Emittance measurement in laser plasma accelerators

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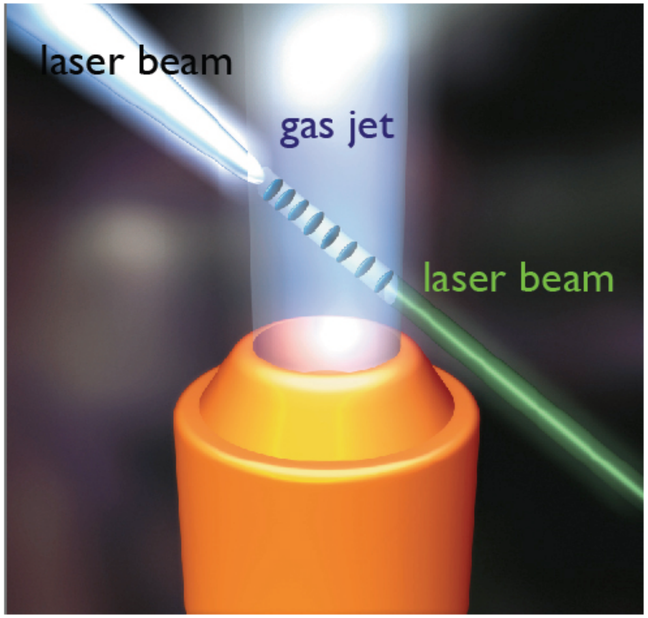
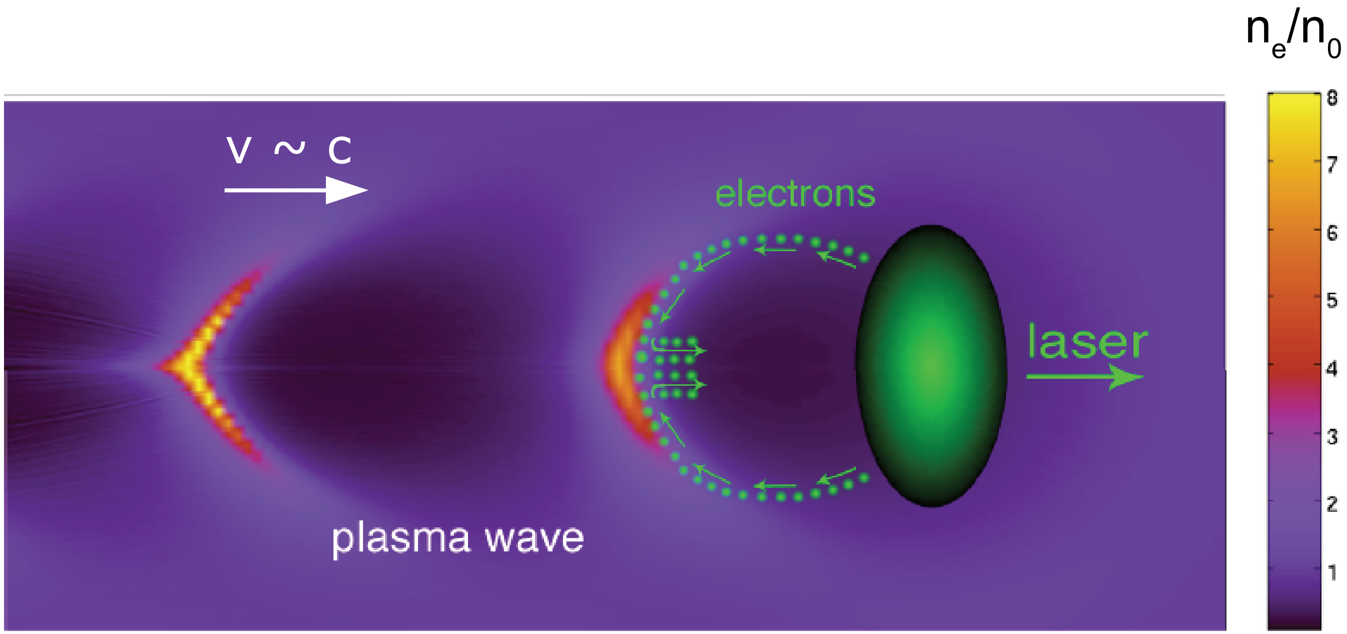
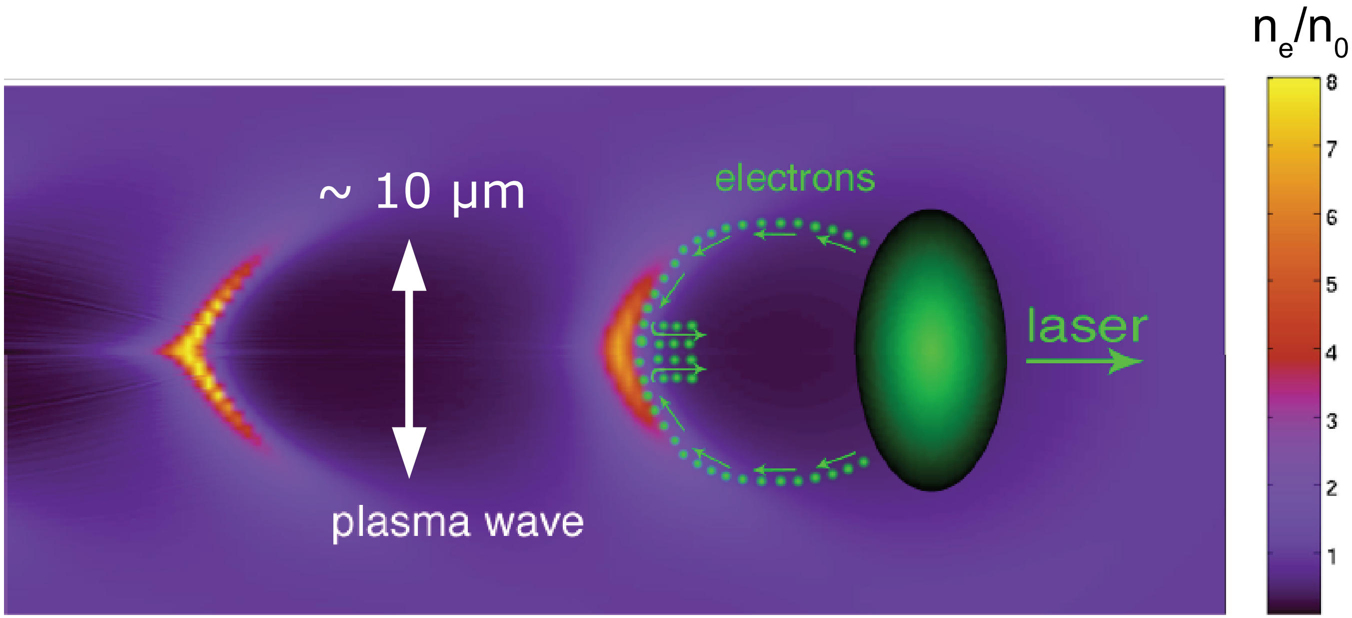
Laser-plasma acceleration





- The rising edge of the laser ionizes the gas and creates a plasma.
- The laser pulse triggers electric fields ~ 100 GV/m in its wake.
- Trapped electrons are accelerated up to ~ 100 MeV – 1 GeV within few mm.

Laser-plasma acceleration



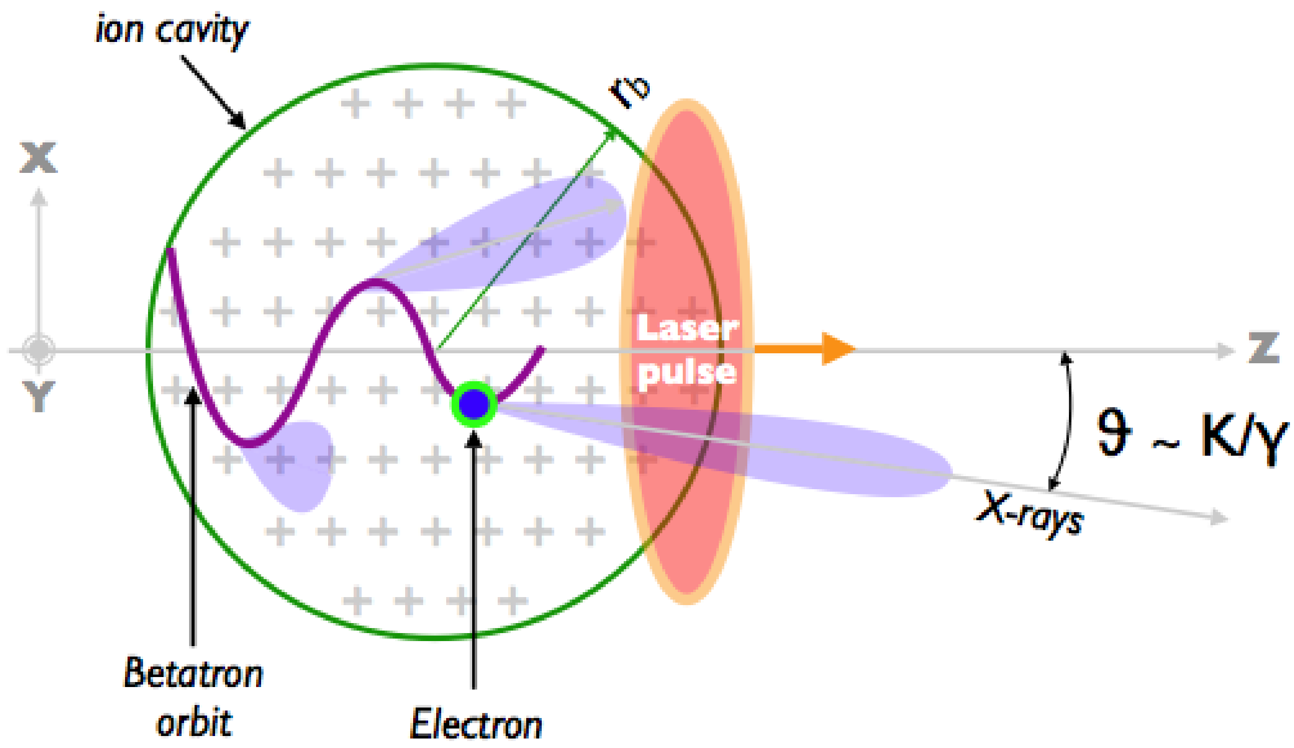
Main properties :

- Up to 1 GeV electron energy.
- ~ 10-100 pC charge, fs duration, $\Delta E/E$ ~ 1-10%.
- Transverse size ~ µm, divergence ~ mrad

  ➡ ε_N ~ π.mm.mrad.

