Status of the emittance measurement paper

The beams have been measured - but are they *matched*?

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LDS

The thrust of the paper

1. Beams were created for the demonstration of cooling at $(3, 6, 10)$ mm $\otimes$ $(140, 200, 240)$ MeV/c
   - A reconstruction technique was developed to identify the particle species and measure the individual phase space vector $(x, p_x, y, p_y; p_z)$ of every particle in the beam
   - Their properties p_z, ϵ_{n}, (β_x, α_x), (β_y, α_y) were measured
   - Simulations agree with the data
2. The matching is non-trivial: $\beta_x \neq \beta_y$, $\alpha_x \neq \alpha_y$ and $D_x \neq 0$
   - This was investigated by simulating the passage of the beams muon by muon in the cooling channel
3. The beams are matched and suitable for an ionization cooling demonstration

Phase space reconstruction at TOF1

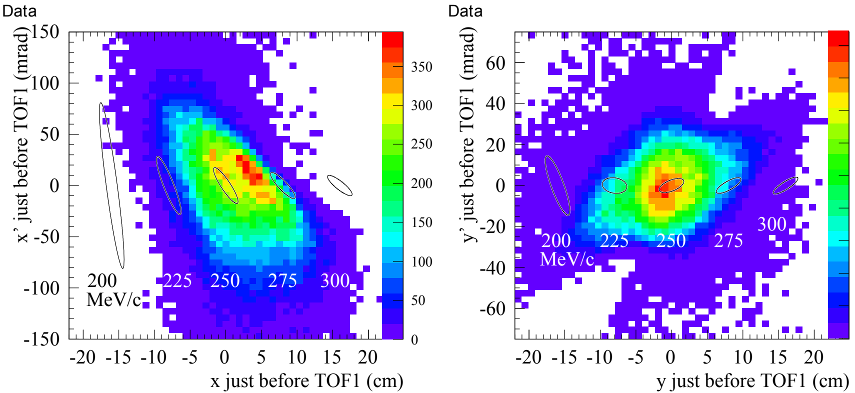
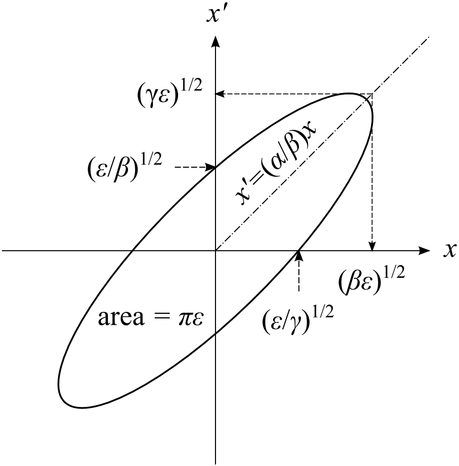


Figure: The transverse trace space of the base line (6 mm, 200 MeV/c) μ^- beam at TOF1. 1σ error ellipses have been superimposed and labelled by their corresponding p_z. They are displaced for clarity; the size and orientation of the error ellipse are invariant in trace space.

Emittance and Twiss parameters



The beams are parameterized separately in horizontal and vertical trace space via their covariance matrices:

$$\Sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xx'} \\ \sigma_{xx'} & \sigma_{x'x'} \end{pmatrix} = \begin{pmatrix} \epsilon\beta & -\epsilon\alpha \\ -\epsilon\alpha & \epsilon\gamma \end{pmatrix}$$

Agreement with Monte Carlo

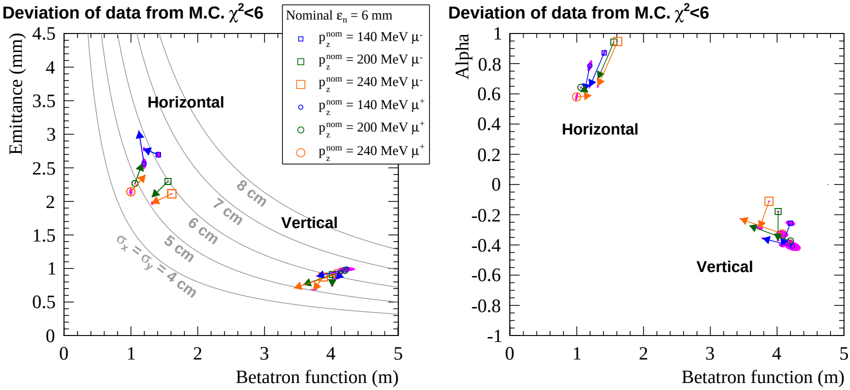
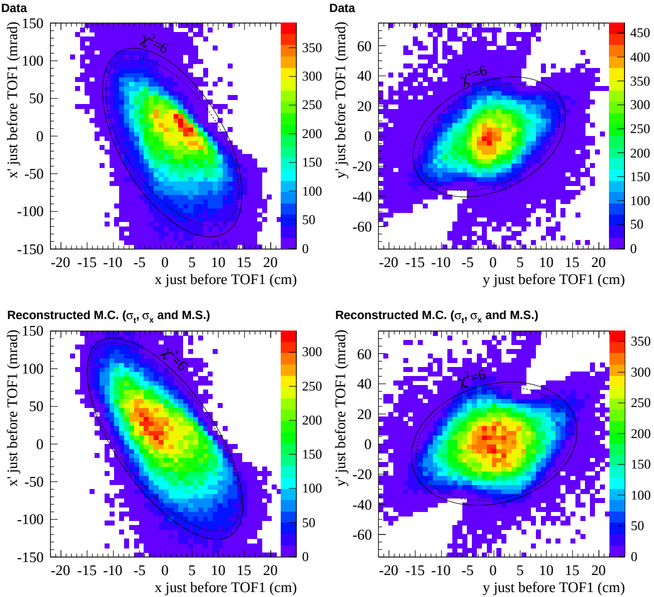


Figure: (β, ϵ) and (β, α) of reconstructed data and the reconstructed resolution smeared $\epsilon_n = 6$ mm beam simulations calculated after a cut at $\chi^2 = 6$. The arrows point from reconstructed simulation to data.

Before and after the $\chi^2 < 6$ cut



Optical measurements for all the M0 beams

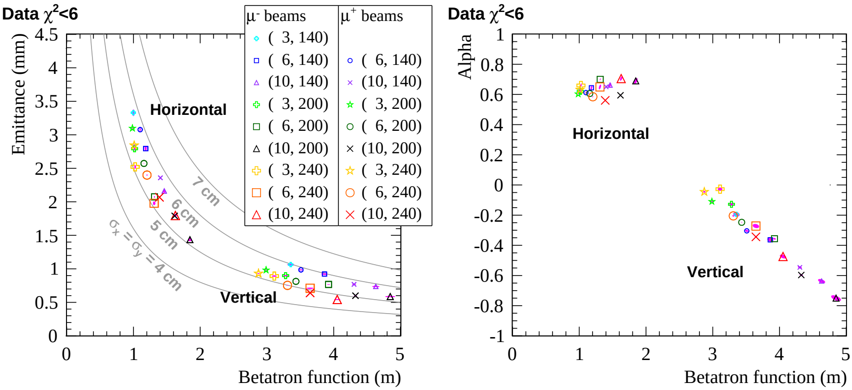


Figure: (β, ϵ) and (β, α) of the re-scaled TURTLE emittance-momentum matrix beams measured in Step I of the MICE experiment. A $\chi^2 = 6$ cut (95% in the Gaussian approximation) has been applied to remove the effect of high amplitude outliers as they are difficult to model and have a disproportionate effect on estimated statistical parameters.

