[B_s(t = 0) \to \bar{\mu}\mu]$

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But with new measurements of $\Delta\Gamma_s$ (incl. sign) from LHCb and CDF, DØ

$\Rightarrow$ experiments actually measure time-integrated Br: [De Bruyn et al. arXiv:1204.1737]

$$Br[B_s \to \bar{\mu}\mu] \equiv \frac{1}{2}\int_0^\infty dt \left(\Gamma[B_s(t) \to \bar{\mu}\mu] + \Gamma[\overline{B}_s(t) \to \bar{\mu}\mu]\right)$$

$$= \frac{1 + y_s \cdot \mathcal{A}_{\Delta\Gamma}}{1 - y_s^2} Br[B_s(t=0) \to \bar{\mu}\mu]$$

with (LHCb '11)

$$y_s = \frac{\Delta\Gamma_s}{2\Gamma_s} = 0.088 \pm 0.014$$

and

$\Rightarrow$ in SM $\quad \mathcal{A}_{\Delta\Gamma}|_{\rm SM} = +1$

$\Rightarrow$ beyond $\mathcal{A}_{\Delta\Gamma} \in [-1, +1]$ $\to$ depends on NP !!!

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In SM for example

$$Br[B_s \to \bar{\mu}\mu]_{\rm SM} = (3.53 \pm 0.38) \times 10^{-9}$$

[Mahmoudi/Neshatpour/Orloff arXiv:1205.1845]

largest uncertainties from

$f_{B_s} = (234 \pm 10)$ MeV $\to 9\,\%$

$V_{ts} \to 5\,\%$

B_s lifetime $\to 2\,\%$

Remark on $Br[B_s \to \mu^+\mu^-]$

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$\Rightarrow$ beyond $\mathcal{A}_{\Delta\Gamma} \in [-1, +1]$ $\to$ depends on NP !!!

. . . or using precise ΔM_s measurement to substitute f_{B_s} (and assuming SM) [Buras hep-ph/0303060]

$$Br[B_s \to \bar{\mu}\mu]_{\mathrm{SM}} = \frac{(3.1 \pm 0.2) \times 10^{-9}}{0.91 \pm 0.01} = (3.4 \pm 0.2) \times 10^{-9}$$

[Buras/Girrbach arXiv:1204.5064]

Goodness of fit & Bayes factor

[Beaujean/CB/van Dyk/Wacker arXiv:1205.1838]

sgn(C_7, C_9, C_{10})	best-fit-point	log(MAP)	goodness-of-fit				log(Z)
			$T_{\rm like}$	$p_{\rm like}$	$T_{\rm pull}$	$p_{\rm pull}$	
$(-,\,+,\,-)$	$(-0.295, 3.73, -4.14)$	424.31	402.40	59%	48.8	74%	385.1
$(+,\,-,\,+)$	$(0.418, -4.64, 3.99)$	424.20	402.32	58%	48.9	74%	385.0
$(-,\,-,\,+)$	$(-0.392, -3.09, 3.19)$	403.72	387.70	0.8%	76.8	3%	363.8
$(+,\,+,\,-)$	$(0.557, 2.25, -3.24)$	399.70	384.66	0.2%	82.9	1%	360.1
SM: $(-,\,+,\,-)$	$(-0.327, 4.28, -4.15)$	$430.56^{\dagger}$	402.30	69%	49.0	82%	392.4

MAP = maximum a posteriori

Z = local evidence $= \int d\vec{\theta}\, d\vec{\nu}\, P(D|\theta,\nu)\cdot P(\theta,\nu) =$ "likelihood $\times$ prior"

$\Rightarrow$ 2 methods to derive p-values from 2 statistics $T_{\rm like}$ and $T_{\rm pull}$:

indicate good fit: $p \sim (60-75)\%$

$\Rightarrow$ model comparison: SM = fixed values of Wilson coefficients $\Leftrightarrow$ SM-like solution

Bayes factor: $B = \exp(392.4 - 385.1) \approx 1500$ in favor of the simpler model

Strategy 1) Altmannshofer/Paradisi/Straub arXiv:1111.1257

update in Altmannshofer/Straub arXiv:1206.0273

$\Rightarrow$ based on MCMC + Bayesian inference

$\Rightarrow$ included data from

- $B \to X_s \gamma$: Br, $A_{\rm CP}$,
  $B \to K^* \gamma$: S
- $B \to X_s \bar{\ell}\ell$: Br,
  $B \to K \bar{\ell}\ell$: Br,
  $B \to K^* \bar{\ell}\ell$: Br, $A_{\rm FB}$, F_L, S_3, A_{im},
  $B_s \to \bar{\mu}\mu$: Br

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  $B \to K^* \bar{\ell}\ell$: Br, $A_{\rm FB}$, F_L, S_3, A_{im},
  $B_s \to \bar{\mu}\mu$: Br

$\Rightarrow$ model-indep. NP (real or complex)

- $C_{7,7',9,9',10,10'}$ (in varying stages)
- Z-penguin + $C_{7,7'}$
  $\Rightarrow$ relates $b \to s \bar{\ell}\ell$ and $b \to s \bar{\nu}\nu$
- $(C_S - C_{S'})$, $(C_P - C_{P'})$

