# Lattice QCD and flavor physics 



Aida X. El-Khadra (UIUC)

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## Outline

Q Introduction and Motivation

- Kaons and pions
- leptonic and SL decays $V_{u s}, V_{u s} / V_{u d}$

$$
f_{K} / f_{\pi} \text { and } f_{+}^{\vec{K} \rightarrow \pi}(0)
$$

- neutral $K$ mixing and nonleptonic decays $B_{K} \& \epsilon_{K}$

$$
K \rightarrow \pi \pi \& \epsilon_{K}^{\prime} / \epsilon_{K}
$$

- $B \& D$ mesons
- leptonic $B \& D$ decays $f_{B}, f_{B_{s}}, f_{B_{s}} / f_{B}, f_{D}, f_{D_{s}}, f_{D_{s}} / f_{D}$
- neutral $B$ mixing $f_{B} \sqrt{B_{B}}, f_{B_{s}} \sqrt{B_{B_{s}}}, \xi$
- heavy to light decays $B \rightarrow \pi \ell \nu$ \& $V_{u b}$

$$
\begin{aligned}
& D \rightarrow K(\pi) \ell \nu \& V_{c s(d)} \\
& B \rightarrow K \ell^{+} \ell^{-}
\end{aligned}
$$

- $B$ to $D$ or $D^{*}$ decays $B \rightarrow D^{(*)} \ell \nu \& V_{c b}$

$$
B_{s} \rightarrow D_{s} \ell \nu / B \rightarrow D \ell \nu \& B_{s} \rightarrow \mu^{+} \mu-, \quad B \rightarrow D \tau \ell \nu
$$

© Conclusions \& outlook
© Appendix \& glossary

## Why Lattice QCD?

Laiho, Lunghi \& Van de Water (Phys.Rev.D81:034503,2010)


Error bands are (still) dominated by theory errors, in particular due to hadronic matrix elements.

## Why Lattice QCD?...cont'd

generic weak process involving hadrons:
(experiment) $=($ known $) \times($ CKM elements $) \times($ had. matrix element $)$

$$
\begin{aligned}
& \Delta m_{d(s)} \\
& \frac{d \Gamma(B \rightarrow \pi \ell \nu)}{d q^{2}}, \frac{d \Gamma(D \rightarrow K \ell \nu)}{d q^{2}}, \ldots \\
& \frac{d \Gamma\left(B \rightarrow D^{(*)} \ell \nu\right.}{d \omega} \\
& R(D)=\frac{\operatorname{Br}(B \rightarrow D \tau \nu)}{\operatorname{Br}(B \rightarrow D \ell \nu)}
\end{aligned}
$$



## Lattice QCD

parameterize the ME in terms of form factors, decay constants, bag parameters, ...

## Why Lattice QCD?...cont'd

## example:

$$
B-\bar{B} \text { mixing }
$$




## Introduction to Lattice QCD

$$
\langle\mathcal{O}\rangle \sim \int \mathcal{D} \psi \mathcal{D} \bar{\psi} \mathcal{D} A \mathcal{O}(\psi, \bar{\psi}, A) e^{-S} \quad S=\int d^{4} x\left[\bar{\psi}(\not D+m) \psi+\frac{1}{4}\left(F_{\mu \nu}^{a}\right)^{2}\right]
$$

use monte carlo methods (importance sampling) to evaluate the integral.
Note: integrating over the fermion fields leaves $\operatorname{det}(D D+m)$ in the integrand. the correlation functions, $\mathcal{O}$, are then written in terms of $(D+m)^{-1}$ and gluon fields

## steps of a lattice QCD calculation:

1. generate gluon field configurations according to $\operatorname{det}(I D+m) e^{-S}$
2. calculate quark propagators, $\left(D D+m_{q}\right)^{-1}$, for each valence quark flavor and source point
3. tie together quark propagators into hadronic correlation functions (usually 2 or 3pt functions)
4. statistical analysis to extract hadron masses, energies, hadronic matrix elements, .... from correlation functions
5. systematic error analysis

## errors, errors, errors, ...

$\checkmark$ statistical errors: from monte carlo integration also need to include errors from fit procedures
$\rangle$ finite lattice spacing, $a:\langle\mathcal{O}\rangle^{\text {lat }}=\langle\mathcal{O}\rangle^{\text {cont }}+O(a p)^{n}$ take continuum limit:
computational effort grows like $\sim(L / a)^{5-6}$
L
-7
$\checkmark$ finite volume: keep $\mathrm{m}_{\pi} L>4$
$>m_{l}$ dependence: chiral extrapolation
in numerical simulations, $m_{l}>m_{u d}$ but very recently ensembles with $m_{l}=m_{u d}$ (i.e. at physical value!)
$\Rightarrow$ use chiral perturbation theory to extrapolate or interpolate