4D transverse phase space

- The covariance matrix of the phase space vector (x, p_x, y, p_y) is usually parameterized as:

$$\begin{pmatrix} \epsilon_\mathrm{n}\beta_\perp m_\mu c/p_z & -m_\mu c\epsilon_\mathrm{n}\alpha_\perp & 0 & -m_\mu c\epsilon_\mathrm{n}(\beta_\perp\kappa - \mathcal{L}) \\ & m_\mu c p_z \epsilon_\mathrm{n}\gamma_\perp & m_\mu c\epsilon_\mathrm{n}(\beta_\perp\kappa - \mathcal{L}) & 0 \\ & & \epsilon_\mathrm{n}\beta_\perp m_\mu c/p_z & m_\mu c\epsilon_\mathrm{n}\alpha_\perp \\ & & & m_\mu c p_z \epsilon_\mathrm{n}\gamma_\perp \end{pmatrix}$$

  - x-y symmetric
- Hill's equation evolves according to the focusing strength $\kappa(z)$

$$2\beta_\perp\beta_\perp'' - (\beta_\perp')^2 + 4\beta_\perp^2\kappa^2 - 4 = 0$$

  - A Gaussian approximation - doen't model scraping by default
- A beam is matched when $\beta\kappa = 1$
- Re-scale $\kappa(z) = \frac{1}{2}qB_z(z)/p_z$ to get the same $\beta_\perp(z)$ at different p_z
  - e.g. the $p_z = 140$ MeV/c beams

Working back from the spectrometer

- In the same way, Kevin's (6 mm, 200 MeV/c) base line quadrupole beam line optics also may also adapted by re-scaling $k(z) = q(\mathrm{d}B_y(z)/\mathrm{d}x)/p_z$ to get the same $\beta(z)$ at different p_z
  $\Rightarrow$ **Marco's M0 optics**
- This is **not the same** as working back from $\beta\kappa = 1$ in the spectrometer (see Marco's famous table on next slide)

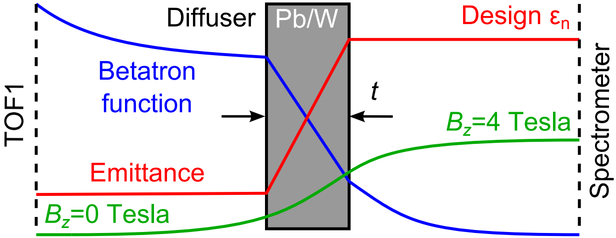


Figure: A sketch of the principle of emittance inflation in the diffuser showing the interface between a region of quadrupole focusing and a region of solenoidal focusing.

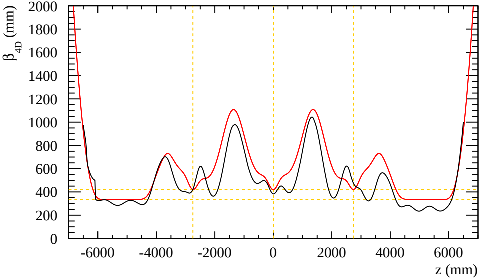
ϵ_n after the diffuser		p_z in the absorbers: 140 MeV/c	200 MeV/c	240 MeV/c
3 mm	t	0 mm	0 mm	0 mm
	p_z	151 MeV/c	207 MeV/c	245 MeV/c
	$\alpha_\perp$	0.2	0.1	0.1
	$\beta_\perp$	56 cm	36 cm	42 cm
6 mm	t	5.0 mm	7.5 mm	7.5 mm
	p_z	148 MeV/c	215 MeV/c	256 MeV/c
	$\alpha_\perp$	0.3	0.2	0.2
	$\beta_\perp$	113 cm	78 cm	80 cm
10 mm	t	10.0 mm	15.5 mm	15.5 mm
	p_z	164 MeV/c	229 MeV/c	267 MeV/c
	$\alpha_\perp$	0.6	0.4	0.3
	$\beta_\perp$	198 cm	131 cm	129 cm

Table: The lead diffuser thickness t and the momentum p_z and transverse Twiss parameters $\alpha_\perp$ and $\beta_\perp$ which are required immediately upstream of the diffuser to generate the emittance and momentum corresponding to each beam, while fulfilling the matching requirements.

Studying whether the beams are matched

- We've sliced the base line beam:

(1 MeV/c slices, 200 MeV/c to 300MeV/c, jumps of 10 MeV/c)



- However we must simulate realistic **broad p_z-distributions**
- In the following slides beams of 2000 muons with a Gaussian p_z-distribution with RMS 10 MeV/c have been simulated from just before TOF1 to just after AFC1 (15% - 50% transmission)
  - This sidesteps the tricky issue of RF matching on crest
  - We can nevertheless evaluate the matching in the spectrometer and in absorber focus coil 1

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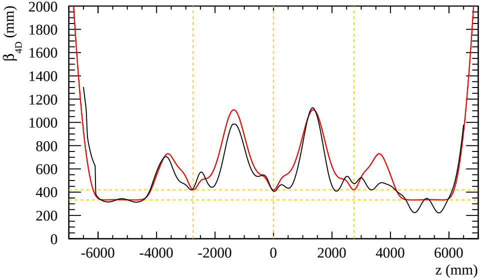

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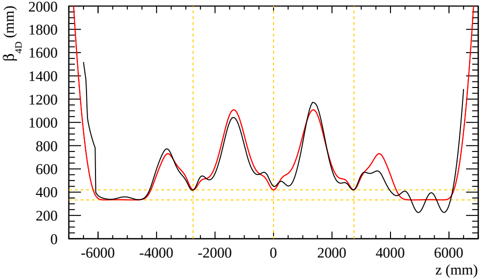

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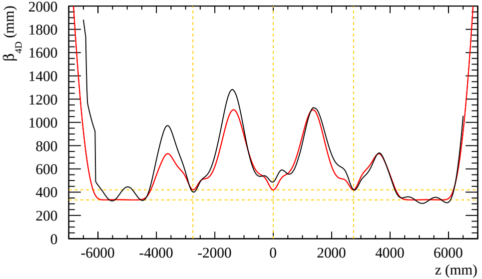

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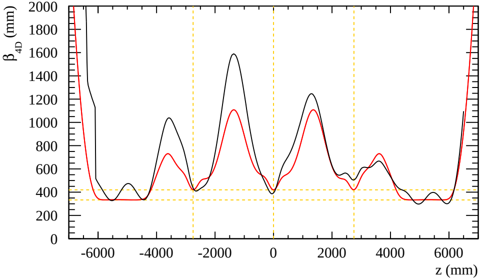

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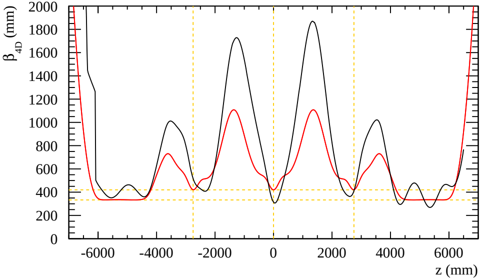

