ACOT Theory Issues & Developments

Heavy Quarks
NNLO & beyond ...

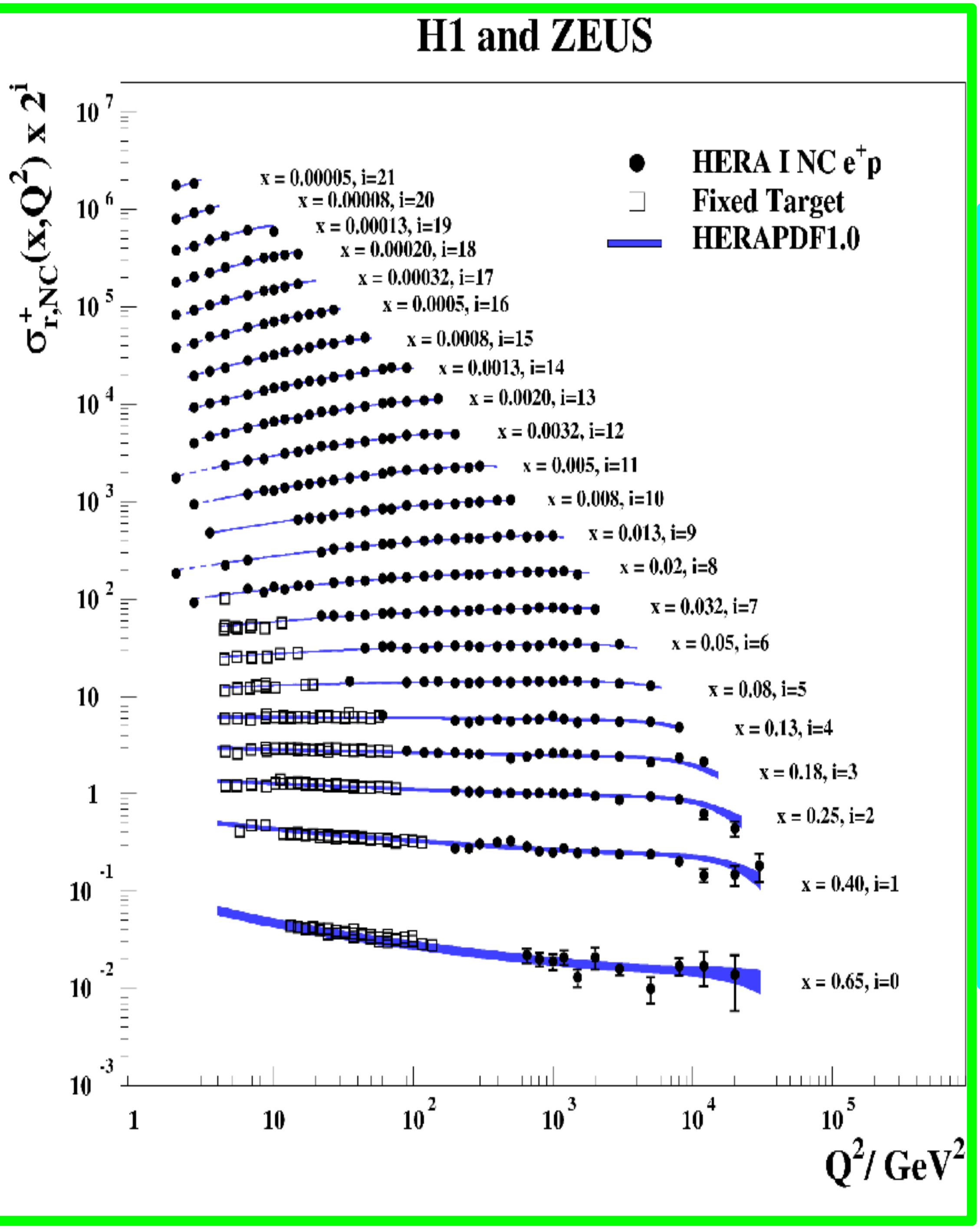
Fred Olness

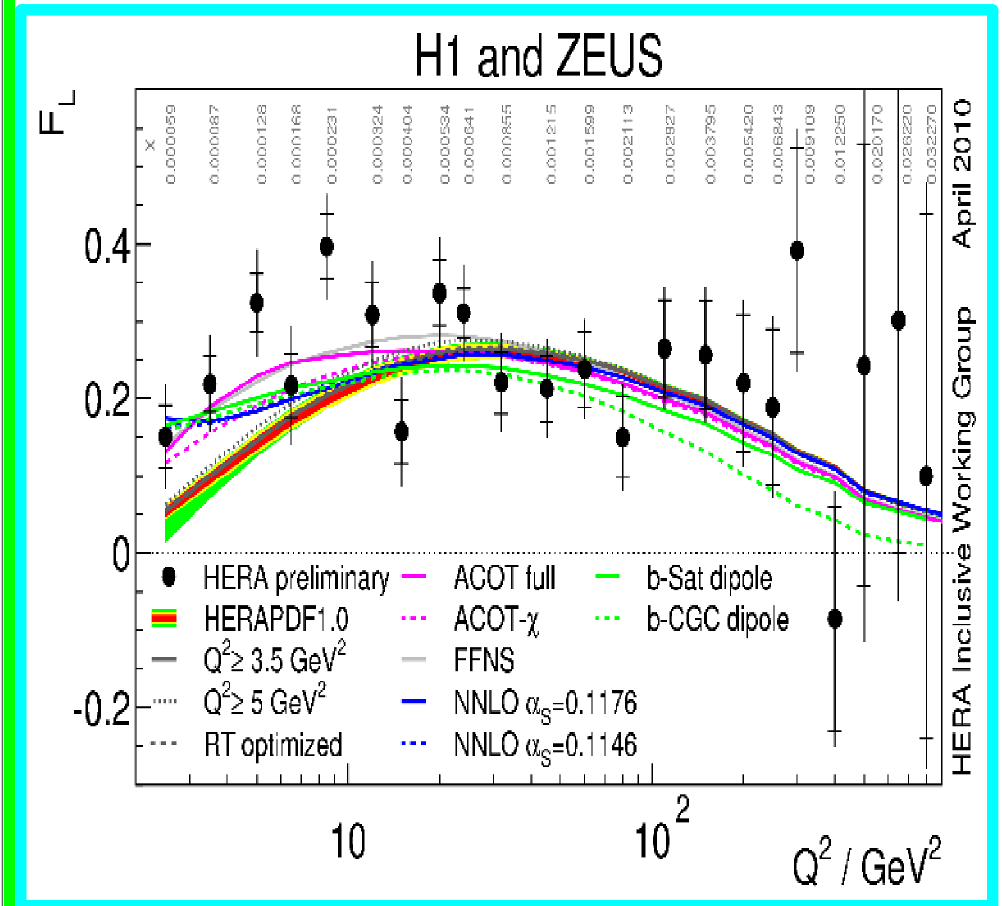
SMU

Conspirators:
T.P. Stavreva, I Schienbein, A. Kusina,
J.-Y. Yu, K. Kovarik, P. Nadolsky, M. Guzzi
J. Owens, J. Morfin, C. Keppel, D. Soper ...

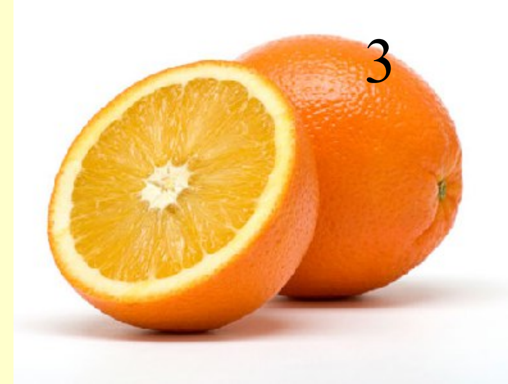
HERA-Fitter
13 February 2012

HERA Measurements: Essential ingredient for PDF analysis





Les Houches Comparative Studies



We have made progress in addressing how to compute heavy quarks.
Recent efforts by many groups

The Cast:

ACOT & S-ACOT Codes
Used in CTEQ4HQ, 5HQ, 6HQ

Aivazis, Collins, Olness, Tung,
Phys.Rev.D50:3102-3118,1994.

S-ACOT
CTEQ 6.5 & 6.6

Tung, Lai, Belyaev, Pumplin, Stump, Yuan,
JHEP 0702:053,2007.
Nadolsky, Tung, Phys.Rev.D79:113014,2009.