Strategy 1) Altmannshofer/Paradisi/Straub arXiv:1111.1257

update in Altmannshofer/Straub arXiv:1206.0273

⇒ individual constraints at 95 %

$S[B \to K^*\gamma]$

$Br[B \to X_s\gamma]$

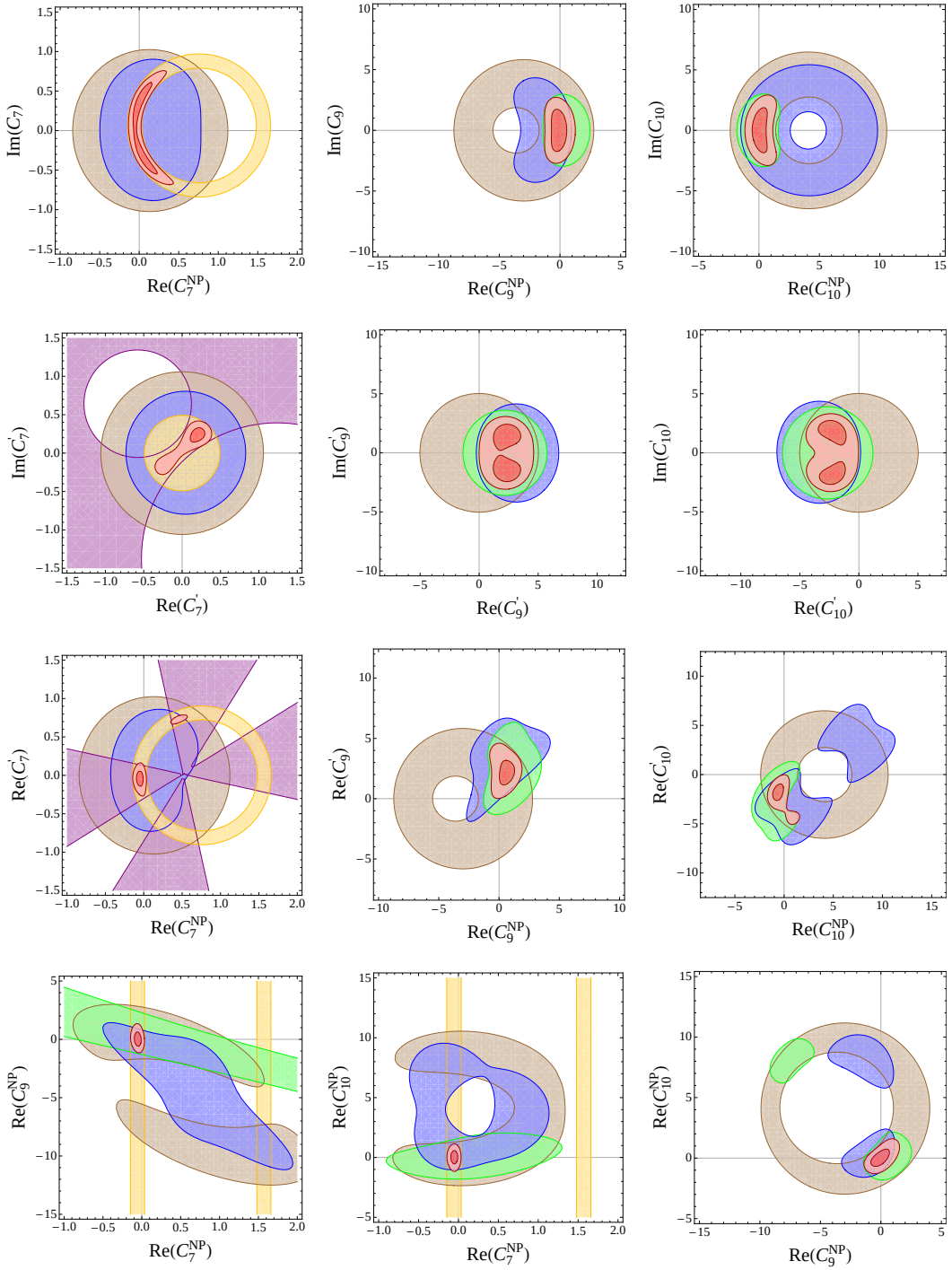
lo+hi-q^2 $Br[B \to X_s\bar{\ell}\ell]$

lo-q^2 $B \to K^*\bar{\ell}\ell$

hi-q^2 $B \to K^*\bar{\ell}\ell$

combined constraints: 68 % (95 %)

here in 2 parameter scenarios from arXiv:1111.1257 ⇒



Strategy 1) Altmannshofer/Paradisi/Straub arXiv:1111.1257

update in Altmannshofer/Straub arXiv:1206.0273

$\Rightarrow$ predictions of unmeasured observables

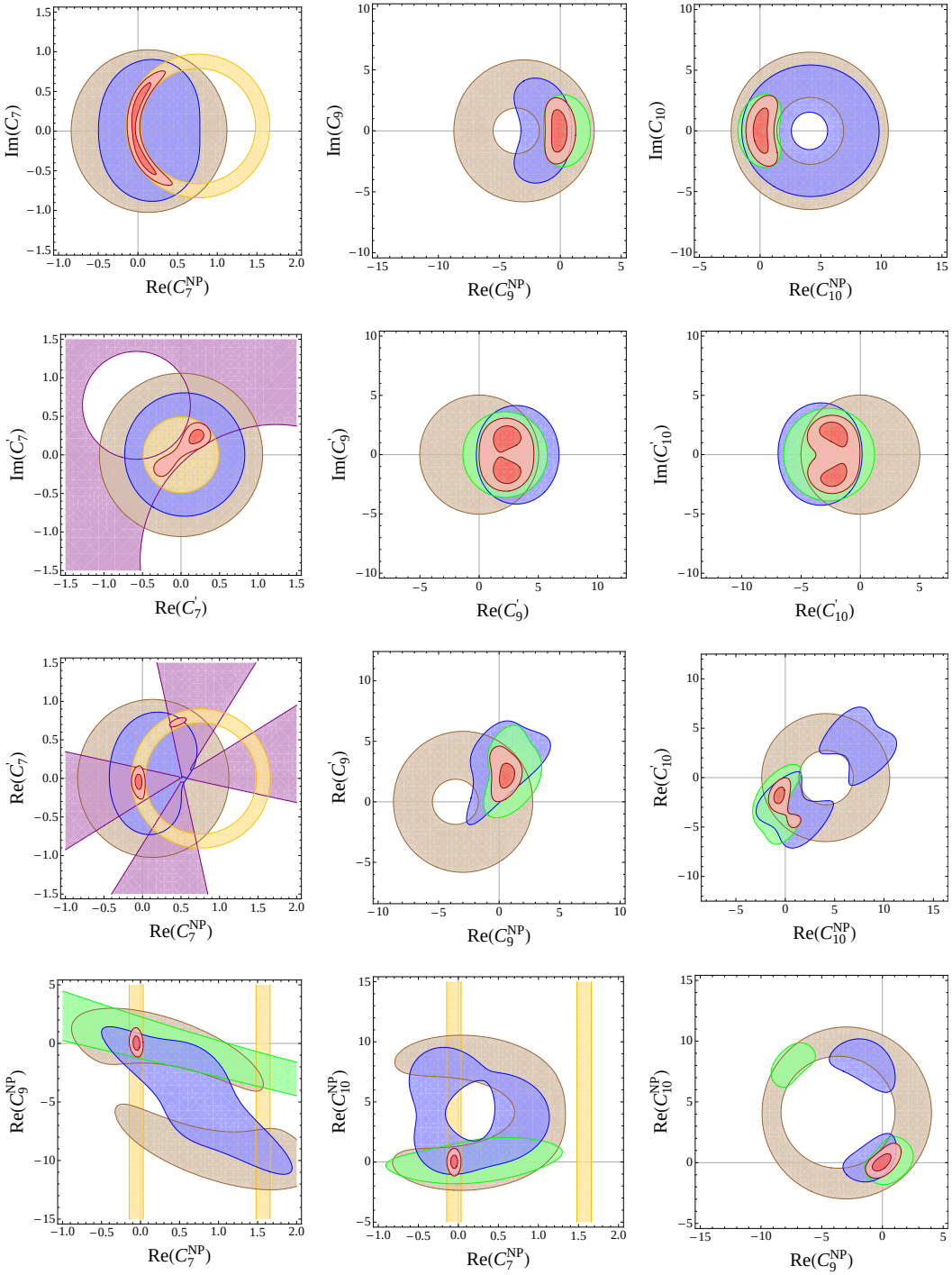
- still large T-odd CP-asymmetries

  at low-q^2:

$$|\langle A_7\rangle_{[1,6]}| < 35\,\%$$
$$|\langle A_8\rangle_{[1,6]}| < 21\,\%$$
$$|\langle A_9\rangle_{[1,6]}| < 13\,\%$$

  at high-q^2:

$$|\langle A_8\rangle_{[14,16]}| < 12\,\%$$
$$|\langle A_9\rangle_{[14,16]}| < 20\,\%$$



Strategy 1) Altmannshofer/Paradisi/Straub arXiv:1111.1257

update in Altmannshofer/Straub arXiv:1206.0273

Similar analysis

- Descotes-Genon/Ghosh/Matias/Ramon arXiv:1104.3342
- CB/Hiller/van Dyk arXiv:1105.0376
- CB/Hiller/van Dyk/Wacker arXiv:1111.2558
- Becirevic/Kosnik/Mescia/Schneider arXiv:1205.5811
- Becirevic/Kou/Le Yaounac/Tayduganov arXiv:1206.1502
- Hurth/Mahmoudi arXiv:1207.0688
- Descotes-Genon/Matias/Ramon/Virto arXiv:1207.2753
- ...