Because of the large divergence, conventional emittance measurement techniques are not suitable.

Betatron emission in laser accelerator



Longitudinal force *Transverse force*

$$\frac{d\vec{p}}{dt} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

$$\vec{F}_{\perp} = -\frac{1}{2} m \alpha \omega_p^2 r \vec{e}_r$$

➡ ***Harmonic oscillator in the transverse direction***

A. Rousse *et al.* Phys. Rev. Lett. **93**, 135005 (2004)

Betatron emission in laser accelerator

Betatron oscillations for an adiabatic acceleration

$$\hat{x}(t) = \sqrt{\hat{J}} \sin\left[\int_0^t \omega_\beta(t')dt' + \phi\right]$$

$$\hat{p}_x(t) = \sqrt{\hat{J}} \cos\left[\int_0^t \omega_\beta(t')dt' + \phi\right]$$

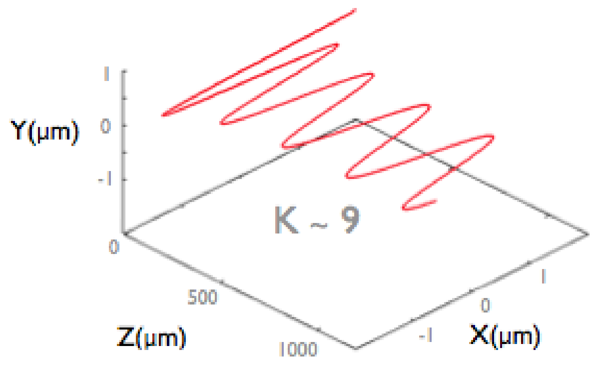
where

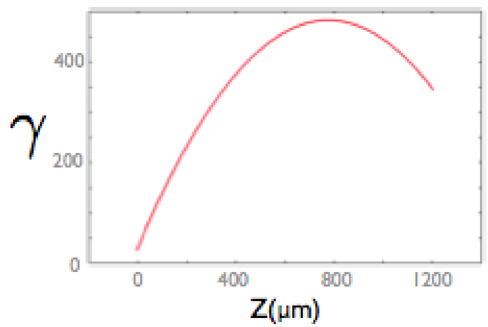
$$\hat{x} = \left(\frac{\alpha\gamma}{8}\right)^{1/4} k_p x$$

$$\hat{p}_x = \frac{p_x}{(2\alpha\gamma)^{1/4} mc}$$

$$\hat{J} = \frac{k_p J}{mc} = \hat{x}^2 + \hat{p}_x^2$$

➡ Sinusoidal oscillations with a time varying frequency $\omega_\beta = \sqrt{\alpha \omega_p^2 / 2\gamma(t)}$

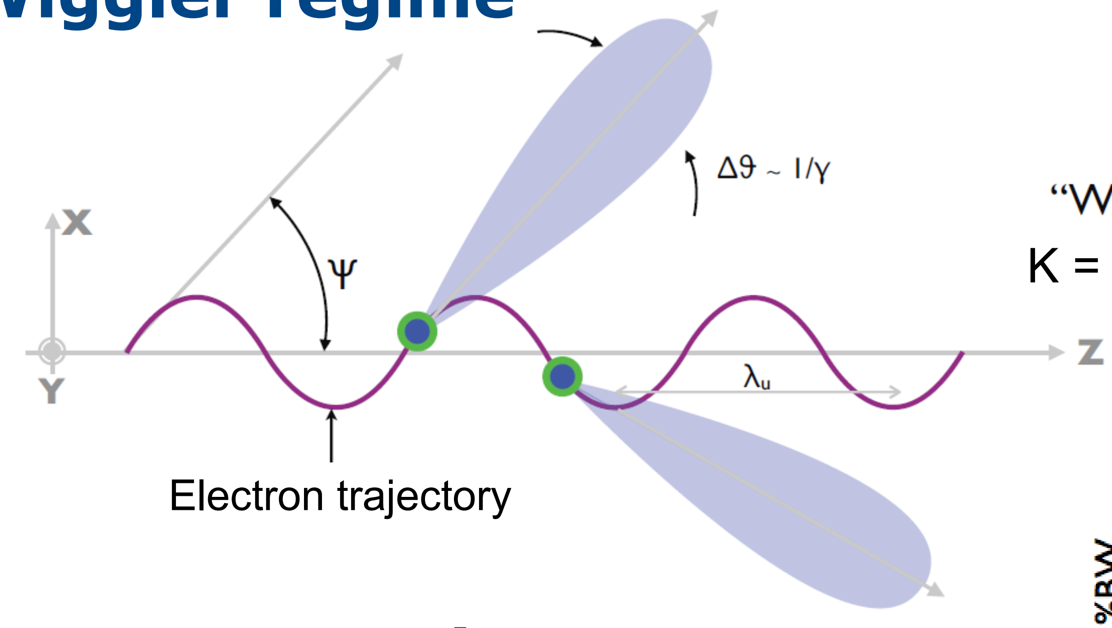




Plasma cavity = undulator

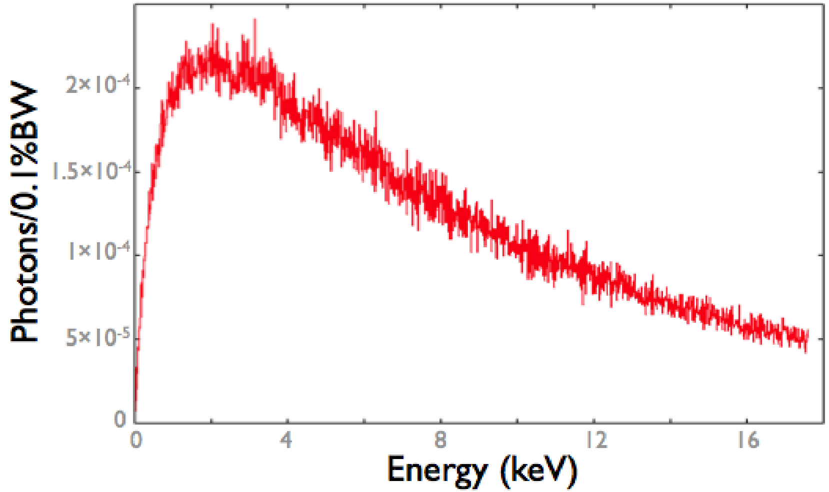
Betatron emission in laser accelerator

Wiggler regime



"Wiggler"

K = γ ψ >> 1



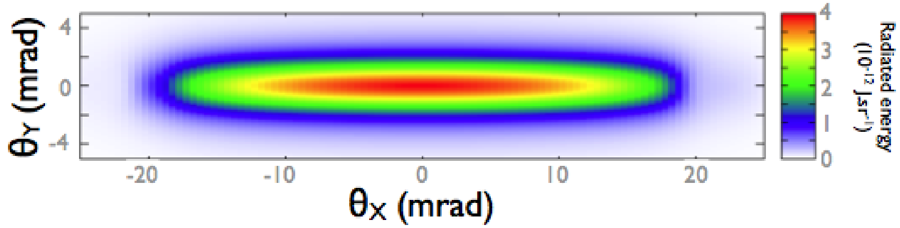
X-ray properties

$$E_c = \frac{3\hbar\omega_p^2}{4c}\,\gamma^2\alpha r_\beta$$

$$\theta = \frac{p_\beta}{\gamma mc} = \frac{k_p}{\sqrt{2}}\,\gamma^{-1/2}\alpha^{1/2}r_\beta$$

$$r_\beta = \sqrt{\hat{J}}\left(\frac{\alpha\gamma}{8}\right)^{-1/4}k_p^{-1}$$

$$p_\beta = \sqrt{\hat{J}}\,(2\alpha\gamma)^{1/4}\,mc$$



Principle of emittance measurement

Three variables (γ, α, ε_N) ➡ three measurements

- Electron spectrum ➡ γ
- X-ray angular profile ➡ $p_\beta/(\gamma m c) \propto \epsilon_N^{1/2} \alpha^{1/4} \gamma^{-3/4}$
- X-ray spectrum ➡ $\gamma^2 \alpha r_\beta \propto \epsilon_N^{1/2} \gamma^{7/4} \alpha^{3/4}$
- X-ray source size ➡ $r_\beta \propto \epsilon_N^{1/2} (\alpha \gamma)^{-1/4}$

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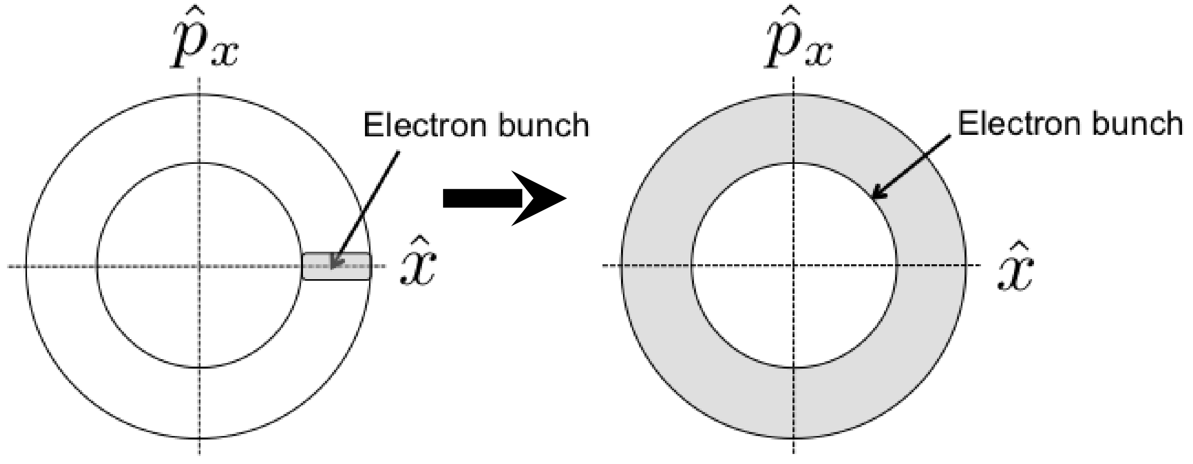
- Symmetrized emittance = non-coherent upper limit of the normalized emittance.



