A common view of MC@NLO and POWHEG

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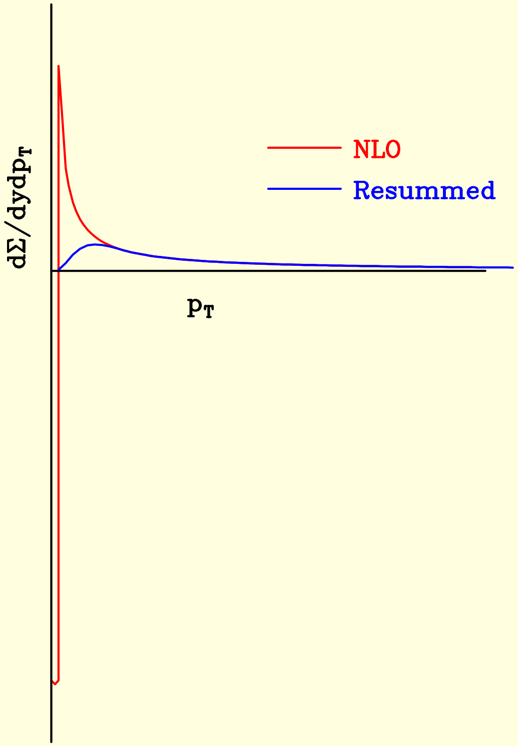
NLO+PS

Two approaches to QCD calculations:

- Fixed order calculations. Aims:
  - Give prediction for inclusive observables at fixed order
  - No model for exclusive final state
- Resummed/Shower. Aims:
  - Predictions in exclusive limits by resumming dominant contributions to all orders in perturbation theory
  - Can be used to model exclusive final states

NLO+PS approaches aim at getting the benefits of both approaches in a unified context.

Example: Higgs production



NLO result: divergent distribution at low p_{T};
Negative divergent spike at $p_{\mathrm{T}}=0$, so that

$$\int \frac{d\sigma^{\mathrm{NLO}}}{dy\,dp_{\mathrm{T}}}\,\mathrm{d}p_{\mathrm{T}}=\frac{d\sigma^{\mathrm{NLO}}}{dy}$$

MC resummed result: smooth Sudakov shape at small p_{T}, all positive.

$$\int \frac{d\sigma^{\mathrm{res}}}{dy\,dp_{\mathrm{T}}}\,\mathrm{d}p_{\mathrm{T}}=\frac{d\sigma^{\mathrm{B}}}{dy}$$

Magnitude of NLO corrections all concentrated at $p_{\mathrm{T}}=0$ at NLO.
In NLO+PS, NLO corrections must be spread in the finite p_{T} region.

MC@NLO and POWHEG

We define

$$\Phi = \Phi(\Phi_{\rm B}, \Phi_{\rm rad}), \quad {\rm d}\Phi = {\rm d}\Phi_{\rm B}\, {\rm d}\Phi_{\rm rad}, \; R = R^{S\,(\text{singular})} + R^{F\,(\text{finite})}$$

$$d\sigma^{\rm NLO+PS} = \bar{B}^S(\Phi_{\rm B})\left[\Delta^S_{q_0}(\Phi_{\rm B}) + \Delta^S_{k_{\rm T}}(\Phi_{\rm B})\frac{R^S(\Phi_{\rm B},\Phi_{\rm rad})}{B(\Phi_{\rm B})}\right] + R^F(\Phi_{\rm B},\Phi_{\rm rad})$$

$$\bar{B}^S(\Phi_{\rm B}) = B(\Phi_{\rm B}) + \underbrace{V(\Phi_{\rm B}) + \int R^S(\Phi_{\rm B},\Phi_{\rm rad})\,{\rm d}\Phi_{\rm rad}}_{\text{Individually divergent; finite sum}}$$

$$\Delta^S_{p_{\rm T}}(\Phi_{\rm B}) = \exp\left[-\int_{k_{\rm T}\geqslant p_{\rm T}} \frac{R^S(\Phi_{\rm B},\Phi_{\rm rad})}{B(\Phi_{\rm B})}\,{\rm d}\Phi_{\rm rad}\right]$$

In MC@NLO $R^S = R^{\rm MC}$. Hardest emission formula (squared bracket) for angular ordered showers derived in P.N.2004. For transverse momentum ordered showers, hardest emission=first emission.

In POWHEG $R^S = FR$, with $F \leqslant 1$, and $F \to 1$ in the singular region.