Thorne-Roberts (TR')
MSTW Fits

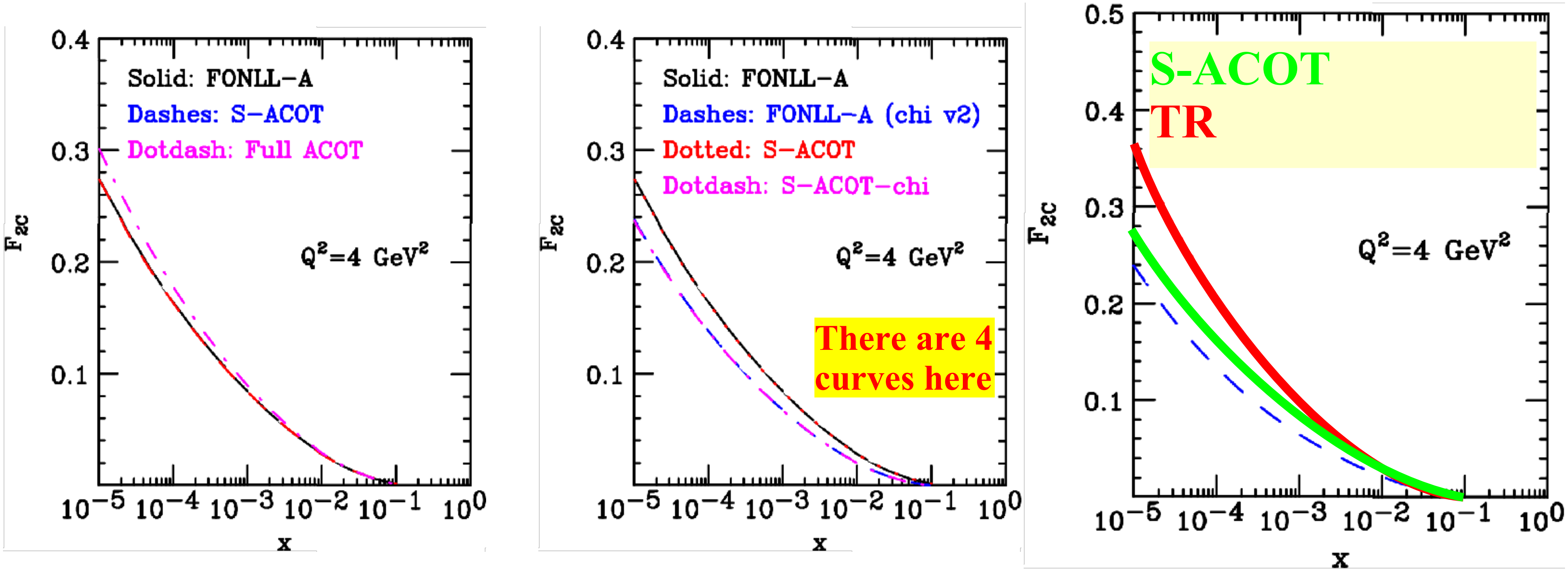
Thorne, Phys.Rev.D73:054019,2006.

FONLL:
Used in NNPDF Fits

Forte, Laenen, Nason, Rojo,
Nucl.Phys.B834:116-162,2010.

ABKM:

Blumlein, Klein, Moch
Phys.Rev.D81:014032,2010



ACOT & S-ACOT essentially identical

... scheme differences are higher order

FONNL & S-ACOT

Numerically similar

MSTW09

We can quantify <u>theoretical</u> scheme diffferences

The SM and NLO Multileg Working Group: Summary report.
J. Rojo, et al., e-Print: arXiv:1003.1241 [hep-ph]

Differences understood
Should reduce at higher order

ACOT

ACOT Features

ACOT: m$\rightarrow$ 0 limit yields MS-Bar

with no finite renormalization

Based on the Collins-Wilczek-Zee (CWZ) Renormalization Scheme

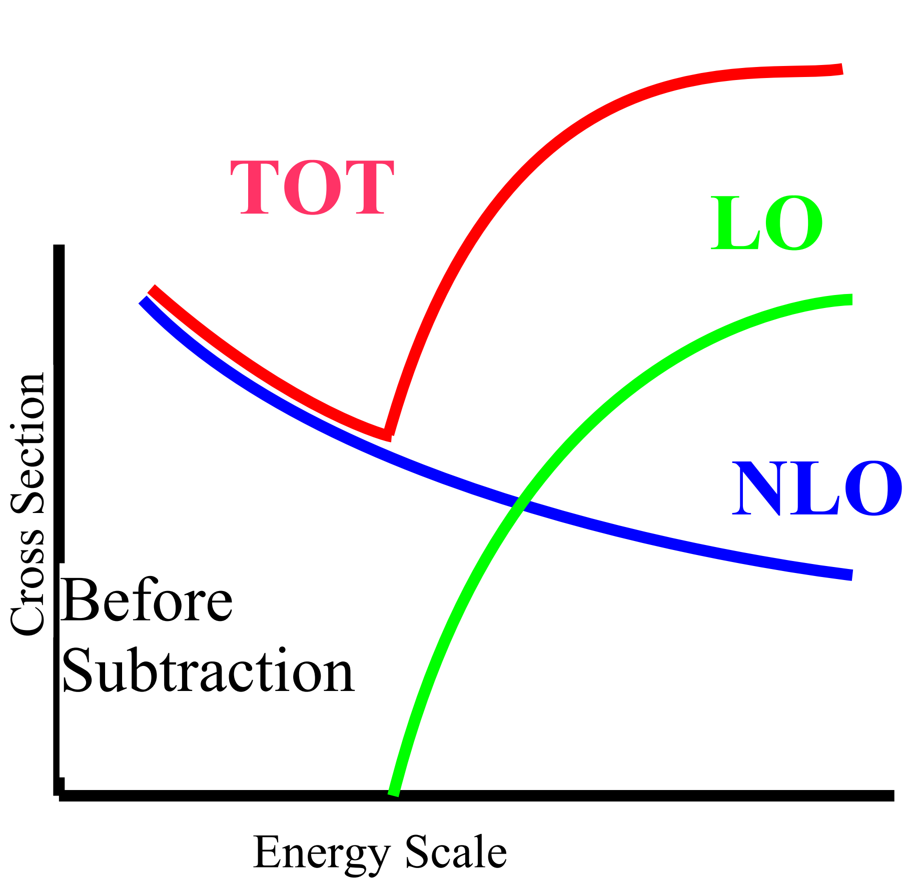
... *hence, extensible to all orders*

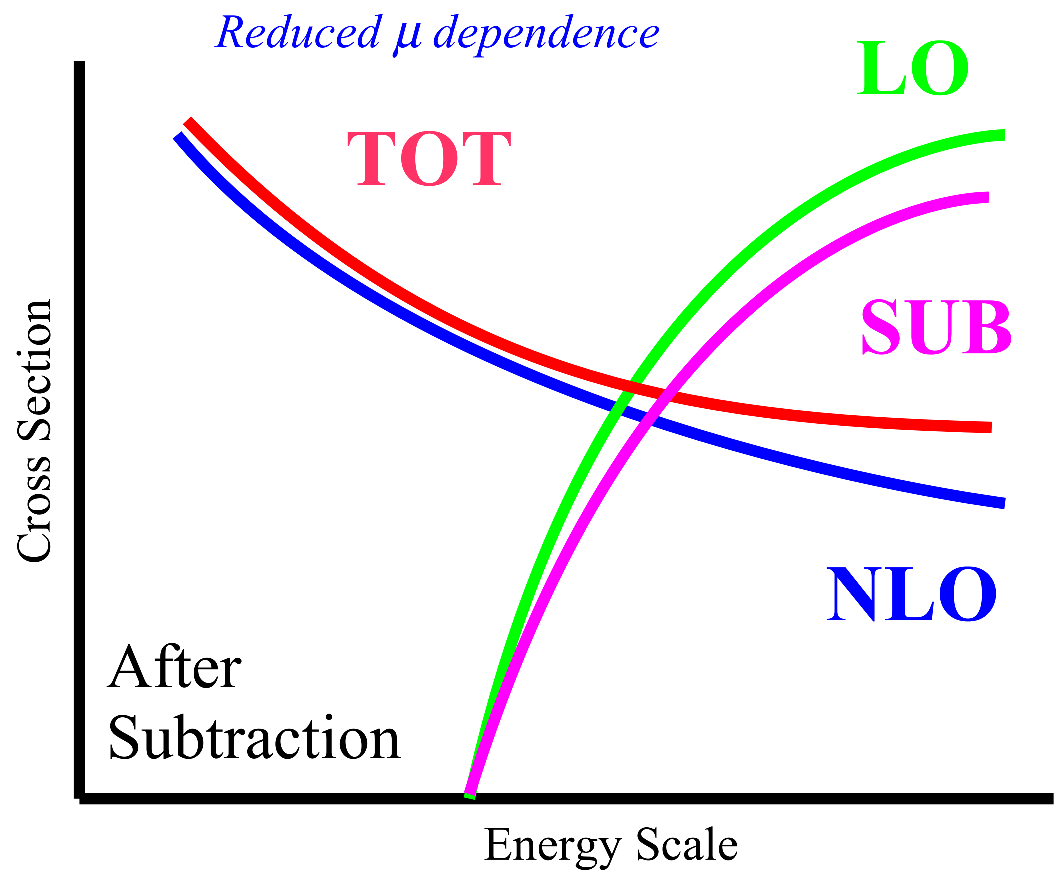
DGLAP kernels & PDF evolution are pure MS-Bar