Data for $B \to K^* + \ell^+\ell^-$: data in 6 q^2-bins for $\langle Br \rangle$, $\langle A_{\rm FB} \rangle$, $\langle F_L \rangle$

angular analysis in each q^2-bin in θ_ℓ, θ_K

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_K} = \frac{3}{2}F_L\cos^2\theta_K + \frac{3}{4}(1-F_L)\sin^2\theta_K$$

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{3}{4}F_L\sin^2\theta_\ell + \frac{3}{8}(1-F_L)(1+\cos^2\theta_\ell) + A_{\rm FB}\cos\theta_\ell$$

$\Rightarrow$ fitted F_L and $A_{\rm FB}$

Data for $B \to K^* + \ell^+\ell^-$: data in 6 q^2-bins for $\langle Br \rangle$, $\langle A_{\rm FB} \rangle$, $\langle F_L \rangle$

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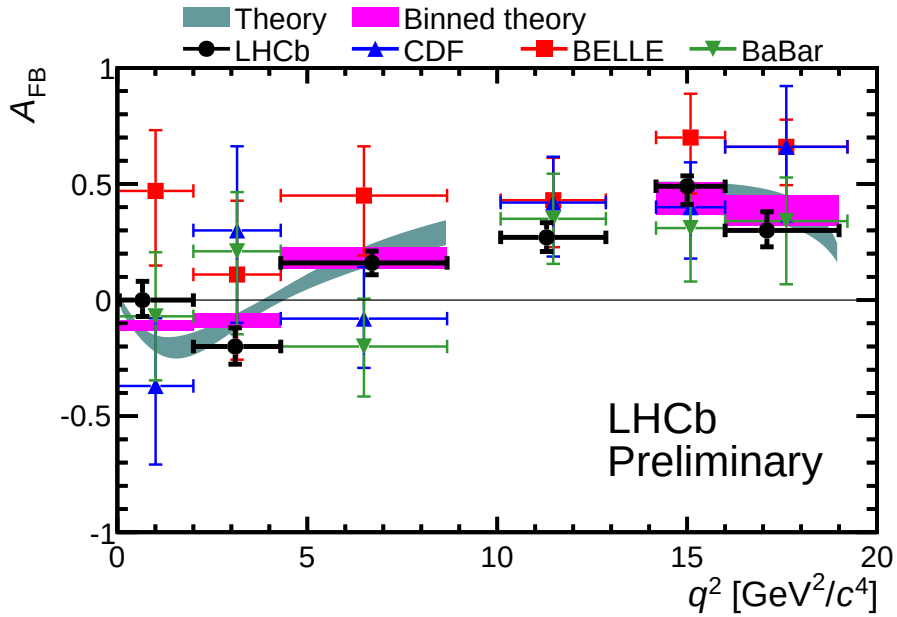
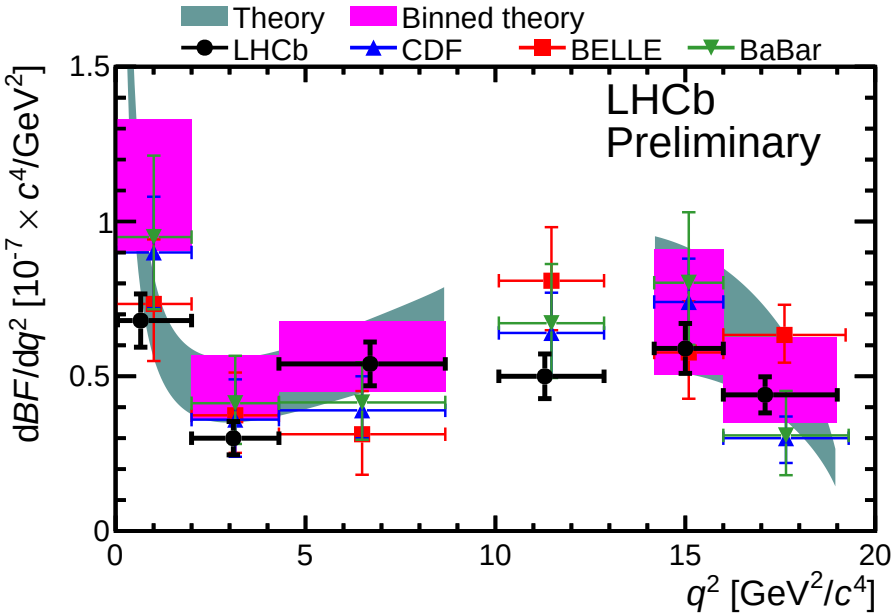
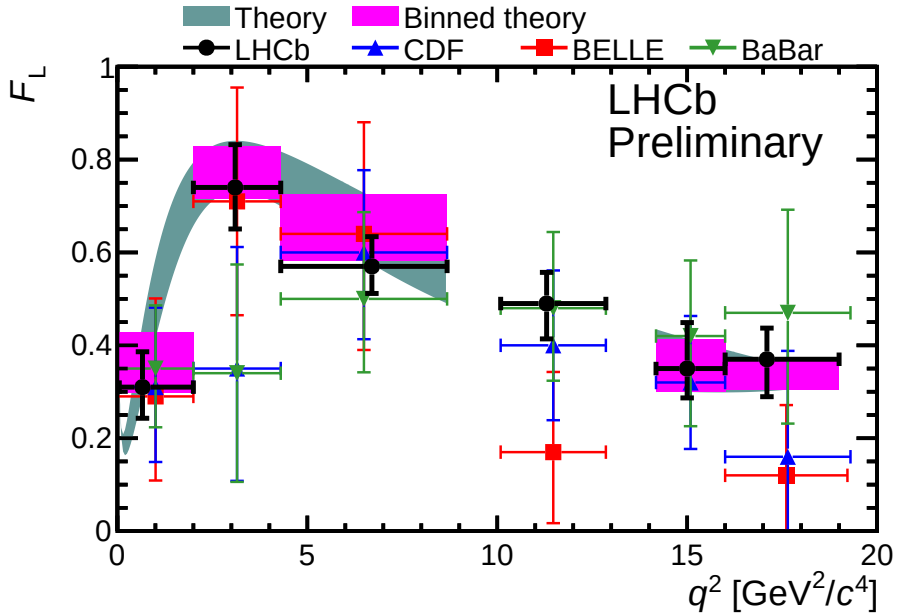
$\Rightarrow$ fitted F_L and $A_{\rm FB}$

[SM-predictions: CB/Hiller/van Dyk arXiv:1105.0376]







Data for $B \to K^* + \ell^+\ell^-$: data in 6 q^2-bins for $\langle S_3 \rangle$, $\langle A_{im} \rangle$

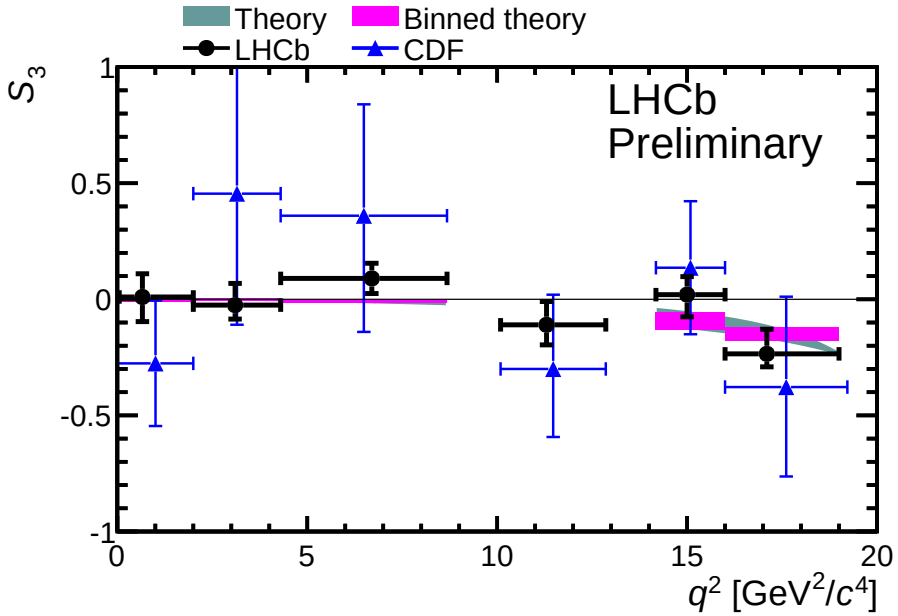
additional measurement of S_3 (or $A_T^{(2)}$) and A_{im} from CDF and LHCb