Proof of NLO accuracy (Frixione, Oleari, P.N. 2007):

$$\langle O \rangle = \int \mathrm{d}\Phi_{\rm B}\, \bar{B}^S \left[\Delta^S_{q_0} O(\Phi_{\rm B}) + \int \mathrm{d}\Phi_{\rm rad} \Delta^S_{k_{\rm T}} \frac{R^S}{B} O(\Phi) \right] + \int \mathrm{d}\Phi\, R^F O(\Phi)$$

$$= \int \mathrm{d}\Phi_{\rm B}\, \bar{B}^S \underbrace{\left[\Delta^S_{q_0} O(\Phi_{\rm B}) + \int \mathrm{d}\Phi_{\rm rad} \Delta^S_{k_{\rm T}} \frac{R^S}{B} O(\Phi_B) \right]}_{= O(\Phi_{\rm B})\ \text{because of unitarity}} +$$

$$+ \int \mathrm{d}\Phi\, \bar{B}^S \Delta^S_{k_{\rm T}} \frac{R^S}{B} [O(\Phi) - O(\Phi_{\rm B})] + \int \mathrm{d}\Phi\, R^F O(\Phi)$$

Since $O(\Phi) - O(\Phi_{\rm B})$ kills the singular region, the red terms yield $1 + \mathcal{O}(\alpha_S)$;
Replacing μ_F, μ_R in R^S with the same scale used in $\bar{B}$ also yields $1 + \mathcal{O}(\alpha_S)$;

So:

$$\langle O\rangle = \int \mathrm{d}\Phi_{\mathrm{B}}\, \bar{B}^S O(\Phi_{\mathrm{B}}) + \int \mathrm{d}\Phi\, R^S[O(\Phi) - O(\Phi_{\mathrm{B}})] + \int \mathrm{d}\Phi\, R^F O(\Phi)$$

$$= \int \mathrm{d}\Phi_{\mathrm{B}} \left[\bar{B}^S + V\right] O(\Phi_{\mathrm{B}}) + \int \mathrm{d}\Phi\, R^S O(\Phi) + \int \mathrm{d}\Phi\, R^F O(\Phi)$$

where now an appropriate IR regulator is used in V and R^S.

Notice: by restricting R^S more and more we recover exactly the NLO result, with no further higher order effects!

For example: $R^S = \theta(k_{\mathrm{cut}} - k_T) R$, as $k_{\mathrm{cut}} \to 0$

$$\lim_{k_{\mathrm{cut}} \to 0} \int \mathrm{d}\Phi\, \bar{B}^S \Delta^S_{k_{\mathrm{T}}} \frac{R^S}{B} [O(\Phi) - O(\Phi_{\mathrm{B}})] = 0$$

and this is the only term where we had to neglect higher order terms.

But we CANNOT do this if we want an NLO+PS result; as $k_{\mathrm{cut}} \to 0$ the Sudakov region becomes squeezed and distorted, even to a point when positivity is lost.

In an NLO+PS implementation, visible differences with respect to the pure-fixed order result will be present, due to

- The $\Delta^S_{k_\mathrm{T}}$ factor, dropped in the NLO-accuracy derivation
- The $\bar{B}^S/B$ factor, also dropped
- Scales issues in R^S

The $\Delta^S_{k_\mathrm{T}}$ factor yields resummation improved results at NLO.
It is < 1; it always reduces the transverse momentum spectrum of radiation with respect to the pure NLO result

The $\bar{B}^S/B$ factor spreads the K–factor over the finite p_T region.
The spreading of the K factor depends upon the R^S/R^F separation.

