Thorne–Roberts GM-VFNS for DIS structure functions

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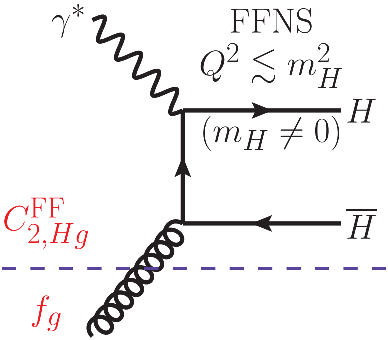
Outline of talk

Disclaimer

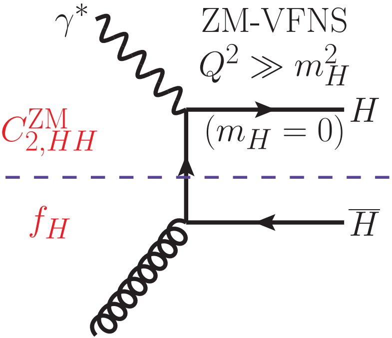
- This is a review talk by a non-expert mostly based on original work by Robert Thorne.
- See in particular:
  R. S. Thorne, "*The Effect of Changes of Variable Flavour Number Scheme on PDFs and Predicted Cross Sections*", arXiv:1201.6180 [extended version of PoS DIS **2010** (2010) 053, arXiv:1006.5925].

Heavy-quark ($H = c, b$) contribution to F_2



Fixed **f**lavour **n**umber **s**cheme

- No heavy-quark PDF.
- Includes $\mathcal{O}(m_H^2/Q^2)$ terms.
- No resummation of $\alpha_S \ln(Q^2/m_H^2)$ terms.



Zero-**m**ass **v**ariable **f**lavour **n**umber **s**cheme

- Use heavy-quark PDF.
- Mass dependence neglected.
- Resums $\alpha_S \ln(Q^2/m_H^2)$ terms similar to light quarks.

General-mass variable flavour number scheme (GM-VFNS)

See review by R. S. Thorne and W.-K. Tung [arXiv:0809.0714].

- Interpolate between two well-defined regions: FFNS for $Q^2 \leq m_H^2$, ZM-VFNS for $Q^2 \gg m_H^2$.
- Define by demanding all-orders equivalence of the $n_f = n$ (FFNS) and $n_f = n+1$ (VFNS) flavour descriptions for $Q^2 > m_H^2$:

$$F_i(x,Q^2) = \sum_k C_{i,k}^{\mathrm{FF},n}(Q^2/m_H^2) \otimes f_k^n(Q^2)$$

$$= \sum_j C_{i,j}^{\mathrm{VF},n+1}(Q^2/m_H^2) \otimes f_j^{n+1}(Q^2)$$

$$\equiv \sum_{j,k} C_{i,j}^{\mathrm{VF},n+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2) \otimes f_k^n(Q^2)$$

$$\Rightarrow \quad C_{i,k}^{\mathrm{FF},n}(Q^2/m_H^2) = \sum_j C_{i,j}^{\mathrm{VF},n+1}(Q^2/m_H^2) \otimes A_{jk}(Q^2/m_H^2)$$

- But $C_{i,j}^{\mathrm{VF},n_f,(m)}$ is only uniquely defined in massless limit at each perturbative order "m" $\Rightarrow$ ambiguous up to $\mathcal{O}(m_H^2/Q^2)$ terms (can redistribute between different perturbative orders).

Choice of GM-VFNS in MRST/MSTW global fits

- MRST 1998–2004 used the Thorne–Roberts (TR) scheme [hep-ph/9709442]: demand $\partial F_2^H/\partial \ln Q^2$ continuous at $Q^2 = m_H^2$.
- PDFs are discontinuous at NNLO in a VFNS [Buza *et al.* '96]:

$$f_j^{n_f+1}(m_H^2) = f_j^{n_f}(m_H^2) + \alpha_S^2 \sum_k A_{jk}^{(2)}(1) \otimes f_k^{n_f}(m_H^2),$$

  but neglected in MRST 2001–2004 NNLO analyses.
- Structure functions at NNLO are then discontinuous at $Q^2 = m_H^2$ in ZM-VFNS, but should be continuous in GM-VFNS.
- Original TR scheme technically difficult to implement at NNLO.
- Instead, R. S. Thorne [hep-ph/0601245] redefined simpler GM-VFNS (denoted TR′) for use up to NNLO. Adopted elements of "ACOT(χ)" [Tung, Kretzer, Schmidt, hep-ph/0110247]:

$$C_{2,HH}^{\mathrm{VF},n_f,(m)}(z, Q^2/m_H^2) = C_{2,HH}^{\mathrm{ZM},n_f,(m)}(z/x_{\mathrm{max}}), \quad x_{\mathrm{max}} \equiv \frac{Q^2}{Q^2 + 4m_H^2}$$