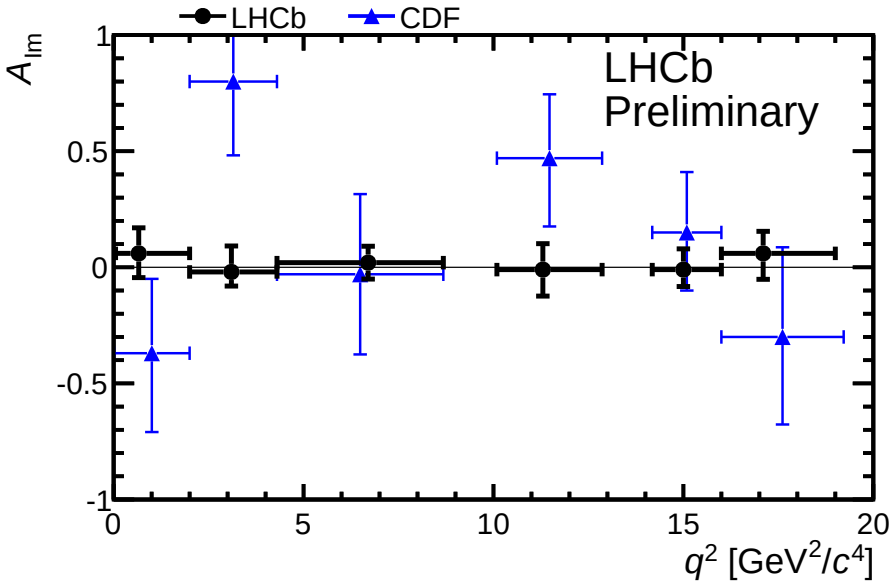
$$\frac{2\pi}{(\Gamma + \bar{\Gamma})} \frac{d(\Gamma + \bar{\Gamma})}{d\phi} = 1 + S_3 \cos 2\phi + A_{im} \sin 2\phi$$

with

$$S_3 = \frac{J_3 + \bar{J}_3}{\Gamma + \bar{\Gamma}} = \frac{1}{2}(1 - F_L)\, A_T^{(2)}, \qquad A_{im} = \frac{J_9 - \bar{J}_9}{\Gamma + \bar{\Gamma}}$$

(since J_9 CP-odd, the CP-asymmetry $\sim (J_9 - \bar{J}_9)$ from untagged B-sample)

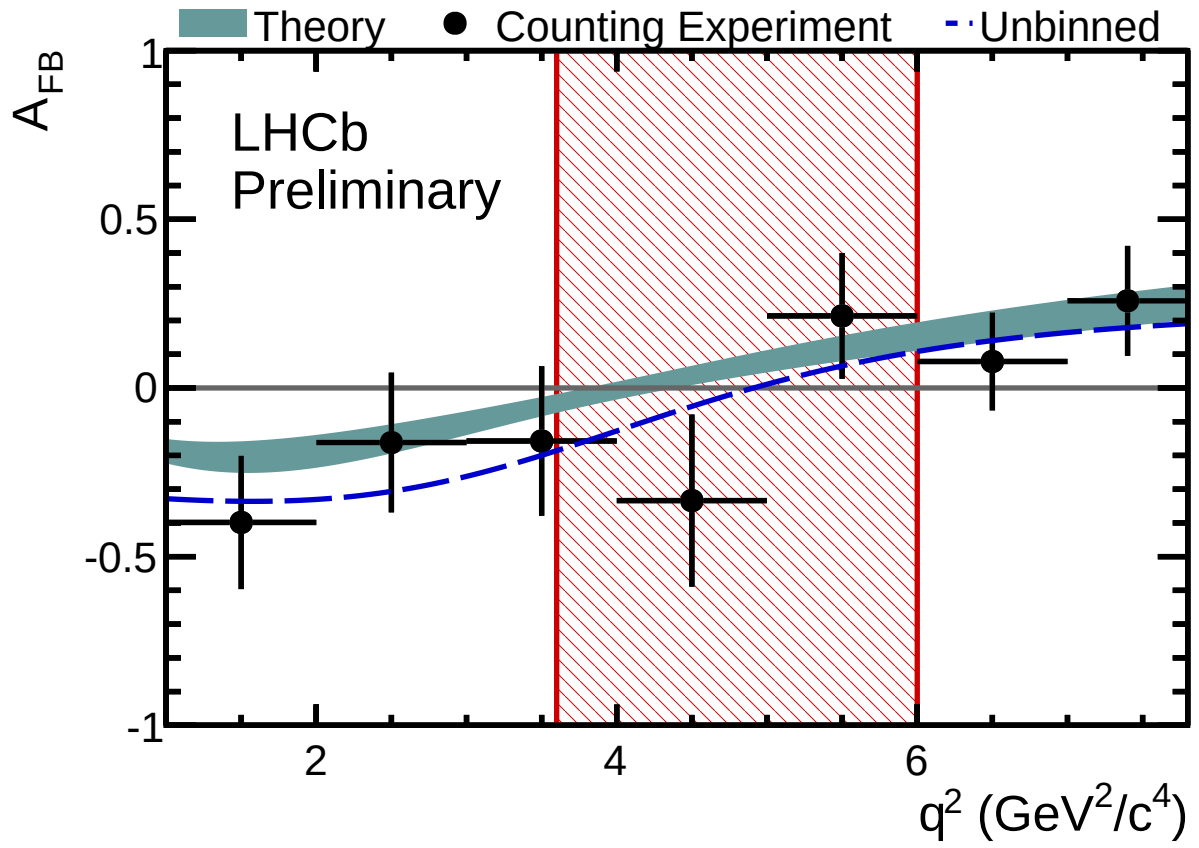




Data for $B \to K^* + \ell^+\ell^-$:

Zero-crossing of $A_{\rm FB}$ in low-q^2 region: [LHCb Collaboration LHCb-CONF-2012-008]

finer q^2-binning than previously: bin-width $= 1$ GeV^2



Measurement:

$q_0^2 = (4.9\,^{+1.1}_{-1.3})$ GeV^2

Theory (SM):

$q_0^2 = (4.0...4.3 \pm 0.3)$ GeV^2

[Beneke/Feldmann/Seidel hep-ph/0412400]
[Ali/Kramer/Zhu hep-ph/0601034]
[CB/Hiller/van Dyk/Wacker arXiv:1111.2558]

$B \to K + \ell^+\ell^-$: 3-body decay → 2 kinematic variables

$$\frac{1}{(\mathrm{d}\Gamma/\mathrm{d}q^2)}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell} = \frac{3}{4}\left[1 - F_H\right]\sin^2\theta_\ell + \frac{1}{2}F_H + A_{\mathrm{FB}}\cos\theta_\ell$$

3 observables × CP-conj: $\mathrm{d}Br/\mathrm{d}q^2$, $A_{\mathrm{FB}}(q^2)$, $F_H(q^2)$

