Threshold Corrections at NLO in SMEFT

Livia Maskos

IPPP Durham

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Based on A. Biekötter, LM, and B. Pecjak (2601.15901)

Outline

Motivation

- Program to calculate NLO SMEFT:
  - Process-dependent: cross-sections, decay rates
    Haisch et al. (2507.21768), Bellafronte et al. (2508.14966)
  - Universal, Lagrangian level: RG, matching, threshold corrections
    Jenkins et al. (1308.2627/1310.4838/1312.2014), Aebischer et al. (2502.14030)
- Crucial for consistent RG analyses using NLO SMEFT corrections
  Jenkins et al. (1308.2627)
- NNLO SM threshold corrections calculated Kniehl et al. (1503.02138)

Threshold Corrections

What are threshold corrections?

Expressing symmetric phase parameters of the Lagrangian as a function of broken-phase (experimentally measured) parameters

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Lagrangian Parameters

$$p_i^{\mathrm{sym.}}(\mu) \equiv p_i^{\mathrm{sym.}} \in \{g_1, g_2, \lambda, \mu_H^2, y_t, g_3\}$$

$$p_i^{\mathrm{broken}} \in \{M_W, M_Z, v_\sigma, m_H, m_t, g_s(\mu)\}$$

$$v_\sigma \in \{v_\mu, v_\alpha\} \qquad v_\mu = \left(\sqrt{2}G_F\right)^{-\frac{1}{2}} \qquad v_\alpha = \frac{2M_W s_w}{\sqrt{4\pi\alpha(M_Z)}}$$

Scheme	Inputs
v_μ	$G_F,\ M_W,\ M_Z$
v_α	$\alpha(M_Z),\ M_W,\ M_Z$

Threshold Corrections in the SM

What are threshold corrections?

At tree level (a few examples):

$$g_2 = \frac{2M_W}{v_\sigma}\,, \qquad \lambda = \frac{m_H^2}{2v_\sigma^2}\,, \qquad y_t = \frac{\sqrt{2}m_t}{v_\sigma}$$

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$\Rightarrow$ Radiative corrections

In the case of λ:

$$\lambda_0 = \frac{m_{H,0}^2}{2v_{\sigma,0}^2} \quad \Rightarrow \quad \lambda^{\overline{\mathrm{MS}}}(\mu) = \frac{m_H^{2\,\mathrm{O.S.}}}{2v_\sigma^{2\,\mathrm{O.S.}}}[1 + \delta\lambda(p_i^{\mathrm{broken}}, \mu)]$$

What are threshold corrections?

At tree level (a few examples):

⇒ R

In the

$2v_{\sigma,0}^2$ $2v_\sigma^{2\,\text{O.S.}}$

$\overline{\text{MS}}$ Scheme

- Renormalised parameter depends on scale μ
- Convenient for RG running

On-Shell Scheme

- Renormalised parameter scale-independent
- Physical inputs at the EW scale
- μ-dependence in counterterms

Threshold Corrections in the SM

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Threshold corrections are gauge-independent and finite functions of on-shell renormalisation CT.

A recap of SMEFT

The SMEFT Lagrangian

$$\mathcal{L} = \mathcal{L}^{(4)} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \mathcal{L}^{(8)} + \dots$$

In our work, $d = 6$:

$$\mathcal{L} = \mathcal{L}^{(4)} + \mathcal{L}^{(6)}; \quad \mathcal{L}^{(6)} = \sum_i C_i(\mu)\, Q_i(\mu)$$

Examples of Dimension-6 Operators - Warsaw Basis (arXiv:1008.4884)

$\mathcal{O}_{HD} = (H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$	$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{HG} = (H^\dagger H) G^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}_{uH} = (H^\dagger H)(\bar{q}_L u_R \tilde{H}) + \text{h.c.}$

Threshold Corrections with SMEFT

We define those expansion coefficients:

$$\lambda = \frac{m_H^2}{2v_\sigma^2}\left[1 + \frac{1}{v_\sigma^2}\delta\lambda^{(4,1,\sigma)} + v_\sigma^2\delta\lambda^{(6,0,\sigma)} + \delta\lambda^{(6,1,\sigma)}\right]$$

Reminder:

$$p_i^{\mathrm{sym.}}(\mu) \equiv p_i^{\mathrm{sym.}} \in \{g_1, g_2, \lambda, \mu_H^2, y_t, g_3\} \equiv p_i^{\overline{\mathrm{MS}}}$$

$$p_i^{\mathrm{broken}} \in \{M_W, M_Z, v_\sigma, m_H, m_t, g_s(\mu)\} \equiv p_i^{\mathrm{O.S.}}$$