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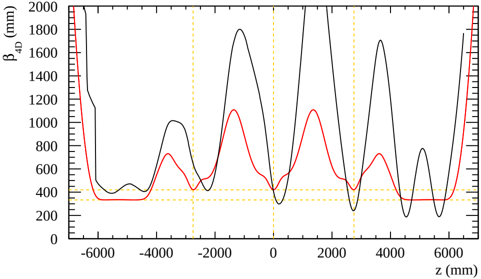

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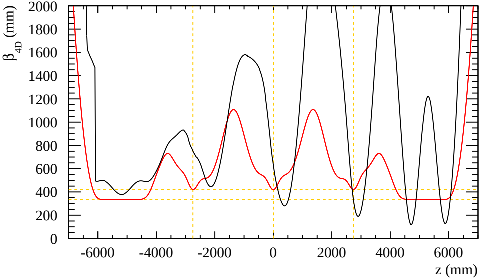

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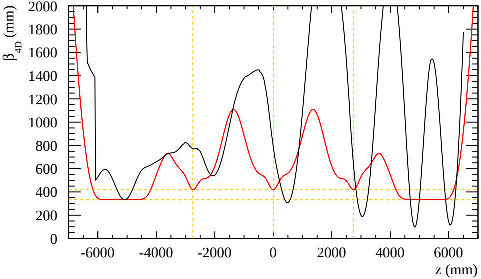

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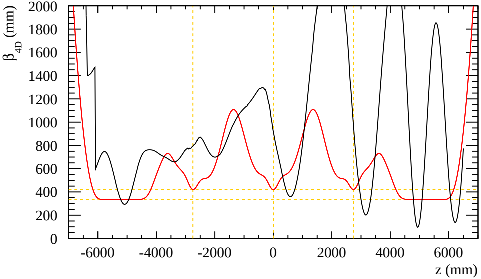

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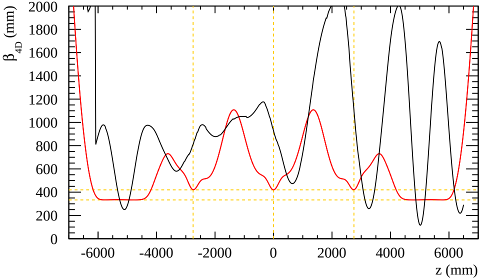

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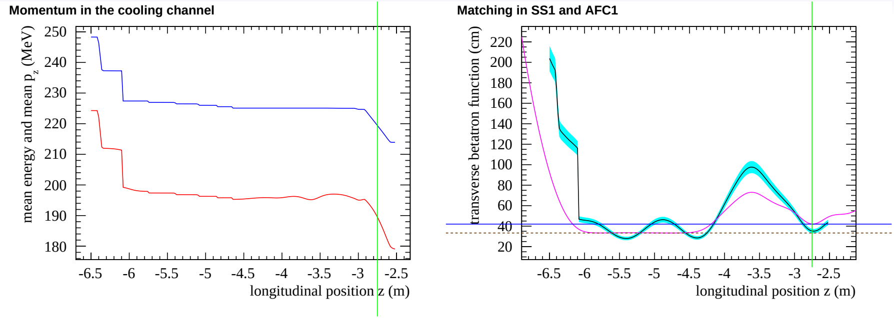
e.g. (6 mm, 200 MeV/c) M0 μ^- beam



Key to the plots

- $\langle E \rangle$ and $\langle p_z \rangle$ are drawn from just upstream of TOF1 to **AFC1**
- Black curves show the measured $\beta_\perp$ and ϵ_n with **statistical error**
- The (monochromatic) **matched betatron function** is drawn for reference
- Lines plot matched $\beta_\perp$ in the spectrometer (33 cm) and AFC (42 cm)

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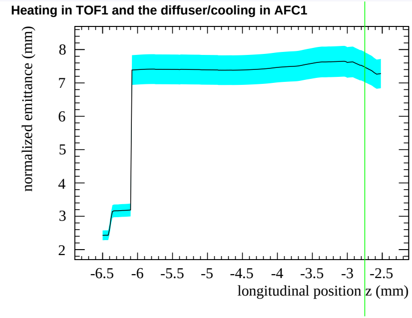

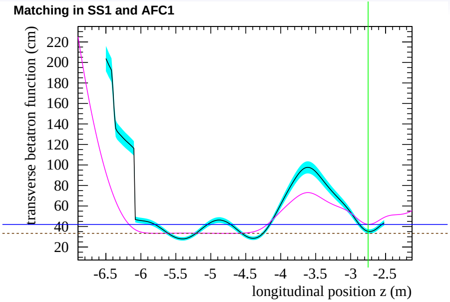

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N.B. Matching quadrupoles to solenoids

- Complication: the initial beams do not have *x*-*y* symmetry: let

$$\beta_\perp \equiv \frac{p_z}{m_\mu c \epsilon_{\mathrm{n}}} \left(\frac{\sigma_{xx} + \sigma_{yy}}{2} \right)$$

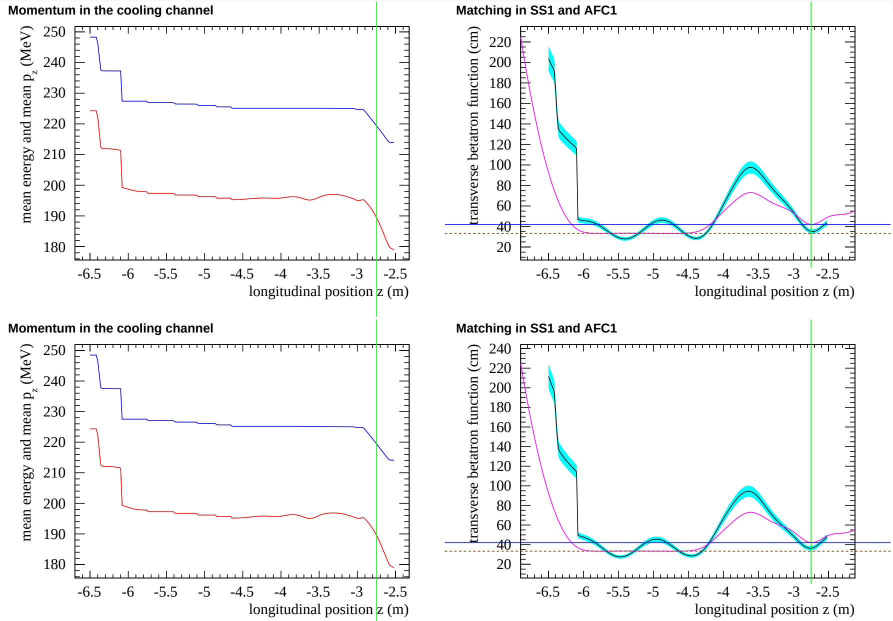
- Monochromatic approximations connect the two parameterizations

$$\epsilon_{\mathrm{n}} = \frac{p_z}{m_\mu} \sqrt{\epsilon_x \epsilon_y}$$

$$\beta_\perp = \tfrac{1}{2} \left(\sqrt{\frac{\epsilon_x}{\epsilon_y}} \beta_x + \sqrt{\frac{\epsilon_y}{\epsilon_x}} \beta_y \right)$$

- In addition, the matched Hill's equation solution is itself a monochromatic approximation!
- Also note: upstream spectrometer has $p_z = 207$ MeV/c, downstream 193 MeV/c - this disrupts the matching too

