Attractor mechanism : Trio of effective black hole potential, entropy functional and Routhian

Sourav Roychowdhury

CQUeST, Sogang University

Collaboration with Arghya Chattopadhyay (University of Puerto Rico Mayagüez, USA) and Alessio Marrani (University of Hertfordshire, UK)

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Black Hole Thermodynamics - from Semiclassical to Quantum

Kyung Hee University, Republic of Korea

Introduction & Motivation

- The Attractor mechanism is one of the most intriguing features of extremal black hole solutions in SUGRA : providing dynamical explanation for the universality of extremal BH entropy .

- Originally discovered in ungauged $\mathcal{N} = 2, D = 4$ SUGRA by Ferrara, Gibbons, & Kallosh and extended by many others .

- For extremal, spherically symmetric, static, asymptotically flat BH, the scalar fields of the theory depend on the radial direction only and exhibits an attractor point at the horizon (unique) of BH $\longrightarrow$ the values of the scalar fields at BH horizon are independent of their asymptotic values and completely fixed by conserved electromagnetic charges of BH : No hair property .

Introduction & Motivation

- In order to study the radial behaviour of scalar fields we need to construct an one dimensional effective Lagrangian (under the assumption of static, asymptotically flat, extremal & spherically symmetric BH) .

- One dimensional effective Lagrangian contains with the warp factor entering the 4d BH metric, scalar field (moduli) and a positive definite effective black hole potential V_{BH} .

- The critical points of $V_{BH} \rightarrow$ horizon values of scalars and Hamiltonian constraint relates the critical value $V_{BH,Hor}$ to Bekenstein-Hawking entropy .

  : **S. Ferrara, G.W. Gibbons, & R. Kallosh ; 1997** .

Introduction & Motivation

- A. Sen introduced entropy functional formalism based on universal emergence of AdS_2 factor at near horizon geometry for extremal BH $\longrightarrow$ compute entropy of various classes of extremal black hole by extremizing a Legendre transformation of near horizon Lagrangian density .

- Extremum value of the entropy functional gives entropy of BH .

- Entropy functional formalism is a very powerful framework for higher-derivative theory, precise comparison between macroscopic entropy and microstates counting, also for supersymmetric setup .

  : **A. Sen ; 2005** .

Questions we asked

- How do we get one-dimensional effective action from the higher-dimensional seed action of the theory ??
- The equivalence between FGK framework & Entropy functional framework ??
- Connection between black hole potential at horizon, entropy functional with BH entropy ??

Outline

- FGK framework and one dimensional effective action
- One dimensional effective action through Routhian formulation
- Attractor mechanism and entropy
- Entropy functional framework
- Connection between Routhian formulation and entropy functional approach
- Summary, conclusion & future directions

Black holes in ungauged $(1+3)$d SUGRA

- General action of Maxwell-Einstein-scalar with no scalar potential in 4d (with a slight restriction of generality, we do consider complex scalar fields)

$$\begin{aligned}\mathcal{S} \;\; = \int d^4x\sqrt{-g}\Big[\;\; & -\frac{R}{2} + \frac{1}{2}g^{\mu\nu}\,G_{a\bar{a}}(z^a,\bar{z}^{\bar{a}})\partial_\mu z^a \partial_\nu \bar{z}^{\bar{a}} \\ & + \frac{1}{4}\mu_{\Lambda\Sigma}(z^a,\bar{z}^{\bar{a}})F^{\Lambda}_{\mu\nu}F^{\Sigma\mu\nu} \\ & + \frac{1}{4}\nu_{\Lambda\Sigma}(z^a,\bar{z}^{\bar{a}})F^{\Lambda}_{\mu\nu}\big(\star\, F^{\Sigma\mu\nu}\big)\Big] \;, \end{aligned} \tag{1}$$

- Extremal, static, spherically symmetric & asymptotically flat solution

$$ds^2 = e^{2U(\tau)}dt^2 - e^{-2U(\tau)}\left(\frac{d\tau^2}{\tau^4} + \frac{1}{\tau^2}d\Omega_2^2(\theta,\phi)\right) \;, \tag{2}$$

being the only non-vanishing components of the Abelian 2-form field strengths $F^{\Lambda}_{\tau t}$, $F^{\Lambda}_{\theta\phi}$ and $z^a = z^a(\tau)$.