Threshold Corrections in SMEFT

Before EWSB:

$$\mathcal{L}_{\text{sym}} = (D_\mu H)^\dagger (D^\mu H) + C_H (H^\dagger H)\Box(H^\dagger H) + C_{HD}(H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$$

$$V(H) = \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 - C_H (H^\dagger H)^3 \quad \Rightarrow \quad v_T \equiv \left(1 + \frac{3C_H v^2}{8\lambda}\right) v$$

After EWSB:

$$\mathcal{L}_{\text{broken}} \supset \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_H^2 h^2 + \ldots$$

$$m_H^2 = 2\lambda v_T^2 \left(1 - \frac{3C_H v_T^2}{2\lambda} + 2C_{H\Box} v_T^2 - \frac{1}{2}C_{HD} v_T^2\right)$$

Threshold Corrections in SMEFT

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Leading to:

$$\lambda = \frac{m_H^2}{2 v_T^2}\left(1 + v_T^2 \left(\frac{3 v_T^2}{m_H^2} C_H + \frac{1}{2} C_{HD} - 2 C_{H\Box}\right)\right)$$

Threshold Corrections in SMEFT

$\Rightarrow$ Radiative corrections $\Rightarrow$ Renormalisation procedure to one-loop

$$\lambda_0 = \frac{m_{H,0}^2}{2v_{T,0}^2}\left(1 + v_{T,0}^2\left(\frac{3v_{T,0}^2}{m_{H,0}^2}C_{H,0} + \frac{1}{2}C_{HD,0} - 2C_{H\Box,0}\right)\right)$$

Threshold Corrections in SMEFT

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For a generic renormalised parameter X: $\quad X_0 = X\,(1 + \Delta X)$

For Wilson coefficients: $\quad C_{i,0} = C_i + \Delta C_i$

For broken-phase parameters renormalised on-shell, split the counterterms into divergent and finite pieces in the dimensional regulator

$$\Delta X^{\rm O.S.} = \frac{\Delta X^{\rm O.S.}_{\rm div.}}{\epsilon} + \delta X^{\rm O.S.} \qquad\qquad X^{\rm O.S.} \in \{M_W, M_Z, v_\sigma, m_t, m_H\}$$

Threshold Corrections in SMEFT

Reminder:

$$\lambda = \frac{m_H^2}{2v_\sigma^2}\left[1 + \frac{1}{v_\sigma^2}\delta\lambda^{(4,1,\sigma)} + v_\sigma^2\delta\lambda^{(6,0,\sigma)} + \delta\lambda^{(6,1,\sigma)}\right]$$

For λ:

$$\lambda = \frac{m_H^2}{2v_\sigma^2}[1 + v_\sigma^2(\frac{3v_\sigma^2}{m_H^2}C_H - 2C_{H,\mathrm{kin}} - \delta v_\sigma^{(6,0)} + \frac{1}{v}(2\,\delta m_H^{(4,1)} - \delta v_\sigma^{(4,1)})$$

$$+ 2\,\delta m_H^{(6,1)} - \delta v_\sigma^{(6,1)} - 4\,\delta v_\sigma^{(6,0)}\,\delta m_H^{(4,1)} + \frac{3v_\sigma^2}{m_H^2}\delta v_\sigma^{(4,1)}C_H - 4\,\delta m_H^{(4,1)}C_{H,\mathrm{kin}}]$$

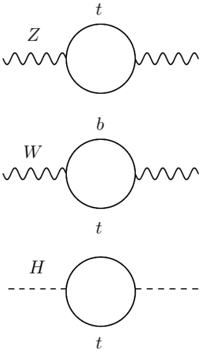
Have calculated all the counterterms $\delta v_\sigma^{(6,0)}$, $\delta m_H^{(6,1)}$...