(6 mm, 200 MeV/c) **M0 beams, μ^- top, μ^+ bottom**



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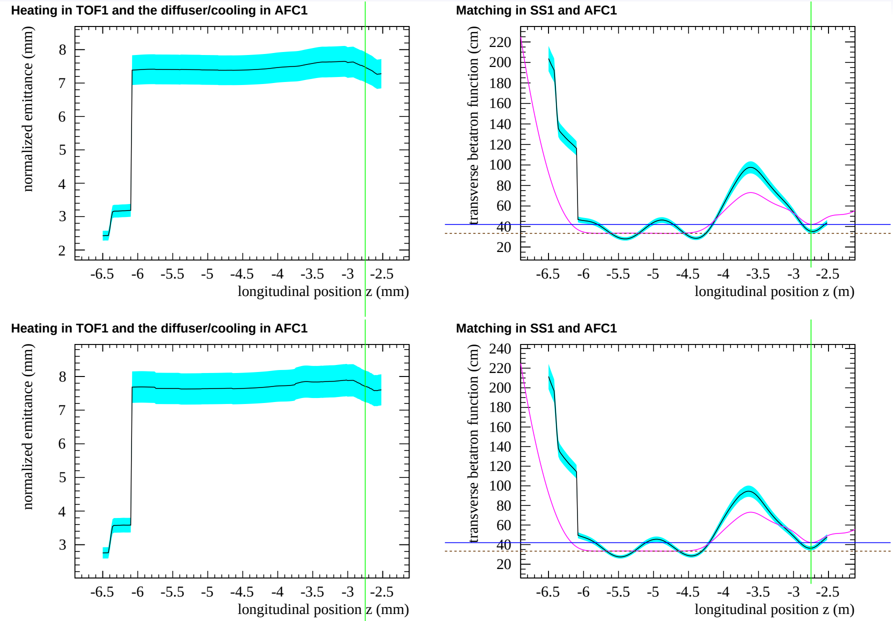
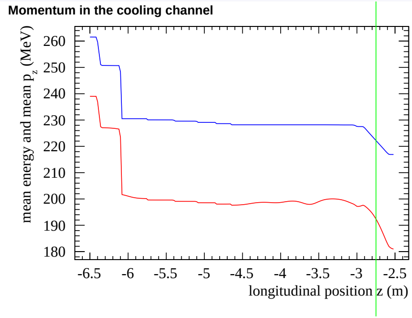
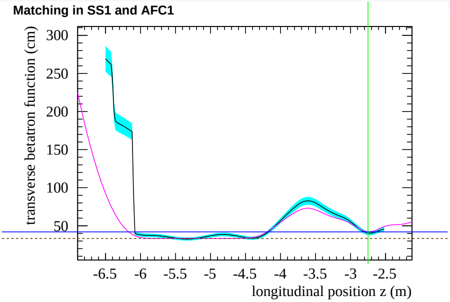


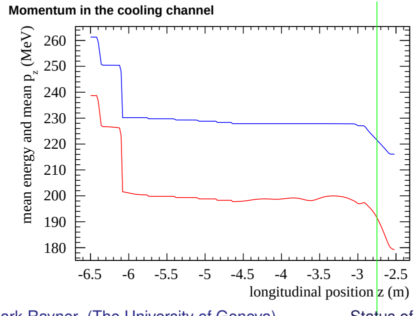
- We see encouraging similarity between μ^- and μ^+ beams
  - Suggests the calculation is quite stable
- These base line beams were optimized by Kevin and Marco
- Marco re-scaled these currents to get approximately matched non-baseline emittance-momentum matrix elements

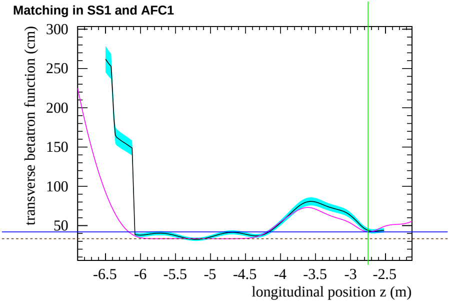
  (Marco's 'magic' tables)
  - Must vary diffuser thickness t to generate $\epsilon_\mathrm{n} = 3$ mm and 10 mm
  - This changes p_z in the quadruples
  - Work backwards from $\beta\kappa = 1$ in the spectrometer to dipole 2
- Should give the correct p_z and ϵ_n upstream of the diffuser
  - Bear in mind that Marco had scant knowledge of the initial ϵ_n
- Must hope that $\beta_\perp$ and $\alpha_\perp$ are acceptably matched
  - i.e. The optics have not been optimized!

(10 mm, 200 MeV/c) M0 beams, μ^- top, μ^+ bottom

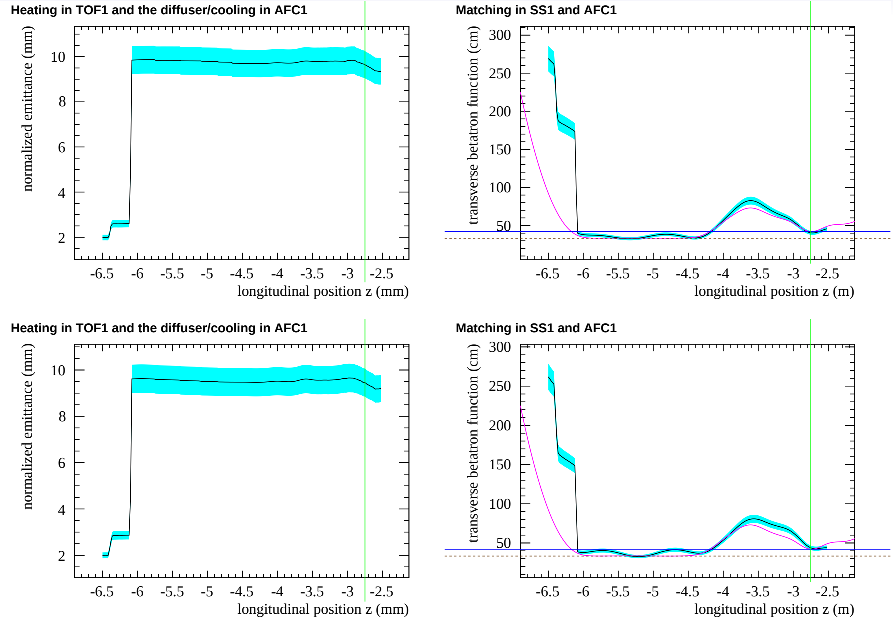




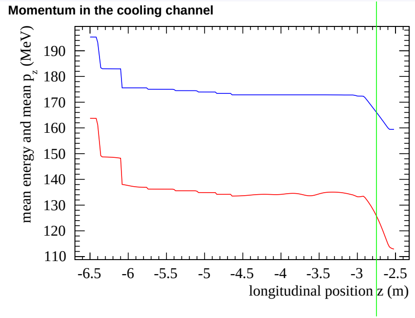


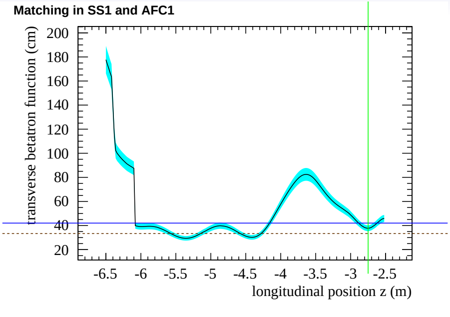


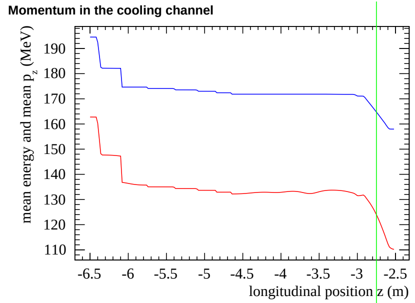
(10 mm, 200 MeV/c) **M0 beams, μ^- top, μ^+ bottom**