$a(\mathrm{fm})$

$\checkmark n_{f}$ dependence: realistic sea quark effects: use $n_{f}=2+1$ or $n_{f}=2+1+1$
renormalization: $\left\langle J_{\mu}^{\text {cont }}\right\rangle=Z^{\text {lat }}\left\langle J_{\mu}^{\text {lat }}\right\rangle$
$\Rightarrow$ use lattice perturbation theory: $Z=z^{(0)}+z^{(1)} \alpha_{s}+z^{(2)} \alpha_{s}^{2}+O\left(\alpha_{s}^{3}\right)$ need to include PT errors
$\Rightarrow$ use nonrenormalized operators where possible
$\Rightarrow$ or use nonperturbative methods

## LQCD: Current status

plot by C. Hoelbling (based on Rev. Mod. Phys. 84 (2012) 449)


Different groups use different actions \& methods see glossary in Appendix
stable (or almost stable) hadrons, masses and amplitudes with no more than one initial (final) state hadron, for example:

## Focus on "easy" LQCD calculations

stable (or almost stable) hadrons, masses and amplitudes with no more than one initial (final) state hadron, for example:

- $\pi, K, D, D_{s}, B, B_{s}$ mesons
masses, decay constants, weak matrix elements for mixing, semileptonic and rare decay form factors
- charmonium and bottomonium ( $\left.\eta_{c}, J / \psi, h_{c}, \ldots, \eta_{b}, \mathrm{Y}(1 \mathrm{~S}), \mathrm{Y}(2 \mathrm{~S}), ..\right)$ states below open $D / B$ threshold
masses, leptonic widths, electromagnetic matrix elements

This list includes most of the important quantities for CKM physics. Excluded are $\rho, K^{*}$ mesons and other resonances.

## Lattice QCD program relevant to CKM elements

$$
\begin{aligned}
& V_{u d} \underset{V_{u s}}{ } \quad \begin{array}{c}
V_{u b} \\
K \rightarrow \pi l v
\end{array} \quad B \rightarrow \pi l v \\
& \pi \rightarrow \mu \nu, K \rightarrow \mu \nu \\
& V_{c d} \\
& V_{c s} \\
& D \rightarrow \pi l v \\
& D \rightarrow K l v \\
& B \rightarrow D, D^{*} l v \\
& D \rightarrow l v \quad D_{s} \rightarrow l v \\
& V_{t d} \\
& V_{t s} \\
& B^{0}-\overline{B^{0}} \text { mixing } B_{s}-\overline{B_{s}} \\
& K^{0}-\overline{K^{0}} \\
& V_{t b}
\end{aligned}
$$

## Strategy

- Lattice QCD action has the same free parameters as continuum QCD: quark masses and $\alpha_{s}$
- use experimentally measured hadron masses as input, for example: $\pi, K, D_{s}, B_{s}$ mesons for $u, d, s, c, b$ quark masses
- need an experimental input to determine the lattice spacing $(a)$ in GeV : $2 \mathrm{~S}-1 \mathrm{~S}$ splitting in Y system, $f_{\pi}, \Xi$ mass, $\ldots$ this also determines $\alpha_{s}$
- lattice QCD calculations of all other quantities should agree with experiment ...


## LQCD Achievements

MILC+HPQCD+FNAL (Phys. Rev. Lett. 92:022001,2004)
lattice QCD/experiment

see Appendix for a (partial) list of other LQCD achievements

## LQCD Achievements: Hadron spectrum


$\pi \ldots$ :.$\Omega$ :
$D, B$ : Fermilab, HPQCD, Mohler-Woloshyn

## LQCD Achievements: $f_{D s}$ time history

A. Kronfeld (Annu. Rev. Part. \& Nucl. Sci, arXiv:1203.1204)

see Appendix for other LQCD predictions

## progress in last $\sim 5$ years



## I will focus on reliable LQCD results that can be used for testing the SM!

new results reported at Lattice 2012


## Lattice Averages

- We now have reliable \& independent lattice results from different lattice groups using different methods for an increasing number of quantities $\Rightarrow$ need averages $\Rightarrow$ inputs into UT fits
- two efforts:

1. FLAG-1 (Flavianet Lattice Averaging Group)

Colangelo, et al (Eur. Phys. J. C71 (2011) 1695, http://itpwiki.unibe.ch/flag) 12 people (EU) light quark quantities only
2. LLV (Laiho, Lunghi, Van de Water)
(Phys.Rev.D81:034503,2010, http://latticeaverages.org/)
light and heavy quark quantities

+ UT fits with lattice averages as input


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+ UT fits with lattice averages as input


## $=$ FLAG -2 (Flavor Lattice Averaging Group)

28 people (EU, US, Japan) representing all big lattice collaborations light and heavy quark quantities
$1^{\text {st }}$ review at end of 2012

## Kaons and pions

- leptonic and semileptonic decay

$$
\left.\begin{array}{l}
f_{K} / f_{\pi} \\
f_{+}^{K \rightarrow \pi}(0)
\end{array}\right\} \quad V_{u s}, V_{u s} / V_{u d}
$$

$\uparrow$ mixing and nonleptonic decay

$$
\begin{aligned}
& B_{K} \& \epsilon_{K} \\
& K \rightarrow \pi \pi \& \epsilon_{K}^{\prime} / \epsilon_{K}
\end{aligned}
$$

## leptonic $K, \pi, D$, and $B$ meson decays

example: $B \rightarrow \tau \nu$

$\Gamma(B \rightarrow \tau \nu)=($ known $) \times\left(\left|V_{u b}\right|^{2}\right) \times f_{B}^{2}$

Same for $K, \pi, D$ mesons
use exp. combined with LQCD input for:

- determination of CKM element
- constraints on new physics ( $D, B$ )


## Kaon and pion decay constant ratio


$f_{K} / f_{\pi}$
$(\sim 0.4 \%$ error $)$
new results presented at Lattice 2012 not yet included.

FLAG-2 update (G. Colangelo at Lattice 2012)

## semileptonic $K, D$, and $B$ decays



## Form factor for $K \rightarrow \pi \ell \nu$

FLAG-2 update (G. Colangelo at Lattice 2012)

| $\begin{aligned} & \underset{\sim}{+} \\ & \underset{Z}{Z} \end{aligned}$ |  | JLQCD 11 <br> RBC/UKQCD 10 <br> RBC/UKQCD 07 <br> our estimate for $\mathrm{N}_{\mathrm{f}}=2+1$ |
| :---: | :---: | :---: |
| $\begin{aligned} & \mathbb{N} \\ & Z \end{aligned}$ |  | ETM 10D <br> ETM 09A <br> QCDSF 07 <br> RBC 06 <br> JLQCD 05 <br> JLQCD 05 <br> our estimate for $\mathrm{N}_{\mathrm{f}}=2$ |
| $\frac{\stackrel{U}{\#}}{\frac{U}{U}}$ |  | Kastner 08 <br> Cirigliano 05 <br> Jamin 04 <br> Bijnens 03 <br> LR 84 |

$$
f_{+}^{K \rightarrow \pi}(0)
$$

- new results by FNAL/MILC, RBC/UKQCD, JLQCD @ Lattice 2012.
- FNAL/MILC (Gamiz @ Lattice 2012): preliminary results with physical light quark masses


## $V_{u d}$ and $V_{u s}$

FLAG-2 update (G. Colangelo at Lattice 2012)

$\left|\frac{V_{u s}}{V_{u d}}\right| \frac{f_{K}}{f_{\pi}}=0.2758(5)$
(M. Antonelli, Eur. Phys. J. C (2010)69, 399)
$\left|V_{u s}\right| f_{+}(0)=0.2163(5)$
$\Rightarrow$ unitarity test of $1^{\text {st }}$ row of CKM matrix at $0.1 \%$ level

## neutral $K, B$, and $B_{s}$ meson mixing

example:

$$
B_{d}^{0}-\overline{B_{d}^{0}} \text { mixing }
$$



$$
\frac{\Delta M_{s}}{\Delta M_{d}}=\frac{m_{B_{s}}}{m_{B d}} \times\left|\frac{V_{t s}}{V_{t d}}\right|^{2} \times \xi^{2} \quad \text { with } \quad \xi \equiv \frac{f_{B_{s}} \sqrt{B_{B_{s}}}}{f_{B_{d}} \sqrt{B_{B_{d}}}}
$$

- many groups also calculate BSM mixing parameters
- for kaon mixing, just need to calculate the bag parameter(s).