Checks:

- UV divergent parts cancel and answer is gauge invariant
- μ-dependence on LHS and RHS match

Numerical Analysis at the EW scale

Numerical Analysis at the EW scale

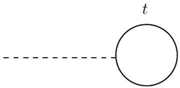


- Top-quark loop generic size:

$$\frac{N_c}{16\pi^2}\frac{m_t^2}{v_\sigma^2} = \frac{\Delta\rho_t^{(4,1)}}{v_\sigma^2} \approx 1\%$$

- Tadpoles scale like:

$$\frac{\Delta\rho_t^{\mathrm{tad}(4,1)}}{v_\sigma^2} \approx \frac{m_t^2}{m_H^2}\Delta\rho_t^{(4,1)} \approx 2\Delta\rho_t^{(4,1)}$$



- Renormalisation of s_w^2 gives enhanced correction: $\frac{c_w^2}{s_w^2}\frac{\Delta\rho_t^{(4,1)}}{v_\sigma^2} \approx 3.5\%$

Numerical Analysis at the EW scale

At the EW $\mu = M_Z$ scale

Using experimental values (e.g. $M_H = 125.20$ GeV) as input:

$$g_1^{v_\alpha(4,0)} = \frac{\sqrt{4\pi\alpha(M_Z)}}{c_w} = 0.3542\,, \qquad g_1^{v_\mu(4,0)} = \frac{2M_Z s_w}{v_\mu} = 0.3499$$

$$\frac{g_1^{v_\alpha}}{g_1^{v_\alpha(4,0)}} = 1.000 + v_\alpha^2\Big[-(1\times 1.07)C_{HB} - (0.25\times 1.08)C_{HD} + \dots\Big]$$

$$\frac{g_1^{v_\mu}}{g_1^{v_\mu(4,0)}} = 1.024 + v_\mu^2\Big[-(1.87\times 1.03)C_{HWB} - (1.12\times 1.05)C_{HD} + \dots\Big]$$

Numerical Analysis at the EW scale

At the EW $\mu = M_Z$ scale

Using experimental values (e.g. $M_H = 125.20$ GeV) as input:

$$\lambda^{v_\alpha (4,0)} = \frac{m_H^2}{2v_\alpha^2} = 0.1325\,, \qquad \lambda^{v_\mu (4,0)} = \frac{m_H^2}{2v_\mu^2} = 0.1293$$

$$\frac{\lambda^{v_\alpha}}{\lambda^{v_\alpha (4,0)}} = 1.073 + v_\alpha^2 \Big[(11.3 \times 1.07) C_H + (3.73 \times 1.21) C_{HWB} + (2.24 \times 1.17) C_{HD} - (2 \times 1.00) C_{H\Box} - 0.377 C_{\underset{33}{uH}} - 0.159 C_{\underset{33}{Hu}} + 0.135 C^{(1)}_{\underset{33}{Hq}} - 0.109 C^{(3)}_{\underset{33}{Hq}} + \dots \Big]\,,$$

$$\frac{\lambda^{v_\mu}}{\lambda^{v_\mu (4,0)}} = 1.119 + v_\mu^2 \Big[(11.6 \times 1.02) C_H - (2 \times 1.00) C_{H\Box} - (1 \times 1.27) \Big(C^{(3)}_{\underset{11}{Hl}} + C^{(3)}_{\underset{22}{Hl}} \Big) + (1 \times 1.24) C_{\underset{1221}{ll}} + (0.5 \times 1.00) C_{HD} - 0.373 C_{\underset{33}{uH}} - 0.029 C^{(3)}_{\underset{33}{Hq}} + \dots \Big]\,.$$

→ **Loop corrections of order $\mathcal{O}(10\%)$**

Threshold Corrections in RGEs

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The bottom-up approach

At fixed order:

$$p_i^{\mathrm{sym.}}(\Lambda) = p_i^{\mathrm{sym.}}(\mu_{EW}) + \dot{p}_i^{\mathrm{sym.}}(\mu_{EW}) \ln\left(\frac{\Lambda}{\mu_{EW}}\right)$$

$$C_i(\Lambda) = c_i(\mu_{EW}) + \dot{C}_i(\mu_{EW}) \ln\left(\frac{\Lambda}{\mu_{EW}}\right)$$

where

$$\dot{p}_i^{\mathrm{sym.}}(\mu_{EW}) \equiv \dot{p}_i^{\mathrm{sym.}}\left(p_i^{\mathrm{sym.}}(\mu_{EW}), c_i(\mu_{EW})\right)$$
$$\dot{C}_i^{\mathrm{sym.}}(\mu_{EW}) \equiv \dot{C}_i^{\mathrm{sym.}}\left(p_i^{\mathrm{sym.}}(\mu_{EW}), c_i(\mu_{EW})\right)$$

- c_i are fixed initial conditions (at the EW scale)
- $\dot{C}_i$ include the Wilson coefficients that will be generated