Equation of motion and black hole potential

- Equation of motion of metric function $U(\tau)$ and moduli field $z^a(\tau)$:

$$\begin{aligned} \ddot{U}(\tau) &= e^{2U(\tau)} V_{BH} \ , \\ \ddot{z}^a(\tau) + \Gamma^a_{bc} \dot{z}^a(\tau) \dot{z}^b(\tau) &= e^{2U(\tau)} G^{a\bar{b}} \frac{\partial V_{BH}}{\partial \bar{z}^{\bar{b}}} \ , \\ \dot{U}(\tau)^2 + \frac{1}{2} G_{a\bar{a}} \dot{z}^a(\tau) \dot{\bar{z}}^{\bar{a}}(\tau) - e^{2U(\tau)} V_{BH} &= 0 \ . \end{aligned} \tag{3}$$

- Black hole potential (positive definite) :

$$V_{BH} = -\frac{1}{2} \begin{pmatrix} p^\Lambda & q_\Lambda \end{pmatrix} \begin{pmatrix} \left(\mu + \nu \mu^{-1} \nu\right)_{\Lambda\Sigma} & \left(\nu \mu^{-1}\right)^\Sigma_\Lambda \\ \left(\mu^{-1} \nu\right)^\Lambda_\Sigma & \left(\mu^{-1}\right)^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^\Sigma \\ q_\Sigma \end{pmatrix} . \tag{4}$$

FGK framework and one-dimensional effective action

- V_{BH} is a functional of the moduli scalars $z^a, \bar{z}^{\bar{a}}$.
- The last equation is a constraint, can be derived from Einstein equation of the 4d action .
- It can be checked (and it was actually guessed by FGK!) that EOM of the metric function $U(\tau)$ and the moduli fields $z^a, \bar{z}^{\bar{a}}$ can be read from the one-dimensional effective action

$$\mathfrak{S}_{eff} = \int d\tau \left(\dot{U}(\tau)^2 + \frac{1}{2} G_{a\bar{a}} \dot{z}^a(\tau) \dot{\bar{z}}^{\bar{a}}(\tau) + e^{2U(\tau)} V_{BH} \right) , \qquad (5)$$

the role of the time is played by the radial coordinate τ .

S. Ferrara, G.W. Gibbons, & R. Kallosh ; 1997

4d action to one-dimensional effective action : Naïve approach

- Express field strength and dual field strength in terms of potentials :

$$F^{\Lambda}_{\tau t} = \partial_\tau \psi^{\Lambda} \; ; \; \mathcal{G}_{\Lambda \tau t} = \partial_\tau \chi_{\Lambda} \; ,$$

$$\mathcal{G}_{\Lambda \mu\nu} = -\epsilon_{\mu\nu\rho\sigma} \frac{\partial \mathcal{S}}{\partial F^{\Lambda}_{\rho\sigma}} = -\mu_{\Lambda\Sigma} \big(\star F \big)^{\Sigma}_{\mu\nu} + \nu_{\Sigma\Lambda} F^{\Sigma}_{\mu\nu} \; . \tag{6}$$

$\psi^{\Lambda} \rightarrow$ electric potential, $\chi_{\Lambda} \rightarrow$ magnetic potential .

- Integrating angular parts, the one-dimensional effective Lagrangian

$$\mathcal{L}_{eff} \; = -\dot{U}(\tau)^2 - \tfrac{1}{2} G_{a\bar{a}} \dot{z}^a(\tau) \dot{\bar{z}}^{\bar{a}}(\tau) + e^{2U(\tau)} \tilde{V}_{BH} \; . \tag{7}$$

Naïve approach $\rightarrow$ Wrong expression of black hole potential

- The expression of $\tilde{V}_{BH}$

$$\tilde{V}_{BH} = \frac{1}{4}\begin{pmatrix}\tilde{p}^{\Lambda} & \tilde{q}_{\Lambda}\end{pmatrix}\begin{pmatrix}-\left(\mu+\nu\mu^{-1}\nu\right)_{\Lambda\Sigma} & -\left(\nu\mu^{-1}\right)^{\Sigma}_{\Lambda} \\ \left(\mu^{-1}\nu\right)^{\Lambda}_{\Sigma} & \left(\mu^{-1}\right)^{\Lambda\Sigma}\end{pmatrix}\begin{pmatrix}\tilde{p}^{\Sigma} \\ \tilde{q}_{\Sigma}\end{pmatrix}. \quad (8)$$

with $\tilde{p}^{\Lambda} = \frac{\partial\mathcal{L}}{\partial(\partial_\tau\chi_\Lambda)}$; $\tilde{q}_{\Lambda} = \frac{\partial\mathcal{L}}{\partial(\partial_\tau\psi^\Lambda)}$.

