Matrix element merging in PYTHIA8

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(Lund University)

NLO and parton showers miniWorkshop,
CERN, February 27, 2012

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The old story...

When having to describe soft and hard jets together, we need the virtues of both fixed order and resummation, since

- ME accurate to fixed order far away from phase space boundaries, but breaks down close to boundary, e.g. in infrared region.
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- This introduces another problem: Cut dependence.
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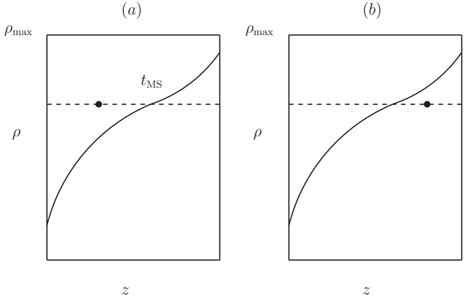
- This means reweighting the matrix element with α_s factors (for running α_s in the PS), PDF ratios (for backward evolution) and no-emission probabilities (since there are no emissions above the scale of the first emission).

One jet above ρ_c



Take (a) from +1 jet matrix element $|\mathcal{M}_{S_{+1}}|^2$. Reweight with the PS weight, i.e. count this state with weight

$$\left[x_1 f_1(x_1,\mu_1)\alpha_s(\mu_R)\,|\mathcal{M}_{S_{+1}}|^2\right] d\Phi_1^{\mathrm{ME}} \times w_{Path} \times \frac{x_0 f_0(x_0,\rho_0)}{x_1 f_1(x_1,\mu_1)}$$
$$\times \quad \frac{\alpha_s(\rho_1)}{\alpha_s(\mu_R)} \frac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)} \Pi_{S_{+0}}(x_0,\rho_0,\rho_1)\Pi_{S_{+1}}(x_1,\rho_1,\rho_c)$$

Take (b) from +0 jet matrix element $|\mathcal{M}_{S_{+0}}|^2$, with one shower splitting, i.e. with weight

$$\left[x_0 f_0(x_0,\mu_0)\,|\mathcal{M}_{S_{+0}}|^2\right] d\Phi_0^{\mathrm{ME}} \times \frac{x_0 f_0(x_0,\rho_0)}{x_0 f_0(x_0,\mu_0)}$$
$$\times \quad \alpha_s(\rho_1)\frac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)} P\left(\frac{x_0}{x_1}\right) d\Phi_1^{\mathrm{PS}} \Pi_{S_{+0}}(x_0,\rho_0,\rho_1)\Pi_{S_{+1}}(x_1,\rho_1,\rho_c)$$

One jet above ρ_c

Combining this, the merged approximation to the inclusive cross section is

$$
\begin{aligned}
d\sigma^{\text{ME1PS}} \quad = \quad & x_0 f_0(x_0, \rho_0) \Big\{ \\
& \tfrac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)} \left|\mathcal{M}_{S_{+1,me}}\right|^2 d\Phi_1^{\text{ME}} \Theta(t(S_{+1,me}) - t_{\text{MS}}) \\
& w_{Path} \alpha_{\text{s}}(\rho_1) \\
& \Pi_{S_{+0,rec}}(x_0, \rho_0, \rho_1) \Pi_{S_{+1,me}}(x_1, \rho_1, \rho_c) \\
+ \quad & \tfrac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)} \left|\mathcal{M}_{S_{+0,me}}\right|^2 d\Phi_0^{\text{ME}} \\
& \alpha_{\text{s}}(\rho_1) P\left(\tfrac{x_0}{x_1}\right) d\Phi_1^{\text{PS}} \Theta(t_{\text{MS}} - t(S_{+1,ps})) \\
& \Pi_{S_{+0,me}}(x_0, \rho_0, \rho_1) \Pi_{S_{+1,rec}}(x_1, \rho_1, \rho_c) \Big\}
\end{aligned}
$$