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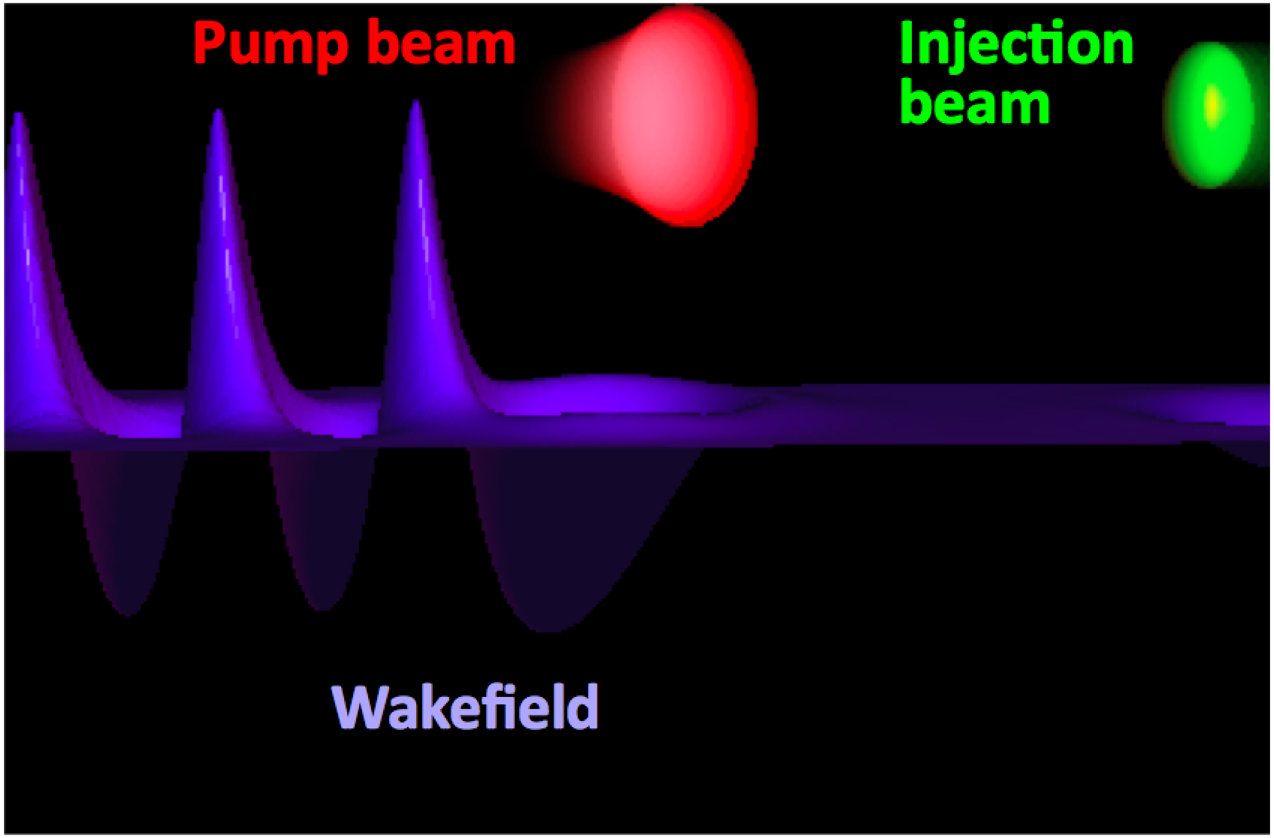
A precise measure requires a good stability and a control over the acceleration

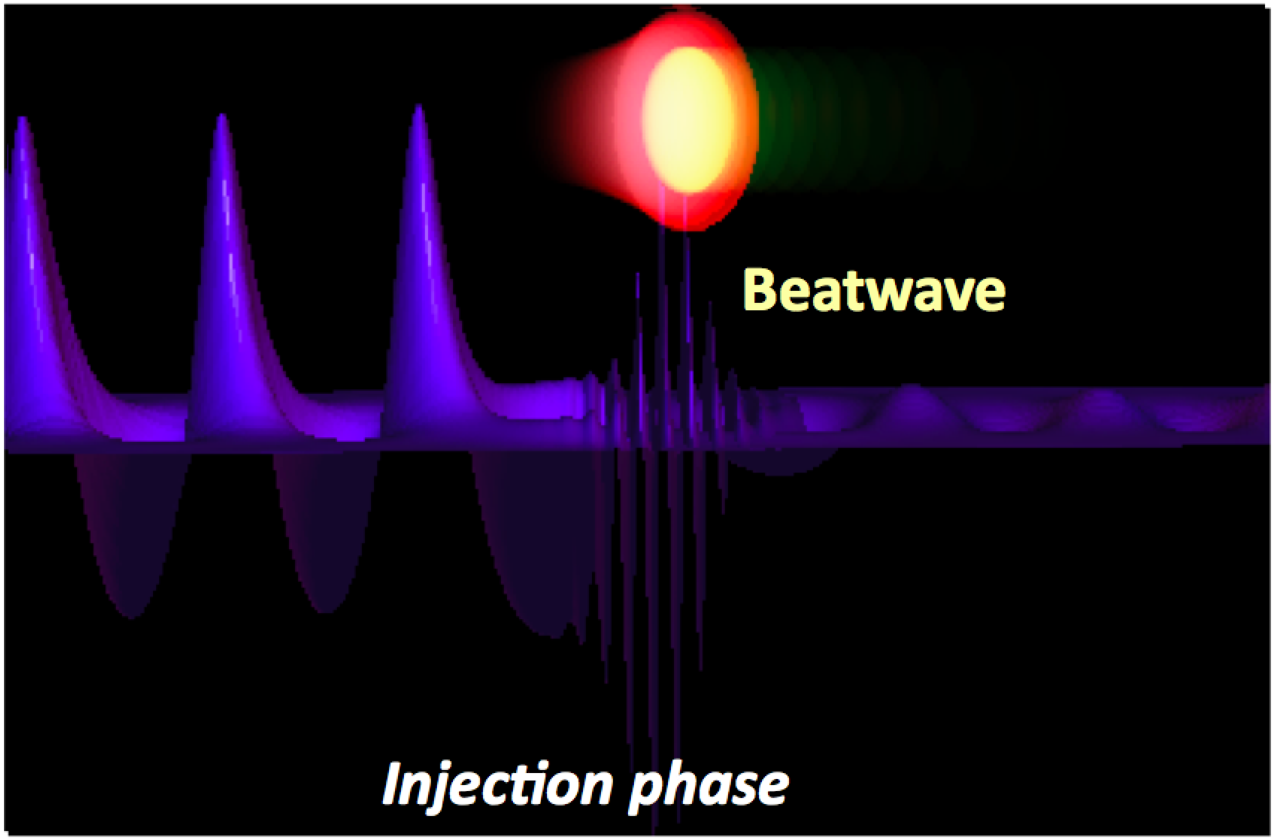
Colliding injection



- Pump beam ➡ accelerating structure.
- Injection beam ➡ local injection.