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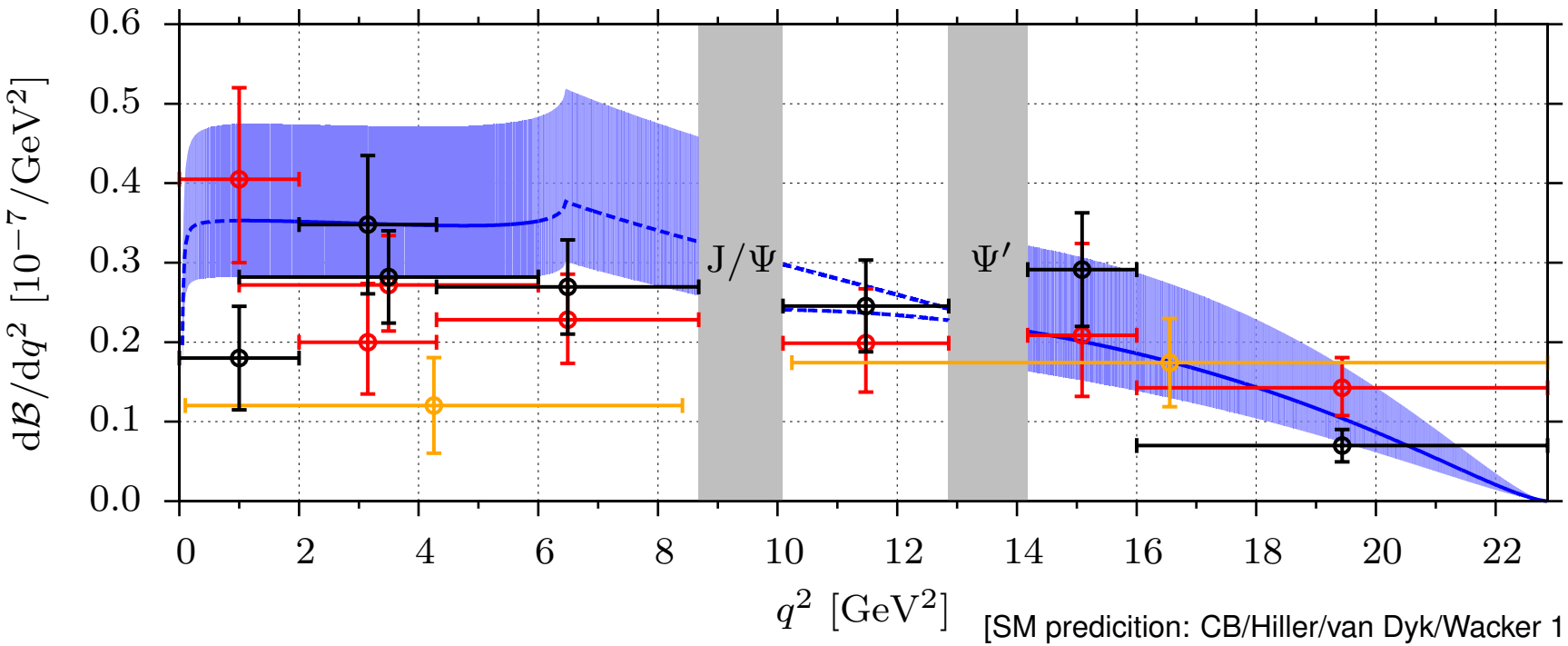
3 observables × CP-conj: $\mathrm{d}Br/\mathrm{d}q^2$, $A_{\mathrm{FB}}(q^2)$, $F_H(q^2)$

data in 6 q^2-bins for: $\langle Br \rangle$ from [BaBar] [Belle] [CDF]



[SM predicition: CB/Hiller/van Dyk/Wacker 1111.2558]

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3 observables × CP-conj: $\mathrm{d}Br/\mathrm{d}q^2$, $A_{\mathrm{FB}}(q^2)$, $F_H(q^2)$

also measured:

- 6 q^2-bins lepton forward-backward asymmetry: $\langle A_{\mathrm{FB}}\rangle$
- 6 q^2-bins isospin asymmetry:

$$\langle A_I\rangle = \frac{(\tau_{B^\pm}/\tau_{B^0})\langle Br[B^0 \to K^0\bar{\ell}\ell]\rangle - \langle Br[B^\pm \to K^\pm\bar{\ell}\ell]\rangle}{(\tau_{B^\pm}/\tau_{B^0})\langle Br[B^0 \to K^0\bar{\ell}\ell]\rangle - \langle Br[B^\pm \to K^\pm\bar{\ell}\ell]\rangle}$$

$B \to K + \ell^+\ell^-$: 3-body decay → 2 kinematic variables

$$\frac{1}{(\mathrm{d}\Gamma/\mathrm{d}q^2)}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell} = \frac{3}{4}\left[1 - F_H\right]\sin^2\theta_\ell + \frac{1}{2}F_H + A_{\mathrm{FB}}\cos\theta_\ell$$

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. . . and improved measurements of

- exclusive $b \to s\gamma$: $B \to K^*\gamma$, $B_s \to \phi\gamma$ (LHCb)
- leptonic $B_s \to \mu^+\mu^-$ and related (LHCb, CMS, ATLAS)
- inclusive $b \to s\gamma$ and $b \to s\ell^+\ell^-$ (Belle II, SuperB)

q^2-Integrated Observables

Experimental measurements of observables P always imply binning in kinematical variables x, i.e.

$$\langle P \rangle_{[x_{min},\, x_{max}]} \equiv \int_{x_{min}}^{x_{max}} \mathrm{d}x\, P(x)$$

Assume, that angular observables $J_i(q^2)$ are measured in experiment for certain q^2 binning (omitting q^2-interval boundaries)

$$\langle J_i \rangle = \int_{q^2_{min}}^{q^2_{max}} \mathrm{d}q^2\, J_i(q^2)$$

and "transversity observables" are then determined as follows (for example)

$$\langle A_T^{(3)} \rangle = \sqrt{\frac{4\langle J_4 \rangle^2 + \langle J_7 \rangle^2}{-2\langle J_{2c} \rangle \langle 2J_{2s} + J_3 \rangle}}$$

$\longrightarrow$ This has to accounted for in theoretical predictions !!!

Measuring Angular Observables

likely that exp. results only in some q^2-integrated bins: $\langle \ldots \rangle = \int_{q^2_{min}}^{q^2_{max}} \mathrm{d}q^2 \ldots$,
then use some (quasi-) single-diff. distributions in θ_ℓ, θ_{K*}, ϕ

- $$\frac{\mathrm{d}\langle\Gamma\rangle}{\mathrm{d}\phi} = \frac{1}{2\pi}\left\{\langle\Gamma\rangle + \langle J_3\rangle\cos 2\phi + \langle J_9\rangle\sin 2\phi\right\}$$

- 2 bins in $\cos\theta_{K*}$

$$\frac{\mathrm{d}\left\langle A_{\theta_{K*}}\right\rangle}{\mathrm{d}\phi} \equiv \int_{-1}^{1}\mathrm{d}\cos\theta_\ell\left[\int_0^1 - \int_{-1}^{0}\right]\mathrm{d}\cos\theta_{K*}\,\frac{\mathrm{d}^3\langle\Gamma\rangle}{\mathrm{d}\cos\theta_{K*}\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\phi}$$

$$= \frac{3}{16}\left\{\langle J_5\rangle\cos\phi + \langle J_7\rangle\sin\phi\right\}$$

- (2 bins in $\cos\theta_{K*}$) + (2 bins in $\cos\theta_\ell$)

$$\frac{\mathrm{d}\left\langle A_{\theta_{K*},\theta_\ell}\right\rangle}{\mathrm{d}\phi} \equiv \left[\int_0^1 - \int_{-1}^{0}\right]\mathrm{d}\cos\theta_\ell\,\frac{\mathrm{d}^2\left\langle A_{\theta_{K*}}\right\rangle}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\phi} = \frac{1}{2\pi}\left\{\langle J_4\rangle\cos\phi + \langle J_8\rangle\sin\phi\right\}$$

Low-q^2 = Large Recoil

QCD Factorisation (QCDF)

[Beneke/Feldmann/Seidel hep-ph/0106067, hep-ph/0412400]

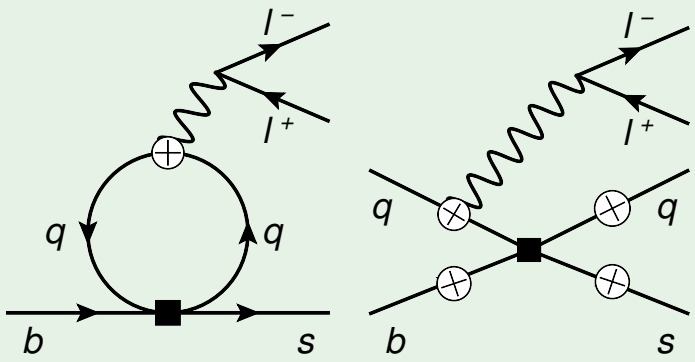
= (large recoil + heavy quark) limit [also Soft Collinear ET (SCET)]

$$\left\langle \bar{\ell}\ell\, K_a^* \,\middle|\, H_{\rm eff}^{(i)} \,\middle|\, B \right\rangle \sim$$

$$C_a^{(i)} \times \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K*} + \mathcal{O}(\Lambda_{\rm QCD}/m_b)$$