Lessons from one jet above ρ_c

The dependence on the cut t_{MS} vanishes if

$$\left|\mathcal{M}_{S_{+1,me}}\right|^2 d\Phi_1^{\text{ME}} w_{Path} = \left|\mathcal{M}_{S_{+0,me}}\right|^2 d\Phi_0^{\text{ME}} P\left(\frac{x_0}{x_1}\right) d\Phi_1^{\text{PS}} \quad (1)$$

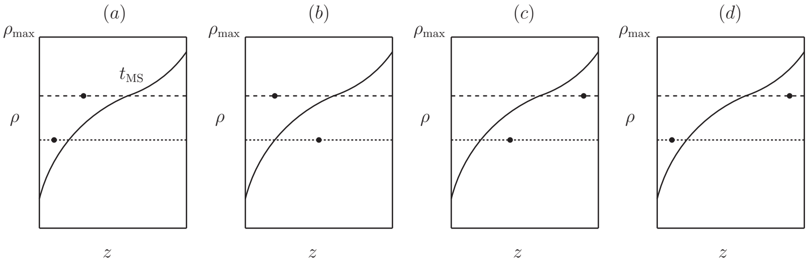
and

$$\begin{aligned} & \left[\Pi_{S_{+0,rec}}(x_0,\rho_0,\rho_1)\Pi_{S_{+1,me}}(x_1,\rho_1,\rho_c)\right] \\ = \quad & \left[\Pi_{S_{+0,me}}(x_0,\rho_0,\rho_1)\Pi_{S_{+1,ps}}(x_1,\rho_1,\rho_c)\right] \end{aligned} \quad (2)$$

This means:

- For (1), make the PS splitting kernels and the PS phase space resemble the ME as closely as possible.
- For (2), get state S_{+1} as correct as possible by using (inverted) parton shower momentum mapping. Use identical shower (routines!) to produce no-emission probabilities $\Pi_{S_{+i}}$.

Two jets above ρ_c: Where the differences are...



(a) Taken from the ME +2 jet sample, no information on merging scale needed

(b) Taken from the ME +1 jet sample, with a shower veto on the first emission

(c) Taken from the ME +0 jet sample, with a shower veto on the first emission

(d) Taken from the ME +0 jet sample, with a shower veto on the first emission. In truncated showers taken from ME +1 sample.

Sum of (c) and (d) in CKKW-L:

$$x_0 f_0(x_0,\rho_0)\left|\mathcal{M}_{S_{+0}}\right|^2 d\Phi_0^{\mathsf{ME}}$$
$$\frac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)}\alpha_{\mathsf{s}}(\rho_1)P\left(\frac{x_0}{x_1}\right)d\Phi_1^{\mathsf{PS}}\Theta(t_{\mathsf{MS}}-t(S_{+1}))\Pi_{S_{+0}}(x_0,\rho_0,\rho_1)$$
$$\frac{x_2 f_2(x_2,\rho_2)}{x_1 f_1(x_1,\rho_2)}\alpha_{\mathsf{s}}(\rho_2)P\left(\frac{x_1}{x_2}\right)d\Phi_{1'}^{\mathsf{PS}}\Pi_{S_{+1}}(x_1,\rho_1,\rho_2)\Pi_{S_{+2}}(x_2,\rho_2,\rho_c)$$

Sum of (c) and (d) in the truncated showering approach:

$$x_0 f_0(x_0,\rho_0)\left|\mathcal{M}_{S_{+0}}\right|^2 d\Phi_0^{\mathsf{ME}}$$
$$\frac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)}\alpha_{\mathsf{s}}(\rho_1)P\left(\frac{x_0}{x_1}\right)d\Phi_1^{\mathsf{PS}}\Theta(t_{\mathsf{MS}}-t(S_{+1}))\Pi_{S_{+0}}(x_0,\rho_0,\rho_1)$$
$$\frac{x_2 f_2(x_2,\rho_2)}{x_1 f_1(x_1,\rho_2)}\alpha_{\mathsf{s}}(\rho_2)P\left(\frac{x_1}{x_2}\right)d\Phi_1^{\mathsf{PS}}\Theta(t_{\mathsf{MS}}-t(S_{+2}))\Pi_{S_{+1}}(x_1,\rho_1,\rho_2)$$
$$\Pi_{S_{+2}}(x_2,\rho_2,\rho_c)$$
$$+\quad x_0 f_0(x_0,\rho_0)\left|\mathcal{M}_{S_{+1'}}\right|^2 d\Phi_1^{\mathsf{ME}}\Theta(t(S_{+1'})-t_{\mathsf{MS}})$$
$$w_{Path}\alpha_{\mathsf{s}}(\rho_2)$$
$$\frac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)}\alpha_{\mathsf{s}}(\rho_1)P^{\mathsf{TS}}\left(\frac{x_0}{x_1}\right)d\Phi_1^{\mathsf{TS}}\Theta(t_{\mathsf{MS}}-t(S_{+1}))\Pi_{S_{+0}}^{\mathsf{TS}}(x_0,\rho_0,\rho_1)$$
$$\Pi_{S_{+1}}^{\mathsf{TS}}(x_1,\rho_1,\rho_2)\Pi_{S_{+1''}}(x_2,\rho_2,\rho_c)$$

Lessons from two jets above ρ_c

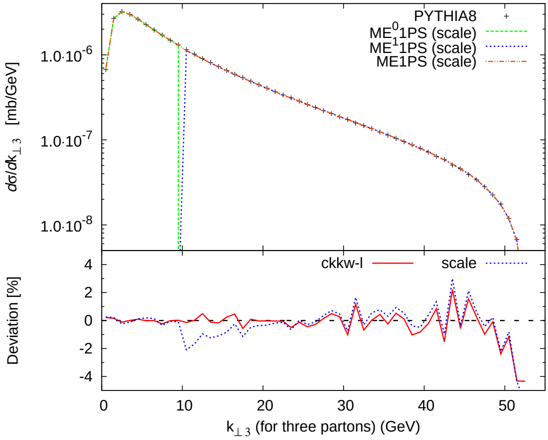
- Cut dependence still cancels to accuracy of the shower if

$$
\begin{aligned}
& \left[\Pi_{S_{+0,rec}}(x_0,\rho_0,\rho_1)\Pi_{S_{+1,rec}}(x_1,\rho_1,\rho_2)\Pi_{S_{+2,me}}(x_2,\rho_2,\rho_c)\right] \\
= \quad & \left[\Pi_{S_{+0,rec}}(x_0,\rho_0,\rho_1)\Pi_{S_{+1,me}}(x_1,\rho_1,\rho_2)\Pi_{S_{+2,ps}}(x_2,\rho_2,\rho_c)\right] \\
= \quad & \left[\Pi_{S_{+0,me}}(x_0,\rho_0,\rho_1)\Pi_{S_{+1,ps}}(x_1,\rho_1,\rho_2)\Pi_{S_{+2,ps}}(x_2,\rho_2,\rho_c)\right] \\
= \quad & \left[\Pi^{\mathrm{TS}}_{S_{+0,rec}}(x_0,\rho_0,\rho_1)\Pi^{\mathrm{TS}}_{S_{+1,ts}}(x_1,\rho_1,\rho_2)\Pi_{S_{+1'',me'}}(x_2,\rho_2,\rho_c)\right]
\end{aligned}
$$

- "Accuracy of the shower" is determined by splitting kernels AND phase space constraints.
- More samples $\Rightarrow$ More tricky to reduce the cut dependence.
- Having many more paths allows for different strategies for picking reclustered states $\Rightarrow$ Might be used to minimise cut dependence (?).

For me, the real question is: What do we want to do, i.e. what do we call an "improvement", and what do we call "unitarity violation"?