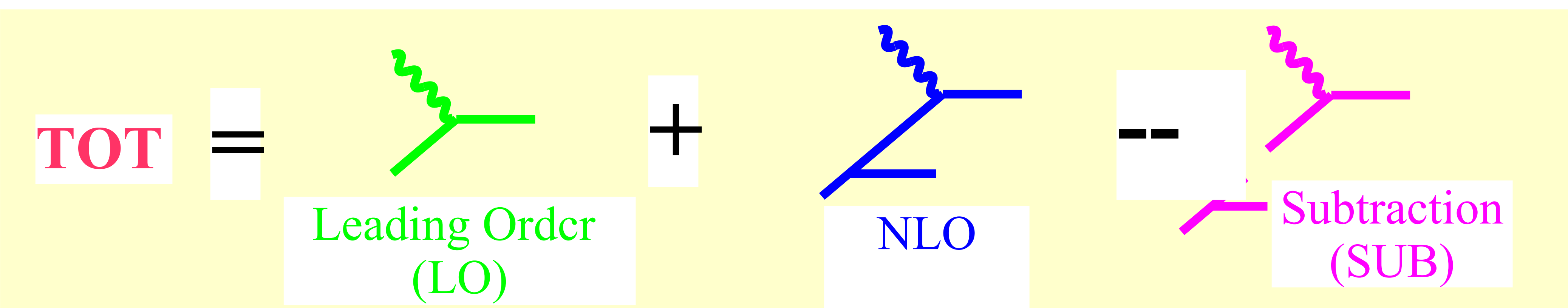
Definition of Subtractions analogous to MS-Bar

The minimal extension of MS-Bar scheme

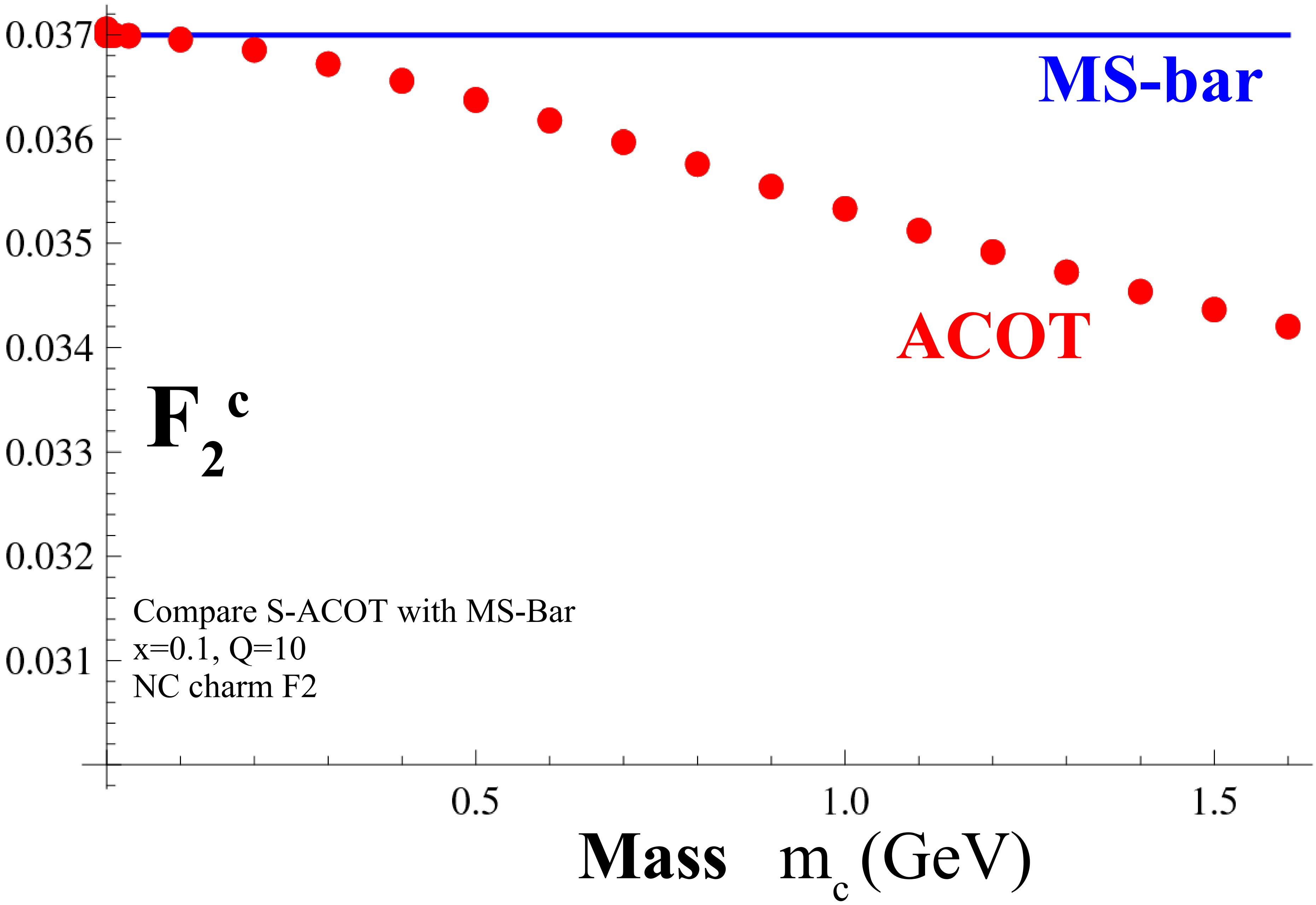
Multi-Scale Problem Resolved: … from low to high scales