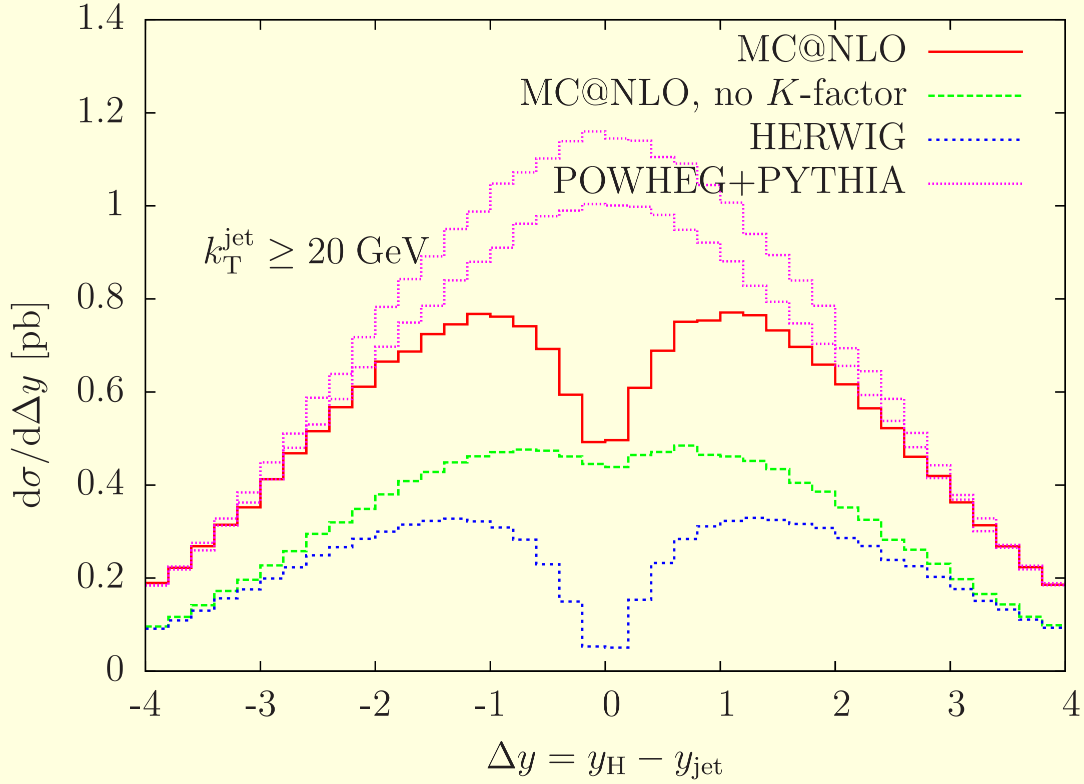
Experience in comparing MC@NLO and POWHEG results (various papers from the POWHEG BOX coll. and from Herwig++ team) has shown that all important differences between MC@NLO and POWHEG can be tracked back to the role of the $\bar{B}^S/B$ factor, and to scale choice issues.

Exponentiation in Δ_{k_T} does not seem to yield important differences; this is understood as due to the fact that the integral in

$$\Delta^S_{p_\mathrm{T}}(\Phi_\mathrm{B}) = \exp\left[- \int_{k_\mathrm{T} \geqslant p_\mathrm{T}} \frac{R^S(\Phi_\mathrm{B}, \Phi_\mathrm{rad})}{B(\Phi_\mathrm{B})} \, \mathrm{d}\Phi_\mathrm{rad} \right]$$

is dominated by the region of soft k_T, where all R^S agree.

Distribution in the rapidity difference between the Higgs and the hardest jet;
If $\bar{B}^S$ is replaced by B in `MC@NLO` the dip goes away:

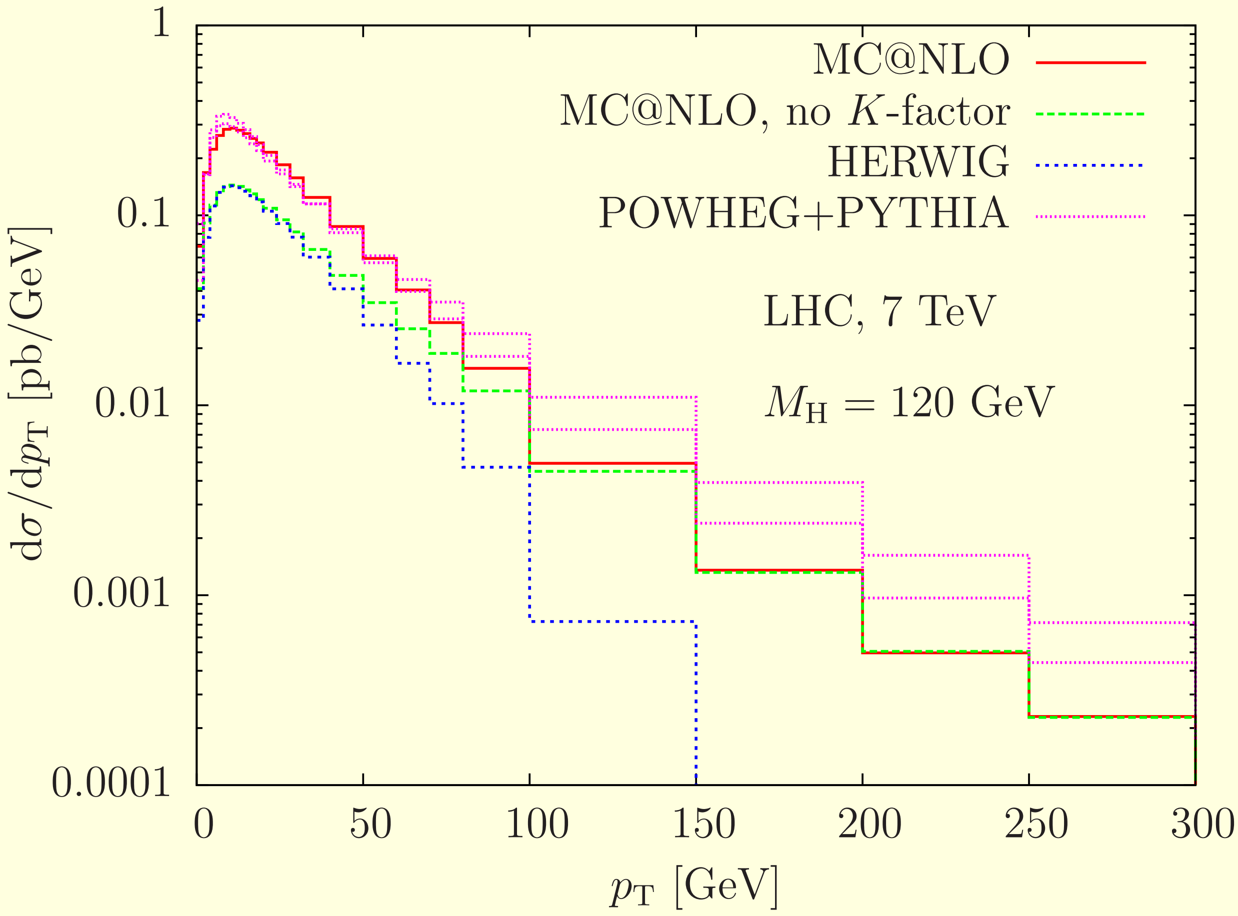


Alioli, Oleari, Re, P.N. 2008; P.N. 2009; Hamilton, Richardson, Tully, 2009; Webber, P.N. 2012

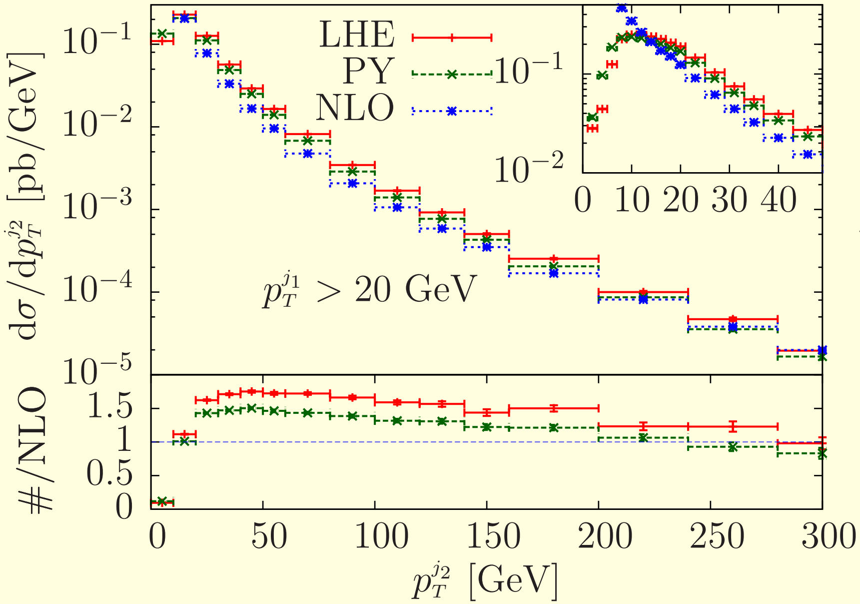
Higgs p_{T} distribution;

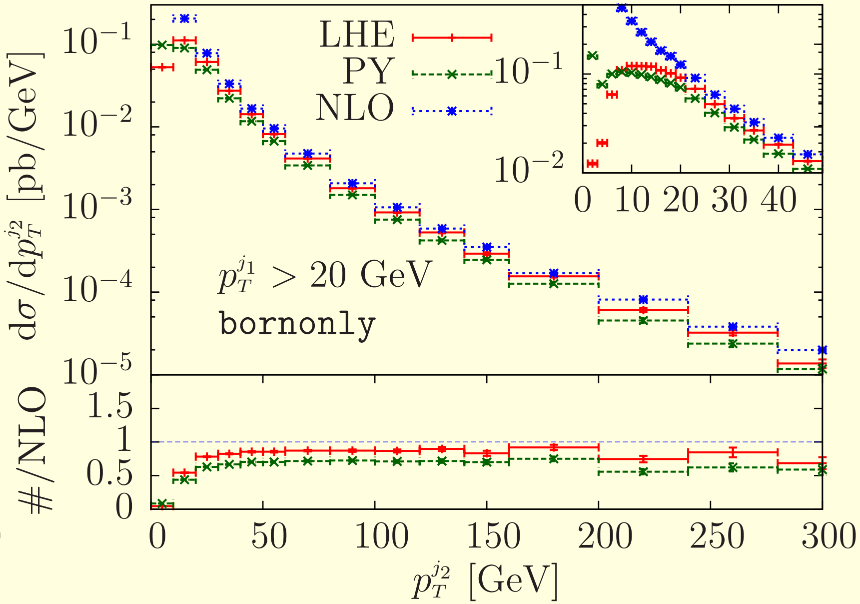
MC@NLO coincides with NLO at large p_{T} (R^F dominated)