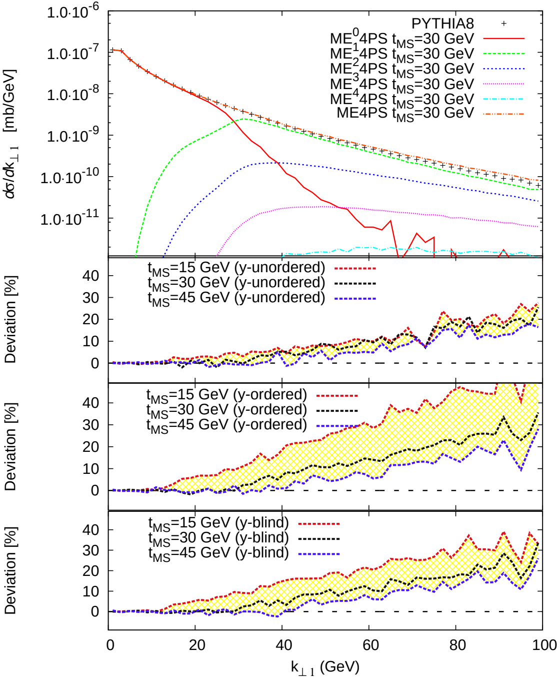
Some examples



Comparison of two prescriptions of choosing the history for $e^+e^- \to 3$ jets:

ckkw-l: Choose probabilistically according to splitting kernels.

scale : Always choose history with lower reconstructed scale.



PYTHIA8
ME0 4PS tMS=30 GeV
ME1 4PS tMS=30 GeV
ME2 4PS tMS=30 GeV
ME3 4PS tMS=30 GeV
ME4 4PS tMS=30 GeV
ME4PS tMS=30 GeV
dσ/dk⊥1 [mb/GeV]
tMS=15 GeV (y-unordered)
tMS=30 GeV (y-unordered)
tMS=45 GeV (y-unordered)
tMS=15 GeV (y-ordered)
tMS=30 GeV (y-ordered)
tMS=45 GeV (y-ordered)
tMS=15 GeV (y-blind)
tMS=30 GeV (y-blind)
tMS=45 GeV (y-blind)
Deviation [%]
k⊥1 (GeV)

Conclusions

- By now, matrix element merging is an old hat.
- In Pythia8, we've recently implemented CKKW-L merging.

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- We believe that these uncertainties can be used to guide NLO multi-jet merging, e.g.
  ...to have less unitarity violations at NLO.
  ...to use histories to construct exclusive NLO x-sections.
  ...to find the PS-$\mathcal{O}(\alpha_s)$-term by comparing with clustered states.

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  ...to have less unitarity violations at NLO.
  ...to use histories to construct exclusive NLO x-sections.
  ...to find the PS-$\mathcal{O}(\alpha_s)$-term by comparing with clustered states.
- At NLO, unitarity violation needs to be smaller (see Leifs talk)

Backup slides

$k_\perp$ used as merging scale

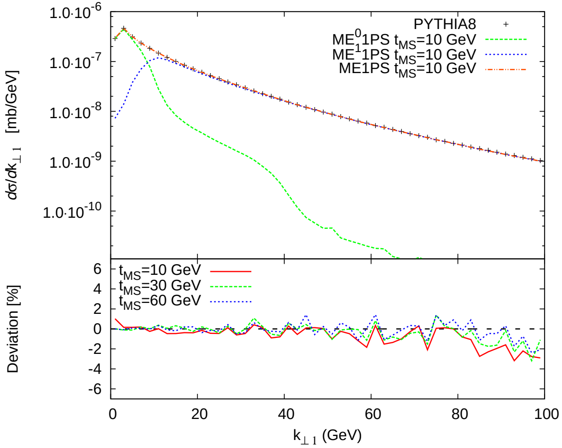

Figure: Transverse momentum of the first jet in W + 1 jet, in pp collisions at $E_{CM} = 7000$ GeV. Jet defined with $k_\perp$ algorithm as implemented in `fastjet`, with $D = 0.4$.