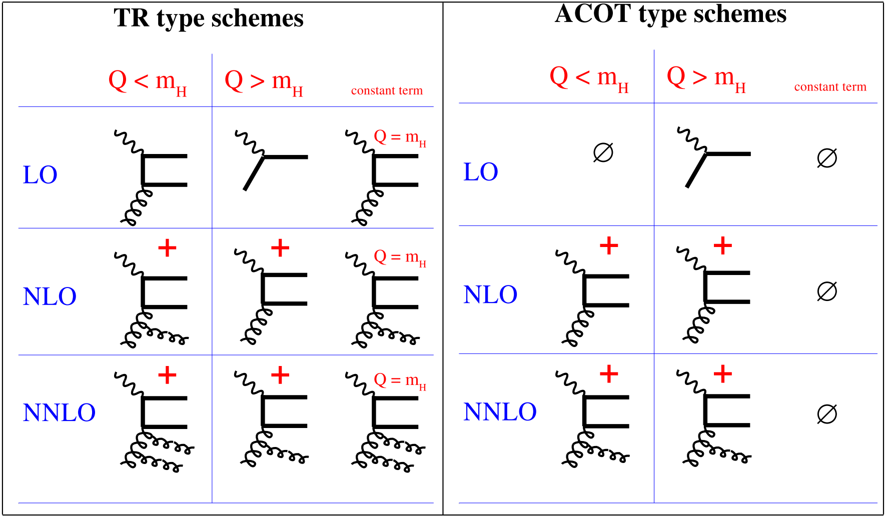
Ordering of the perturbative expansion for F_2^H (TR, TR′)

n-flavour, $Q^2 < m_H^2$	
LO :	$\frac{\alpha_S}{4\pi} C_{2,Hg}^{\mathrm{FF},n,(1)} \otimes g^n$
NLO :	$\left(\frac{\alpha_S}{4\pi}\right)^2 \left(C_{2,Hg}^{\mathrm{FF},n,(2)} \otimes g^n + C_{2,Hq}^{\mathrm{FF},n,(2)} \otimes \Sigma^n \right)$
NNLO :	$\left(\frac{\alpha_S}{4\pi}\right)^3 \sum_j C_{2,Hj}^{\mathrm{FF},n,(3)} \otimes f_j^n$

$(n+1)$-flavour, $Q^2 > m_H^2$	
LO :	$C_{2,HH}^{\mathrm{VF},n+1,(0)} \otimes (H + \overline{H})$
NLO :	$\frac{\alpha_S}{4\pi} \left(C_{2,HH}^{\mathrm{VF},n+1,(1)} \otimes (H + \overline{H}) + C_{2,Hg}^{\mathrm{VF},n+1,(1)} \otimes g^{n+1} \right)$
NNLO :	$\left(\frac{\alpha_S}{4\pi}\right)^2 \sum_j C_{2,Hj}^{\mathrm{VF},n+1,(2)} \otimes f_j^{n+1}$

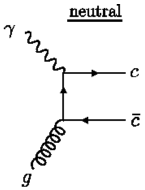
- Maintain continuity at $Q^2 = m_H^2$ by freezing term with highest power of α_S for $Q^2 < m_H^2$ when moving above m_H^2.
- ACOT-type schemes instead use same order of α_S for n- and $(n+1)$-flavours, e.g. $F_2^H = 0$ at LO for $Q^2 < m_H^2$.

Perturbative expansion for F_2^H (TR/TR′ versus ACOT)



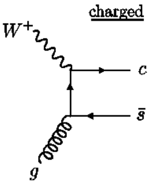
- Figure from F. Olness and I. Schienbein [arXiv:0812.3371].

