

# In-sensitive Unification of Gauge Couplings

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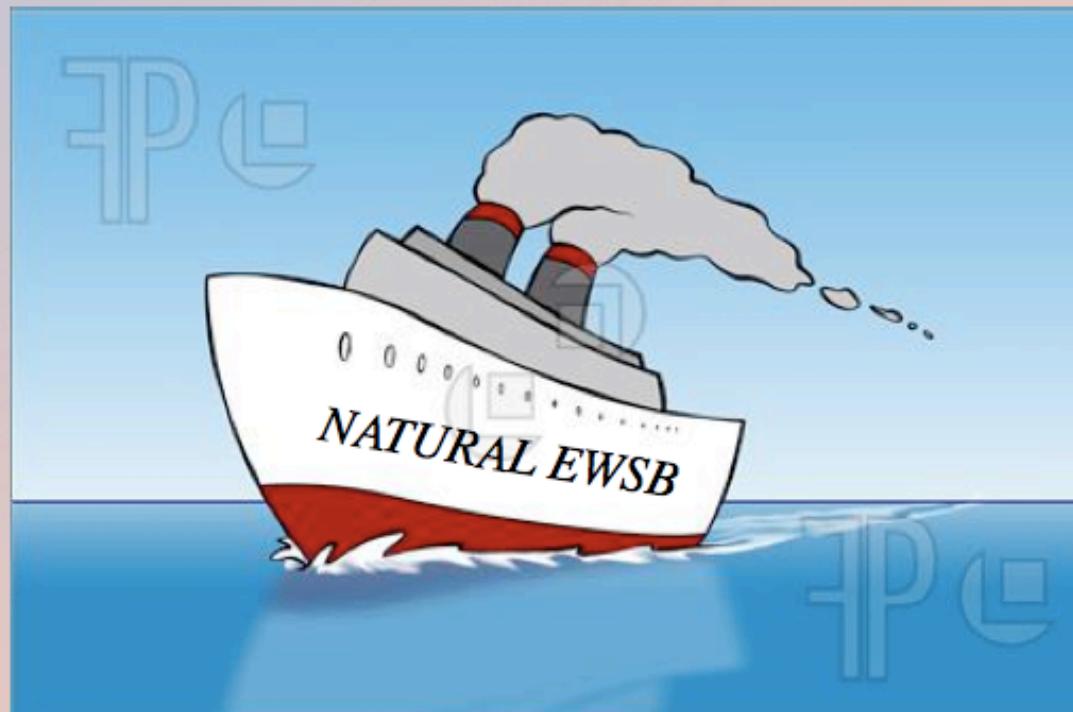
Radovan Dermisek  
*Indiana University, Bloomington*

arXiv:1204.6533 [hep-ph]

**BSM Institute, CERN, June 20, 2012**

# Motivation

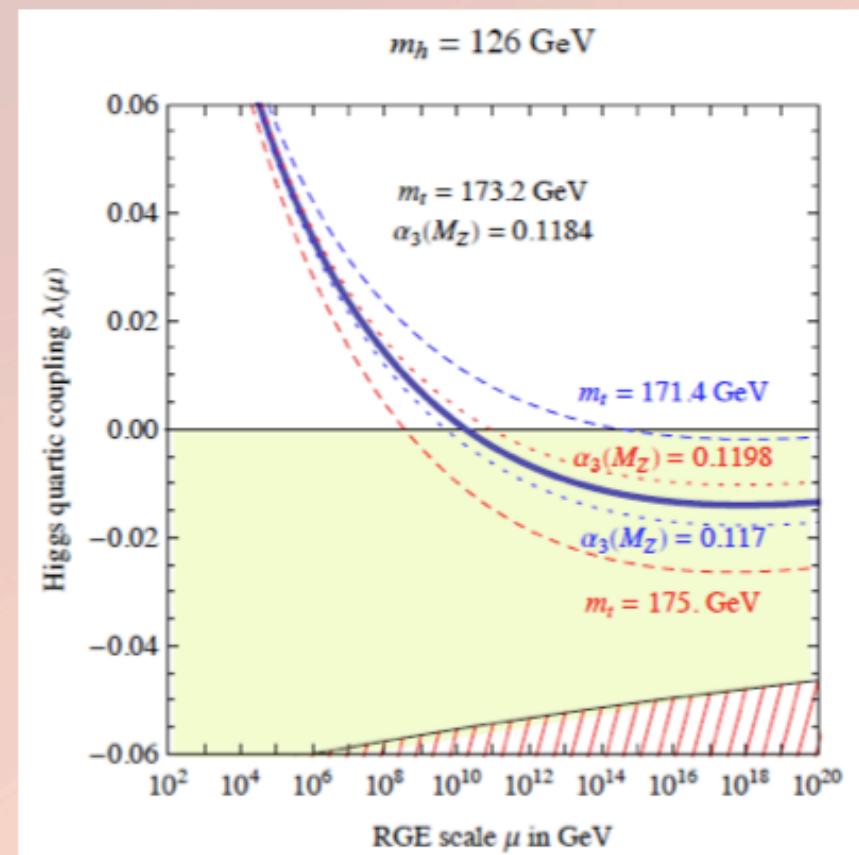
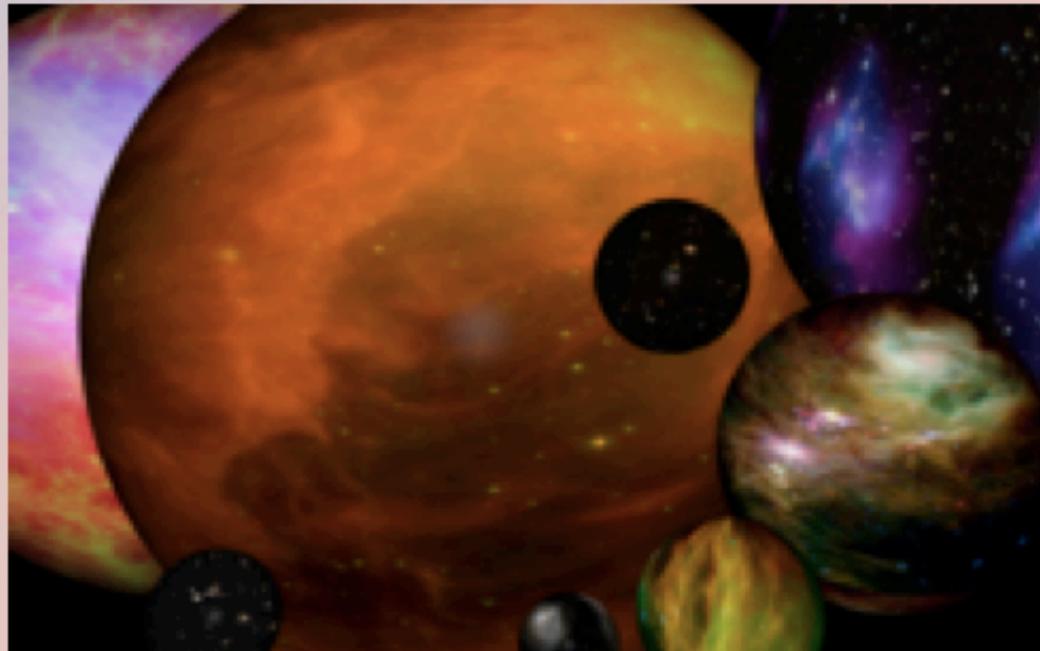
- ◆ Natural EWSB ship has sailed, and SUSY is not on it