## $B_{K}$ and $\epsilon_{K}$

$$
\epsilon_{K}=(\text { known }) \times B_{K} \kappa_{\epsilon} \times\left|V_{c b}\right|^{2} \times \underbrace{\substack{\text { non-local } \\ \text { operator }}}_{\substack{\text { ME of local } \\ \text { operator }}}
$$

## $B_{K}$ and $\epsilon_{K}$



## $B_{K}$

# updates by RBC/UKQCD, SWME, ETMC at Lattice 2012 

also new results for BSM MEs

[^0]
## $B_{K}$ and $\epsilon_{K}$

$$
\epsilon_{K}=(\text { known }) \times B_{K} \kappa_{\epsilon} \times\left|V_{c b}\right|^{2} \times \bar{\eta} \times f\left(\bar{\rho}, \bar{\eta}, V_{c b}, \eta_{i}\right)
$$

Enrico Lunghi (based on arXiv:1204.0791)


Dominant error on $\epsilon_{K}$ is now due to:

1. $V_{c b}$ 2. $\eta_{c c}$ or $\eta_{1}$ (NNLO pert. QCD)

## $K \rightarrow \pi \pi \& \epsilon_{K}^{\prime} / \epsilon_{K}$ : beyond "easy"

## $\Delta I=3 / 2$

- First quantitative results have been obtained by the RBC group at the ~20\% level from a direct calculation at small pion mass
(arXiv:1111.1699, 1111.4889, updated @ Lattice 2012).
- A new method was developed by Laiho \& Van de Water based on combining ChPT (indirect) and direct methods (arXiv:1011.4524), expect $\sim 20 \%$ error

$$
\Delta I=1 / 2
$$

- First calculation using the direct method on small volume and large pion mass with a $25 \%$ statistical error to establish feasibility by RBC group (arXiv:1111.1699, updated @ Lattice 2012).
- goal is obtain results for $\epsilon^{\prime} / \epsilon$ with $\sim 20 \%$ (stat+sys) error


## $B$ and $D$ mesons

$\checkmark$ leptonic $B \& D$ decays

$$
f_{B}, f_{B_{s}}, f_{B_{s}} / f_{B}, f_{D}, f_{D_{s}}, f_{D_{s}} / f_{D}
$$

$\uparrow$ neutral $B$ meson mixing

$$
f_{B} \sqrt{B_{B}}, f_{B_{s}} \sqrt{B_{B_{s}}}, \xi
$$

$\uparrow$ semileptonic $B \& D$ decays

$$
\begin{aligned}
B & \rightarrow \pi \ell \nu \& V_{u b} \quad D \rightarrow K(\pi) \ell \nu \& V_{c s(d)} \\
B & \rightarrow K \ell^{+} \ell^{-} \\
B & \rightarrow D^{(*)} \ell \nu \& V_{c b} \\
B_{s} & \rightarrow D_{s} \ell \nu / B \rightarrow D \ell \nu \& B_{s} \rightarrow \mu^{+} \mu-, \quad B \rightarrow D \tau \ell \nu
\end{aligned}
$$

## $B$ and $B_{s}$ meson decay constants


 Laiho, Lunghi \& Van

- HPQCD 11: uses HISQ b quarks, with extrapolation to physical b quarks mass using $1 / \mathrm{m}$ expansion.
- HPQCD 12: uses NRQCD b quarks, updated @ Lattice 2012.
- FNAL/MILC'11: results based on small data set. Lattice 2012: new results on full data set with $n_{f}=2+1$ coming soon.
- RBC/UKQCD: first results using rel. HQ action.
- $n_{f}=2$ : results by ALPHA, ETMC, updated at Lattice 2012.


## $D$ and $D_{s}$ meson decay constants

Laiho, Lunghi \& Van de Water (Phys.Rev.D81:034503,2010)


## $B$ and $B_{s}$ meson mixing parameters

Laiho, Lunghi \& Van de Water (Phys.Rev.D81:034503,2010)


- FNAL/MILC'11: preliminary results from partial data set for all 5 operators (including BSM) updated at Lattice 2012
- FNAL/MILC 2012: final analysis on small data set
- RBC/UKQCD: first results using rel. HQ action expected $\sim$ one year
- ETMC ( $n_{f}=2$ ): first preliminary results presented at Lattice 2012 also results for $D$ mixing


## semileptonic $D$ and $B$ decays

example: $B \rightarrow \pi \ell \nu$


$$
\left.\frac{d \Gamma(B \rightarrow \pi \ell \nu)}{d q^{2}}=(\text { known }) \times\left|V_{u b}\right|^{2}\right) \times\left|f_{+}\left(q^{2}\right)\right|^{2}
$$

$\star$ normalization $f_{+}(0) \Rightarrow V_{\text {CKM }}$

* shape for $B, D$ 's:
use z-expansion for model-independent parameterization of $q^{2}$ dependence
$\star$ test LQCD with shape, not normalization