Modelling of higher-order massive DIS coefficient functions



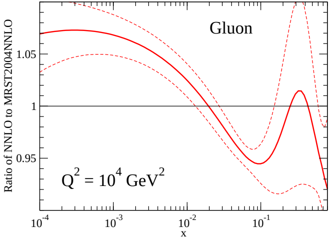
- Massive $\mathcal{O}(\alpha_S^2)$ NC known [Laenen *et al.* '93].
- Massive $\mathcal{O}(\alpha_S^3)$ NC unknown [Blümlein *et al.*, in progress], but needed for GM-VFNS at NNLO.
- Model [Thorne '06] using known leading threshold logarithms [Laenen, Moch '99] and leading $\ln(1/x)$ terms [Catani, Ciafaloni, Hautmann '91]. Variation in free parameters does not lead to a large change.
- Improvement possible with more threshold logs now derived [Lo Presti *et al.*, arXiv:1008.0951].

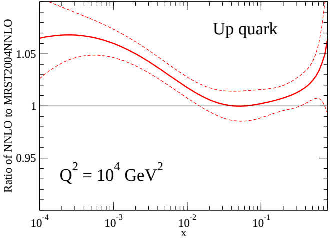
- Massive $\mathcal{O}(\alpha_S^2)$ CC coefficient functions unknown, but needed for GM-VFNS at NLO.
- Model by modifying $\mathcal{O}(\alpha_S^2)$ NC contributions for different threshold behaviour. No attempt made to model $\mathcal{O}(\alpha_S^3)$ CC contribution needed at NNLO.



Impact of consistent GM-VFNS at NNLO in MRST fits

Ratio 2006 to 2004:





MRST 2006 NNLO [arXiv:0706.0459]

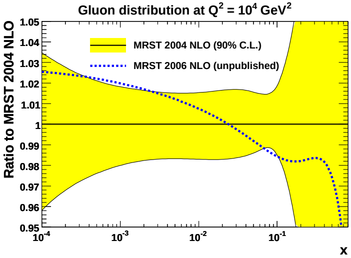
- First implementation of TR′ scheme.
- Increase in low-x PDFs when discontinuities included.
- $\sigma_{W,Z}$ at 14 TeV LHC sensitive to light sea-quark PDFs at $x \sim 0.006$ $\Rightarrow$ 6% increase in $\sigma_{W,Z}$ at LHC.

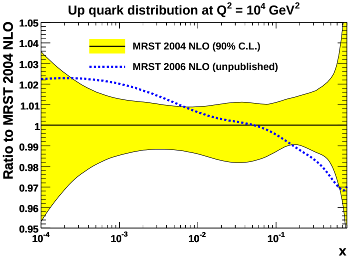
- From CTEQ6.1 NLO (ZM-VFNS) to CTEQ6.5 NLO (GM-VFNS): $\Rightarrow$ 8% increase in $\sigma_{W,Z}$ at LHC.
- From NNPDF2.0 NLO (ZM-VFNS) to NNPDF2.1 NLO (GM-VFNS): $\Rightarrow$ 3% increase in $\sigma_{W,Z}$ at LHC.

A **correction**, not an *uncertainty*.

Scheme dependence of GM-VFNS at NLO in MRST fits

- Change TR → TR′ allowed study of scheme dependence at NLO.
- MRST 2004 (TR) and MRST 2006 (TR′) fits used same data.





- Nearly 3% increase in $\sigma_{W,Z}$ at 14 TeV LHC at NLO. Genuine theory uncertainty: should decrease going to higher orders.
- An **uncertainty**, not a *correction* at NLO.

Variants of the TR′ GM-VFNS [R. S. Thorne, arXiv:1201.6180]

TR′ standard (MSTW08)

- Frozen term ($Q^2 = m_H^2$):

$$\left(\frac{\alpha_S}{4\pi}\right)^m \sum_j C_{2,Hj}^{\mathrm{FF},n_f,(m)}(1) \otimes f_j^{n_f}(m_H^2)$$

- Coefficient function:

$$C_{2,HH}^{\mathrm{VF},n_f,(0)} \propto \delta(z - x_{\max})$$

- Threshold ($W^2 \geq 4m_H^2$):

$$x_{\max} = \left(1 + \frac{4m_H^2}{Q^2}\right)^{-1}$$

- Frozen term **persists** even at asymptotic Q^2.
- Necessary for original TR, but not for TR′.