# Motivation

- ◆ Natural EWSB ship has sailed, and SUSY is not on it
- ◆ Just the standard model?

anthropic, multiverse, ...



Elias-Miro et al., 1112.3022 [hep-ph]

# Motivation

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- ◆ **Natural EWSB ship has sailed, and SUSY is not on it**
- ◆ **Just the standard model? (possible but not satisfactory)**

**Ignoring the hierarchy problem, what simple extensions of the SM provide more satisfactory picture?**

# Motivation and Outline

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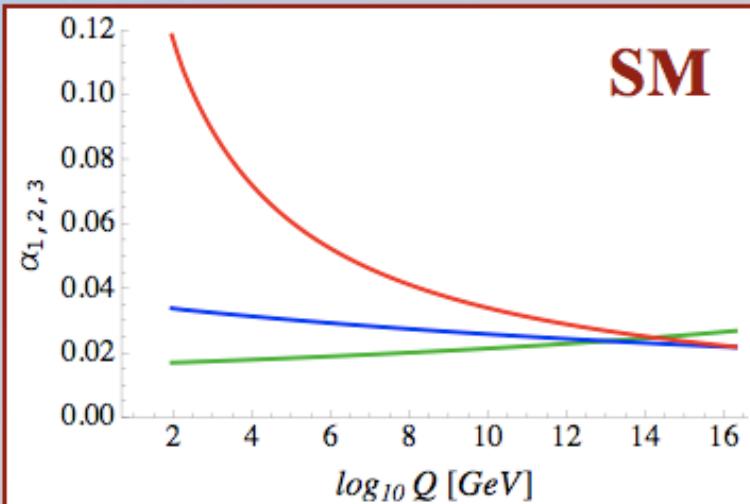
- ◆ Natural EWSB ship has sailed, and SUSY is not on it
- ◆ Just the standard model? (possible but not satisfactory)

Ignoring the hierarchy problem, what simple extensions of the SM provide more satisfactory picture?

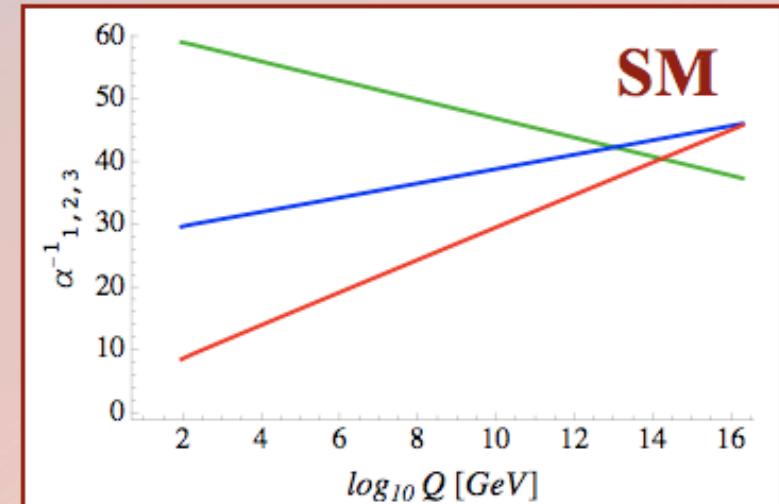
**SM + 3 complete vector-like families (at 1 TeV - 100 TeV):**