J. Faure *et al.* Nature **444,** 737-739 (2006)

Colliding injection



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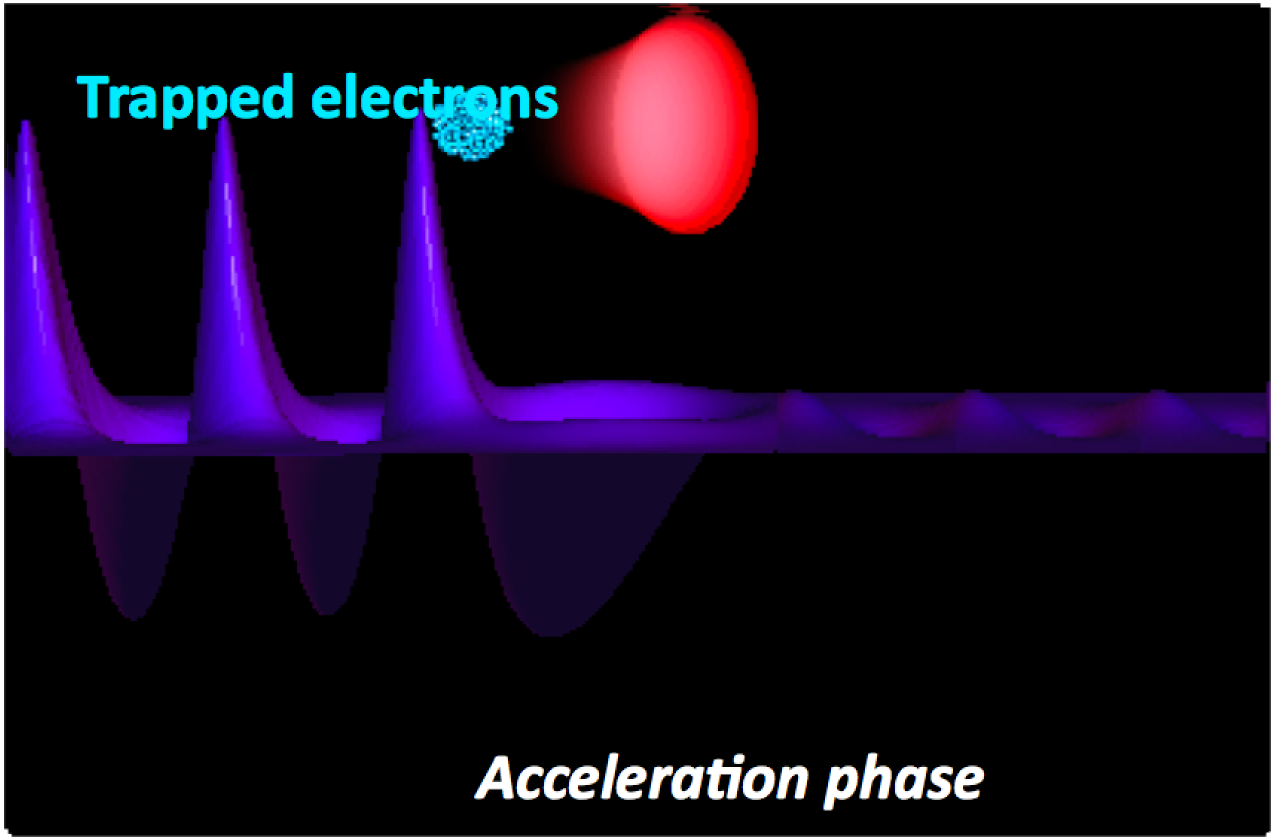
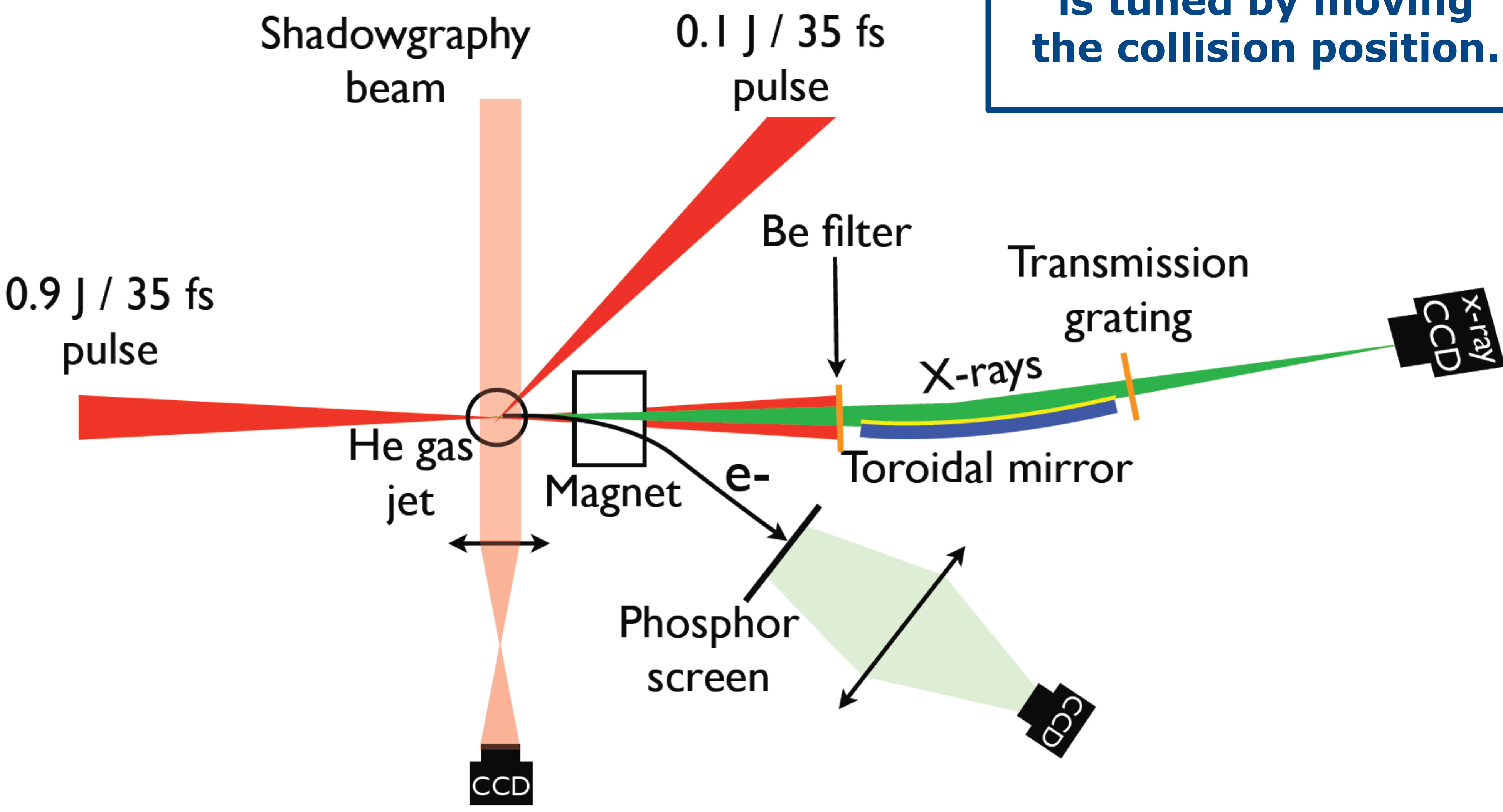


- Pump beam ➡ accelerating structure.
- Injection beam ➡ local injection.
- During the collision, some electrons are heated by the beat-wave ponderomotive force ➡ they gain enough energy to be trapped.

J. Faure *et al.* Nature **444,** 737-739 (2006)

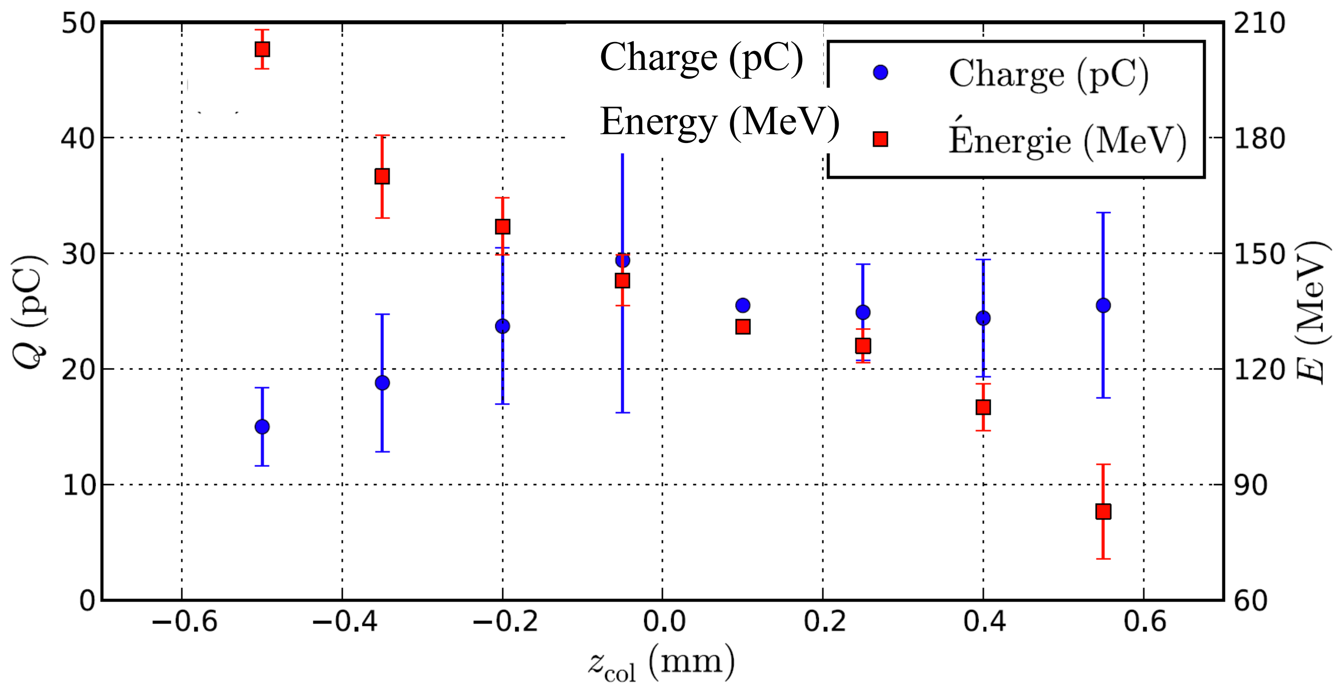
Experimental setup



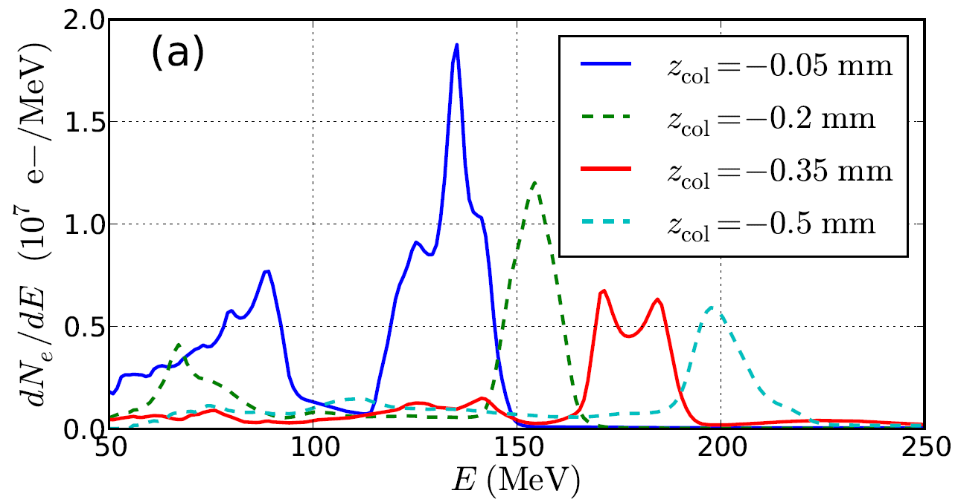
The acceleration length is tuned by moving the collision position.