TR′ variations (4 parameters)

- Allow power-suppression ("a"):

$$\left(\frac{m_H^2}{Q^2}\right)^a \left(\frac{\alpha_S}{4\pi}\right)^m \sum_j C_{2,Hj}^{\mathrm{FF},n_f,(m)}(1) \otimes f_j^{n_f}(m_H^2)$$

- Modify with extra factor ("b", "c"):

$$C_{2,HH}^{\mathrm{VF},n_f,(0)} \propto \left[1 + b\left(\frac{m_H^2}{Q^2}\right)^c\right] \delta(z - x_{\max})$$

- Modify argument of δ-function ("d"):

$$x_{\max} \to \left\{1 + \left[x\left(1 + \frac{4m_H^2}{Q^2}\right)\right]^d \frac{4m_H^2}{Q^2}\right\}^{-1}$$

cf. intermediate-mass (IM) scheme of Nadolsky–Tung [arXiv:0903.2667].

Parameter values of TR′ variations [R. S. Thorne, arXiv:1201.6180]

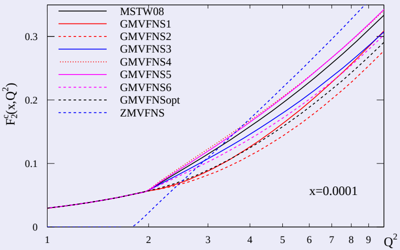
- Perform global fits using same procedure and data sets as MSTW 2008 [arXiv:0901.0002], e.g. uncombined HERA data.

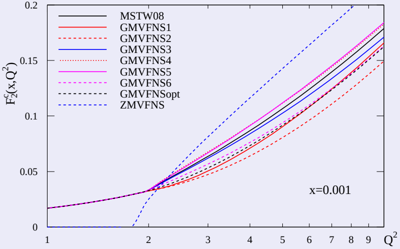
TR′ variant	a	b	c	d
Standard	0	0	0	0
GM-VFNS1	0	-1	1	0
GM-VFNS2	0	-1	0.5	0
GM-VFNS3	1	0	0	0
GM-VFNS4	0	0.3	1	0
GM-VFNS5	0	0	0	0.1
GM-VFNS6	0	0	0	-0.2
Optimal	1	-2/3	1	0

- Parameter values chosen based on sensible behaviour and NLO fit quality ($\Delta\chi^2_{\rm global} \lesssim 20$ and $\Delta\chi^2_{F_2^{\rm charm}} \lesssim 7$).
- "**Optimal**" is smooth at transition ($Q^2 = m_H^2$), reduces to exact ZM-VFNS as $Q^2/m_H^2 \to \infty$, and improves NLO fit quality ($\Delta\chi^2_{\rm global} = -23$ and $\chi^2_{F_2^{\rm charm}} = 107 \to 93$ for 83 points).
- Same parameter values used to study TR′ variations at NNLO.

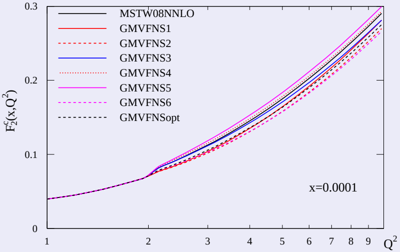
Impact of TR′ variations on F_2^{charm} [R. S. Thorne, arXiv:1201.6180]

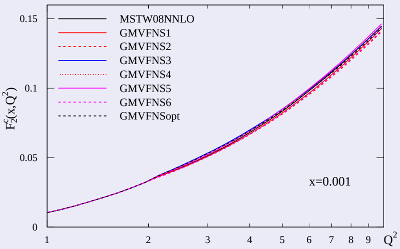
F_2^{charm} at NLO (fixed PDFs)





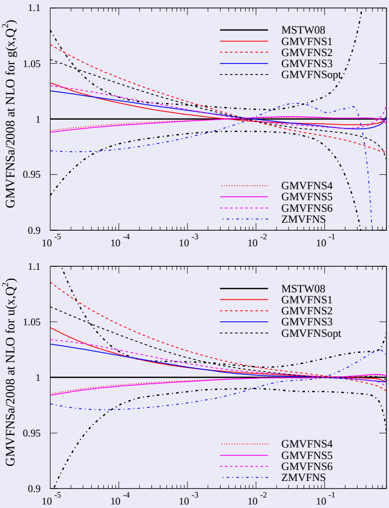
F_2^{charm} at NNLO (fixed PDFs)