- gauge coupling unification  
couplings at the EW scale are highly insensitive to fundamental parameters
- resurrects simple non-supersymmetric GUTs  
easily avoids limits on proton lifetime,  
GUT scale can be identified with string/Planck scale
- the electroweak minimum is stable all the way to the GUT scale

# Gauge couplings in the standard model



$$\begin{aligned}\alpha_3(M_Z)_{exp} &= 0.1184 \\ \alpha_2(M_Z)_{exp} &= 0.03380 \\ \alpha_1(M_Z)_{exp} &= 0.01695 \\ \alpha_{EM}(M_Z) &= 1/127.916 \\ \sin^2 \theta_W &= 0.2313\end{aligned}$$



**RGEs:**

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i \quad t = \ln Q/Q_0$$

$$b_i = (41/10, -19/6, -7)$$

$$b_i = \left( \frac{1}{10} + \frac{4}{3}n_g, -\frac{43}{6} + \frac{4}{3}n_g, -11 + \frac{4}{3}n_g \right)$$

sensitivity

$$\frac{\delta \alpha_3(M_Z)}{\alpha_3(M_Z)} = \frac{\alpha_3(M_Z)}{\alpha_G} \frac{\delta \alpha_G}{\alpha_G}$$

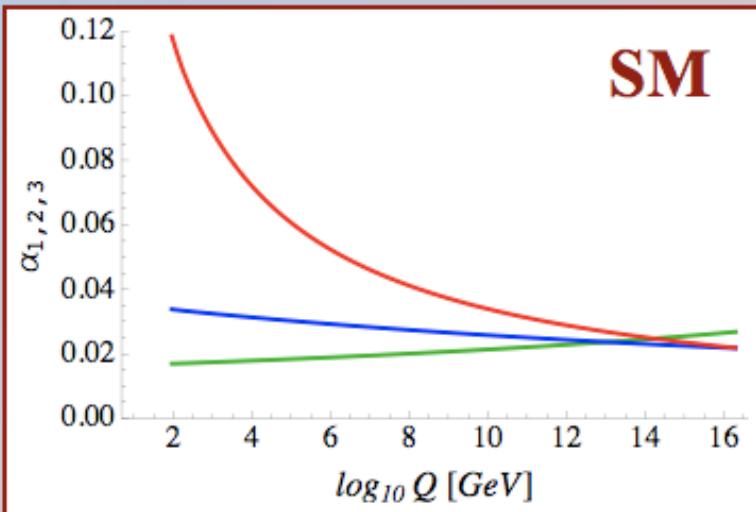
~4

**solution:**

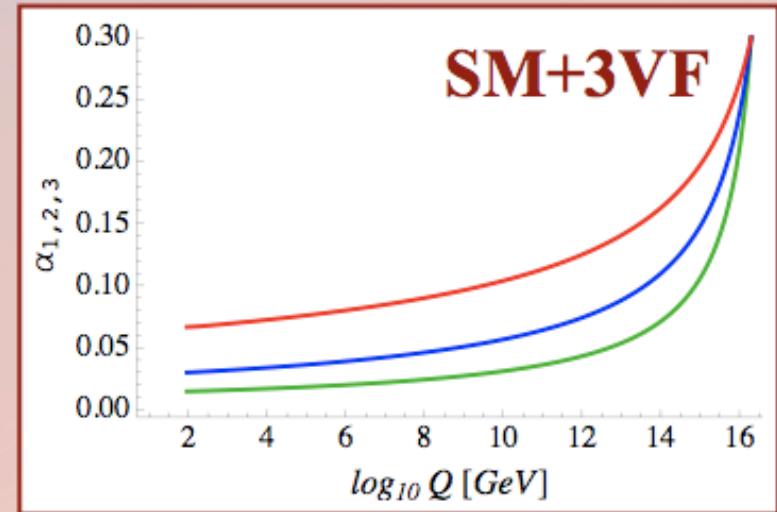
$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \alpha_i^{-1}(M_G)$$

# SM

# SM+3VF

**SM**

$$\begin{aligned}\alpha_3(M_Z)_{exp} &= 0.1184 \\ \alpha_2(M_Z)_{exp} &= 0.03380 \\ \alpha_1(M_Z)_{exp} &= 0.01695 \\ \alpha_{EM}(M_Z) &= 1/127.916 \\ \sin^2 \theta_W &= 0.2313\end{aligned}$$

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$$b_i = (121/10, 29/6, +1)$$