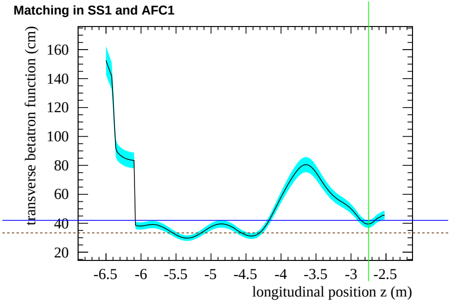


(6 mm, 140 MeV/c) **M0 beams, μ^- top, μ^+ bottom**

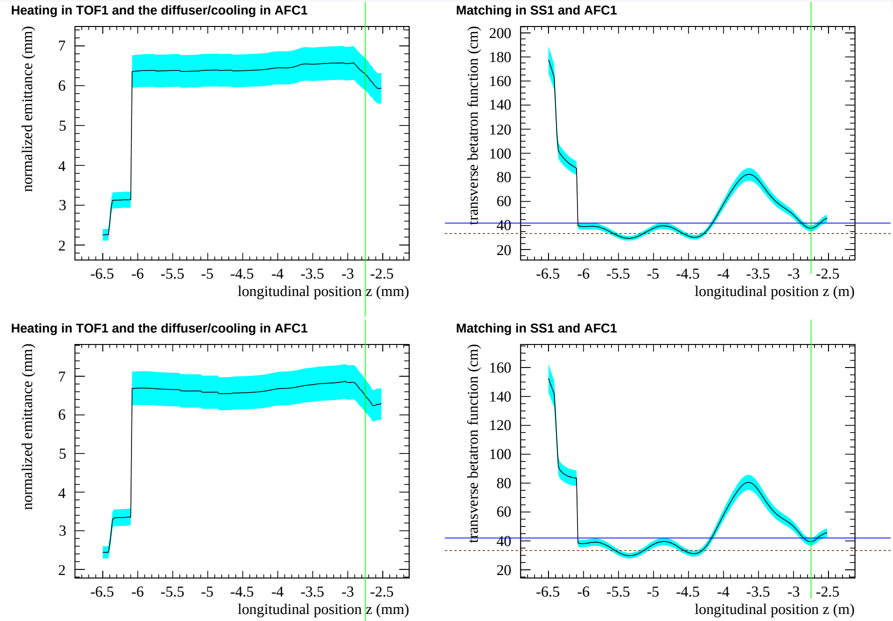




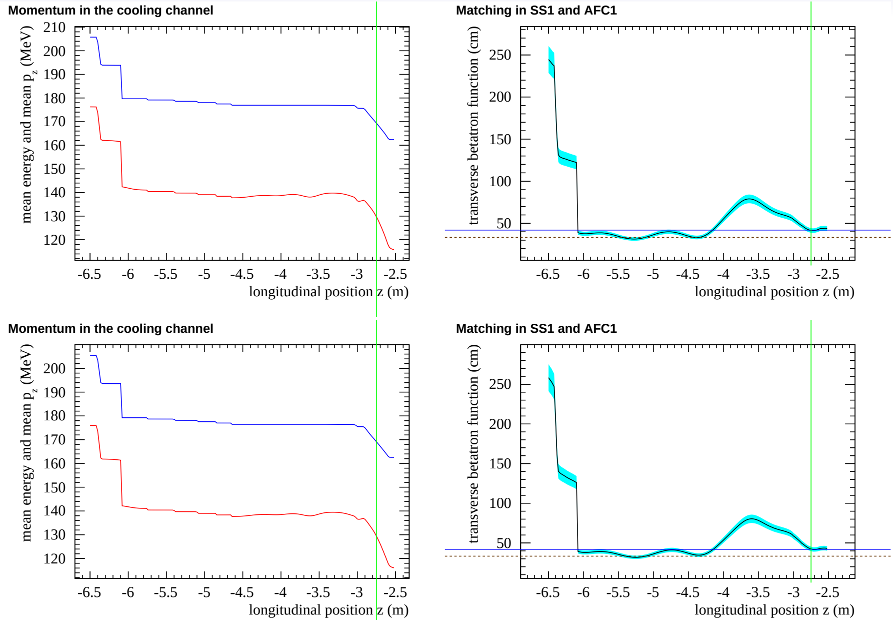




(6 mm, 140 MeV/c) **M0 beams, μ^- top, μ^+ bottom**



($\color{red}{10}$ mm, $\color{blue}{140}$ MeV/c) **M0 beams, μ^- top, μ^+ bottom**



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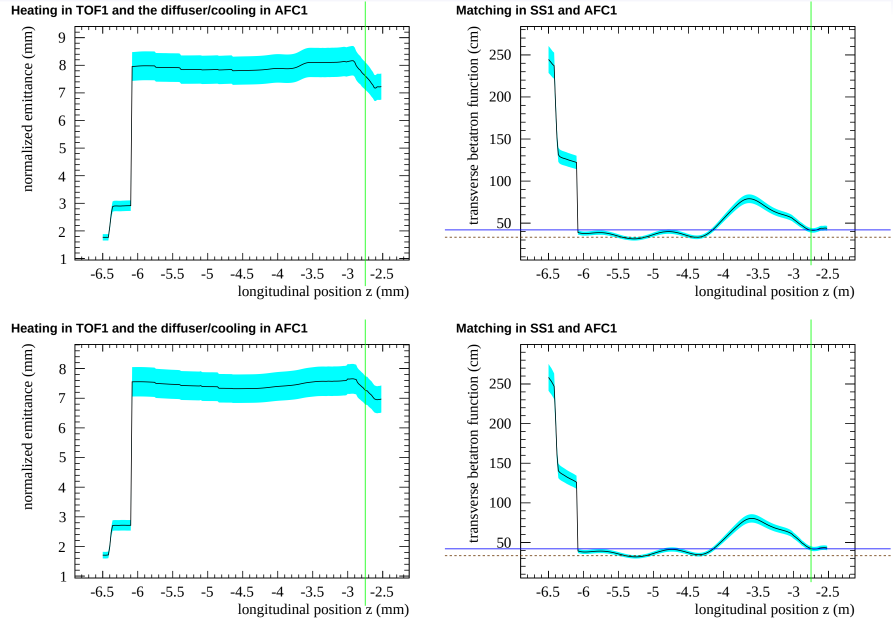
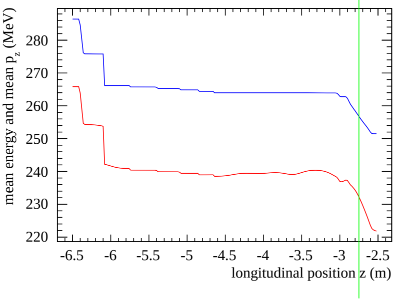