S. Corde *et al.* Phys. Rev. Lett. **107**, 255003 (2011)

First measurement : electron energy

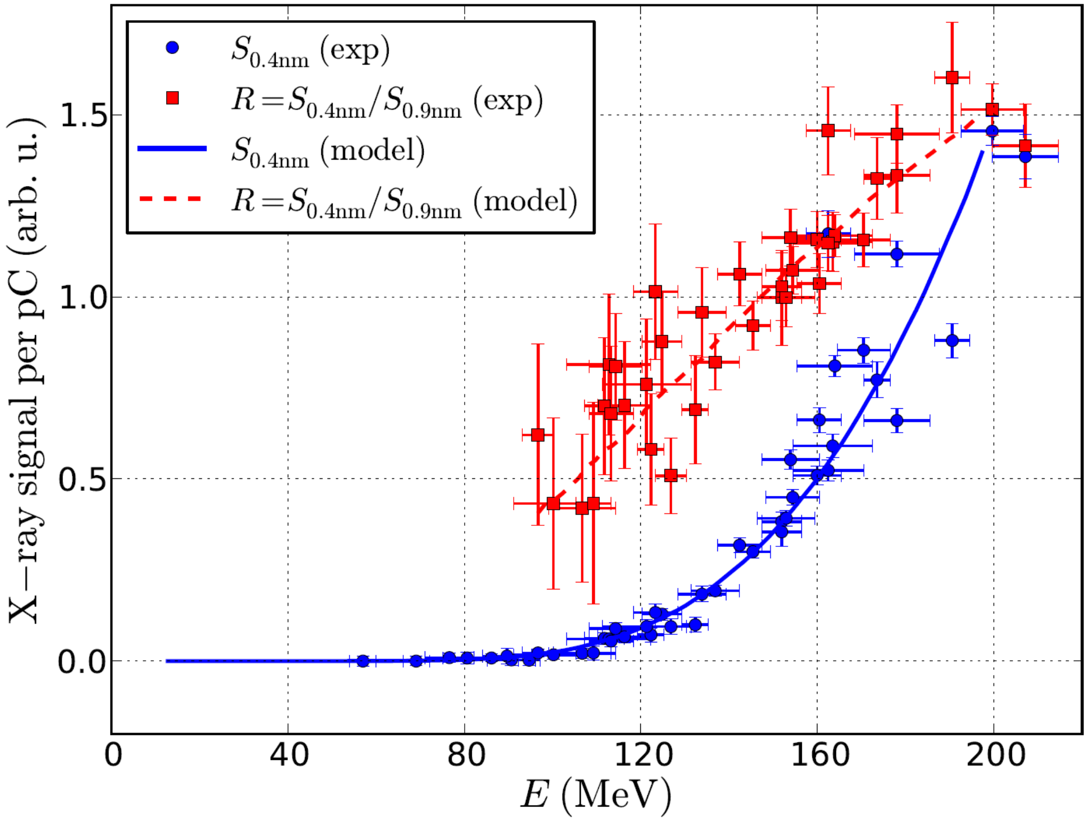


- Charge ~ 25 pC
- Energy ∈ [80,210] MeV



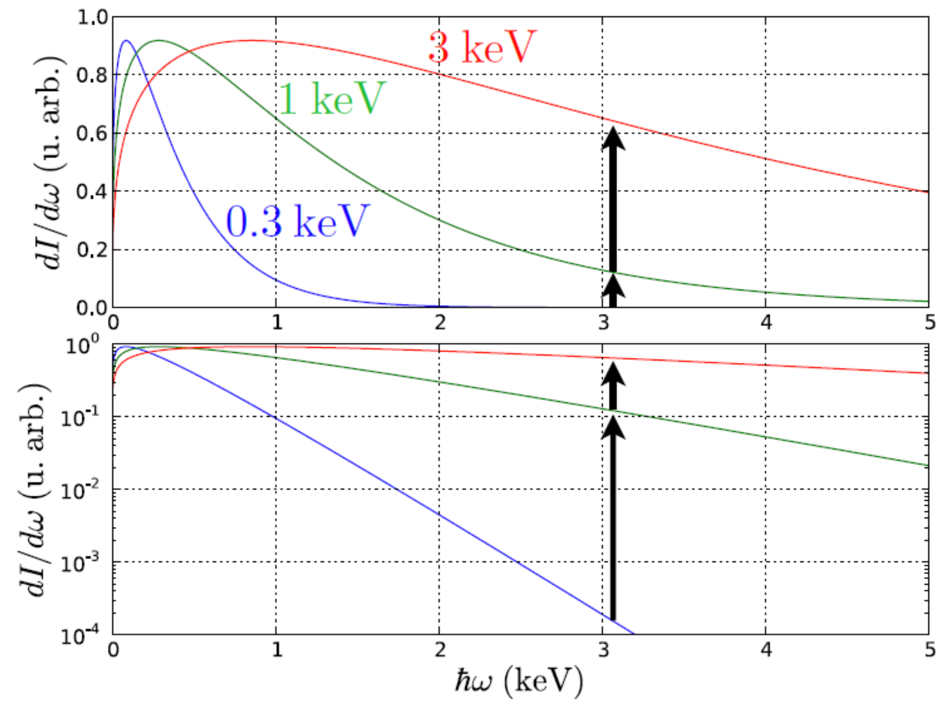
S. Corde *et al.* Phys. Rev. Lett. **107**, 255003 (2011)

Second measurement : X-ray critical energy



$$E_C \propto \epsilon_N^{1/2} \gamma^{7/4} \alpha^{3/4}$$

Critical energy increases with γ.



Good agreement between model an experiment

⬇

validation of the method

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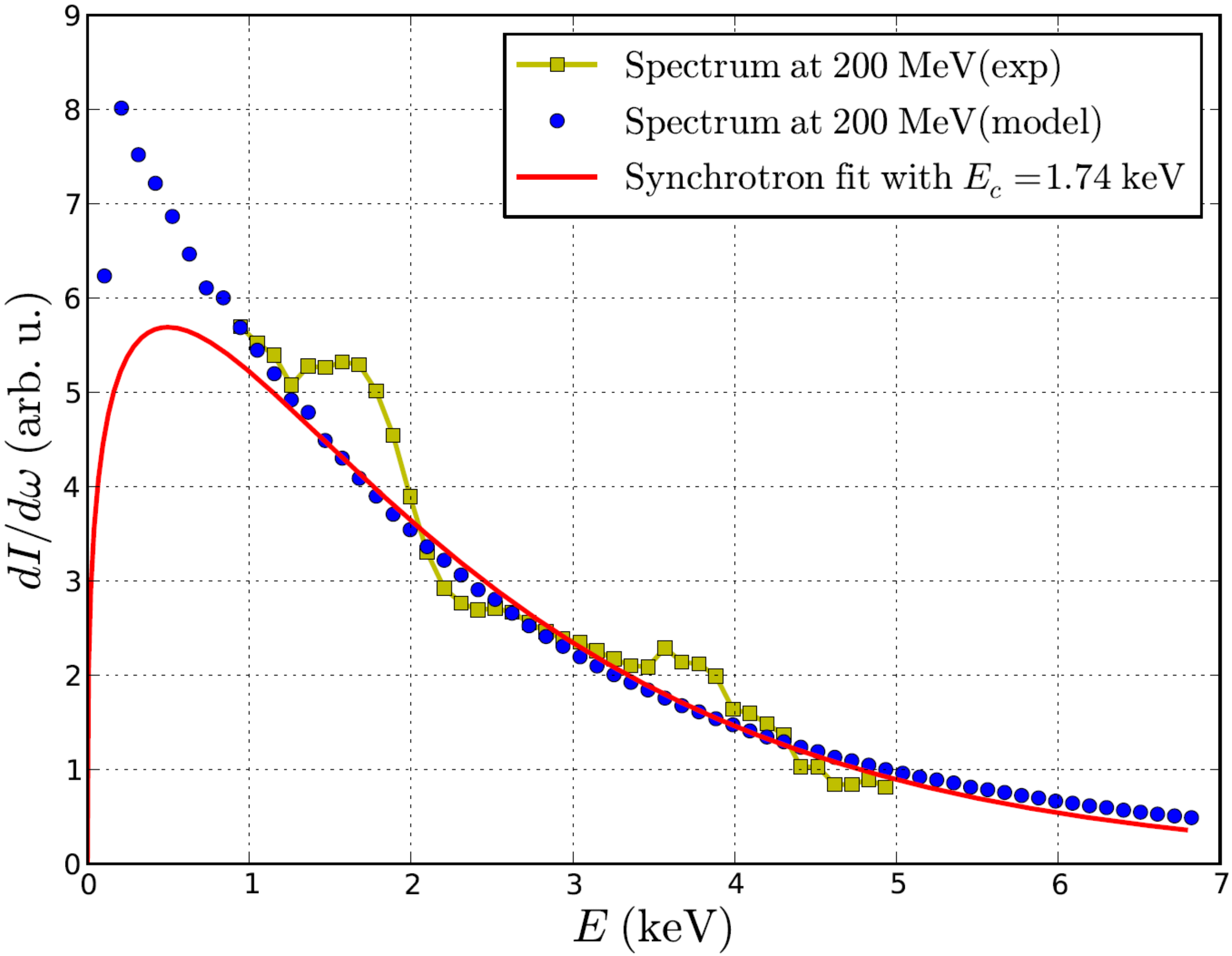
Second measurement : X-ray critical energy