## Form factor for $B \rightarrow \pi \ell \nu \& V_{u b}$

FNAL/MILC (PRD 79, 054507 (2009)) + BaBar (PRL 98, 091801 (2007))



## z-expansion:

$\star$ compare shape between LQCD and exp.
$\star$ combined fit of lattice and exp. data from different recoil regions
$\Rightarrow$ better determination of $V_{u b}$

## Form factors for $B \rightarrow \pi \ell \nu \& V_{u b}$

Laiho, Lunghi \& Van de Water (Phys.Rev.D81:034503,2010)


## Lattice 2012:

- new prelim. results by HPQCD, FNAL/MILC
- RBC/UKQCD: first results using rel. HQ action ~ one year
- ALPHA ( $n_{f}=2$ ): first prelim. results in static limit
cf. PDG'12 (HFAG 2011) (inclusive):

$$
\left|V_{u b}\right|=\left(4.41 \pm 0.15_{-0.17}^{+0.15}\right) \times 10^{-3}
$$

FNAL/MILC, HPQCD:
expect first results for $B_{s} \rightarrow K \ell \nu \sim$ one year

## Form factors for $D \rightarrow K(\pi) \ell \nu \& V_{c s(d)}$

Heechang Na (HPQCD PRD 84 (2011) 114505 \& PRD 82 (2010) 114506)


new method by HPQCD:

- use HISQ action for charm and light quarks
- calculate $f_{0}\left(q^{2}=0\right)$ from scalar current matrix element that doesn't require renormalization
- use kinematic constraint $f_{+}(0)=f_{0}(0)$


## Form factors for $D \rightarrow K(\pi) \ell \nu \& V_{c s(d)}$

Heechang Na (HPQCD PRD 84 (2011) 114505 \& PRD 82 (2010) 114506)


$\Rightarrow$ unitarity test of $2^{\text {nd }}$ row of CKM matrix:

$$
\left|V_{c d}\right|^{2}+\left|V_{c s}\right|^{2}+\left|V_{c b}\right|^{2}=0.976(50)
$$

## Form factors for $D \rightarrow K(\pi) \ell \nu \& V_{c s(d)}$

 Jon Bailey (FNAL/MILC Lattice 2012)

- shape from z-expansion
- partial data set (stat. errors only)

Form factors for $B \rightarrow K \ell^{+} \ell^{-}$

## Ran Zhou (FNAL/MILC 2012)



-FNAL/MILC (Lattice 2012): shape from z-expansion, systematic errors included expect final results this fall

- HPQCD (Lattice 2012): preliminary results for $B \rightarrow K^{*} \ell^{+} \ell^{-}, B_{s} \rightarrow \phi \ell^{+} \ell^{-}, \ldots$ assume $K^{*}$ stable (narrow width approximation)

Form factors for $B \rightarrow D^{(*)} \ell \nu \& V_{c b}$

$$
\begin{aligned}
& \frac{d \Gamma\left(B \rightarrow D^{*} \ell \nu\right)}{d \omega}=(\text { known }) \times\left|V_{c b}\right|^{2} \times\left(\omega^{2}-1\right)^{1 / 2}|\mathcal{F}(\omega)|^{2} \\
& \frac{d \Gamma(B \rightarrow D \ell \nu)}{d \omega}=(\text { known }) \times\left|V_{c b}\right|^{2} \times\left.\left(\omega^{2}-1\right)^{3 / 2} \mathcal{G}(\omega)\right|^{2}
\end{aligned}
$$

at zero recoil (HFAG 2011):

$$
\begin{gathered}
B \rightarrow D^{*} \ell \nu:\left|V_{c b}\right| \mathcal{F}(1)=(35.90 \pm 0.45) \times 10^{-3} \\
B \rightarrow D \ell \nu:\left|V_{c b}\right| \mathcal{G}(1)=(42.6 \pm 1.5) \times 10^{-3}
\end{gathered}
$$

$\Rightarrow$ need form-factors at non-zero recoil for $B \rightarrow D \ell \nu$ to match precision for $V_{c b}$ determination from $B \rightarrow D^{*} \ell \nu$

## Form factors for $B \rightarrow D^{(*)} \ell \nu \& V_{c b}$

Laiho, Lunghi \& Van de Water (Phys.Rev.D81:034503,2010)