Does rapidity as merging scale work?

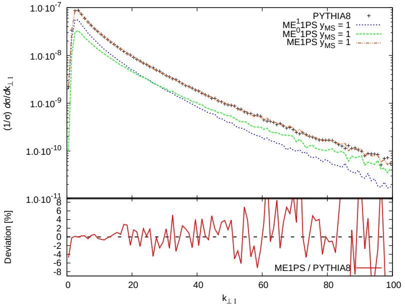

Figure: Transverse momentum and rapidity of the first jet in W + jets in $p\bar{p}$ collisions at $E_{CM} = 1960$ GeV. Rapidity used as merging scale with $y_{MS} = 1.0$. Minimal cut $p_{\perp 1,min} > 2$ GeV applied. Jet defined with $k_\perp$ algorithm as implemented in `fastjet` with $D = 0.4$. Plot produced with CKKW-L implementation in PYTHIA8.

Two jets above ρ_c

In the ME2PS approximation, the inclusive cross section is

$$
\begin{aligned}
d\sigma^{\mathrm{ME2PS}} \quad = \quad & x_0 f_0(x_0,\rho_0)\Big\{ \\
& |\mathcal{M}_{S_{+2}}|^2\, d\Phi_2^{\mathrm{ME}} \Theta(t(S_{+2}) - t_{\mathrm{MS}}) w_{Path,2} \alpha_{\mathrm{s}}(\rho_1)\alpha_{\mathrm{s}}(\rho_2) \\
& \tfrac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)} \Pi_{S_{+0}}(x_0,\rho_0,\rho_1) \\
& \tfrac{x_2 f_2(x_2,\rho_2)}{x_1 f_1(x_1,\rho_2)} \Pi_{S_{+1}}(x_1,\rho_1,\rho_2)\Pi_{S_{+2}}(x_2,\rho_2,\rho_c) \\
& + |\mathcal{M}_{S_{+1}}|^2\, d\Phi_1^{\mathrm{ME}} \Theta(t(S_{+1}) - t_{\mathrm{MS}}) w_{Path,1} \alpha_{\mathrm{s}}(\rho_1) \\
& \tfrac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)} \Pi_{S_{+0}}(x_0,\rho_0,\rho_1) \\
& P\left(\tfrac{x_1}{x_2}\right) d\Phi_{1'}^{\mathrm{PS}} \tfrac{x_2 f_2(x_2,\rho_2)}{x_1 f_1(x_1,\rho_2)} \Pi_{S_{+1}}(x_1,\rho_1,\rho_2) \Theta(t_{\mathrm{MS}} - t(S_{+2})) \\
& \Pi_{S_{+2}}(x_2,\rho_2,\rho_c) \\
& + |\mathcal{M}_{S_{+0}}|^2\, d\Phi_0^{\mathrm{ME}} \alpha_{\mathrm{s}}(\rho_1)\alpha_{\mathrm{s}}(\rho_2) \\
& P\left(\tfrac{x_0}{x_1}\right) d\Phi_1^{\mathrm{PS}} \tfrac{x_1 f_1(x_1,\rho_1)}{x_0 f_0(x_0,\rho_1)} \Pi_{S_{+0}}(x_0,\rho_0,\rho_1) \Theta(t_{\mathrm{MS}} - t(S_{+1})) \\
& P\left(\tfrac{x_1}{x_2}\right) d\Phi_{1'}^{\mathrm{PS}} \tfrac{x_2 f_2(x_2,\rho_2)}{x_1 f_1(x_1,\rho_2)} \Pi_{S_{+2}}(x_1,\rho_1,\rho_2)\Pi_{S_{+2}}(x_2,\rho_2,\rho_c) \quad \Big\}
\end{aligned}
$$