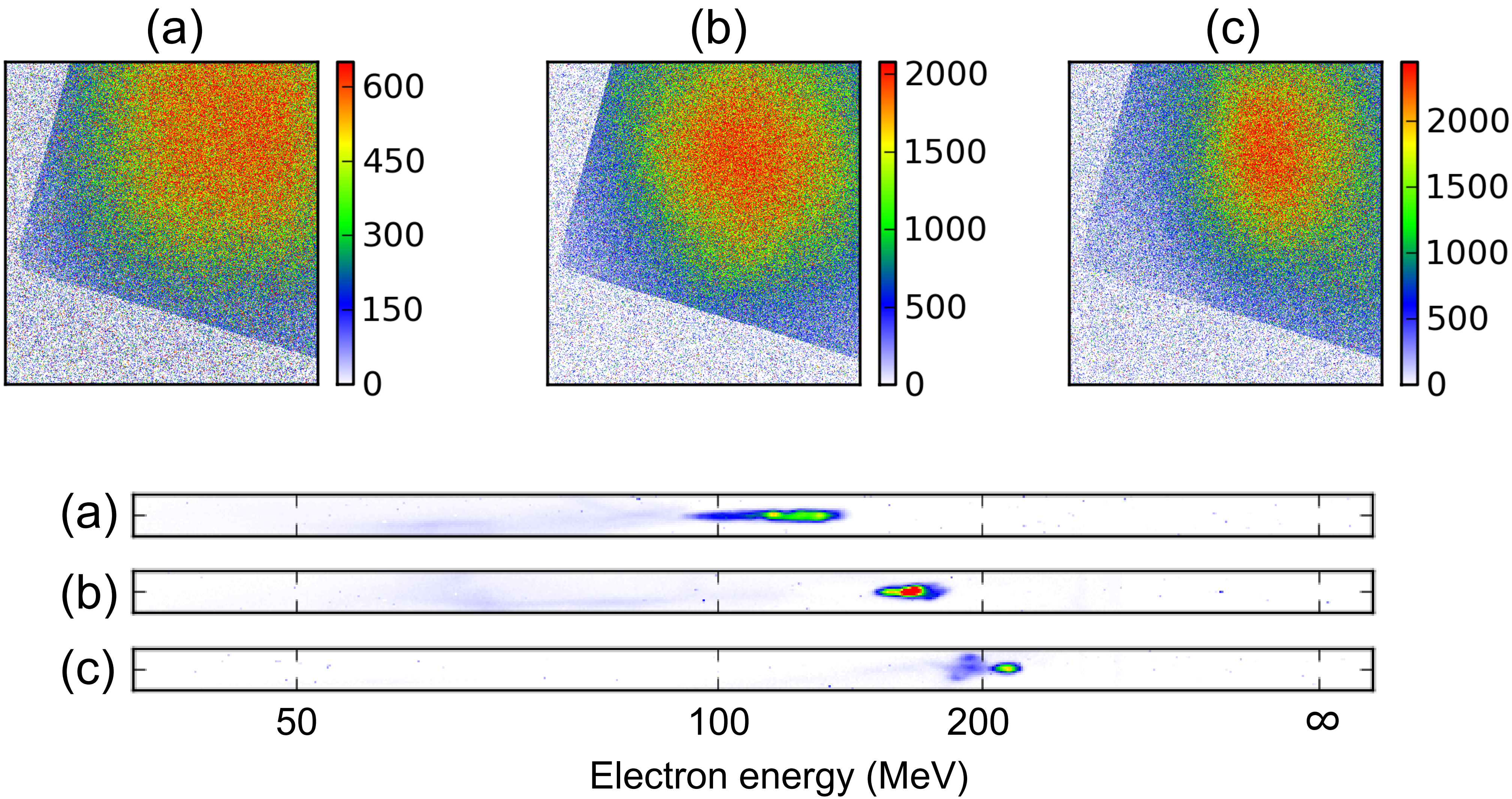


E_C = 1.7±0.5 KeV ➡ ε_N = (0.53 ± 0.36) $\alpha^{-3/2}$ π.mm.mrad

S. Corde *et al.* Phys. Rev. Lett. **107**, 255003 (2011)

Third measurement : X-ray divergence

$\theta \propto \epsilon_N^{1/2} \alpha^{1/4} \gamma^{-3/4}$ ➡ divergence decreases when γ increases



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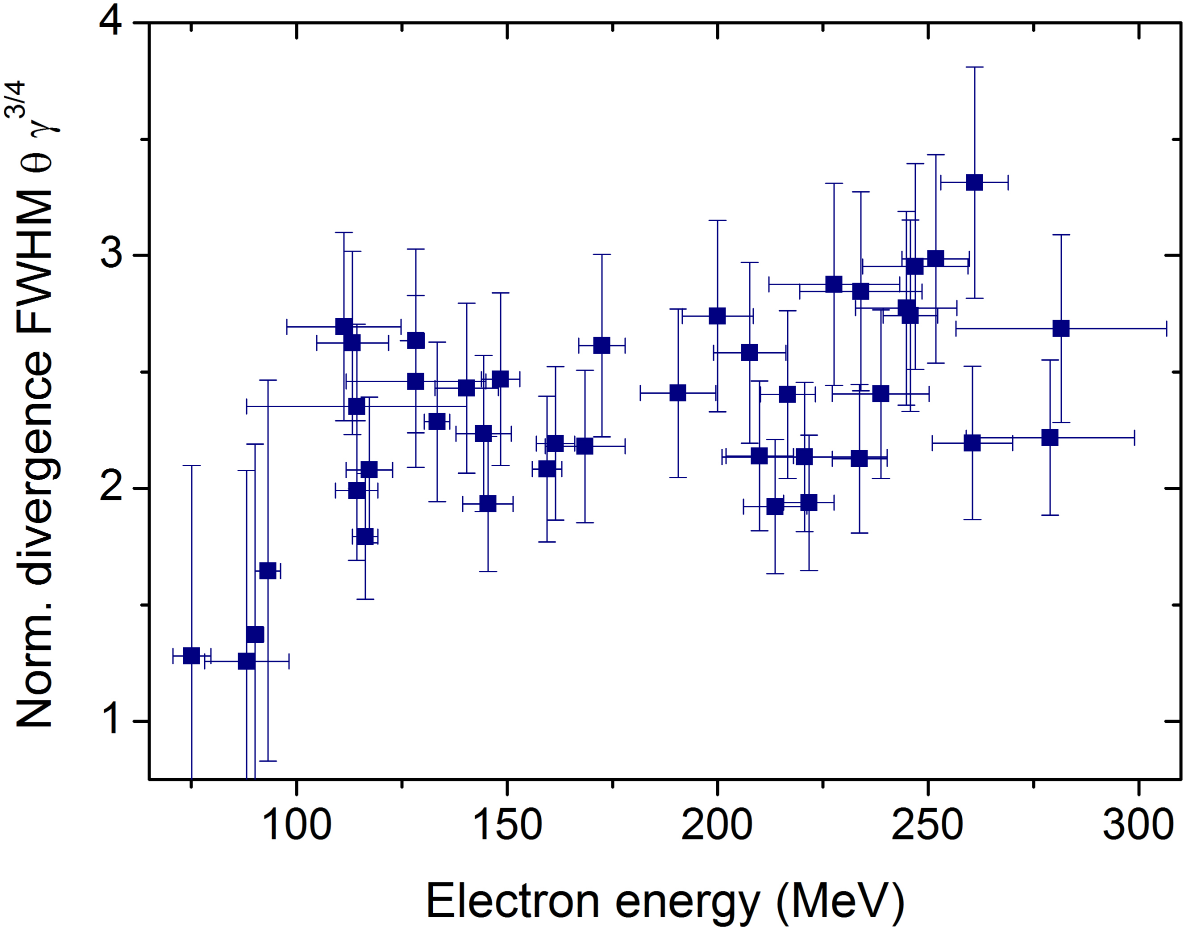


The normalized divergence $\theta\gamma^{3/4}$ increases

Influence of the angular momentum

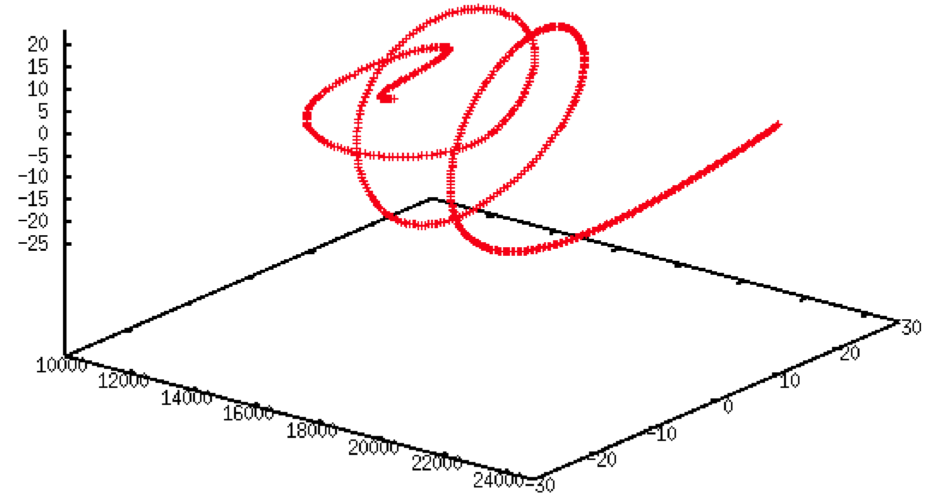
$$\hat{x}(t) = \sqrt{\hat{J}_x} \sin\left[\int_0^t \omega_\beta(t')dt' + \phi_x\right]$$

$$\hat{p}_x(t) = \sqrt{\hat{J}_x} \cos\left[\int_0^t \omega_\beta(t')dt' + \phi_x\right]$$

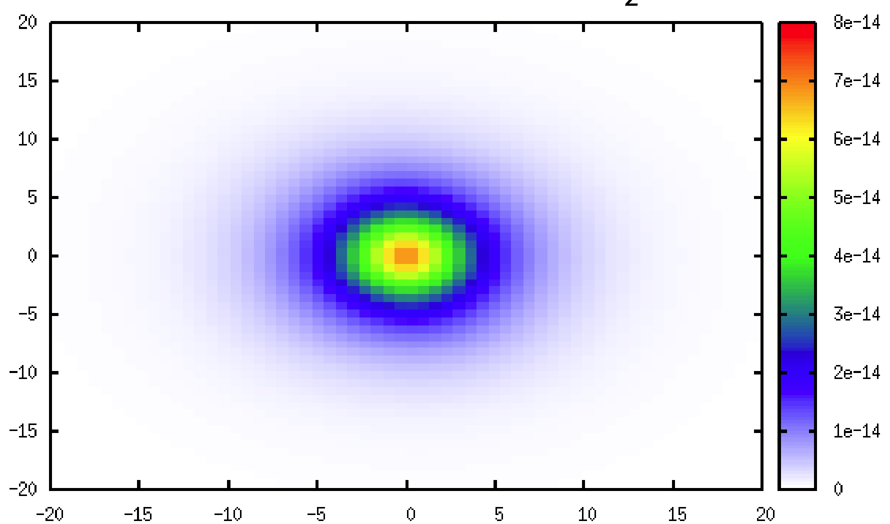
$$\hat{y}(t) = \sqrt{\hat{J}_y} \sin\left[\int_0^t \omega_\beta(t')dt' + \phi_y\right]$$

$$\hat{p}_y(t) = \sqrt{\hat{J}_y} \cos\left[\int_0^t \omega_\beta(t')dt' + \phi_y\right]$$