$\left|V_{c b}\right|=(39.5 \pm 1.0) \times 10^{-3}$
cf. PDG'12 (HFAG) (inclusive):

$$
\left|V_{c b}\right|=(41.9 \pm 0.7) \times 10^{-3}
$$

Form factor ratio $R(D)=\operatorname{Br}(B \rightarrow D \tau \nu) / \operatorname{Br}(B \rightarrow D \ell \nu)$

$$
\frac{d \Gamma}{d q^{2}}=\left|V_{c b}\right|^{2} \times\left(\operatorname{known}\left(q^{2}\right)\right) \times\left[f_{+}^{2}\left(q^{2}\right)+(\text { known }) \times f_{0}^{2}\left(q^{2}\right) \times m_{\ell}^{2}\right]
$$

BaBar (V. Lüth, FPCP 2012, arXiv:1205.5442):
$R(D)=\frac{\operatorname{Br}(B \rightarrow D \tau \nu)}{\operatorname{Br}(B \rightarrow D \ell \nu)}=0.440(71)$
to compare to SM we need the form factors at non-zero recoil.

Form factor ratio $R(D)=\operatorname{Br}(B \rightarrow D \tau \nu) / \operatorname{Br}(B \rightarrow D \ell \nu)$
FNAL/MILC (arXiv:1206.4992, PRL)


## 2HDM II with FNAL/MILC

 form factors (band includes sys. error)FNAL/MILC form factors: from partial data set used in arXiv:1202.6346

2 HDM II with form factors using quenched LQCD, HQS, kinematic constraints, ...

- similar estimate for $R(D)_{\text {sm }}$ by Becirevic, Kosnik, Tayduganov (arXiv: 1206.4977)
- $R\left(D^{*}\right)$ : need four form factors, larger discrepancy with SM

Form factor ratio $B_{s} \rightarrow D_{s} \ell \nu / B \rightarrow D \ell \nu \& B_{s} \rightarrow \mu^{+} \mu-$

- LHCb measures the rare Bs decay using a normalization channel

$$
\operatorname{Br}\left(B_{s} \rightarrow \mu^{+} \mu^{-}\right)=\operatorname{Br}\left(B_{d} \rightarrow X\right) \frac{f_{d}}{f_{s}} \frac{\epsilon_{X}}{\epsilon_{\mu \mu}} \frac{N_{\mu \mu}}{N_{X}}
$$

$\Rightarrow$ They need to know $f_{s} / f_{d}$

- new strategy: determine $f_{s} / f_{d}$ from hadronic decay ratio

$$
\operatorname{BR}\left(\bar{B}_{s}^{0} \rightarrow D_{s}^{+} \pi^{-}\right) / \mathrm{BR}\left(\overline{\bar{B}}^{0} \rightarrow D^{+} K^{-}\right)
$$

using factorization (Fleischer et al, arXiv:1004.3984):

$$
\left(\frac{f_{s}}{f_{d}}=0.0743 \times \frac{\tau_{B^{0}}}{\tau_{B_{s}^{0}}} \times\left[\frac{\epsilon_{D K}}{\epsilon_{D_{s} \pi}} \frac{N_{D_{s} \pi}}{N_{D K}}\right] \times \frac{1}{\mathcal{N}_{a} \mathcal{N}_{F}} \quad \text { with } \quad \mathcal{N}_{a}=\left[\frac{a_{1}^{(s)}\left(D_{s}^{+} \pi^{-}\right)}{a_{1}^{(d)}\left(D^{+} K^{-}\right)}\right]^{2}\right.
$$

and $\quad \mathcal{N}_{F}=\left[\frac{f_{0}^{(s)}\left(M_{\pi}^{2}\right)}{f_{0}^{(d)}\left(M_{K}^{2}\right)}\right]^{2}$

Form factor ratio $B_{s} \rightarrow D_{s} \ell \nu / B \rightarrow D \ell \nu \& B_{s} \rightarrow \mu^{+} \mu-$

Daping Du (FNAL/MILC, J. Bailey et al, arXiv:1202.6346) calculate

$$
\frac{f_{0}^{(s)}\left(M_{\pi}^{2}\right)}{f_{0}^{(d)}\left(M_{K}^{2}\right)}=1.046(44)(15)
$$

using a subset of the full FNAL/MILC data set.
comparison with BaBar 2010 data (arXiv:0904.4063):


Form factor ratio $B_{s} \rightarrow D_{s} \ell \nu / B \rightarrow D \ell \nu \& B_{s} \rightarrow \mu^{+} \mu-$
Daping Du (FNAL/MILC, J. Bailey et al, arXiv:1202.6346) calculate

$$
\frac{f_{0}^{(s)}\left(M_{\pi}^{2}\right)}{f_{0}^{(d)}\left(M_{K}^{2}\right)}=1.046(44)(15)
$$

using a subset of the full MILC/FNAL data set.