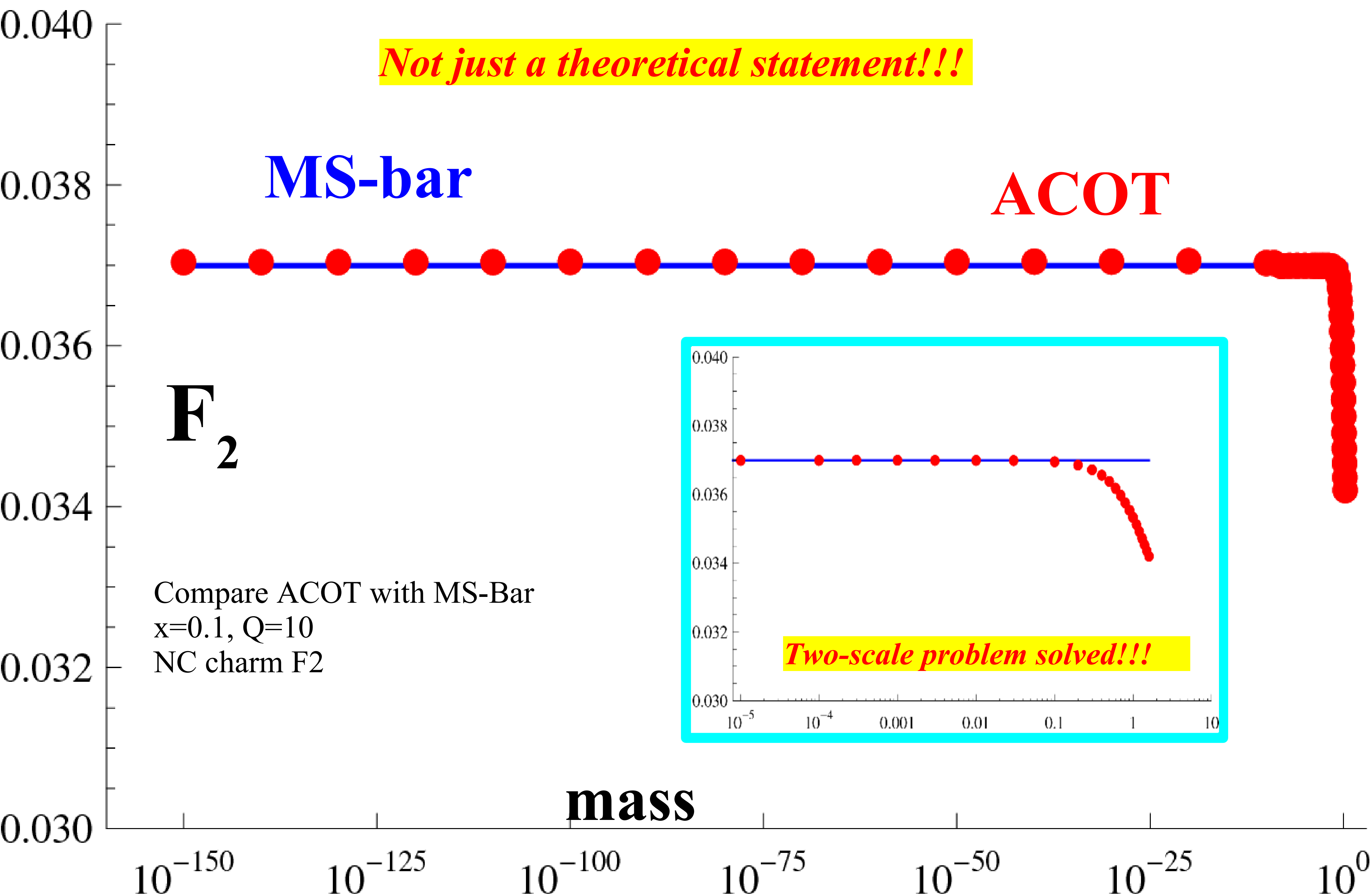




ACOT $m \to 0$ limit yields MS-Bar: *No finite renormalization*



ACOT m→ 0 limit yields MS-Bar: *No finite renormalization*



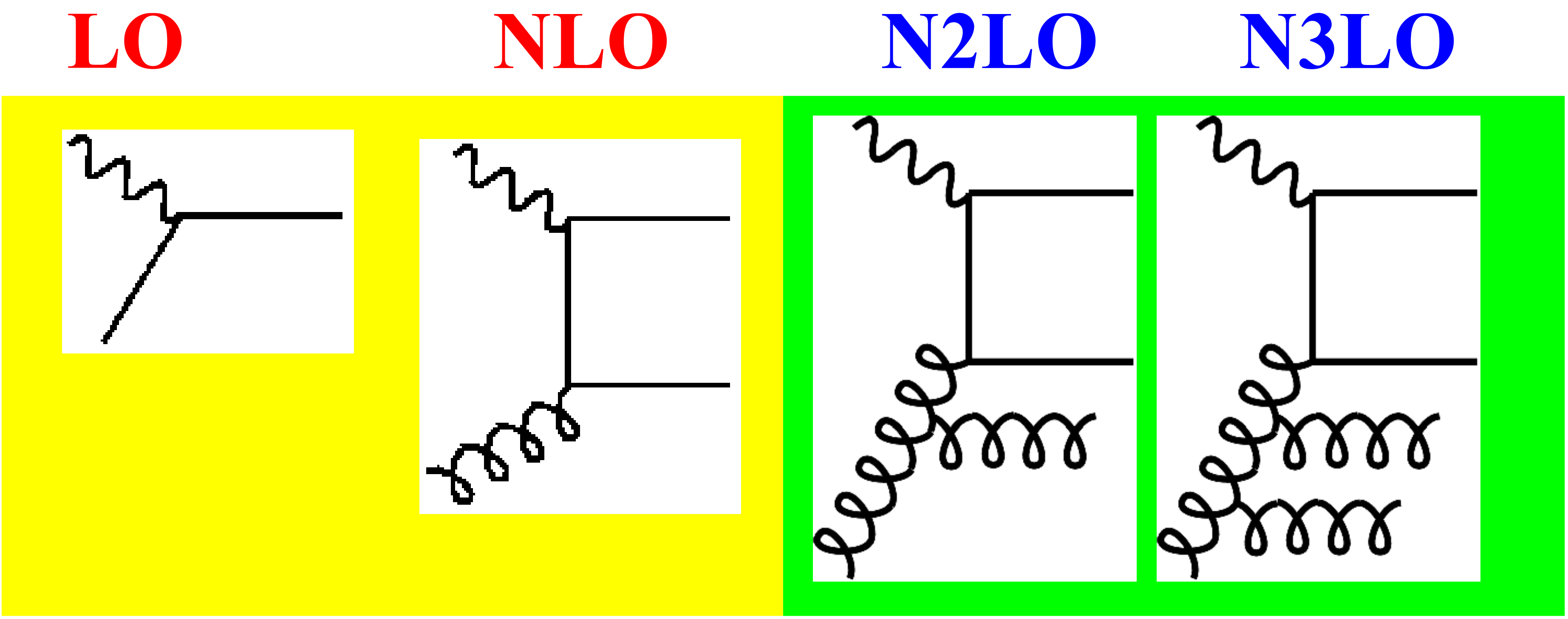
ACOT m→ 0 limit cross check with QCDNUM at NLO

```
PATCH FOR TESTING: HMASS=0=   0.938000023
  IHADRON: SET TO HADRON= 1
  GZ and ZZ are for testing
  print x,q, ratios:   0.00319999992  12.2474487  1.00092636   1.0012981
  print x,q, ratios:   0.00499999989  12.2474487  1.00098575   1.00126809
  print x,q, ratios:   0.00800000038  12.2474487  1.00106943   1.00153596
  print x,q, ratios:   0.00319999992  14.1421356  1.00092542   1.00125357
  print x,q, ratios:   0.00499999989  14.1421356  1.00097202   1.00121532
  print x,q, ratios:   0.00800000038  14.1421356  1.00104411   1.00146055
  print x,q, ratios:   0.0130000003   14.1421356  1.00107382   1.0013549
  print x,q, ratios:   0.0199999996   14.1421356  1.00114663   1.0014694
  print x,q, ratios:   0.0320000015   14.1421356  1.00119237   1.00152525
  print x,q, ratios:   0.0500000007   14.1421356  1.00117963   1.00131561
  print x,q, ratios:   0.0799999982   14.1421356  1.00098036   1.00123239
  print x,q, ratios:   0.00499999989  15.8113883  1.00095999   1.00117694
  print x,q, ratios:   0.00800000038  15.8113883  1.0010229    1.00140587
  print x,q, ratios:   0.0130000003   15.8113883  1.00104124   1.00130353
                       0.0199999996   15.8113883  1.00110599   1.00140934
                       0.0320000015   15.8113883  1.00114419   1.00146259
                       0.0500000007   15.8113883  1.00113621   1.00125726
                       0.0799999982   15.8113883  1.00095108   1.0011996
                       0.129999995    15.8113883  1.00055001   1.00103563
  print x,q, ratios:   0.25           15.8113883  0.99929117   1.0000816
  print x,q, ratios:   0.400000006    15.8113883  0.997267345  0.998376607
  print x,q, ratios:   0.00499999989  17.3205081  1.00094852   1.00114569
  print x,q, ratios:   0.00800000038  17.3205081  1.00100525   1.00136309
  print x,q, ratios:   0.0130000003   17.3205081  1.00101481   1.00118502
  print x,q, ratios:   0.0199999996   17.3205081  1.00107357   1.00136459
  print x,q, ratios:   0.0320000015   17.3205081  1.00110601   1.00140262
```

NLO Check with QCDNUM

~1E-3

ACOT Extension to Higher Orders



Full ACOT

Extensible to any order

$$\sigma = f(\xi(x, m_{ps}), Q) \otimes \hat{\sigma}(m_{dyn})$$

$$\xi(x, m_{ps}) = x \left(1 + \left[\frac{n\, m_{ps}}{Q} \right]^2 \right)$$

$$n = \{0, 1, 2\}$$

Distinguish
"phase space" mass
from
"dynamic" mass

Strategy To Very Mass Effects Under Control

STEP 1:

Look at F_2^C to see effect of individual terms:

i.e. LO, NLO, SUB

Examine effect of mass for both:

a) phase space mass

b) dynamic mass

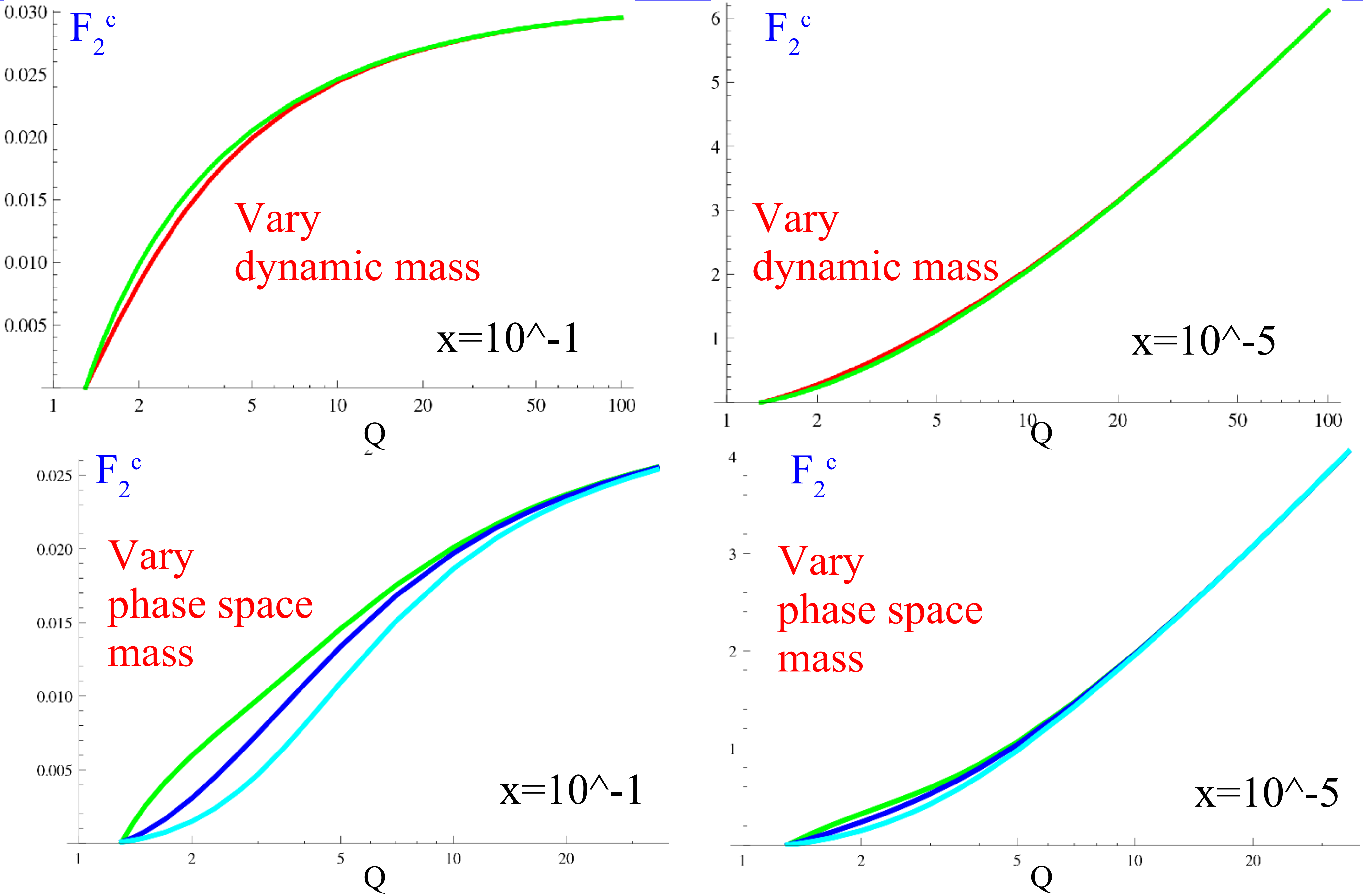
STEP 2:

Look at inclusive F_2 and F_L

STEP 1

$$F_2^{charm}$$

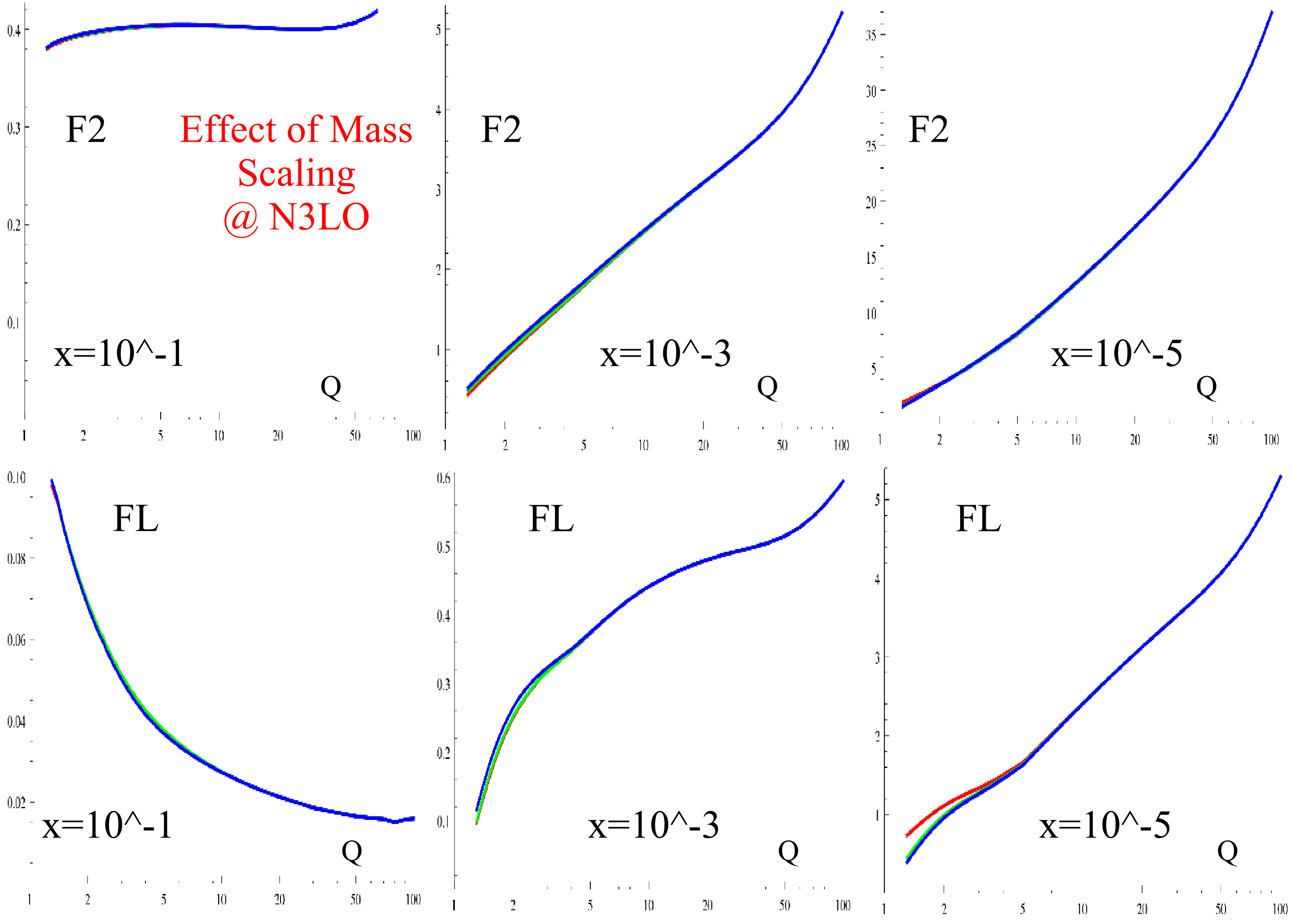
Two Types of Mass Depencence @ NLO: "dynamic" & "phase space"



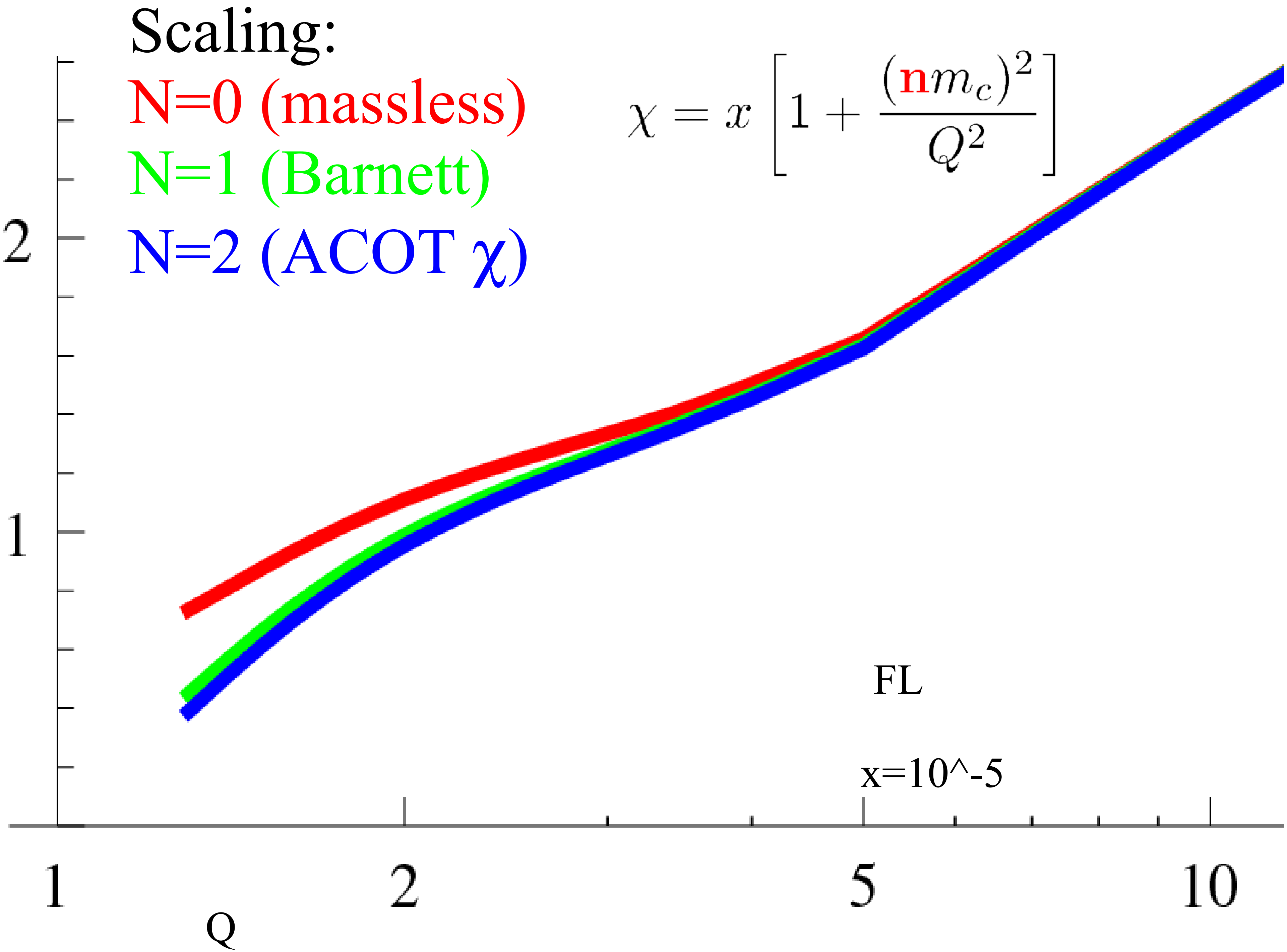
"phase space" mass yields larger variation. Not a proof, but ....