- V_{BH} and $\tilde{V}_{BH}$ differs by signs in front of the vector coupling terms in diagonal along with off diagonal entities and $\tilde{V}_{BH}$ is NOT positive definite .

- The above approach leads to wrong expression of black hole potential, hence discarded !

Routhian formulation

- Field strength $F^{\Lambda}_{\tau t} = \partial_\tau \psi^{\Lambda}$ behaves like momentum, not the $F^{\Lambda}_{\theta\phi}$ component .
- Write down the 4d dimensional Lagrangian in terms of the radial derivative of the potential ψ^{Λ} (: $\partial_\tau \psi^{\Lambda}$) .
- Take the Legendre transformation of the 4d Lagrangian with respect to $\partial_\tau \psi^{\Lambda}$.
- Routhian

$$\mathcal{R} = \partial_\tau \psi^{\Lambda} \frac{\partial \mathcal{L}}{\partial(\partial_\tau \psi^{\Lambda})} - \mathcal{L} \ . \tag{9}$$

Routhian formulation $\rightarrow$ correct V_{BH}

- Routhian density

$$\mathcal{R} = \dot{U}(\tau)^2 + \frac{1}{2} G_{a\bar{a}} \dot{z}^a(\tau) \dot{\bar{z}}^{\bar{a}}(\tau) + e^{2U(\tau)} V_{BH} \ . \tag{10}$$

with the correct positive definite expression of V_{BH}

$$V_{BH} = -\frac{1}{2} \begin{pmatrix} p^\Lambda & q_\Lambda \end{pmatrix} \begin{pmatrix} \left(\mu + \nu\mu^{-1}\nu\right)_{\Lambda\Sigma} & \left(\nu\mu^{-1}\right)^\Sigma_\Lambda \\ \left(\mu^{-1}\nu\right)^\Lambda_\Sigma & \left(\mu^{-1}\right)^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^\Sigma \\ q_\Sigma \end{pmatrix} \ , \tag{11}$$

with $p^\Lambda = \frac{\partial \mathcal{R}}{\partial(\partial_\tau \chi_\Lambda)}$; $q_\Lambda = \frac{\partial \mathcal{R}}{\partial(\partial_\tau \psi^\Lambda)}$ and $V_{BH} \geq 0$.

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Routhian formulation gives one dimensional effective action

- One can view Routhian as an effective Lagrangian where the radial coordinate τ plays as time .
- The effective one dimensional action of the BH described by FGK is actually our Routhian of the four-dimensional action . This puts FGK one-dimensional effective Lagrangian on solid foundations, in the context of classical mechanics .
- The Routhians with respect to other momentum like quantities do NOT provide correct 1d FGK action . Our choice/prescription is the only one giving rise to the correct one.

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Hamiltonian constraint

- The Routhian does not have any explicit τ dependence $\rightarrow$ reparametrisation invariance .
- τ is an arbitrary parameter not time. Hence in this picture the Hamiltonian must generate reparametrisation rather than time evolution .
- Unlike standard mechanics the system does not evolve in a traditional time-dependent manner, but constrained to satisfy the equations at all points along $\tau \rightarrow$ Hamiltonian to be zero .
- Hamiltonian constraint :

$$\mathcal{H} = p_U \dot{U} + p_a \dot{z}^a + p_{\bar{a}} \dot{\bar{z}}^{\bar{a}} - \mathcal{R} = 0 = \dot{U}^2 + \frac{1}{2} G_{a\bar{a}} \dot{z}^a \dot{\bar{z}}^{\bar{a}} - e^{2U} V_{BH} \ . \quad (12)$$

$\rightarrow$ constraint in FGK EOM .