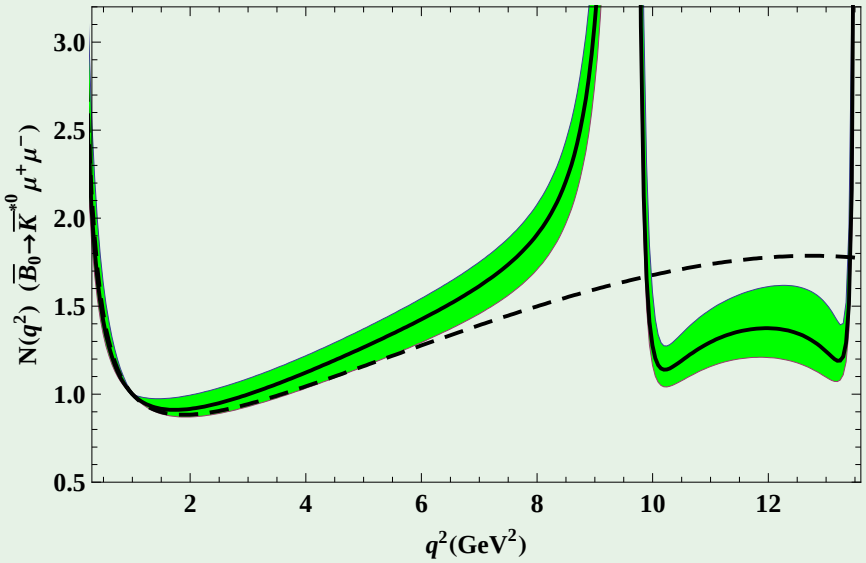
$C_a^{(i)}$, $T_a^{(i)}$: perturbative kernels in α_s ($a = \perp, \|$, $i = u, t$)

ϕ_B, $\phi_{a,K*}$: B– and K_a^*–distribution amplitudes



$c\bar{c}$-contributions

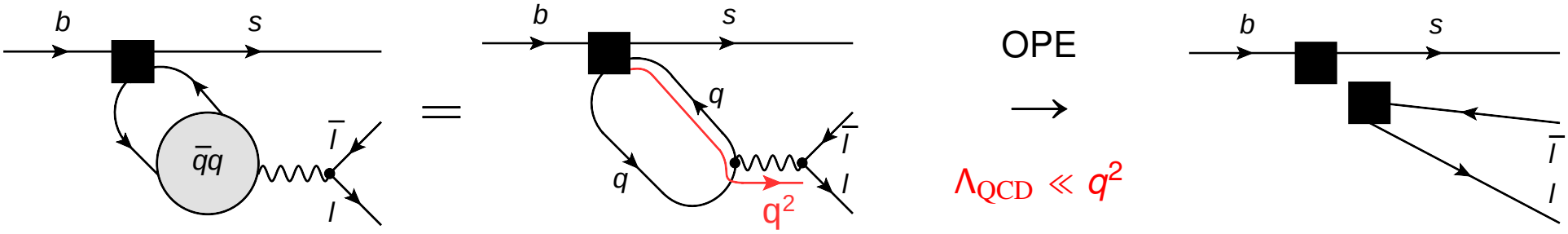
[Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945]



- OPE near light-cone incl. soft-gluon emission (non-local operator) for $q^2 \leqslant 4$ GeV$^2 \ll 4m_c^2$
- hadronic dispersion relation using measured $B \to K^{(*)}(\bar{c}c)$ amplitudes at $q^2 \geqslant 4$ GeV^2
- $B \to K^{(*)}$ form factors from LCSR
- up to (15-20) % in rate for $1 < q^2 < 6$ GeV^2

High-q^2 = Low Recoil

Hard momentum transfer ($q^2 \sim M_B^2$) through $(\bar{q}q) \to \bar{\ell}\ell$ allows local OPE



$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{L}^{\rm eff}(0), j_\mu^{\rm em}(x)\} | \bar{B}\rangle\, [\bar{\ell}\gamma^\mu \ell]$$

$$= \left(\sum_a \mathcal{C}_{3a} \mathcal{Q}_{3a}^\mu + \sum_b \mathcal{C}_{5b} \mathcal{Q}_{5b}^\mu + \sum_c \mathcal{C}_{6c} \mathcal{Q}_{6c}^\mu + \mathcal{O}(dim > 6) \right) [\bar{\ell}\gamma_\mu \ell]$$

Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118

Leading $dim = 3$ operators: $\langle \bar{K}^* | \mathcal{Q}_{3,a} | \bar{B}\rangle \sim$ usual $B \to K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$

$$\mathcal{Q}_{3,1}^\mu = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) [\bar{s}\, \gamma_\nu (1-\gamma_5)\, b] \qquad \to \qquad C_9 \to C_9^{\rm eff}, \qquad (V, A_{1,2})$$

$$\mathcal{Q}_{3,2}^\mu = \frac{i m_b}{q^2}\, q_\nu\, [\bar{s}\, \sigma_{\nu\mu} (1+\gamma_5)\, b] \qquad \to \qquad C_7 \to C_7^{\rm eff}, \qquad (T_{1,2,3})$$

High-q^2 = Low Recoil

[Beylich/Buchalla/Feldmann arXiv:1101.5118]

$dim = 3$	α_s matching corrections are also known
$m_s \neq 0$	2 additional $dim = 3$ operators, suppressed with $\alpha_s m_s/m_b \sim 0.5$ %, NO new form factors
$dim = 4$	absent
$dim = 5$	suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2$ %, explicite estimate @ $q^2 = 15$ GeV^2: < 1%
$dim = 6$	suppressed by $(\Lambda_{\rm QCD}/m_b)^3 \sim 0.2$ % and small QCD-penguin's: $C_{3,4,5,6}$ spectator quark effects: from weak annihilation
beyond OPE	duality violating effects • based on Shifman model for c-quark correlator + fit to recent BES data • ± 2 % for integrated rate $q^2 > 15$ GeV^2

$\Rightarrow$ OPE of exclusive $B \to K^{(*)}\ell^+\ell^-$ predicts small sub-leading contributions !!!

BUT, still missing $B \to K^{(*)}$ form factors @ high-q^2
for predictions of angular observables J_i

High-q^2: OPE + HQET

Framework developed by Grinstein/Pirjol hep-ph/0404250

1) OPE in $\Lambda_{\rm QCD}/Q$ with $Q = \{m_b, \sqrt{q^2}\}$ + matching on HQET + expansion in m_c

$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi}{q^2} \sum_{i=1}^{6} \mathcal{C}_i(\mu)\, \mathcal{T}_\alpha^{(i)}(q^2,\mu)\, [\bar{\ell}\gamma^\alpha \ell]$$

$$\mathcal{T}_\alpha^{(i)}(q^2,\mu) = i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{O}_i(0), j_\alpha^{\rm em}(x)\} | \bar{B}\rangle$$
$$= \sum_{k\geqslant -2} \sum_j C_{i,j}^{(k)} \langle \mathcal{Q}_{j,\alpha}^{(k)} \rangle$$

$\mathcal{Q}_{j,\alpha}^{(k)}$	power	$\mathcal{O}(\alpha_s)$
$\mathcal{Q}_{1,2}^{(-2)}$	1	$\alpha_s^0(Q)$
$\mathcal{Q}_{1-5}^{(-1)}$	$\Lambda_{\rm QCD}/Q$	$\alpha_s^1(Q)$
$\mathcal{Q}_{1,2}^{(0)}$	m_c^2/Q^2	$\alpha_s^0(Q)$
$\mathcal{Q}_{j>3}^{(0)}$	${\Lambda_{\rm QCD}}^2/Q^2$	$\alpha_s^0(Q)$
$\mathcal{Q}_i^{(2)}$	m_c^4/Q^4	$\alpha_s^0(Q)$

included,
unc. estimate by naive pwr cont.