$$F_2^{\text{inclusive}}$$

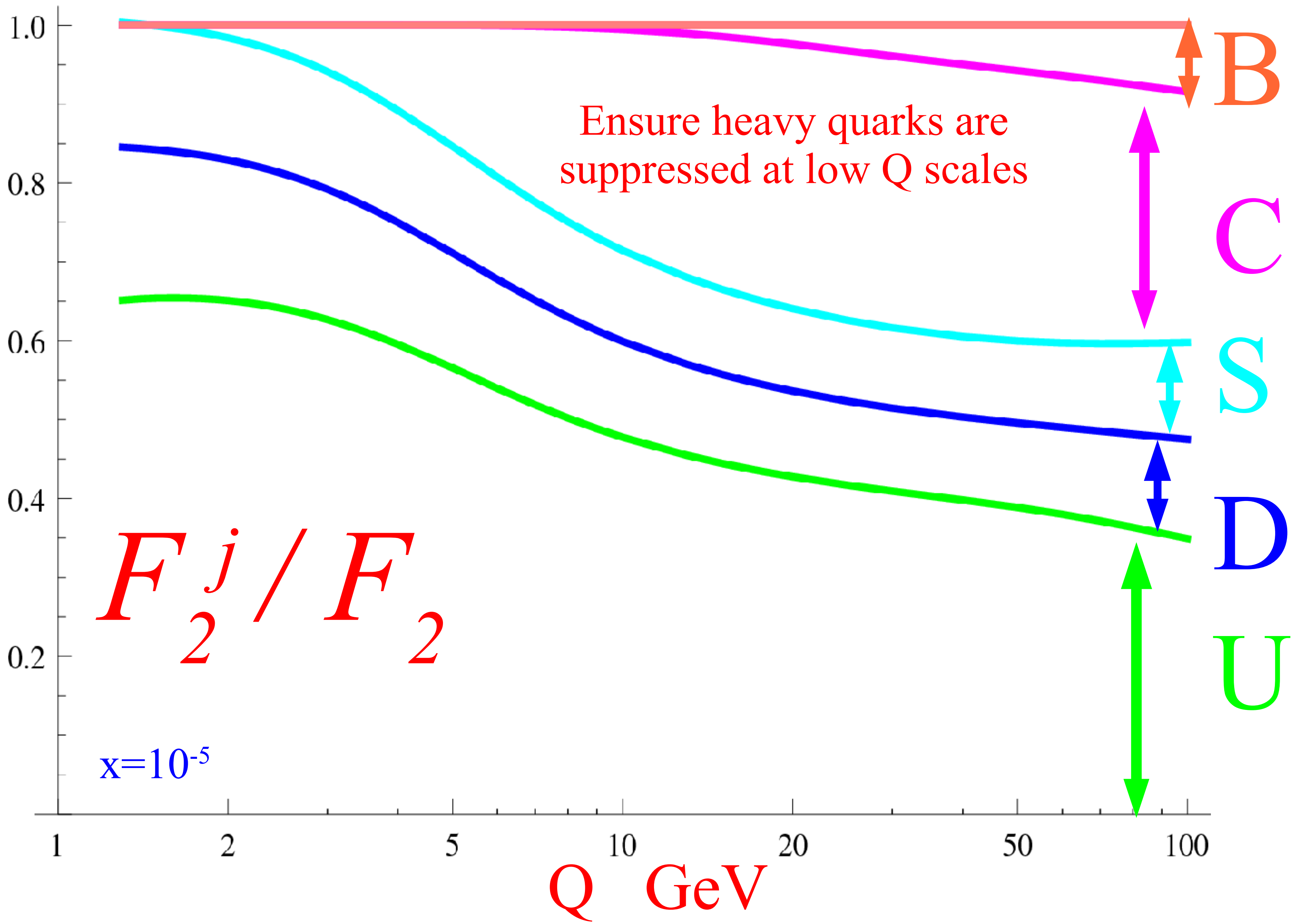
EFFECT OF MASS SCALING @ N3LO (Phase Space Mass)



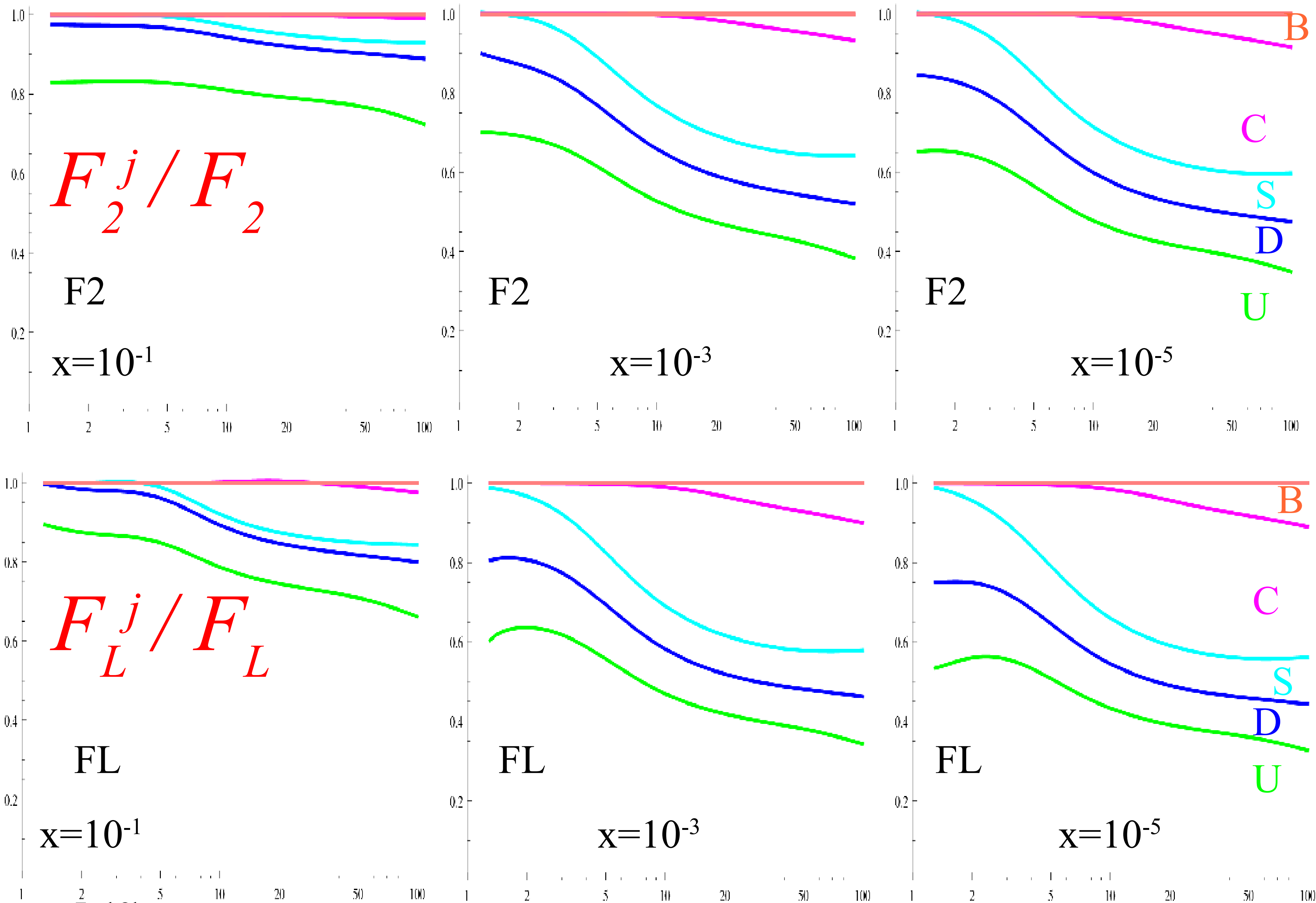
EFFECT OF MASS SCALING @ N3LO (Phase Space Mass)