Equation of motion of Moduli fields

- EOM of scalar field reads from :

$$\partial_\tau \left(\frac{\partial \mathcal{R}}{\partial \left(\partial_\tau \bar{z}^{\bar{a}} \right)} \right) - \frac{\partial \mathcal{R}}{\partial \bar{z}^{\bar{a}}} = 0 \ . \tag{13}$$

- Considering moduli space as a complex Kähler manifold (consistent with $\mathcal{N} = 1$ and $\mathcal{N} = 2$, $D = 4$ SUGRA)

$$G_{a\bar{b}} = \partial_{z^a} \partial_{\bar{z}^{\bar{b}}} K \ . \tag{14}$$

$K \rightarrow$ Kähler potential .

- EOM of z^a

$$\partial_\tau^2 z^a + \Gamma^a_{bc} \partial_\tau z^b \partial_\tau z^c = e^{2U} G^{a\bar{b}} \frac{\partial V_{BH}}{\partial \bar{z}^{\bar{b}}} \ ; \ \Gamma^a_{bc} = G^{a\bar{d}} \partial_{\bar{z}^{\bar{d}}} \partial_{z^b} \partial_{z^c} K \ . \tag{15}$$

Scalar fields at the horizon

- At the horizon (resides at $\tau \sim +\infty$) the metric function behaves

$$e^{-2U}|_{\tau\sim\infty} = \frac{A}{4\pi}\tau^2 \ . \tag{16}$$

  A is the horizon area .

- Considering a coordinate transformation $\omega = \ln \tau$, EOM of moduli field becomes

$$\partial_\omega^2 z^a + \Gamma^a_{bc}\partial_\omega z^b \partial_\omega z^c = G^{a\bar{b}}\frac{\partial V_{BH}}{\partial \bar{z}^{\bar{b}}} \ . \tag{17}$$

- In terms of the new coordinate ω, at fixed time the interval is $ds^2 = r_h^2 d\omega^2 \ \ ; \ r_h^2 = \frac{A}{4\pi}$.
- Therefore the coordinate ω considered as the physical distance from the horizon .

Scalar fields at the horizon & Attractor Mechanism

- The horizon ($\tau \sim +\infty$) corresponds to $\omega \sim +\infty \longrightarrow$ in terms of the proper distance the horizon is infinitely far away : familiar picture of infinite AdS_2 throat of an extremal black hole $\longrightarrow$ scalar field must traverse an infinitely long region to reach the horizon.
- The scalar fields and their derivatives must reach finite values at the horizon .
- Say, as $\omega \to \infty$, $\partial_\omega z^a$ tends to some non zero value . Then from the relation

$$z^a(\omega) = z^a(\omega_0) + \int_{\omega_0}^{\omega} d\omega' \partial_\omega z^a \ . \tag{18}$$

$\longrightarrow$ a non-vanishing velocity integrated over the infinite range of ω would force $z^a \to \infty$. A non-zero $\partial_\omega z^a$ at $\omega \to \infty$ would inevitably make the scalar field diverge as one approaches the horizon.

Attractor Mechanism

- The only way to avoid this divergence is for the field to come to rest at the bottom of the throat $\longrightarrow$ leads the conditions:

$$\begin{aligned} \lim_{\tau\to\infty} z^a(\tau) &= z^a_H = \text{const. ; } |z^a_H| < \infty \ , \\ \lim_{\tau\to\infty} \dot{z}^a(\tau) &= 0 \ . \end{aligned} \tag{19}$$

- An analogous argument by induction yields to conclude that all higher-order derivatives of the scalar fields z^a should also vanish at the horizon .

- Scalar fields take constant values at horizon $\longrightarrow$ horizon behaves an Attractor .

Attractor Mechanism Contd.

- Constant value of moduli fields at the horizon implies

$$\left.\frac{\partial V_{BH}}{\partial \bar{z}^{\bar{b}}}\right|_{z^a = z^a_H} = 0 \ . \tag{20}$$

- The constant values z^a_H of the moduli at the horizon = critical value of V_{BH} .

- The value of z^a_h that solves the above equation is independent of its initial condition $\Rightarrow$ the horizon behaves an attractor point .