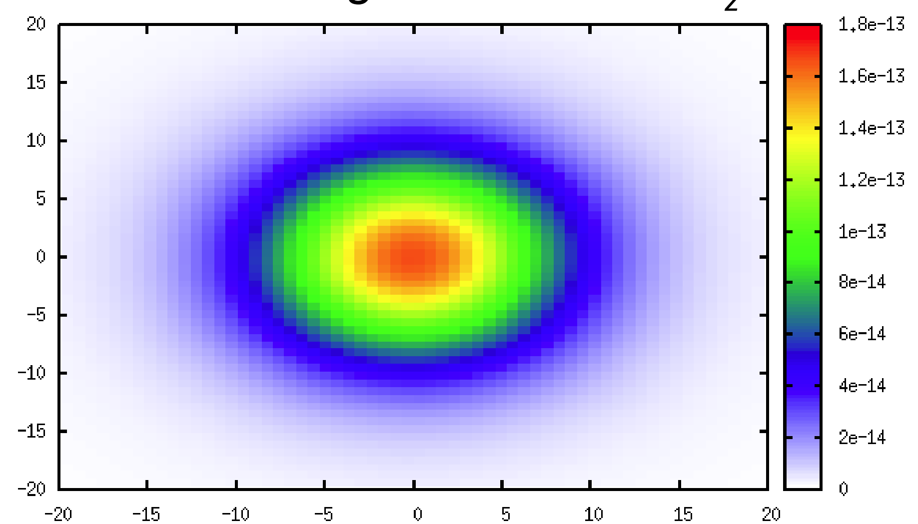
$\Phi_x \neq \Phi_y$ ➡ Non-zero angular momentum L_z



Simul. divergence for L_z=0



Simul. divergence for max. L_z



Angular momentum growth

$L_z(0)=0$ and non-perfectly symmetrical fields

$$F_x = -\alpha m \omega_p^2 x/2$$

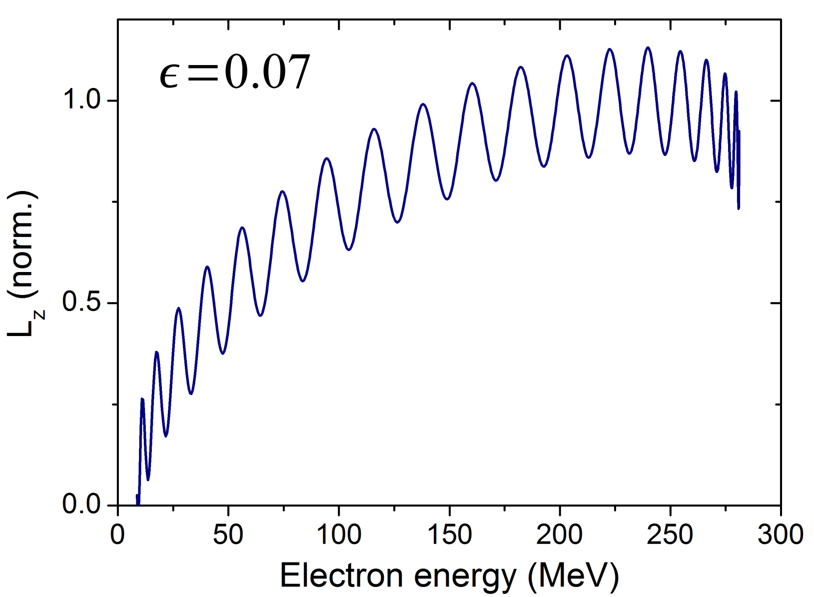
$$F_y = -\alpha(1+\epsilon) m \omega_p^2 x/2$$

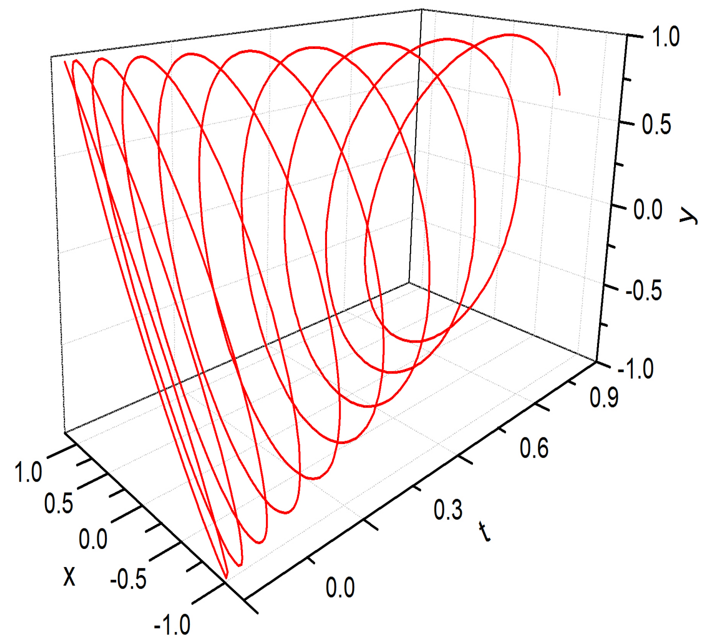
$$\Rightarrow$$

$$\phi_x = \int_0^t \omega_p \sqrt{\alpha/2\gamma(t)}\, dt$$

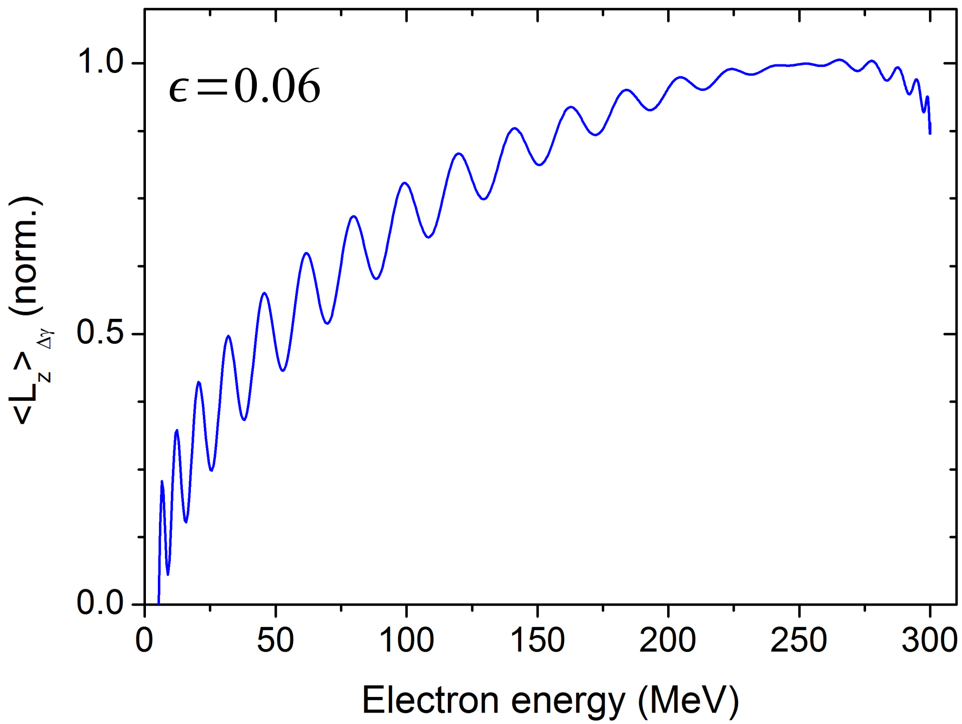
$$\phi_y = \int_0^t \omega_p \sqrt{\alpha(1+\epsilon)/2\gamma(t)}\, dt$$

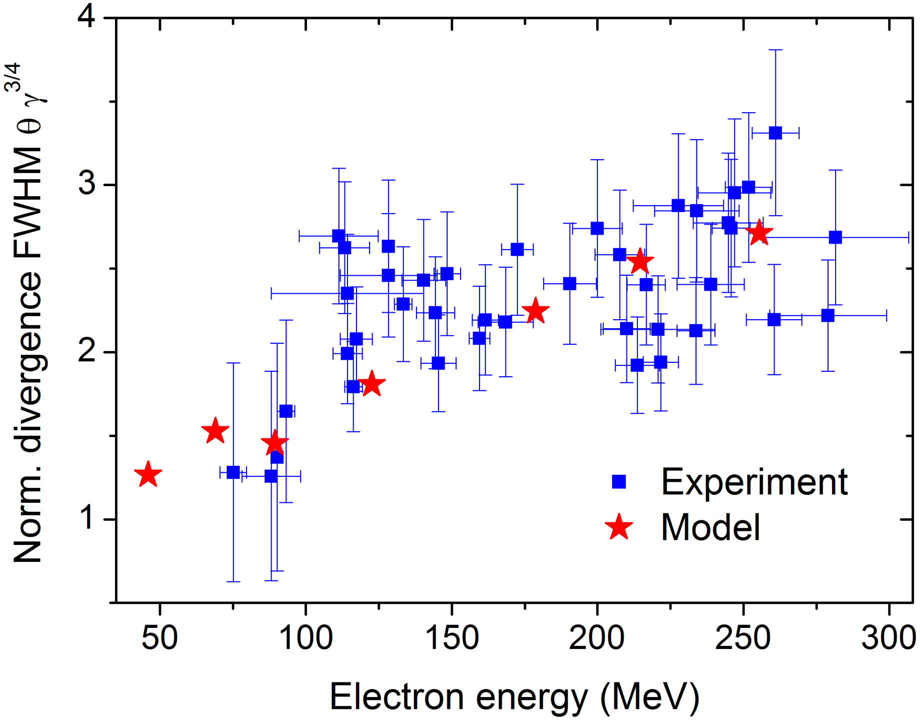
$$L_Z = \sqrt{J_x J_y}\left[(1-\epsilon)\sin(\phi_x - \phi_y) - 2\epsilon \sin(\phi_x + \phi_y)\right]$$