**sensitivity**

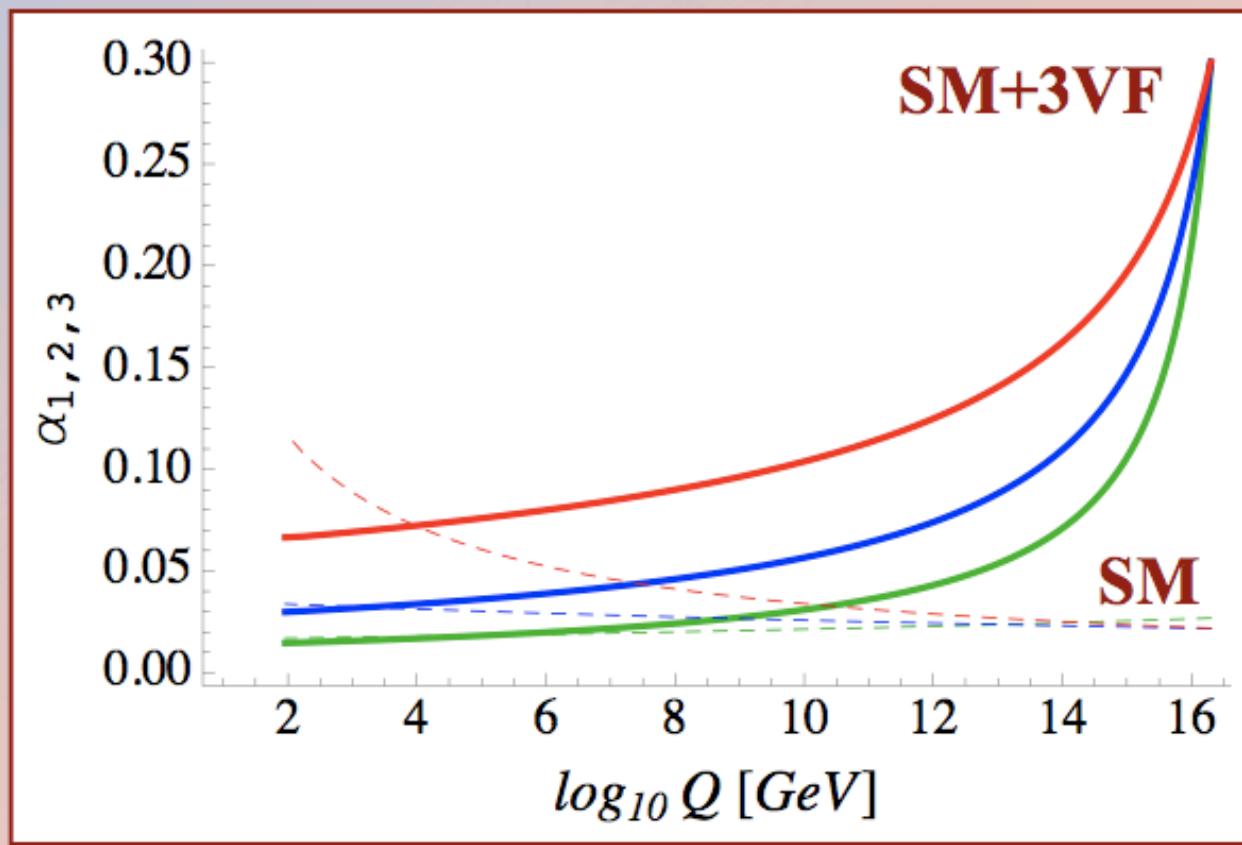
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~4

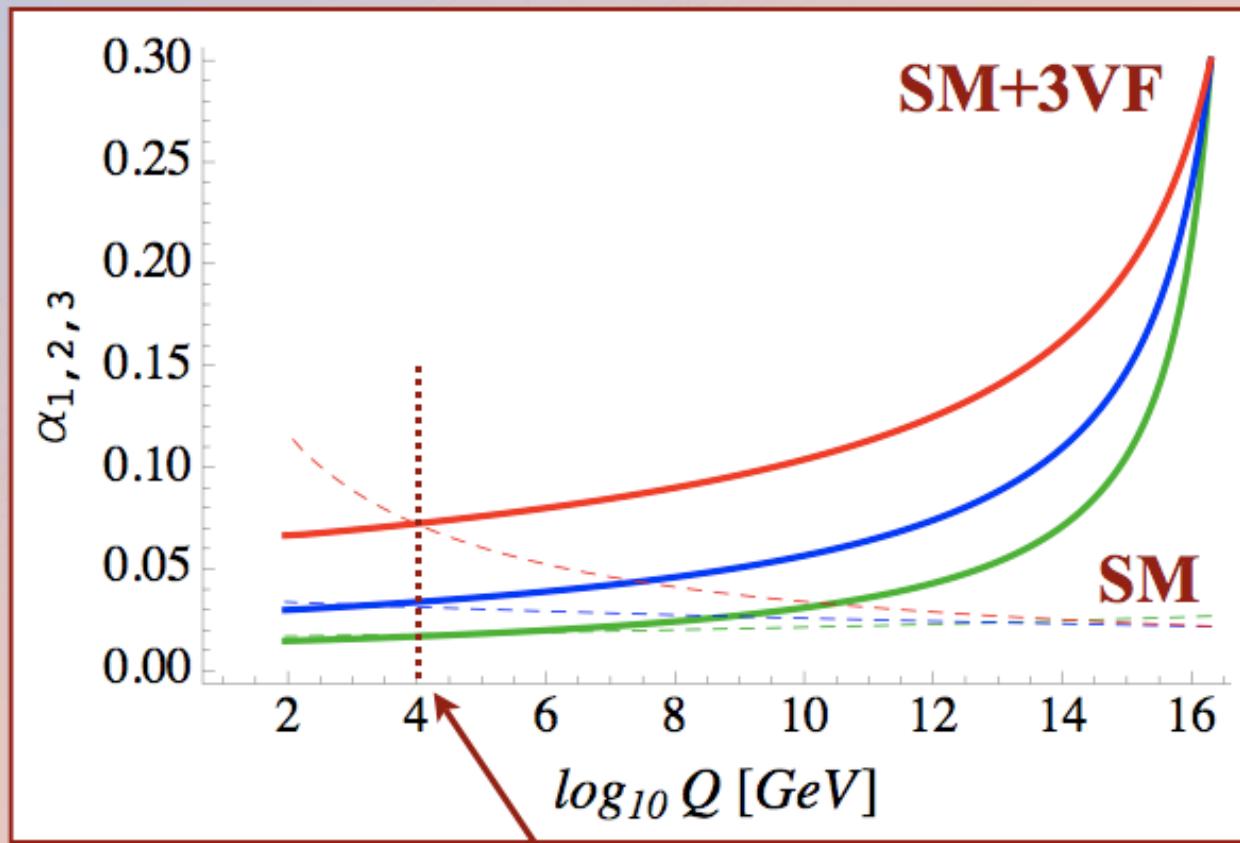
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# SM + 3 vectorlike families