Difficulty at $p_z = 240$ MeV/c

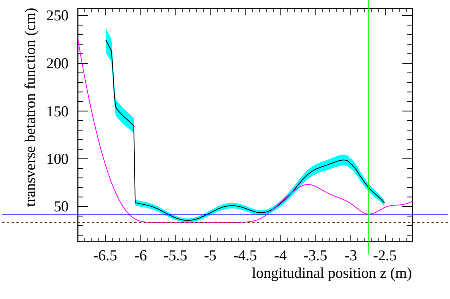
- Unfortunately the spectrometer solenoid is not powerful enough to scale to $(240/200)\kappa_0(z)$
- Left it at the $p_z = 200$ MeV/c matrix element settings
- This is *not* sufficient
  - No focus in absorber 1 and limited cooling
  - Must re-match quadrupoles
  - Must re-optimize matching coils

(6 mm, 240 MeV/c) **M0 beams,** μ^- **top,** μ^+ **bottom**

Momentum in the cooling channel



Matching in SS1 and AFC1



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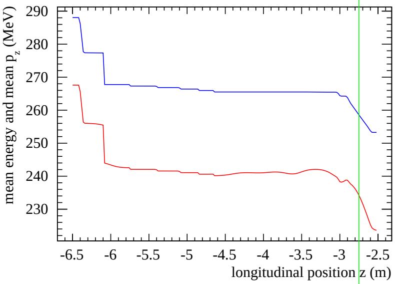

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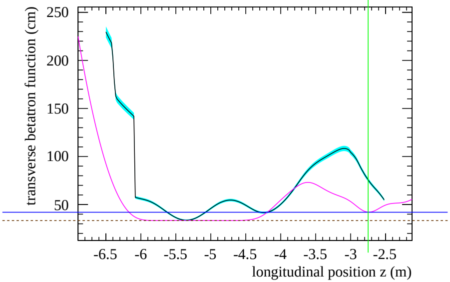
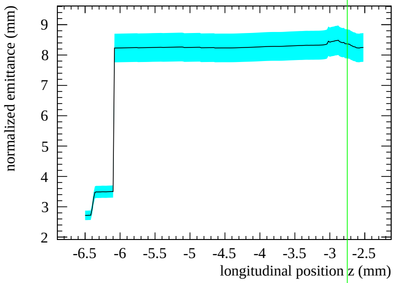


(6 mm, 240 MeV/c) **M0 beams, μ^- top, μ^+ bottom**

Heating in TOF1 and the diffuser/cooling in AFC1



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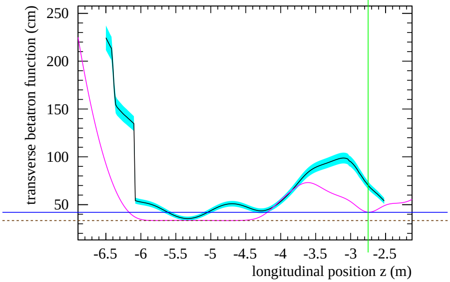

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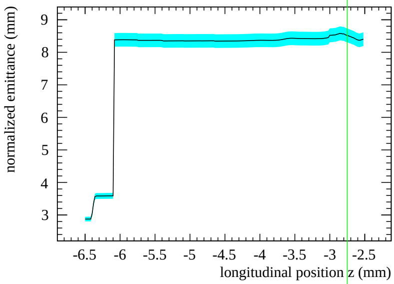

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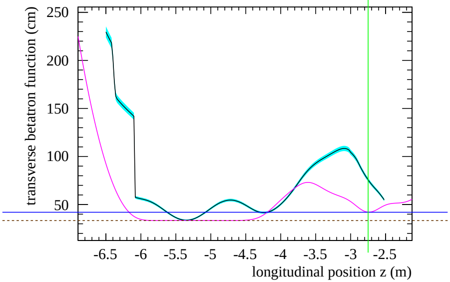
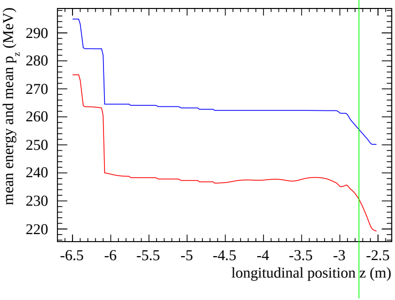


(10 mm, 240 MeV/c) M0 beams, μ^- top, μ^+ bottom

Momentum in the cooling channel



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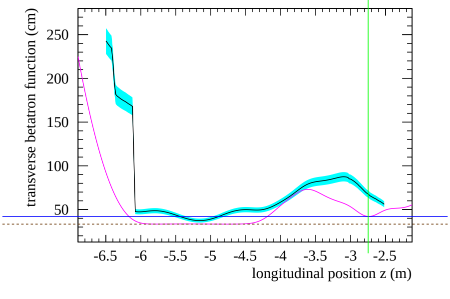

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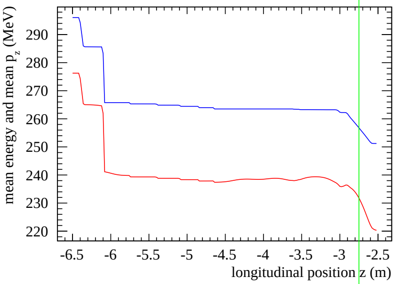

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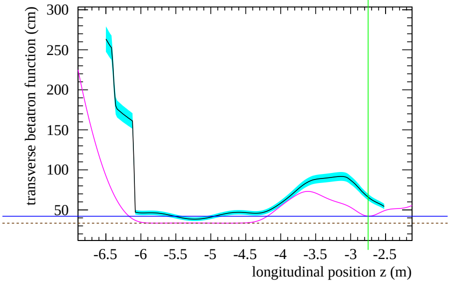

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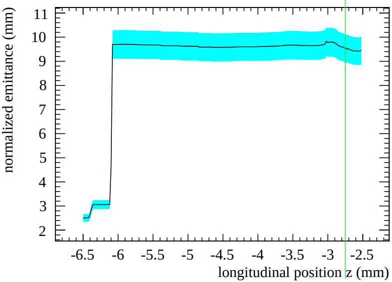

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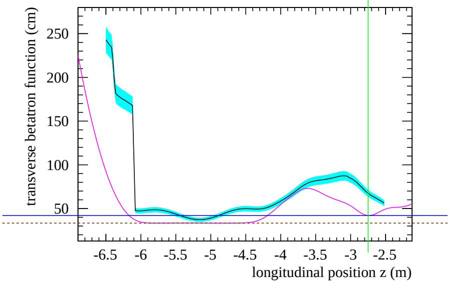

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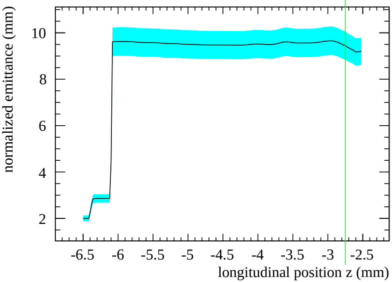

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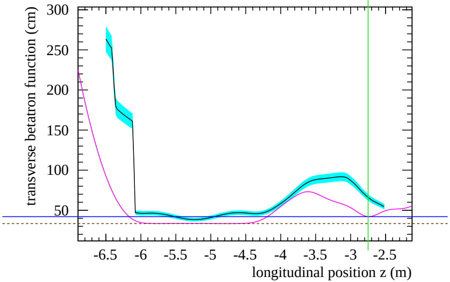
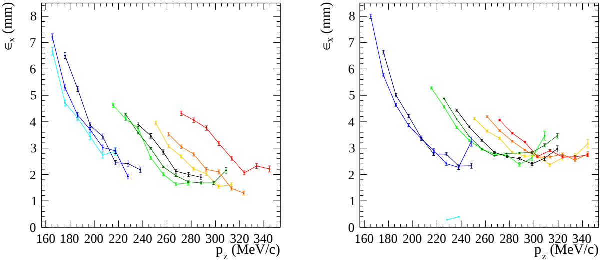