Angular momentum growth





A 4th measurement would be required to get L_z

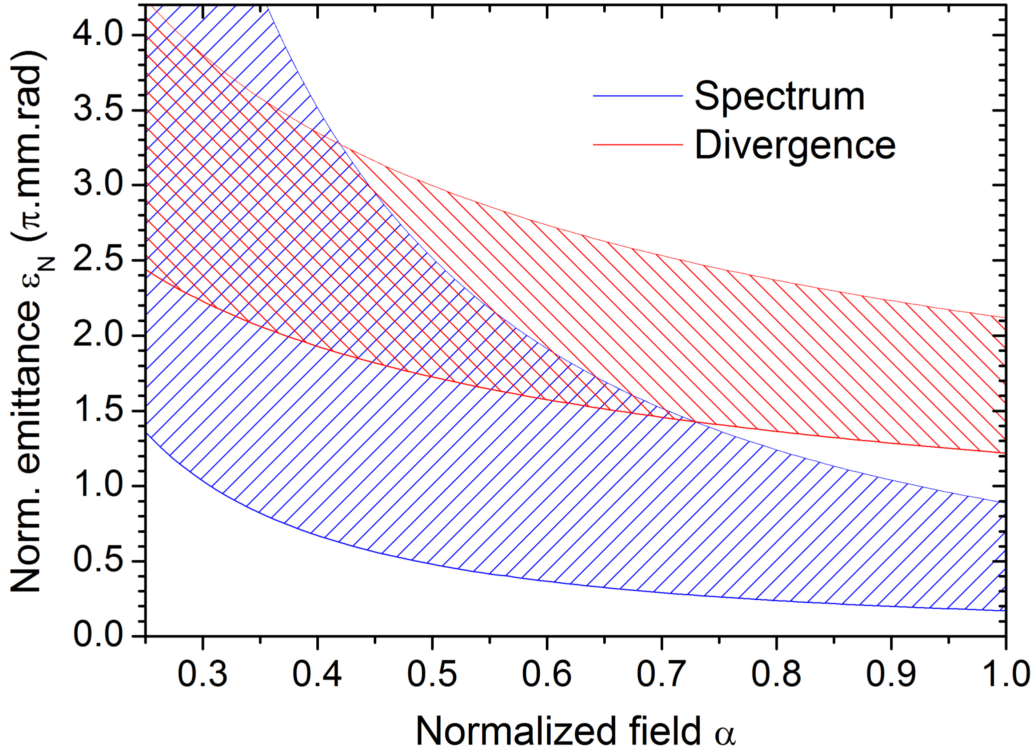
➡ assume that $\gamma^{3/4}\,\theta^{FWHM}$ is max. for E = 260 MeV

For E = 260 MeV, $\gamma^{3/4}\,\theta^{FWHM}$ = 1900 ± 300 mrad

➡ ε_N = (1.7 ± 0.4) $\alpha^{-1/2}$ π.mm.mrad

An estimate of the emittance

- Spectrum ➡ $\varepsilon_N = (0.53 \pm 0.36)\ \alpha^{-3/2}$ π.mm.mrad
- Divergence ➡ $\varepsilon_N = (1.7 \pm 0.4)\ \alpha^{-1/2}$ π.mm.mrad
- Theory ➡ $0.25 \lesssim \alpha \leq 1$



1.4 π.mm.mrad $\lesssim \varepsilon_N \lesssim$ 4.2 π.mm.mrad

Most probable : α = 0.32 and $\varepsilon_N \approx$ 3 π.mm.mrad

Outlook

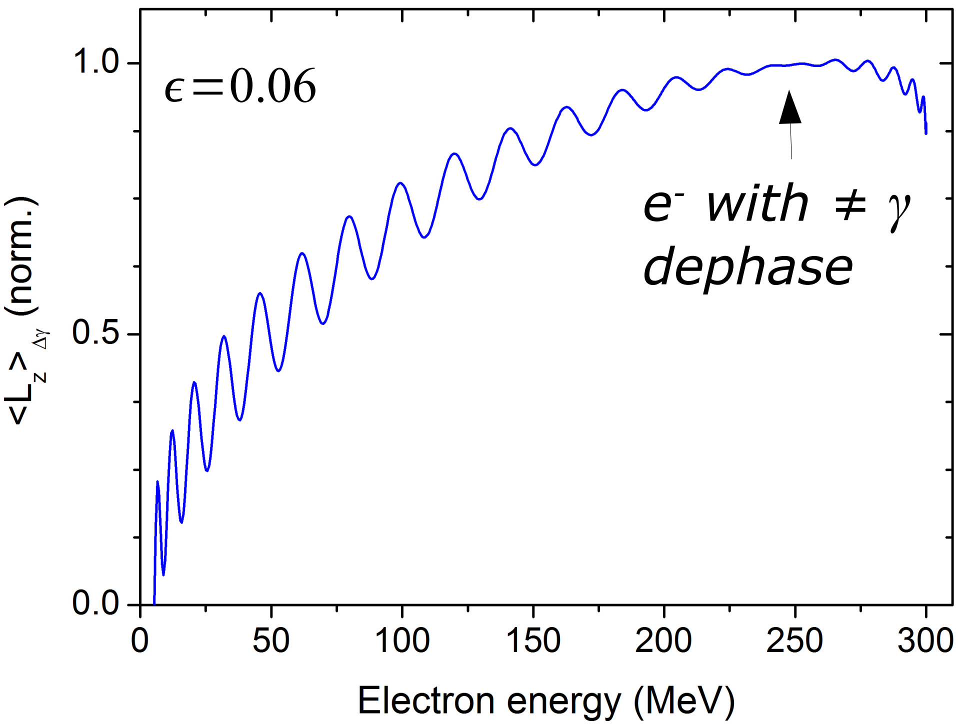
Summary

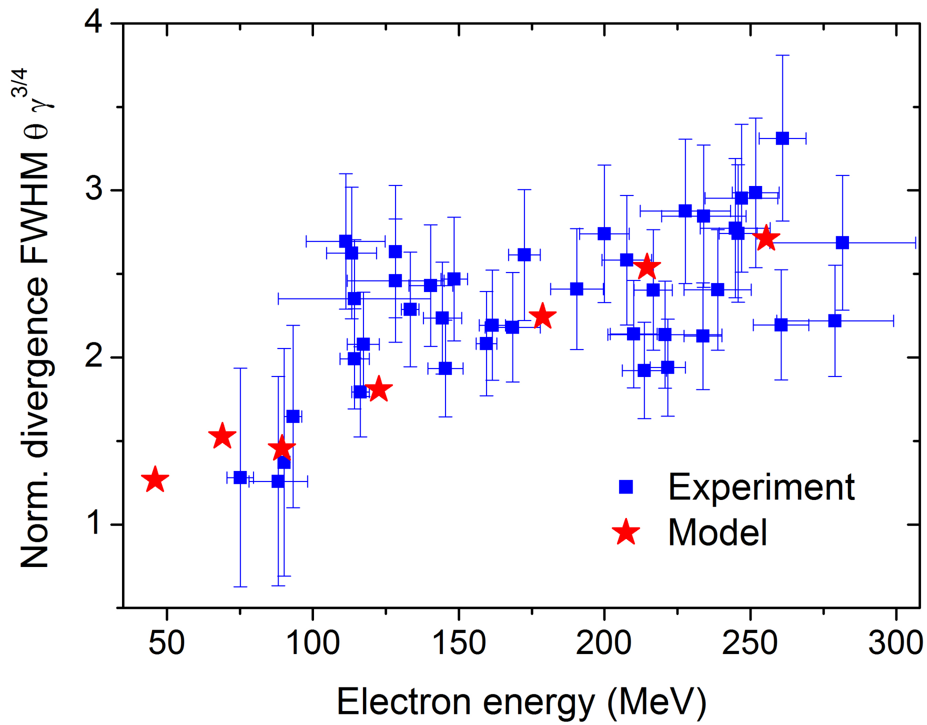
- Proof of principle experiment of betatron based emittance measurement.
- ε_N < 4 π.mm.mrad.
- Method works also for higher electron energy.
- Potentially single shot.

Outlook

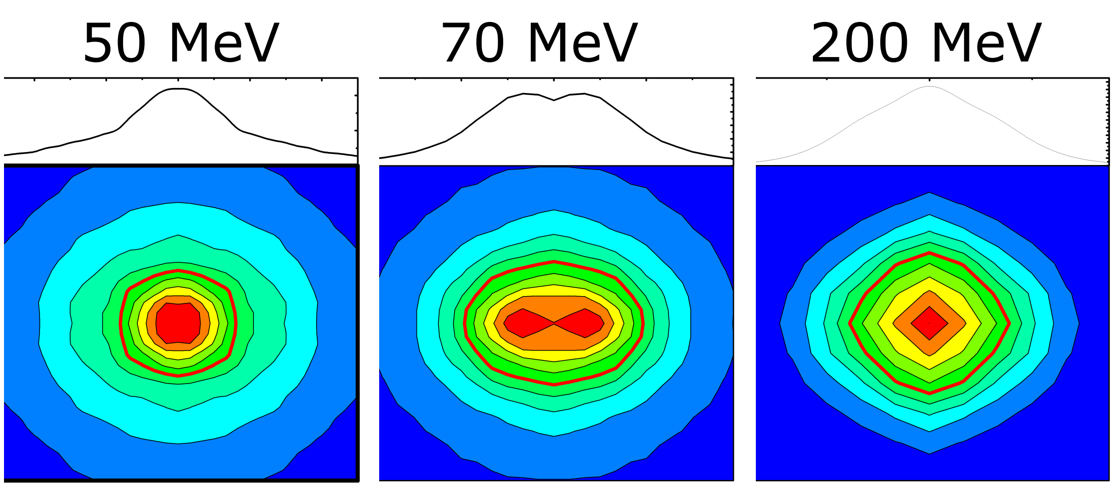
- Reduce the error bars.
- Perform a 4th measurement to get L_z.
- Theoretical study of angular momentum growth.

Angular momentum growth





Model $\gamma^{3/4}\,\theta$



Experimental $\gamma^{3/4}\,\theta$