## Conclusions \& Outlook

- reliable LQCD results for "easy" quantities are here! reliable $=$ complete systematic error budget
- light quarks: several (many) results with different actions and methods, all in reasonable agreement.
- heavy quarks: dominated by HPQCD \& FNAL/MILC, with new results coming from other groups (RBC/UKQCD, ETMC, ALPHA, ...) soon.
- expect a large increase in computational resources (Bluegene Q, Blue Waters, GPU clusters, ....)
- three groups have already generated ensembles with light sea quark masses at their physical values
$\Rightarrow$ expect to see an increasing number of physics results with these and an increasing number of such ensembles
- averages: LLV + FLAG-1 = FLAG-2 $\Rightarrow$ use as inputs to UT fits


## Conclusions \& Outlook

- LQCD is systematically improvable
- most sys. errors are constrained/determined by MC data
- Better precision is still needed in order to maximize the impact of flavor physics experiments
$\Rightarrow$ constrain/discover/understand NP from the precision frontier
- As LQCD errors decrease with better simulations we'll need to include effects that are currently subdominant, for example:
$\star$ isospin breaking
$\star$ EM effects
$\star$ charm sea quarks in progress ...
- Also in progress: Develop methods to reliably calculate quantities that are beyond "easy", for example:
* weak hadronic decays (kaon, D meson, B meson,...)
* non-local operators for $D$ mixing
* weak decays to resonances ( $\mathrm{K}^{*}$, rho, ...)


## Conclusions \& Outlook

- creativity can yield (hard to predict) progress, beyond expected improvements due to increase in computational resources, for example:
- z-expansion for shape of form factors
- development of HISQ action for heavy quarks
- HPQCD's method for calculating decay constants and form factors at $q^{2}=0$ using nonrenormalized currents
- twisted boundary conditions for calculating form factors directly at $q^{2}=0$


## Appendix

- more on LQCD introduction \& achievements
- Glossary of actions:
light quarks heavy quarks
- Glossary of commonly used terms: quenched approximation, full QCD, rooted staggered Chiral PT, ....


## LQCD: Current status



Many different groups, different actions, methods need to increase finite volume as pion mass decreases

## LQCD Achievements: Predictions

Form factor shape for $D \rightarrow K l v$


Form factor shape for $D \rightarrow \pi l v$

(Phys. Rev. Lett. 94:011601, 2005)

- Normalization agrees with experiment plus CKM unitarity
-Prediction of the shape
also: $B_{c}$ mass prediction (HPQCD+FNAL PRL 2005, hep-lat0411027)


# LQCD Achievements: Predictions 

## $f_{D+} \quad D^{+}$meson decay constant

FNAL/MILC (PRL 2005, hep-lat/0506030

## CLEO-c (PRL 2005, hep-ex/0508057)



## Introduction to Lattice QCD


discretize the QCD action (Wilson, ...)
e.g. discrete derivative

$$
\partial_{\mu} \psi(x) \rightarrow \Delta_{\mu} \psi(x)=\frac{1}{2 a}[\psi(x+a \hat{\mu})-\psi(x-a \hat{\mu})]
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in general: $\quad\langle\mathcal{O}\rangle^{\text {lat }}=\langle\mathcal{O}\rangle^{\text {cont }}+O(a p)^{n} \quad n \geq 1$
errors scale with the typical momenta of the particles, e.g. $\left(\Lambda_{\mathrm{QCD}} a\right)^{n}$ for gluons and light quarks $\Rightarrow$ keep $1 / a \ll \Lambda_{\mathrm{QCD}}$
typical lattice spacing $a \leqslant 0.1 \mathrm{fm}$ or $1 / a \gtrsim 2 \mathrm{GeV}$ in practice: need to consider a range of $a$ 's

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Improvement: add more terms to the action to make $n$ large