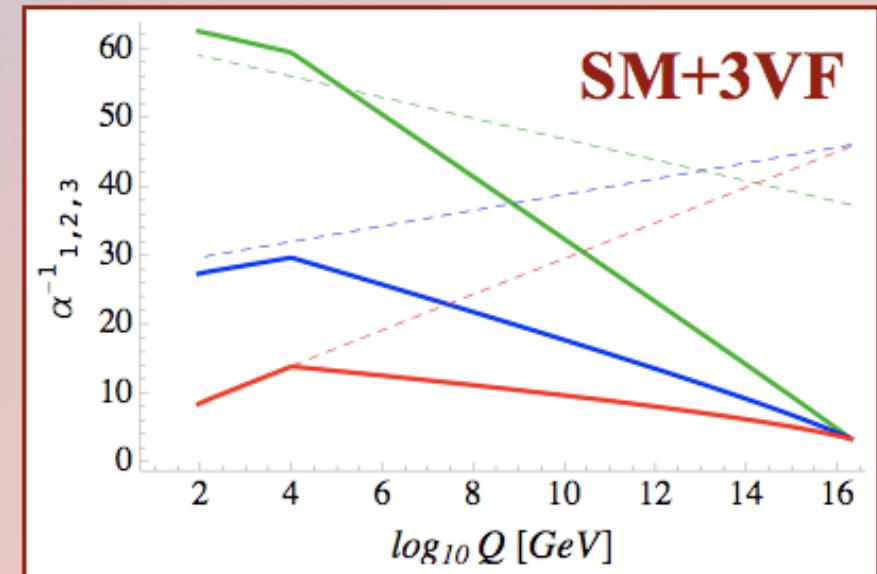
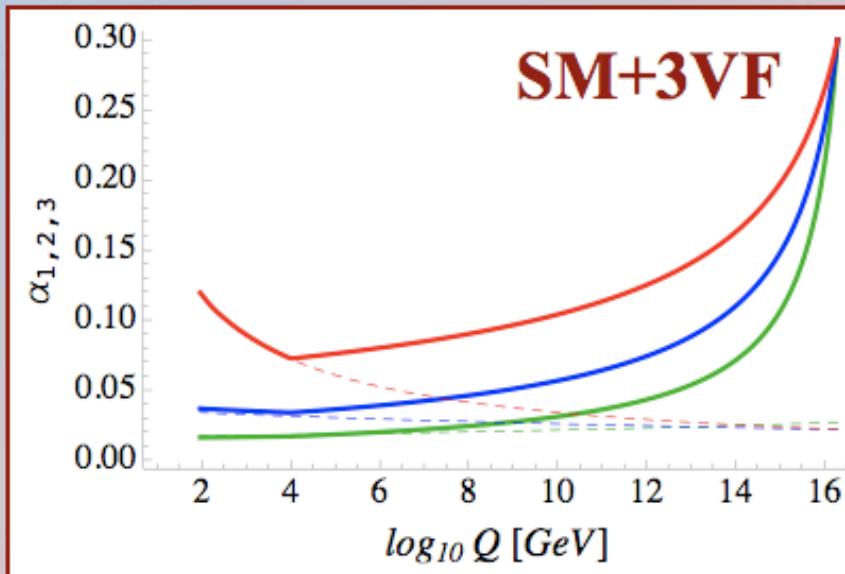


# SM + 3 vectorlike families



Is this a threshold effect?

# SM + 3 vectorlike families at 10 TeV



Exp. values of gauge couplings reproduced within 8%

the only relevant parameters are  $M_G$  and  $M_{VF}$   
predictive, comparable to MSSM unification