S. Ferrara, G.W. Gibbons & R. Kallosh ; 1997
R. Kallosh, N. Sivanandam & M. Soroush ; 2006

Entropy

- To compute entropy we focus on the near horizon region .
- Horizon is a attractor point for z^a with the metric function $e^{-2U}|_{\tau\sim\infty} = \frac{A}{4\pi}\tau^2$.
- Considering Hamiltonian constraint ($\mathcal{H} = 0$) at the horizon

$$\dot{U}^2 = e^{2U} V_{BH} \ , \tag{21}$$

leads the entropy

$$S_{BH} = \frac{A}{4\pi} = \pi V_{BH}|_{horizon} \ . \tag{22}$$

- S_{BH} is fixed by the critical point of V_{BH} and matches with *Bekenstein-Hawking* entropy .

Sen prescription

- Sen prescription: compute entropy from 4d action with static, spherically symmetric solution .

- Focusing on the near horizon geometry and upon changing the coordinate $\tau \to 1/r$ the geometry becomes

$$ds^2 = \frac{1}{v} r^2 dt^2 - v \left[\frac{dr^2}{r^2} + d\Omega_2^2(\theta, \phi) \right] \; ; \; v = \frac{A}{4\pi} = \frac{S}{\pi} \; . \tag{23}$$

- At near horizon, $z^a = \text{const}$, $F^\Lambda_{rt} = e^\Lambda$, $F^\Lambda_{\theta\phi} = -p^\Lambda \sin\theta$.

Sen Prescription Contd.

- Defining the functional $f(z^a, e^\Lambda, p^\Lambda)$ as the evaluation of the $\mathcal{L}$ over angular coordinates on the near horizon

$$f(z^a, e^\Lambda, p^\Lambda) = \int d\theta d\phi \sqrt{-g}\mathcal{L} = -2\pi v \mu_{\Lambda\Sigma} e^\Lambda e^\Sigma + \frac{2\pi}{v}\mu_{\Lambda\Sigma} p^\Lambda p^\Sigma - 4\pi \nu_{\Lambda\Sigma} e^\Lambda p^\Sigma \ . \tag{24}$$

- The magnetic charge is given by

$$q_\Lambda = \frac{1}{4\pi}\frac{\partial f}{\partial e^\Lambda} = -\left(v\mu_{\Lambda\Sigma}e^\Sigma + \nu_{\Lambda\Sigma}p^\Sigma\right) \ . \tag{25}$$

Entropy functional and entropy

- Sen entropy functional

$$\mathcal{E}(z^a, e^\Lambda, p^\Lambda, q_\Lambda) = \frac{v}{4}\left(4\pi e^\Lambda q_\Lambda - f(z^a, e^\Lambda, p^\Lambda)\right), \tag{26}$$

- At the extrema of this function where

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial z^a} = 0 &\implies z^a = \text{constant}, \\ \frac{\partial \mathcal{E}}{\partial e^\Lambda} = 0 &\implies 4\pi q_\Lambda = \frac{\partial f}{\partial e^\Lambda}\ . \end{aligned} \tag{27}$$

- Wald entropy of BH is given by the extremum of $\mathcal{E}$.

A. Sen ; 2005

Entropy functional, black hole potential and entropy

- Using the relation

$$S_W = \mathcal{E}_{extremum} = \pi \, V_{BH}|_{horizon} \, . \tag{28}$$

$\rightarrow$ relates the entropy function ($\mathcal{E}$) with the black hole potential V_{BH} along with the Wald entropy (S_W) of the black hole at their respected extrema.

- At the horizon EOM of z^a are given by the critically condition

$$\frac{\partial \mathcal{E}}{\partial z^a} = 0 = \frac{\partial V_{BH}}{\partial z^a} \tag{29}$$

$\Longrightarrow$ compatible with the FGK EOM .

Entropy functional and Routhian

- Wald entropy can be read as Legendre transformation of $f(z^a, e^\Lambda, p^\Lambda)$, leads to say that the Routhian formulation $(\mathcal{R})$ can relate with $\mathcal{E}$ formulation .