2) HQET FF-relations at sub-leading order + α_s corrections in leading order

$$T_1(q^2) = \kappa\, V(q^2), \qquad T_2(q^2) = \kappa\, A_1(q^2), \qquad T_3(q^2) = \kappa\, A_2(q^2)\frac{M_B^2}{q^2},$$

$$\kappa = \left(1 + \frac{2D_0^{(v)}(\mu)}{C_0^{(v)}(\mu)}\right)\frac{m_b(\mu)}{M_B}$$

can express everything in terms of QCD FF's $V, A_{1,2}$ @ $\mathcal{O}(\alpha_s \Lambda_{\rm QCD}/Q)$!!!

High-q^2: OPE + HQET – Transversity Amplitudes

$$A_\perp^{L,R} = +\left[C^{L,R} + \tilde{r}_a\right] f_\perp\,, \qquad A_\parallel^{L,R} = -\left[C^{L,R} + \tilde{r}_b\right] f_\parallel\,,$$

$$A_0^{L,R} = -C^{L,R} f_0 - N M_B \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 \tilde{r}_b A_1 - \hat{\lambda}\,\tilde{r}_c A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

$\Rightarrow$ Universal short-distance coefficients: $C^{L,R} = C_9^{\rm eff} + \kappa \frac{2 m_b M_B}{q^2} C_7^{\rm eff} \mp C_{10}$

(SM: $C_9 \sim +4$, $C_{10} \sim -4$, $C_7 \sim -0.3$)

known structure of sub-leading corrections [Grinstein/Pirjol hep-ph/0404250]

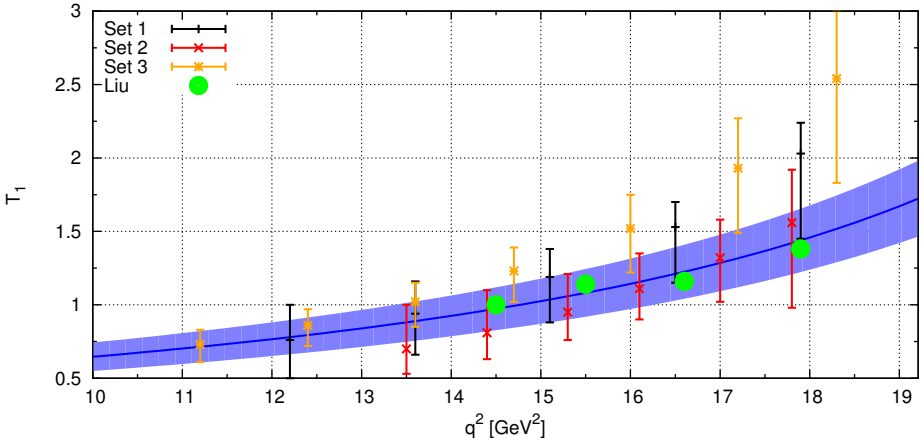
$$\tilde{r}_i \sim \pm \frac{\Lambda_{\rm QCD}}{m_b}\left(C_7^{\rm eff} + \alpha_s(\mu) e^{i\delta_i}\right)\,, \qquad i = a, b, c$$

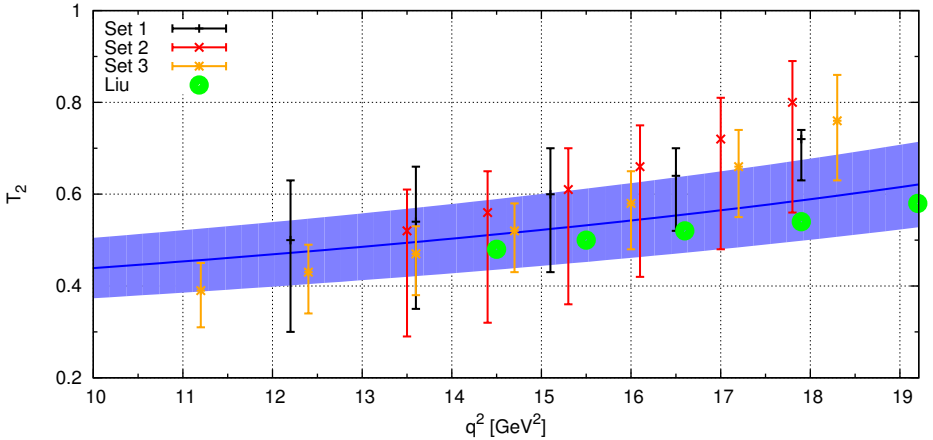
form factors ("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K*}} V, \quad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K*})\,A_1, \quad f_0 = \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

$B \to K^*$ Form factors at high-q^2 ...

... only known from extrapolation of LCSR at low-q^2 $\Rightarrow$ Lattice results desirable





LCSR extrapolation (Ball/Zwicky hep-ph/0412079) of $T_1(q^2)$ and $T_2(q^2)$ to high-q^2 versus quenched Lattice (3 data sets from Becirevic/Lubicz/Mescia hep-ph/0611295)

new unquenched Lattice results to come $\rightarrow$ Liu/Meinel/Hart/Horgan/Müller/Wingate arXiv:0911.2370, arXiv:1101.2726 no final uncertainty estimate yet

Angular observables ($m_\ell = 0$) – in terms of transversity amplitudes

$$4\,J_{2s} = |A_\perp^L|^2 + |A_\parallel^L|^2 + (L \to R), \qquad -J_{2c} = |A_0^L|^2 + |A_0^R|^2,$$

$$2\,J_3 = |A_\perp^L|^2 - |A_\parallel^L|^2 + (L \to R), \qquad \sqrt{2}\,J_4 = \mathrm{Re}\Big[A_0^L A_\parallel^{L\,*} + (L \to R)\Big],$$

$$\frac{J_5}{\sqrt{2}} = \mathrm{Re}\Big[A_0^L A_\perp^{L\,*} - (L \to R)\Big], \qquad \frac{J_{6s}}{2} = \mathrm{Re}\Big[A_\parallel^L A_\perp^{L\,*} - (L \to R)\Big],$$

$$\frac{J_7}{\sqrt{2}} = \mathrm{Im}\Big[A_0^L A_\parallel^{L\,*} - (L \to R)\Big], \qquad \sqrt{2}\,J_8 = \mathrm{Im}\Big[A_0^L A_\perp^{L\,*} + (L \to R)\Big],$$

$$J_9 = \mathrm{Im}\Big[A_\perp^L A_\parallel^{L\,*} + (L \to R)\Big]$$

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$$J_9 = \mathrm{Im}\left[A_\perp^L A_\parallel^{L\,*} + (L \to R)\right]$$

Within SM-basis and $m_\ell = 0 \to$ out of 12 J_i only 8 independent

$$J_{1s} = 3\,J_{2s}, \qquad J_{1c} = -J_{2c}, \qquad J_{6c} = 0,$$

and a 4th (not so trivial) relation [Egede/Hurth/Matias/Ramon/Reece arXiv:1005.0571]

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$$J_9 = \mathrm{Im}\Big[A_\perp^L A_\parallel^{L\,*} + (L \to R)\Big]$$

For example at Large Recoil: J_{2s}, J_3, J_{6s}, $J_9 \sim \xi_\perp \Rightarrow$ ratios have reduced hadronic uncertainty

$$A_T^{(2)} = \frac{J_3}{2\,J_{2s}}, \qquad A_T^{(\mathrm{re})} = \frac{J_{6s}}{4\,J_{2s}}, \qquad A_T^{(\mathrm{im})} = \frac{J_9}{2\,J_{2s}}$$