# Predictions and Sensitivity

**RGEs:**

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i$$

$$b_i = (121/10, 29/6, +1)$$

**solution at 1-loop (good approximation for  $i=1,2$ ):**

$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \alpha_G^{-1} - T_i$$

$\sim 60, 30$        $\sim 3$        $\sim 6$  (for common mass 10 TeV)

$$T_i = \frac{1}{2\pi} \sum_f b_i^f \ln \frac{M_f}{M_Z}$$

**neglecting threshold effect and  $\alpha_G$ :**

$$\frac{\alpha_i(M_Z)}{\alpha_j(M_Z)} \simeq \frac{b_j}{b_i}$$

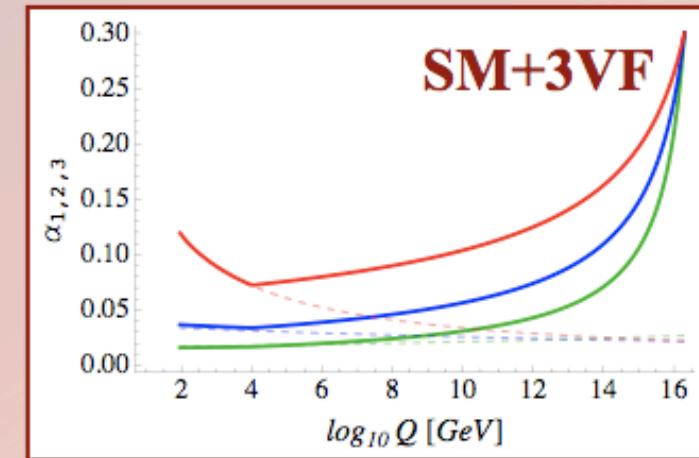
$$\alpha' = \frac{3}{5}\alpha_1, \quad b' = \frac{5}{3}b_1$$

**Parameter free prediction:**

$$\sin^2 \theta_W \equiv \frac{\alpha'}{\alpha_2 + \alpha'} = \frac{b_2}{b_2 + b'} = 0.193$$

$$\alpha_{EM} = \alpha_2 \sin^2 \theta_W$$

Maiani, Parisi, and Petronzio (1978)



# Predictions and Sensitivity

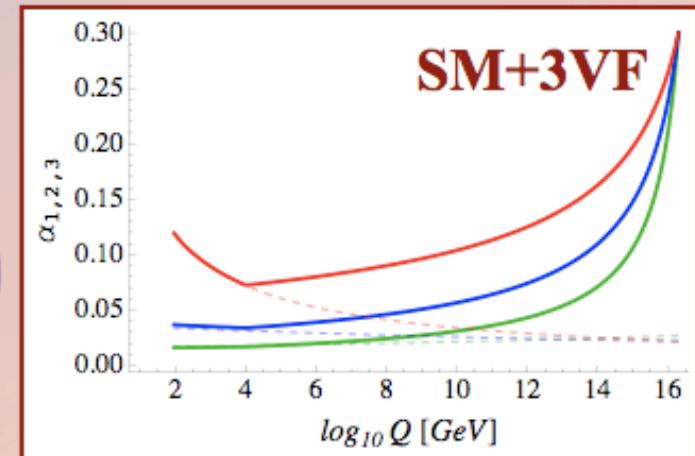
**RGEs:**

$$\frac{d\alpha_i}{dt} = \beta(\alpha_i) = \frac{\alpha_i^2}{2\pi} b_i + \frac{\alpha_i^3}{8\pi^2} B_i + \dots$$

$$b_i = (121/10, 29/6, +1)$$

$$B_3 = -102 + (76/3)n_g = 126$$

**neglecting 1-loop (good approximation for i=3):**



$$\alpha_3^{-1}(M_Z) \simeq \sqrt{\frac{B_3}{4\pi^2} \ln \frac{M_G}{M_Z} + \alpha_G^{-2}} - T_i$$

~8      ~100      ~9      ~6 (for common mass 10 TeV)

$$T_i = \frac{1}{2\pi} \sum_f b_i^f \ln \frac{M_f}{M_Z}$$

**Parameter free prediction:**  
**(neglecting threshold effect and  $\alpha_G$ )**

$$\frac{\alpha_3^2(M_Z)}{\alpha_{EM}(M_Z)} \simeq 2\pi \frac{b_2 + b'}{B_3}$$

**predicts  $\alpha_3 = 0.099$**

1-loop contribution can be added:

$$\alpha_3(M_Z) \rightarrow \frac{\alpha_3(M_Z)}{1 + \frac{1}{3} \frac{\epsilon}{\alpha_3(M_Z)} - \frac{1}{12} \left( \frac{\epsilon}{\alpha_3(M_Z)} \right)^2 + \dots}$$

$\epsilon = 4\pi b_3/B_3$

**predicts  $\alpha_3 = 0.073$**

# Sensitivity to $M_G$ , $\alpha_G$ , and $M_{VF}$

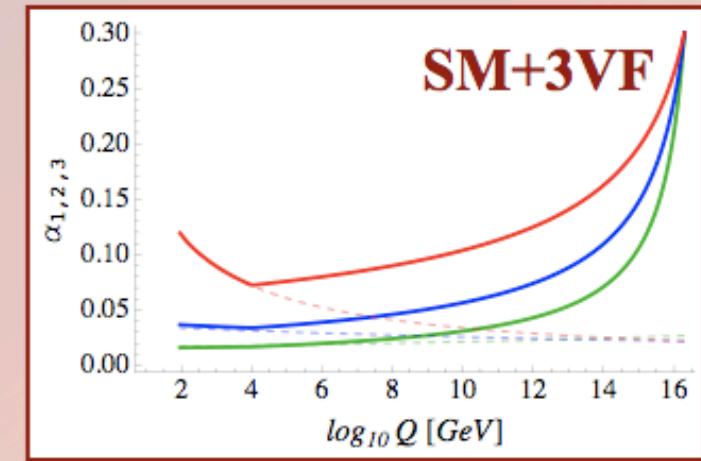
approximate solutions:

$$\alpha_i^{-1}(M_Z) = \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \alpha_G^{-1} - T_i \quad (i=1,2)$$

$\sim 60, 30$        $\sim 3$        $\sim 6$

$$\alpha_3^{-1}(M_Z) \simeq \sqrt{\frac{B_3}{4\pi^2} \ln \frac{M_G}{M_Z} + \alpha_G^{-2}} - T_i$$

$\sim 8$        $\sim 100$        $\sim 9$        $\sim 6$   
 $\sim 14$



$$T_i = \frac{1}{2\pi} \sum_f b_i^f \ln \frac{M_f}{M_Z}$$

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Changing any of the fundamental parameters by a factor of 2 does not modify predicted values of gauge couplings by more than ~10%.