- Instead of $\{p^\Lambda, q_\Lambda\}$ write down $\mathcal{R}$ in terms of $\{e^\Lambda, p^\Lambda\}$

$$\mathcal{R} = \dot{U}^2 + \frac{1}{2} G_{a\bar{a}} \dot{z}^a \dot{\bar{z}}^{\bar{a}} - \frac{1}{2} \mu_{\Lambda\Sigma} \left(e^{-2U} e^\Lambda e^\Sigma + e^{2U} p^\Lambda p^\Sigma \right) \ . \tag{30}$$

- Make a coordinate transformation $\tau = 1/r$ so that

$$\mathfrak{S} = 4\pi \int \mathcal{R}(\tau) d\tau = 4\pi \int \mathfrak{R}(r) dr \ ; \ \mathfrak{R}(r) = -\frac{\mathcal{R}(r)}{r^2} \tag{31}$$

Entropy functional and Routhian Contd.

- At near horizon

$$\mathfrak{R}_{NH} = -r^2 U'^2 - \frac{r^2}{2} G_{a\bar{a}} z'^a \dot{\bar{z}}'^{\bar{a}} + \frac{1}{2}\left(v\ \mu_{\Lambda\Sigma} e^{\Lambda} e^{\Sigma} + \frac{1}{v} \mu_{\Lambda\Sigma} p^{\Lambda} p^{\Sigma} \right) . \tag{32}$$

  $'$ denotes derivative with respect to r coordinate .

- At the horizon ($r = 0$)

$$\mathfrak{R}_{Horizon} = \frac{1}{2}\left(v \mu_{\Lambda\Sigma} e^{\Lambda} e^{\Sigma} + \frac{1}{v} \mu_{\Lambda\Sigma} p^{\Lambda} p^{\Sigma} \right) . \tag{33}$$

- Defining $\mathfrak{R}$ we already integrated out the 4π factor arising from integrating angular parts .

Routhian-Black hole potential-Entropy function-Wald Entropy

- At horizon

$$-v\pi\ \mathfrak{R}_{Horizon} = \pi V_{BH}|_{Horizon}\ . \tag{34}$$

- All together we have

$$-v\pi\ \mathfrak{R}_{Horizon} = \pi V_{BH}|_{Horizon} = \mathcal{E}_{extremum} = S_W \tag{35}$$

$\rightarrow$ a close logical circle with Routhian, the black hole potential, the entropy functional and the Wald entropy .

- The entropy computation through entropy function formulation is equivalent to Routhian framework .

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Summary and Conclusions

- We considered extremal, static, spherically symmetric BH in Maxwell-Einstein-scalar theory in bosonic sector of 4d ungauged SUGRA and discussed a reformulation of the attractor dynamics .
- We showed that standard one-dimensional effective action (introduced by FGK) is a very specific Routhian reduction of the underlying four dimensional covariant theory .
- The Horizon behaves an Attractor and the value of the moduli scalar at horizon is fixed by the critical point of the black hole potential (V_{BH}) .
- Hamiltonian constraint relates the critical value $V_{BH,Hor}$ to Bekenstein-Hawking entropy .

Summary and Conclusions

- The entropy of BH is given by extremum value of the entropy function which is same as the horizon value of black hole potential (up to a multiplicative factor) .

- The criticality conditions of black hole potential, entropy functional and Routhian all coincide and correctly provide the entropy of BH (in the case of two-derivative theory) .

Take away messages

- 1d FGK effective action $\Longrightarrow$ very specific Routhian reduction of the underlying 4d theory .
- BH entropy $\Longrightarrow$ horizon value of V_{BH} .
- Attractor value of Moduli field $\Longrightarrow$ critical point of V_{BH} .
- Routhian $\Longrightarrow$ "off-shell" (away from horizon) generalization of $\mathcal{E}$.
- At the horizon, $\mathcal{R}$, $\mathcal{E}$ and V_{BH} all coincide and gives BH entropy .

Future Directions & Ongoing works

- Higher derivative corrections : Routhian-based derivation of the effective dynamics may provide a systematic extension of the attractor flows beyond two derivatives.
- Beyond spherical symmetry: Rotating Attractor with near horizon geometry $AdS_2 \times S^1$, Multi-center BH, topological solutions .
- Near horizon AdS_2 symmetry with AdS_2/CFT_1 .
- Attractors in gauged SUGRA solutions with Abelian gauging (Fayet-Iliopoulos, STU-model) and non-Abelian gauging .
- Attractors in 5d black rings : dyonic, asymptotically non-flat (extremal) solutions in 5D, with near horizon geometry $AdS_2 \times S^1 \times S^2$.

Key References

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Thank You !