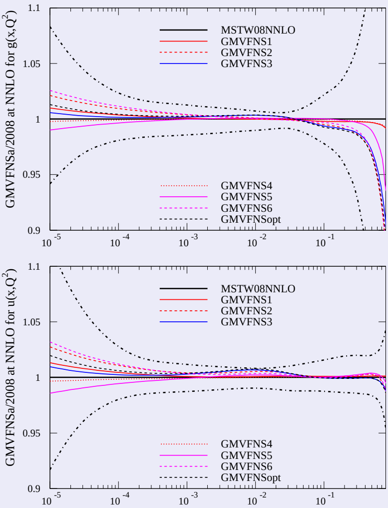


Impact of TR′ variations on PDFs [R. S. Thorne, arXiv:1201.6180]

Effect on g and u at NLO



Effect on g and u at NNLO



Impact of TR′ variations on $\sigma^{\text{NLO}}_{Z,\text{Higgs}}$ [R. S. Thorne, arXiv:1201.6180]

- Predicted cross sections at **NLO** for Z and a 120 GeV Higgs boson at the Tevatron and LHC ($\sqrt{s} = 7$ TeV and 14 TeV):

NLO PDF set	Tevatron		LHC	(7 TeV)	LHC	(14 TeV)
	σ_Z (nb)	σ_{Higgs}(pb)	σ_Z (nb)	σ_{Higgs}(pb)	σ_Z (nb)	σ_{Higgs}(pb)
MSTW2008	7.207	0.7462	27.70	12.41	59.25	40.69
GMvar1	+0.3%	−0.5%	+0.7%	−0.1%	+1.1%	+0.2%
GMvar2	+0.7%	−1.1%	+2.0%	+0.5%	+3.0%	+1.5%
GMvar3	+0.1%	−0.3%	+0.7%	+0.4%	+1.1%	+0.8%
GMvar4	+0.0%	−0.1%	−0.3%	−0.1%	−0.4%	−0.2%
GMvar5	−0.1%	−0.1%	−0.4%	−0.2%	−0.5%	−0.3%
GMvar6	+0.3%	−0.4%	+1.0%	+0.3%	+1.6%	+0.8%
GMvaropt	+0.3%	−1.5%	+1.5%	+0.1%	+2.0%	+0.4%
ZM-VFNS	−0.7%	−1.2%	−1.6%	−1.8%	−3.0%	−3.1%

- Quite significant effect from TR′ GM-VFNS variations at **NLO**, but generally smaller than effect of ZM-VFNS.

Impact of TR′ variations on $\sigma^{\text{NNLO}}_{Z,\text{Higgs}}$ [R. S. Thorne, arXiv:1201.6180]

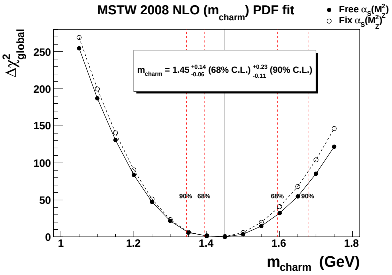
- Predicted cross sections at **NNLO** for Z and a 120 GeV Higgs boson at the Tevatron and LHC ($\sqrt{s} = 7$ TeV and 14 TeV):

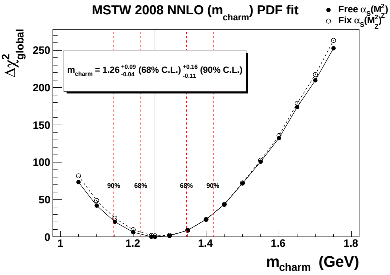
NNLO PDF set	Tevatron		LHC (7 TeV)		LHC (14 TeV)	
	σ_Z (nb)	σ_{Higgs}(pb)	σ_Z (nb)	σ_{Higgs}(pb)	σ_Z (nb)	σ_{Higgs}(pb)
MSTW2008	7.448	0.9550	28.53	15.71	60.93	50.51
GMvar1	+0.1%	−0.5%	+0.1%	−0.3%	+0.1%	−0.2%
GMvar2	+0.3%	−0.8%	+0.2%	+0.1%	+0.5%	+0.1%
GMvar3	+0.4%	−0.1%	+0.4%	+0.6%	+0.5%	+0.7%
GMvar4	+0.0%	−0.2%	−0.1%	−0.1%	+0.1%	−0.1%
GMvar5	+0.1%	−0.3%	−0.1%	−0.2%	−0.2%	−0.2%
GMvar6	+0.1%	−0.9%	+0.0%	−0.5%	+0.3%	−0.2%
GMvaropt	+0.4%	−0.2%	+0.5%	+0.6%	+0.6%	+0.8%

- Much less effect from TR′ GM-VFNS variations at **NNLO**, as expected from a theoretical uncertainty of perturbative origin.