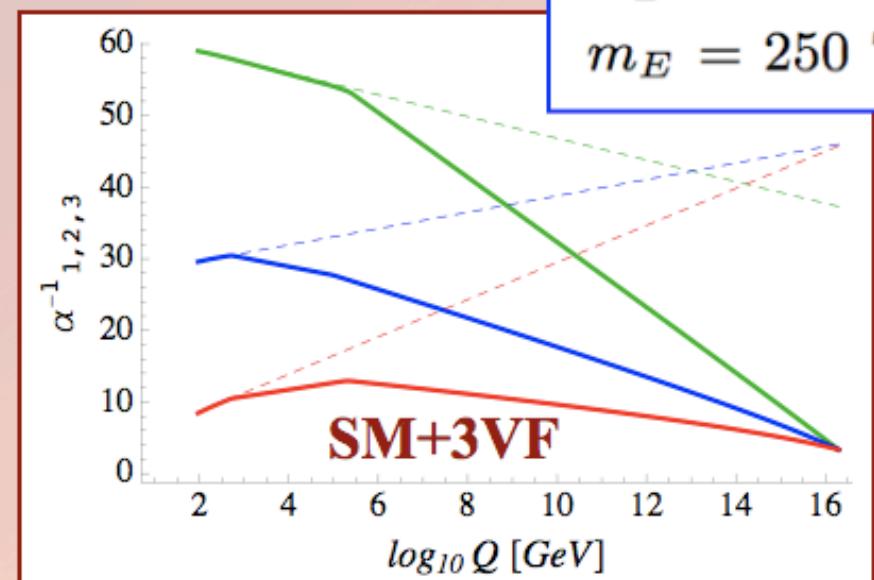
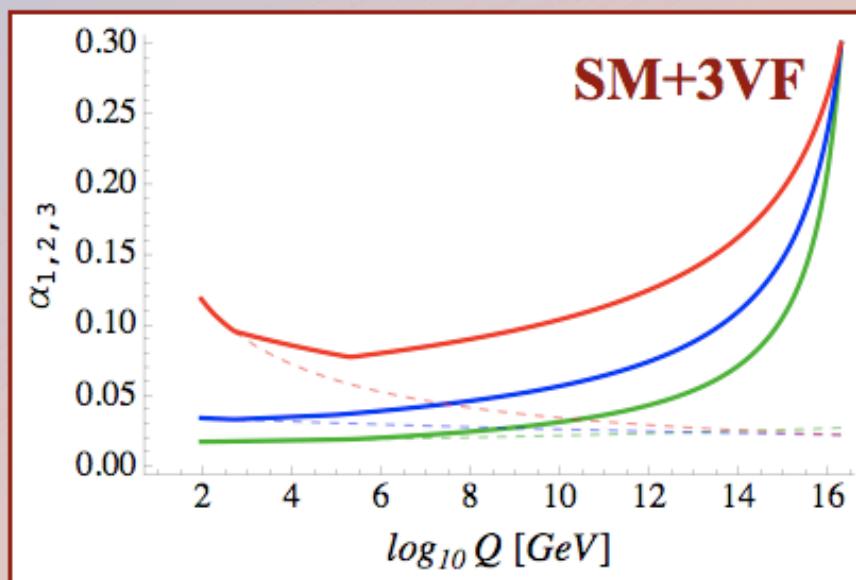
# Realistic example

$$\begin{aligned}\alpha_3(M_Z)_{exp} &= 0.1184 \\ \alpha_2(M_Z)_{exp} &= 0.03380 \\ \alpha_1(M_Z)_{exp} &= 0.01695 \\ \alpha_{EM}(M_Z) &= 1/127.916 \\ \sin^2 \theta_W &= 0.2313\end{aligned}$$

Gauge couplings reproduced (within fractions of exp. uncertainties) for :