[Krüger/Matias hep-ph/0502060, Becirevic/Schneider, arXiv:1106.3283]

Angular observables @ Low Recoil using FF relations

[CB/Hiller/van Dyk arXiv:1006.5013]

$$\frac{4}{3}(2\,J_{2s}+J_3) = 2\,\rho_1\,f_\perp^2, \qquad -\frac{4}{3}J_{2c} = 2\,\rho_1\,f_0^2, \qquad \frac{2\sqrt{2}}{3}J_5 = 4\,\rho_2\,f_0 f_\perp,$$

$$\frac{4}{3}(2\,J_{2s}-J_3) = 2\,\rho_1\,f_\parallel^2, \qquad \frac{4\sqrt{2}}{3}\,J_4 = 2\,\rho_1\,f_0 f_\parallel, \qquad \frac{2}{3}J_{6s} = 4\,\rho_2\,f_\parallel f_\perp,$$

$$J_7 = J_8 = J_9 = 0,$$

$f_{\perp,\parallel,0}$ = form factors

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$$J_7 = J_8 = J_9 = 0,$$

$f_{\perp,\parallel,0}$ = form factors

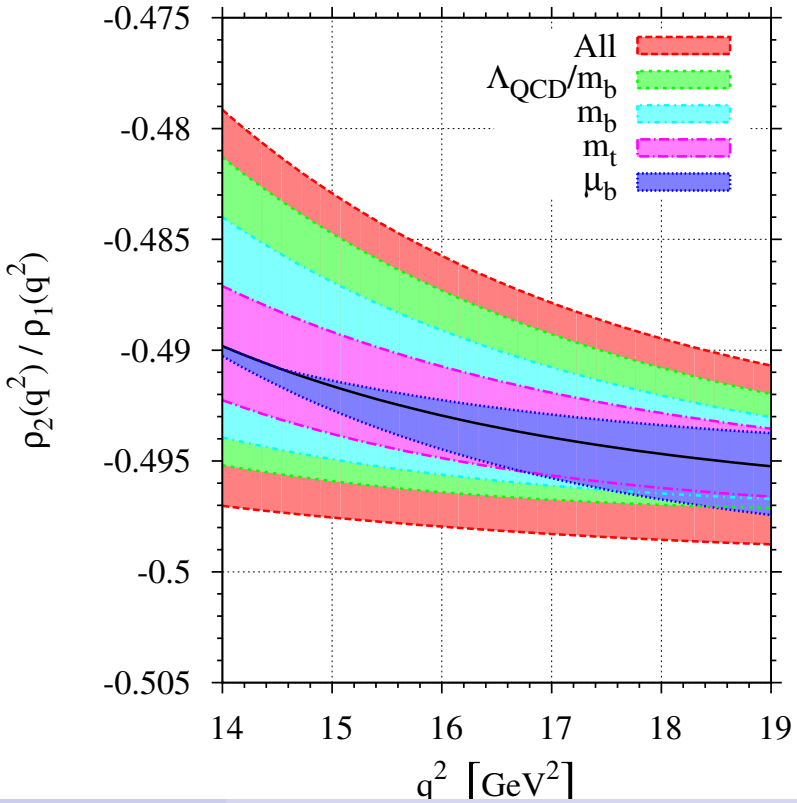
ρ_1 and ρ_2 are largely μ_b-scale independent

$$\rho_1(q^2) \equiv \left| C_9^{\rm eff} + \kappa\frac{2m_b^2}{q^2}C_7^{\rm eff}\right|^2 + |C_{10}|^2\,,$$

$$\rho_2(q^2) \equiv {\rm Re}\left[\left(C_9^{\rm eff} + \kappa\frac{2m_b^2}{q^2}C_7^{\rm eff}\right)C_{10}^*\right]$$

$\kappa(\mu_b)$ radiative QCD-correction to matching of FF relations between QCD and HQET

$\Rightarrow$ accounts for μ_b-dependence of tensor form factors $T_{1,2,3}$



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$$\frac{4}{3}(2\,J_{2s}-J_3) = 2\,\rho_1\,f_\parallel^2, \qquad \frac{4\sqrt{2}}{3}\,J_4 = 2\,\rho_1\,f_0 f_\parallel, \qquad \frac{2}{3}J_{6s} = 4\,\rho_2\,f_\parallel f_\perp,$$

$$J_7 = J_8 = J_9 = 0,$$

$f_{\perp,\parallel,0}$ = form factors

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = 2\,\rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \qquad A_{\mathrm{FB}} = 3\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_L = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \qquad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \qquad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \qquad A_T^{(4)} = 2\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

at low recoil: F_L, $A_T^{(2)}$, $A_T^{(3)}$ are short-distance independent, contrary to large recoil

$\Rightarrow$ could be used to fit form factor shape

Angular observables @ Low Recoil using FF relations [CB/Hiller/van Dyk arXiv:1006.5013]

$$\frac{4}{3}(2\,J_{2s}+J_3) = 2\,\rho_1\,f_\perp^2, \qquad -\frac{4}{3}J_{2c} = 2\,\rho_1\,f_0^2, \qquad \frac{2\sqrt{2}}{3}J_5 = 4\,\rho_2\,f_0 f_\perp,$$

$$\frac{4}{3}(2\,J_{2s}-J_3) = 2\,\rho_1\,f_\parallel^2, \qquad \frac{4\sqrt{2}}{3}\,J_4 = 2\,\rho_1\,f_0 f_\parallel, \qquad \frac{2}{3}J_{6s} = 4\,\rho_2\,f_\parallel f_\perp,$$

$$J_7 = J_8 = J_9 = 0,$$

$f_{\perp,\parallel,0}$ = form factors

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = 2\,\rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \qquad A_{\mathrm{FB}} = 3\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_L = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \qquad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \qquad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \qquad A_T^{(4)} = 2\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

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All relations valid up to sub-leading corrections in $C_7/C_9 \times \Lambda_{\mathrm{QCD}}/m_b$ due to FF relations.
(Later: OPE of 4-quark contributions yield also additional $(\Lambda_{\mathrm{QCD}}/m_b)^2$)

FF-free CP-asymmetries @ low recoil

[CB/Hiller/van Dyk arXiv:1105.0376]

$$a_{\rm CP}^{(1)} = \frac{\rho_1 - \bar\rho_1}{\rho_1 + \bar\rho_1}\,, \qquad a_{\rm CP}^{(2)} = \frac{\frac{\rho_2}{\rho_1} - \frac{\bar\rho_2}{\bar\rho_1}}{\frac{\rho_2}{\rho_1} + \frac{\bar\rho_2}{\bar\rho_1}}\,, \qquad a_{\rm CP}^{(3)} = 2\,\frac{\rho_2 - \bar\rho_2}{\rho_1 + \bar\rho_1}$$

- NLO QCD corrections large $\Rightarrow$ decrease CP-asymmetries
- still, theoretical uncertainties large: dominated by renorm. scale μ_b

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$B_s \to \phi(\to K^+K^-) + \ell^+\ell^-$

- time-integrated $a_{\rm CP}^{mix}$ in $B_s \to \phi(\to K^+K^-) + \bar\ell\ell$ is CP-odd = untagged
- $a_{\rm CP}^{mix}$ depends only on $(\Delta\Gamma_s/\Gamma_s)^2$ $\Rightarrow$ no sensitivity to sign of $\Delta\Gamma_s$
- since $(\Delta\Gamma_s/\Gamma_s)^2 \ll 1$ no significant sensitivity to B_s mixing parameters
  $\Rightarrow$ comparable to $a_{\rm CP}^{(3)}[B \to K^*\ell^+\ell^-]$

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$B \to K\,\ell^+\ell^-$

- @ high-q^2: $A_{\rm CP}[B \to K\,\ell^+\ell^-] = a_{\rm CP}^{(1)}[B \to K^*\ell^+\ell^-]$ in SM operator basis