POWHEG ($R^F = 0$) has uniform K factor also at large p_{T}.

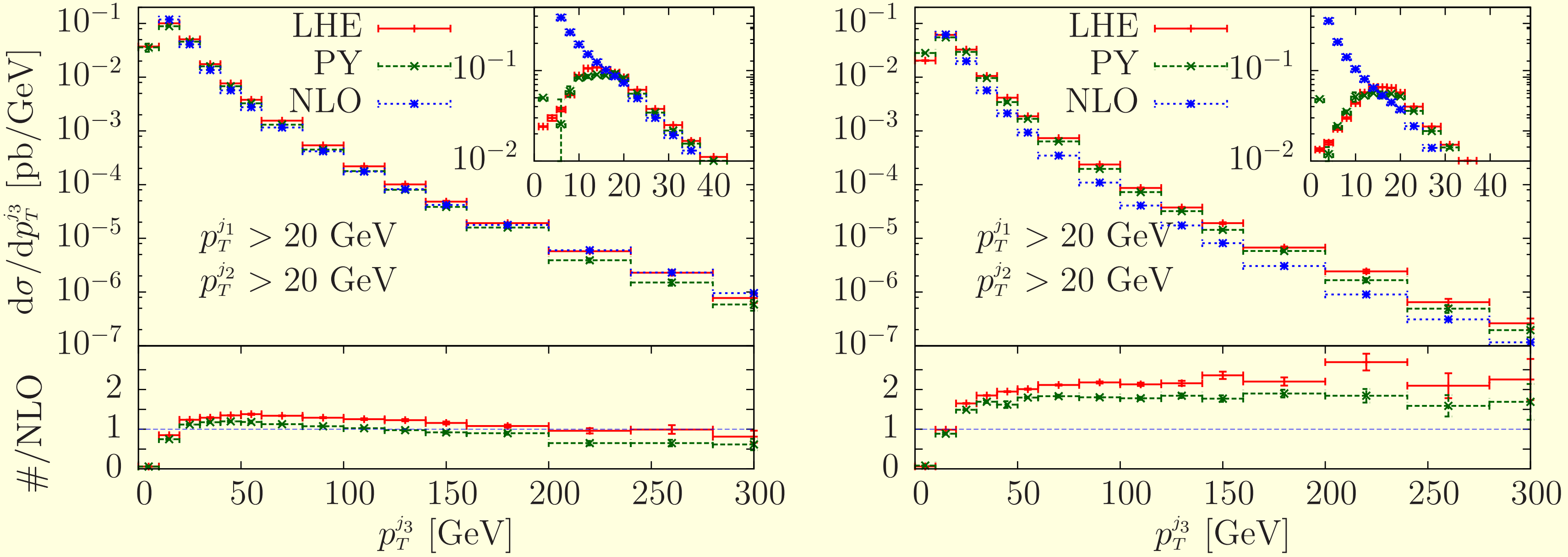


Trend also visible in very complex processes; $H + 1|2\,\mathrm{jets}$,
(Campbell, Ellis, Frederix, Oleari, Williams, P.N., 2012)





Second jet p_{T} in $H + \mathrm{jet}$ production; in the right plot $\bar{B} \to B$
(`bornonly` `POWHEG BOX` option)



Third jet p_{T} in $H + 2\,\mathrm{jets}$;
In the left plot, $\mu_F = \mu_R = M_H$, in the right plot $\mu_F = \mu_R = \hat{H}_{\mathrm{T}}$; the K-factor is near 1 in the fixed scale case, while it is large in the running scale case.

Conclusions

- NLO+PS (MC@NLO and POWHEG) results in relation to fixed order NLO and among each other are well understood since several years.
- Most important differences due to where the NLO K factor is applied Large for large K factors, small otherwise.
- Arguments as to where the K factor should apply are not firmly conclusive (the proof of NLO accuracy holds in all cases). However, the region where the K factor acts should not be too small, in order not to spoil LL and NLL resummation