- Asqtad (improved staggered):
errors: $\sim \mathrm{O}\left(\alpha_{\mathrm{s}} a^{2}\right), \mathrm{O}\left(a^{4}\right)$, but large due to taste-changing interactions
has chiral symmetry; uses square root of the determinant in sea
computationally efficient
- HISQ (Highly Improved Staggered Action): also similar: HYP smeared
errors: $\sim \mathrm{O}\left(\alpha_{s} a^{2}\right), \mathrm{O}\left(a^{4}\right), \times 1 / 3$ smaller than Asqtad
comp. cost: efficicient, $\times 2$ Asqtad
- improved Wilson (Clover, ...): also Stout link smeared
errors: $\sim \mathrm{O}\left(\alpha_{s} a\right)$, if tree-level (tadpole) imp.; $\mathrm{O}\left(a^{2}\right)$ if nonpert. imp.
Wilson term breaks chiral symmetry
comp. cost: $\times 4$ Asqtad for $m_{\text {light }} \sim m_{\text {strange }}$, but less efficient at small quark masses
- twisted mass Wilson (tmQCD):
errors: ~ $\mathrm{O}\left(a^{2}\right)$
twisted mass term for quark masses at chiral limit
comp. cost: $\times 4$ Asqtad
- Domain Wall Fermions (DWF):
errors: $\sim \mathrm{O}\left(a^{2}\right), \mathrm{O}\left(m_{\text {res }} a\right)$
almost exact chiral symmetry; breaking $\sim m_{\text {res }} \sim 3 \times 10^{-3}$
comp. cost: $\times L_{5}$ Asqtad, $L_{5} \sim 16-20$
- Overlap Fermions:
errors: $\sim \mathrm{O}\left(a^{2}\right)$
exact chiral symmetry
comp. cost: $\times 5-10$ DWF
- relativistic HQ actions (Fermilab, Columbia, AKT,...)
start with $O(a)$ improved WIIson action
use HQET to analyze discretization errors
$\Rightarrow$ no errors that increase as power of $a m_{Q}$
- NRQCD
start with effective theory, then discretize
power expansion in $p / m$ or $v$
need to keep lattice spacing > 0
$\Rightarrow$ use highly improved action with negligible discretization errors
needs scaling window to extrapolate errors due to light quark and gluon actions
- HQET/static
$1 / m$ expansion, static limit is the leading term
$1 / m$ corrections $\sim 10 \%$ included in HQET
- HISQ charm

HISQ action has very small discretization errors
$\Rightarrow$ can be used for charm and heavier quarks, keep $a m_{Q}<1$

## Glossary - sea quarks

- quenched approximation: no sea quarks, $n_{f}=0$
$\operatorname{det}(D+m)=$ const. $\Rightarrow$ computational cost reduced by factor $\sim 100-1000$
but systematic errors ~ 10-30\% ( for $\pi$ 's $K$ 's, ... particles without decay thresholds)
- unquenched: $n_{f} \neq 0$
simulation includes sea quarks, $\quad \operatorname{det}(D+m)$ included in integration
- $n_{f}=2$
two degenerate flavors of light quarks (for up and down) in sea, generally with $m_{l}>m_{u d}$ strange quark is still quenched
- $n_{f}=2+1$
two degenerate flavors (for $u$ and $d$ ) plus one heavier sea quark (for $s$ ) with mass $\approx m_{s}{ }^{\text {phys }}$
- $n_{f}=2+1+1$
two degenerate flavors (for $u$ and $d$ ) plus one heavier sea quark (for $s$ ) with mass $\approx m_{s}^{\text {phys }}$ plus one heavy sea quark (for $c$ ) with mass $\approx m_{c}^{\text {phys }}$
- partially quenched: $n_{f} \neq 0$ with $m_{\text {sea }} \neq m_{\text {valence }}$
sea quarks are computationally much more expensive than valence quarks
$\Rightarrow$ one often generates several light valence quarks on each sea quark ensemble
use partially quenched ChPT; extremely useful for determining chiral parameters


## Glossary con'td

- rooted staggered quarks
doubling problem $\Rightarrow 4$ "tastes" (degenerate lattice quark flavors) for every continuum flavor in the sea: $\quad \sqrt{\operatorname{det}(D D+m)} \Rightarrow$ two remaining tastes = two degenerate continuum flavors $(u, d)$ $\sqrt[4]{\operatorname{det}(D+m)} \Rightarrow$ one remaining taste $=$ one flavor $(s)$
- Is rooted staggered lattice QCD = QCD ?
$\sqrt{\operatorname{det}(\not D+m)}$ is nonlocal at $a \neq 0$ (Bernard, Golterman, Shamir)
but there is a lot of evidence that nonlocality $\sim a^{2}$
based on renormalization group analysis (Shamir) and ChPT analysis (Bernard)
also a growing body of numerical checks (Dürr\& Hoelbling, Follana, Hart \& Davies, MILC, ..)
- rooted staggered chiral perturbation theory
accounts for the taste violations in the rooted staggered sea
$\Rightarrow$ includes leading discretization effects, which can then be removed in continuum limit


[^0]:    FLAG-2 update (G. Colangelo at Lattice 2012)