Pathological behaviour of the $\epsilon_n = 3$ mm beams

- The following slides show pathological behaviour for the $\epsilon_n = 3$ mm matrix elements
- The problem could be the illegitimacy of our tacit assumption that a 4D covariance matrix is sufficient to model beams with a broad p_z-distribution (connected to dispersion generated in dipole 2)

left: μ^-, right: μ^+, (3 mm, 140 MeV/c) to (10 mm, 240 MeV/c)



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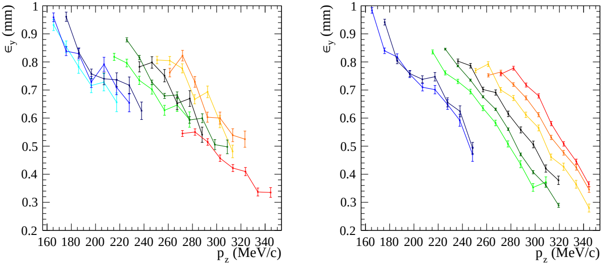

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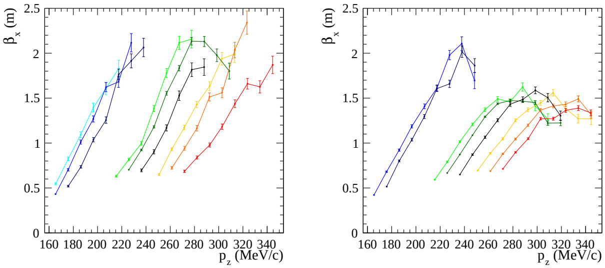

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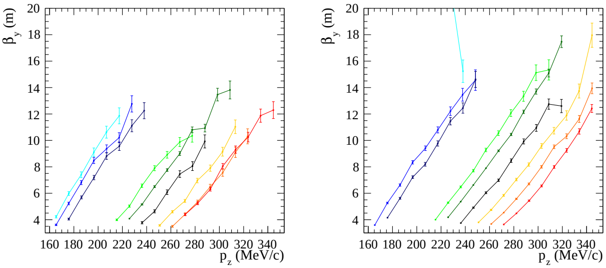

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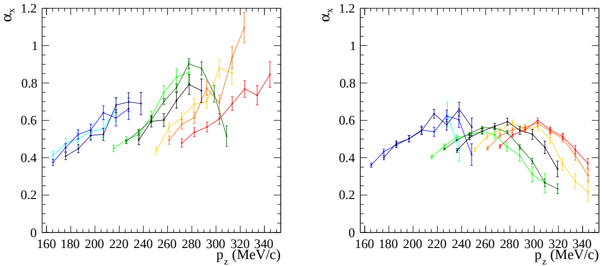

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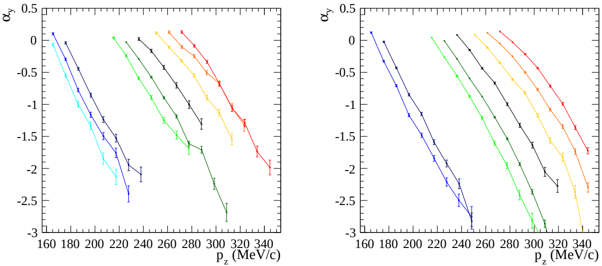

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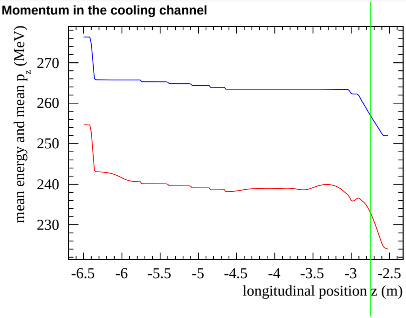
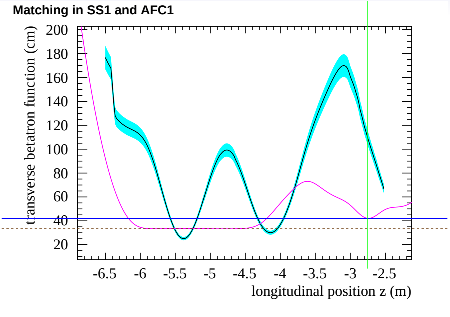
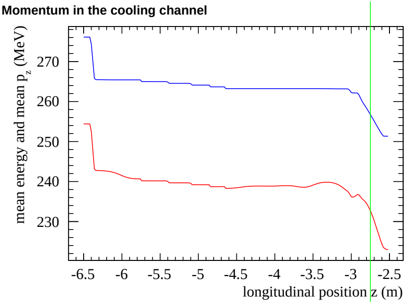
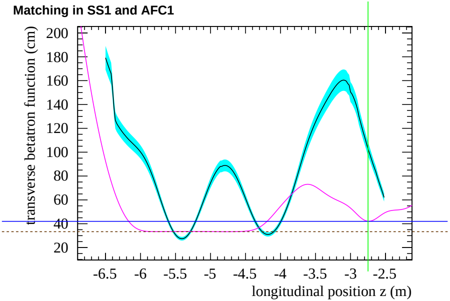
(3 mm, 240 MeV/c) M0 beams, μ^- top, μ^+ bottom









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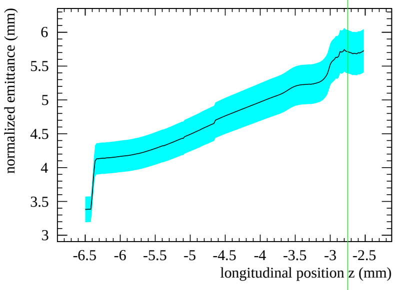

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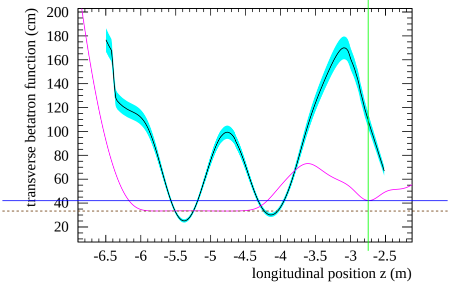

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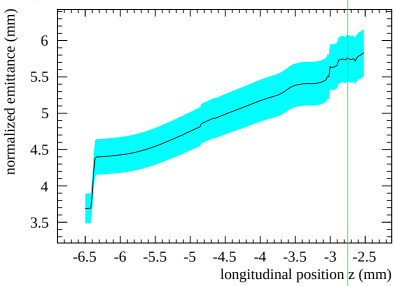