Charm-quark mass dependence using TR′ GM-VFNS

- Default charm-quark pole mass, $m_c = 1.40$ GeV.





- Best-fit values using "standard" TR′ of $m_c = 1.45$ GeV (NLO) and 1.26 GeV (NNLO) [MSTW, arXiv:1007.2624].
- Best-fit values using "optimal" TR′ of $m_c = 1.35$ GeV (NLO) and 1.23 GeV (NNLO) [R. S. Thorne, arXiv:1201.6180].
- cf. NLO best-fit values to combined HERA data (inclusive+$F_2^{\rm charm}$) of $m_c = 1.58$ GeV (standard TR′) and 1.46 GeV (optimal TR′) [H1-prelim-10-143, ZEUS-prel-10-019].

Availability of public code for NC structure functions

http://projects.hepforge.org/mstwpdf/code/code.html

"Corresponding Fortran code for calculating proton (and neutron) structure functions in neutral-current deep-inelastic scattering (only photon exchange), i.e. F_2 and F_L including their heavy quark (c,b) components, is available on request. Note that the previous MRST structure function code, available from HEPDATA, should not be used with the MSTW PDFs due to the different implementation of the general-mass variable flavour number scheme (GM-VFNS).
Update (26th January 2012):
`mstw2008structurefunctions.f` and `example_sf.f` now public."

- `CALL MSTWNC(x,q,ipn,f2,f2c,f2b,fl,flc,flb)`
- Calculates F_2, F_2^c, F_2^b, F_L, F_L^c, F_L^b at LO, NLO, NNLO.
- Corresponds to exact TR′ code used in **MSTW 2008** fits.
- `h1fitter-0.1.0/RT/src/robert_thorne.f` (provided privately by R. S. Thorne) should agree with NLO public code.
- "Standard" NNLO and "optimal" NLO/NNLO also provided privately by R. S. Thorne for a future HERAFitter release.

Limitations of current TR′ implementation in HERAFitter

- Only **photon** contribution to NC DIS structure functions (F_2 and F_L), need to add γZ and Z contributions.
- No standalone TR′ code currently exists for CC DIS, only embedded in fitting code for specific observables included in fit: HERA e^+p, NuTeV/CHORUS νN, CCFR/NuTeV $\nu N \to \mu\mu X$.
- Fixed scale choices $\mu_R^2 = \mu_F^2 = Q^2$ (and flavour matching / transition scale at $\mu^2 = m_H^2$). No possibility for scale variations.
- No target-mass corrections (assume negligible due to data cuts).
- Different variants (e.g. NLO/NNLO or standard/optimal) provided as separate files. Better to have one code with different flags. Non-optimal TR′ variants could also be supplied.
- Convolution integrals done "on the fly" using 96-point Gaussian quadrature: useful to have a "fast" computation *à la* QCDNUM.
- No possibility to study possible "intrinsic" heavy quarks.

Summary: Thorne–Roberts GM-VFNS for DIS

- Choice of GM-VFNS is a source of theoretical uncertainty.
- **Past:** MRST 1998–2004 used TR GM-VFNS [hep-ph/9709442].
- **Present:** MRST 2006 and MSTW 2008 used TR′ [hep-ph/0601245].
- **Future:** recommend "optimal" TR′ GM-VFNS [arXiv:1201.6180].
- "Optimal" TR′ GM-VFNS reduces exactly to FFNS for $Q^2 \leq m_H^2$ and to ZM-VFNS for $Q^2/m_H^2 \to \infty$, with a smooth transition at $Q^2 = m_H^2$. It gives better fit quality at NLO, and less difference between extracted NLO/NNLO $m_c^{\rm pole}$ values.
- GM-VFNS variations can have a significant effect on PDFs and predicted cross sections at **NLO**, but the effect is much reduced (less than 1% for typical cross sections) at **NNLO**.