Case study 1

RG evolution from $\mu_{EW} = M_Z$ to $\mu_{NP} = 1$ TeV

Only 1 non-zero Wilson Coefficient at μ_{EW}

$$c_i(M_Z) = C_{uH}(M_Z) = \frac{1}{1\,\mathrm{TeV}^2}\,.$$

Reminder: $\mathcal{O}_{uH} = \left(H^\dagger H\right)(\bar{q}_L u_R \tilde{H})$

Threshold Corrections in RGEs

$$C_i(\Lambda) = c_i(\mu_{EW}) + \dot{C}_i(\mu_{EW}) \ln\left(\frac{\Lambda}{\mu_{EW}}\right)$$

For our case study $c_i(M_Z) = C_{uH}(M_Z)$**, we require:**

$$16\pi^2 \dot{C}_H = \left(-\frac{9}{2}g_1^2 - \frac{27}{2}g_2^2 + 108\lambda + 18y_t^2\right) C_H + \left(24\lambda y_t - 24y_t^3\right) C_{\substack{uH\\33}}$$

$$16\pi^2 \dot{C}_{\substack{uH\\33}} = \left(-\frac{35}{12}g_1^2 - \frac{27}{4}g_2^2 - 8g_3^2 + 24\lambda + \frac{51}{2}y_t^2\right) C_{\substack{uH\\33}}$$

$\Rightarrow g_1, g_2, \lambda$... are calculated from our threshold corrections

Threshold Corrections in RGEs

$$p_i^{\text{sym.}}(\Lambda) = p_i^{\text{sym.}}(\mu_{EW}) + \dot{p}_i^{\text{sym.}}(\mu_{EW}) \ln\left(\frac{\Lambda}{\mu_{EW}}\right)$$

For the SM parameters:

$$\begin{aligned} \dot{p}_i &= \dot{p}_i^{(4,1)} , \qquad p_i \in \{g_1, g_2, g_3, \mu_H^2\} , \\ \dot{\lambda} &= \dot{\lambda}^{(4,1)} + \frac{3\mu_H^2}{4\pi^2}\left(2C_H + y_t C_{\underset{33}{uH}}\right) , \\ \dot{y}_t &= \dot{y}_t^{(4,1)} + \frac{3\mu_H^2}{8\pi^2} C_{\underset{33}{uH}} . \end{aligned}$$

$\Rightarrow \mu_H^2, y_t$ are calculated from our threshold corrections

Threshold Corrections in RGEs

$$\{g_1, g_2, \lambda, \mu_H^2, y_t, g_3\}$$

$$\frac{p_i^{\mathrm{SM},v_\mu}(\Lambda)}{p_i^{\mathrm{SM},v_\mu}(M_Z)} = \{1.01, 0.980, 0.686, 1.07, 0.890, 0.875\}$$

$$\frac{p_i^{\mathrm{SM}+(6,0),v_\mu}(\Lambda)}{p_i^{\mathrm{SM},v_\mu}(\Lambda)} = \{1.00, 1.00, 0.925, 1.00, 1.04, 1.00\}$$

$$\frac{p_i^{\mathrm{SM}+(6,0)+(6,1),v_\mu}(\Lambda)}{p_i^{\mathrm{SM}+(6,0),v_\mu}(\Lambda)} = \{1.00, 1.00, 0.958, 0.981, 1.00, 1.00\}$$

$\rightarrow$ Adding NLO SMEFT decrease the values of λ and μ_H^2 at the scale Λ by 4% and 2%

Threshold Corrections in RGEs

Case study

RG evolution from $\mu_{EW} = M_Z$ to $\mu_{NP} = 1$ TeV

Only 1 non-zero Wilson Coefficient at μ_{EW}

$$c_i(M_Z) = C_{uH}(M_Z) = \frac{1}{1\,\mathrm{TeV}^2}$$

NLO threshold corrections effect the results for $C_i(\Lambda)$ at below the 1% level

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NLO threshold corrections effect the results for $C_i(\Lambda)$ at below the 1% level

New scenario

2 non-zero Wilson coefficients

$$c_i(M_Z) = \left\{ C_{uH}(M_Z) = \frac{1}{\Lambda^2}\,, C_{uG}(M_Z) = \frac{1}{(0.75\Lambda)^2} \right\}$$

The value of C_{uH} (C_H) at that scale is shifted by 3% (-2%) compared to LO SMEFT

Conclusion

- Complete set of NLO SMEFT threshold corrections in analytic form, in two EW input schemes
- Numerical analysis shows changes $\mathcal{O}$(5-10%) in SM symmetric phase parameters values due to NLO threshold corrections
- Would easily be implemented in RG running programs, such as Wilson and RGESolver J. Aebischer et al. (1804.05033), S. Di Noi et al. (2210.06838)