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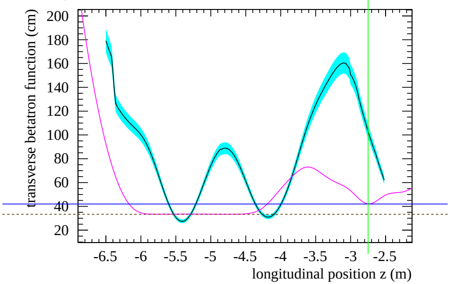
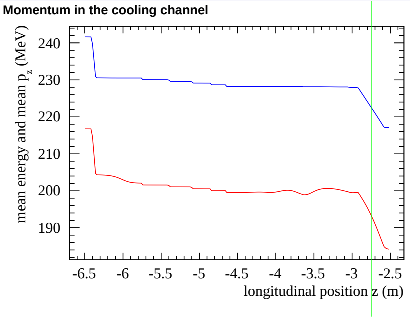
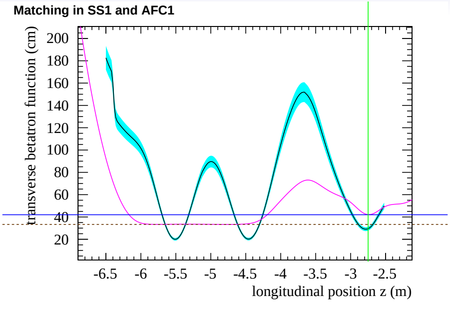
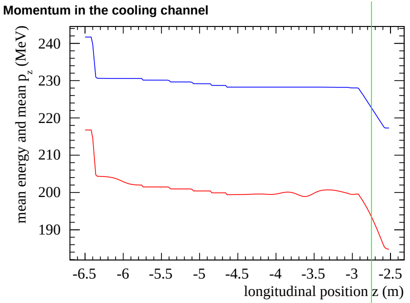
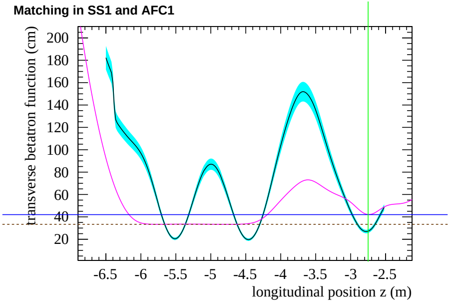


(3 mm, 200 MeV/c) **M0 beams, μ^- top, μ^+ bottom**

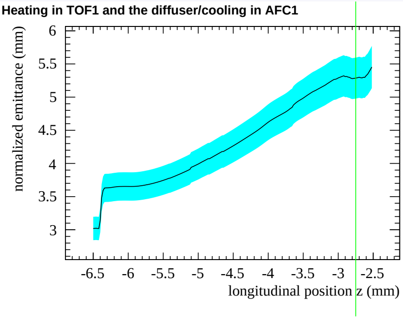


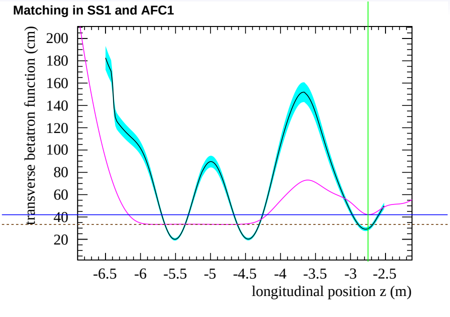


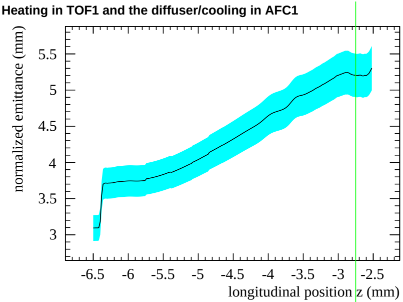


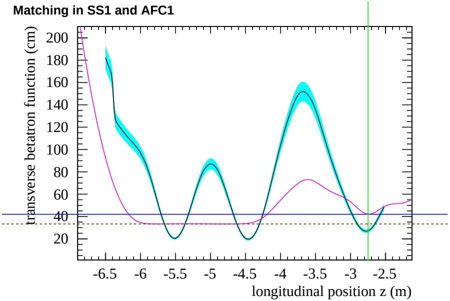


(3 mm, 200 MeV/c) **M0 beams,** μ^- **top,** μ^+ **bottom**

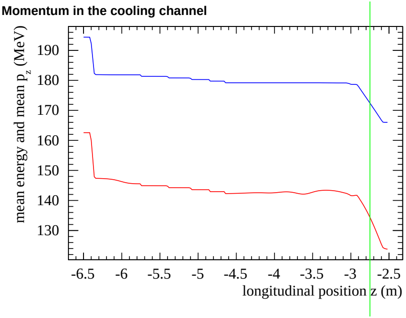


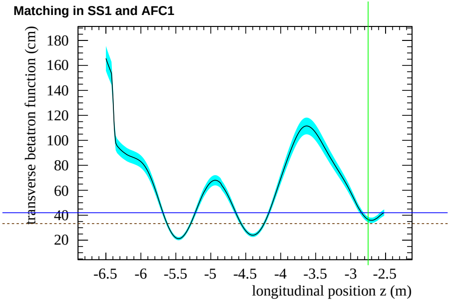






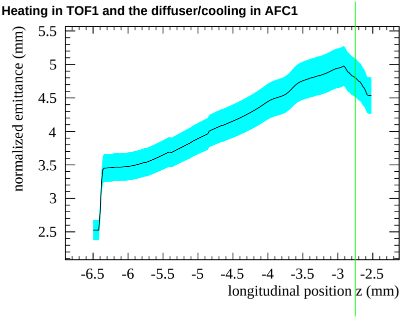
(3 mm, 140 MeV/c) **M0** μ^- **beam**

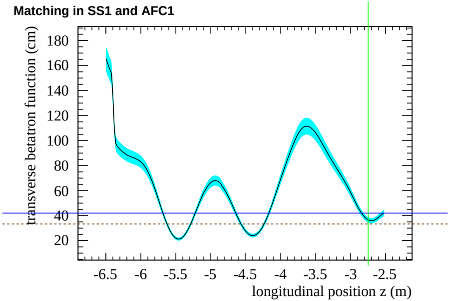




No transmitted muons in the M0 (3 mm, 140 MeV/c) μ^+ data

(3 mm, 140 MeV/c) **M0** μ^- **beam**





No transmitted muons in the M0 (3 mm, 140 MeV/c) μ^+ data

Summary of the simulations

- The $(6, 10)$ mm $\otimes$ $(140, 200)$ MeV/c quartet of beams are not perfectly matched but perform quite well
  - It's difficult to judge the match quality in the spectrometer
  - A better test is whether $\beta_\perp$ has a minimum near 42 cm in the AFC
- The $p_z = 240$ MeV/c elements are not well matched in the simplest scenario outlined here, as the spectrometer solenoid cannot be scaled up to a powerful enough field strength to maintain the optical design
  - Must re-match the quadrupoles and/or the matching coils
- The $\epsilon_\mathrm{n} = 3$ mm elements are currently pathological
  - Significant nonlinear emittance growth
  - Due to variation in $(\epsilon, \beta, \alpha)$ as a function of p_z?

Conclusion

- There is work to do before we can state that the beams are 'matched and suitable for an ionization cooling demonstration'
  - How shall we optimize the quadrupole currents and t?
  - As Marco realized, Monte Carlo is probably required here
  - *How much of this work should be included in the paper?*
- Please contact me if you'd like to use the data or need help running the G4MICE reconstruction application
- Victoria (and I) will continue to take this forward
  - Simulate all the measured muons
  - Optimize the diffuser plate thicknesses and use the new design
  - Optimize the quadrupole and match coil currents
  - Deal with the RF matching