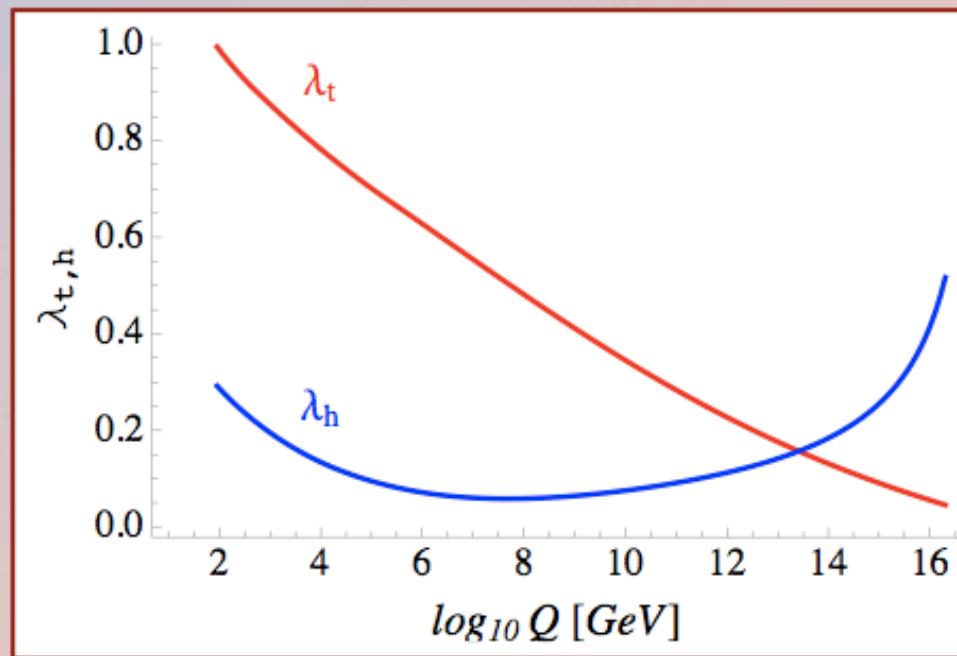
4 sig. figures

$$\begin{aligned}m_Q &= 500 \text{ GeV} \\ m_L &= 95 \text{ TeV} \\ m_U &= 220 \text{ TeV} \\ m_D &= 180 \text{ TeV} \\ m_E &= 250 \text{ TeV}\end{aligned}$$



Many possible solutions!

# Top Yukawa and Higgs quartic couplings



$m_H = 125 \text{ GeV}$

- reduced sensitivity of EW observables to GUT scale values of top Yukawa and Higgs quartic couplings  
(different textures for fermion masses compared to usual GUTs)
- Electroweak minimum is stable!

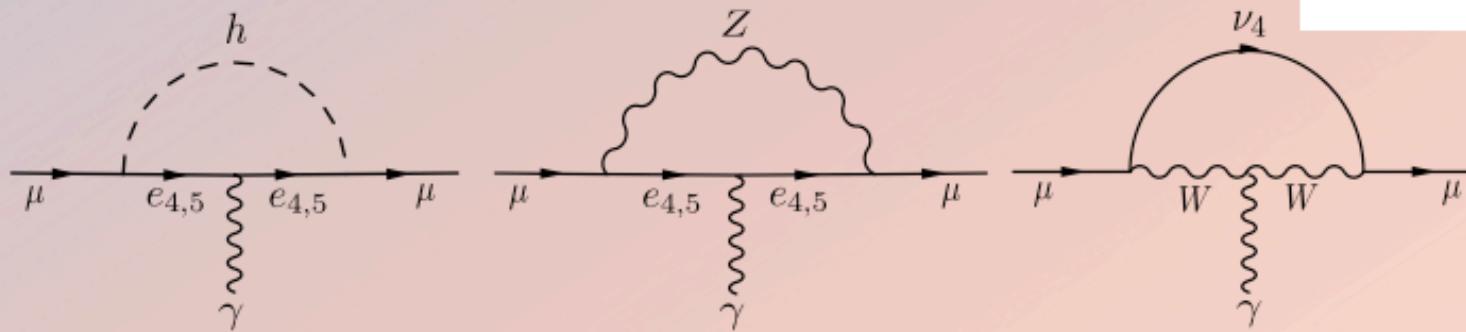
# What else are extra VFs good for?

## ◆ muon g-2

K. Kannike, M. Raidal, D.M. Straub and A. Strumia, 1111.2551 [hep-ph]  
R.D. and A. Raval, in progress

$$\mathcal{L} \supset -\bar{l}_{Li}y_{ij}e_{Rj}H - \bar{l}_{Li}\lambda_i^E E_R H - \bar{L}_L\lambda_j^L e_{Rj}H - \lambda\bar{L}_L E_R H - \bar{\lambda}H^\dagger\bar{E}_L L_R \\ - \mu_L\bar{L}_L L_R - \mu_E\bar{E}_L E_R + h.c.,$$

$$(\bar{e}_{Li}, \bar{L}_L^-, \bar{E}_L) \begin{pmatrix} y_{ij}v & 0 & \lambda_i^E v \\ \lambda_j^L v & \mu_L & \lambda v \\ 0 & \bar{\lambda}v & \mu_E \end{pmatrix} \begin{pmatrix} e_{Rj} \\ L_R^- \\ E_R \end{pmatrix}$$



## ◆ anomalies in Z-pole observables $A_{FB}^b$ and $A_e$

D. Choudhury, T.M.P. Tait and C.E.M. Wagner, hep-ph/0109097  
R.D., S.G. Kim and A. Raval, 1105.0773 [hep-ph], 1201.0315 [hep-ph]

# Conclusions

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- ◆ **3 (or more) pairs of vectorlike families allow for insensitive unification of gauge couplings**  
predictive, comparable to SUSY unification
- ◆ **resurrects simple non-supersymmetric GUTs (proton decay)**  
the GUT scale is adjustable and could be identified with the string or Planck scale
- ◆ **the electroweak minimum is stable all the way to the GUT scale**
- ◆ **some of the extra fermions might be within the reach of the LHC**  
and modify phenomenology of the SM:  
small flavor violation from mixing through Yukawa couplings, contributions in loops, ...