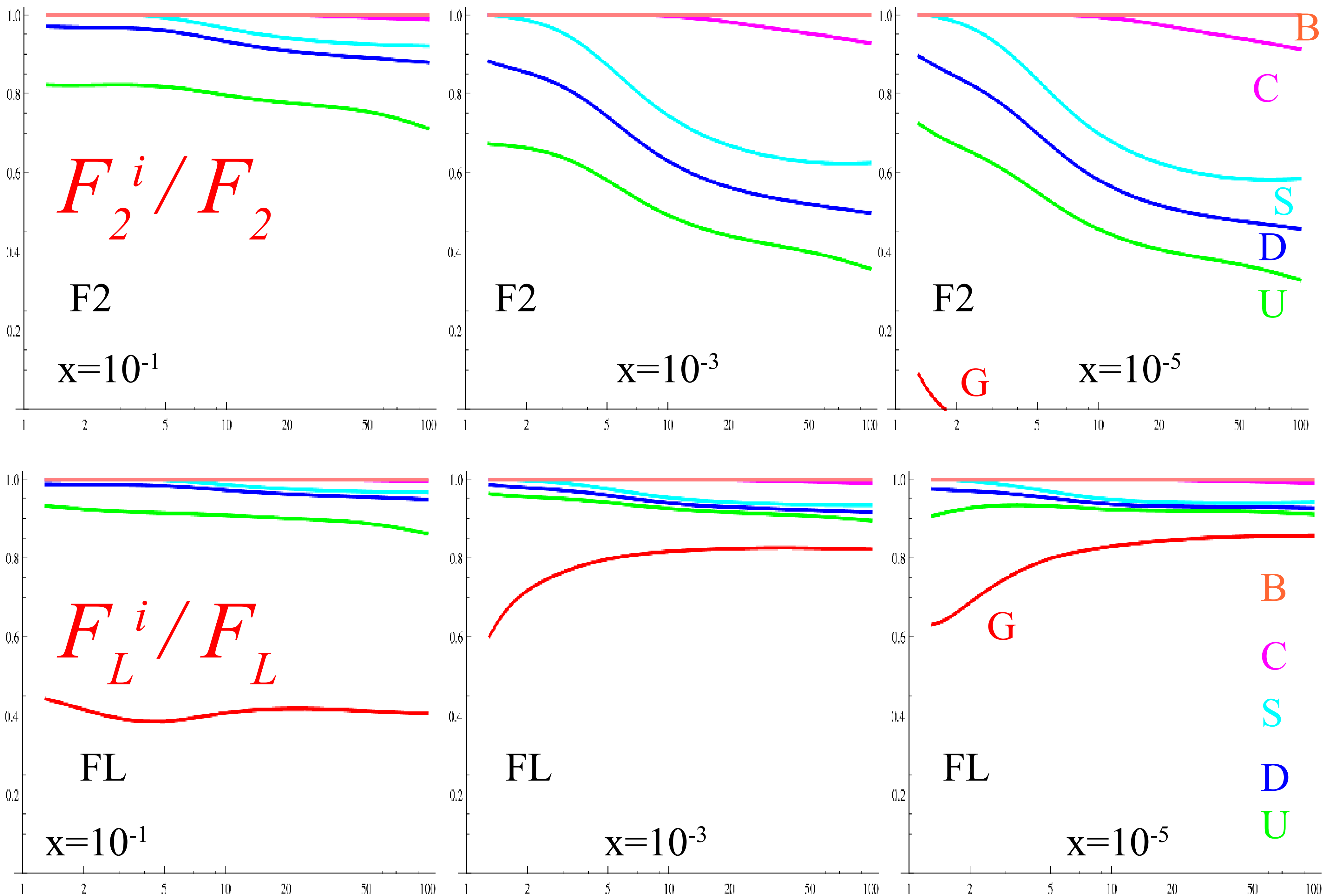
FLAVOR DECOMPOSITION



FLAVOR DECOMPOSITION: Final State Quark:



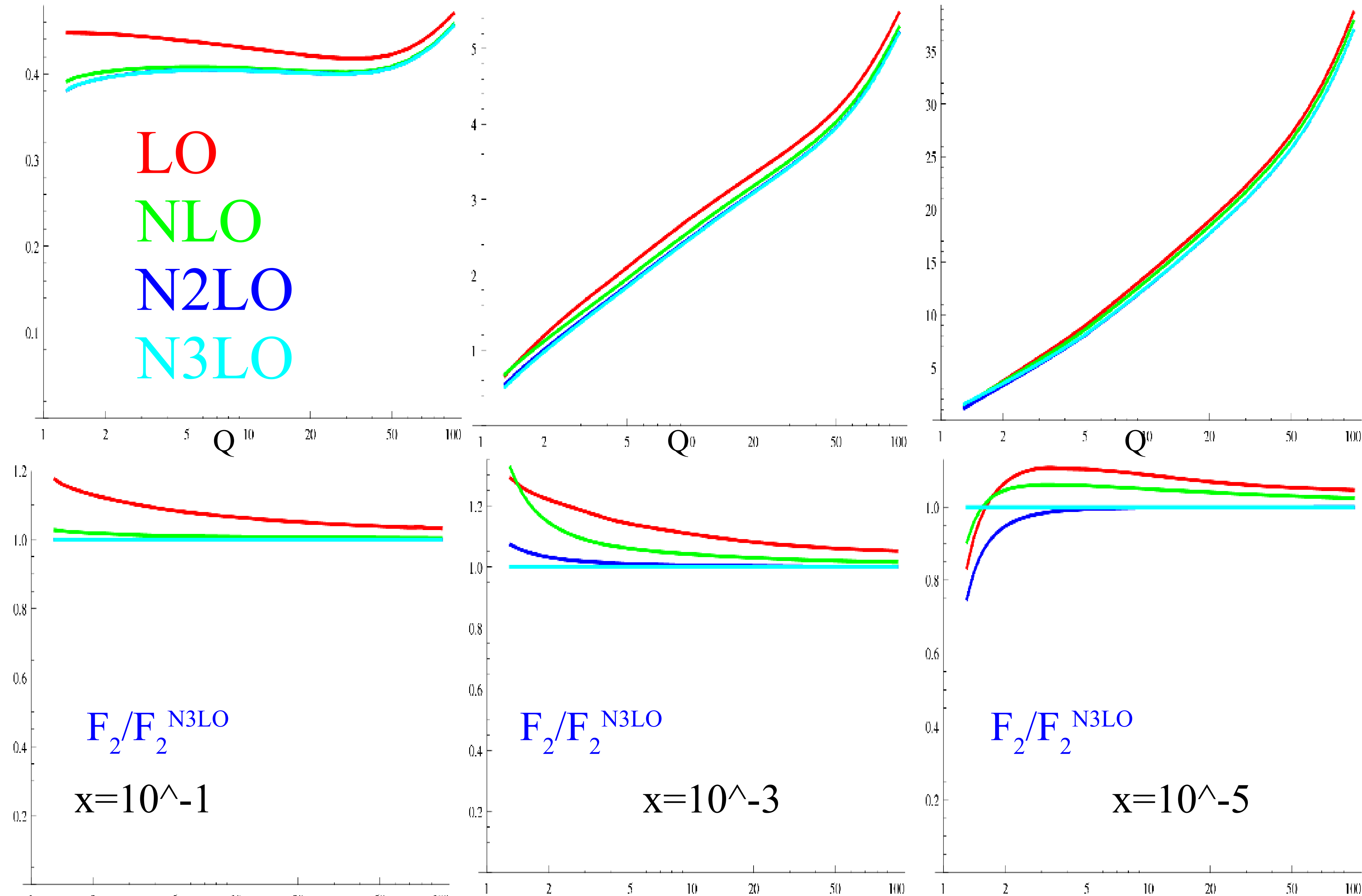
FLAVOR DECOMPOSITION: Initial State Quark:



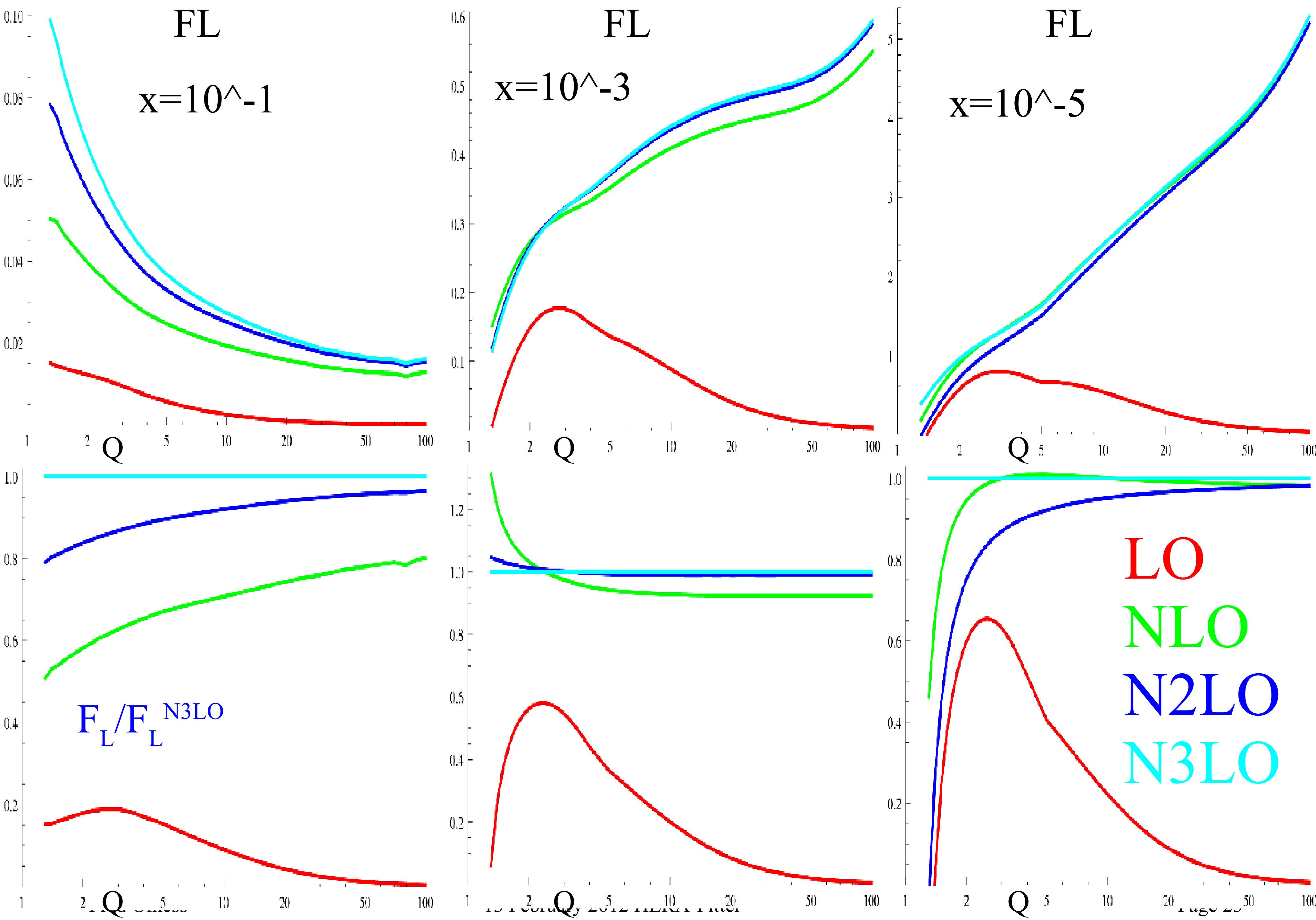
FINAL RESULT

$F_{2,L}$ @ N3LO

F_2 @ N3LO



F_L @ N3LO



Part I CONCLUSIONS

This technique provides an N3LO extension of ACOT

"Phase space" mass is included via rescaling
Appears to be dominant effect

F2:

Very stable.

LO and NLO have full m-dependence

FL:

More complex as NLO corrections are large
N2LO and N3LO terms converge

For inclusive FL, heavy quark terms vanish for low Q;
this moderates mass effects

Fortran Interface

HERA Fitter interface to ACOT programs

```
subroutine SF_ACOT_wrap(
        x_in,
        q2_in,
        f123l_out,
        f123lc_out,
        f123lb_out,
        hfscheme_in,
        icharge_in,
        iflag,
        index,
        UseKFactors)
```

F1, F2, F3, FL are out via f123l_out
f123l_out(1)=F1
f123l_out(2)=F2
f123l_out(3)=F3
f123l_out(4)=FL
same for charm only contribution: f123lc
same for charm only contribution: f123lb

hfscheme_in: NLO massless and massive
icharge_in: 0 NC: photon exchange only
icharge_in: 4 NC: gamma+gammaZ+Z
icharge_in: -1 CC e-
icharge_in: +1 CC e+
iflag: flag from FCN
index: data index - integer
UseKFactors: use of kfactors

The "Interface" Function

Call Fgen123LK_TCB(index, icharge, X, Q, μ, F123L_TotCB)

Data point index

γ/W/Z exchange

F_1	F_2	F_3	F_L
F_1^c	F_2^c	F_3^c	F_L^c
F_1^b	F_2^b	F_3^b	F_L^b

$$\text{K-factor} = \frac{\sigma(\text{Full})}{\sigma(\text{LO-Massive})}$$

$$\sigma(\text{Full}) = \text{K-factor} \times \sigma(\text{LO-Massive})$$

Near Future:
NLO ACOT
+
QCDNUM

Why do we need the K-factor method???

ACOT

$$
\begin{aligned}
\hat{f}_1^Q(\hat{s}_-) &= \frac{8}{\Delta'^2}\Big\{ -\Delta^2(S_+\Sigma_{++} - 2m_1m_2S_-) I_{\xi'} + 2m_1m_2S_-\Big(\frac{1}{\hat{s}_1}[\Delta'^2 + 4m_2^2\Sigma_{++}]\\
&+ 2\Sigma_{+-} - \Sigma_{--} + \frac{\Sigma_{+-} + \hat{s}_1}{2} + \frac{\hat{s}_1 + m_2^2}{\Delta'\hat{s}_1}\left[\Delta'^2 + 2\Sigma_{--}\Sigma_{++} - (m_2^2 - Q^2)\,\hat{s}_1\right] L_{\xi'}\Big)\\
&+ S_+\Big(\frac{-m_2^2\Sigma_{--}}{(\hat{s}_- + m_2^2)\hat{s}_1}(\Delta^2 + 4m_2^2\Sigma_{--}) - \frac{1}{4(\hat{s}_1 + m_2^2)}\Big[3\Sigma_{++}^2\Sigma_{--} + 4m_2^2(10\Sigma_{--}\Sigma_{+-}\\
&- \Sigma_{+-}\Sigma_{-+} - m_1^2\Sigma_{++}) + \hat{s}_1[-7\Sigma_{++}\Sigma_{-+} + 18\Delta^2 - 4m_1^2(7Q^2 - 4m_2^2 + 7m_1^2)]\\
&+ 3\hat{s}_1^2[\Sigma_{--} - 2m_1^2] - \hat{s}_1^3\Big] + \frac{\hat{s}_1 + m_2^2}{2\Delta'}\Big[\frac{-2}{\hat{s}_1}\Sigma_{-+}(\Delta^2 - 2\Sigma_{++}\Sigma_{-+})\\
&+ (4m_1^2m_2^2 - 7\Sigma_{+-}\Sigma_{++}) - 4\Sigma_{+-}\hat{s}_1 - \hat{s}_1^2\Big] L_{\xi'}\Big)\Big\}\\
\hat{f}_2^Q(\hat{s}_-) &= \frac{16}{\Delta'^4}\Big\{ -2\Delta^4 S_+ I_{\xi'} + 2m_1m_2S_-\Big(\frac{\hat{s}_1 + m_2^2}{\Delta'}\left(\Delta'^2 - 6m_1^2Q^2\right) L_{\xi'}\\
&- \frac{\Delta'^2(\hat{s}_- + \Sigma_{+-})}{2(\hat{s}_1 + m_2^2)} + \left(2\Delta'^2 - 3Q^2(\hat{s}_- + \Sigma_{--})\right)\Big) + S_+\Big(-2(\Delta^2 - 6m_1^2Q^2)(\hat{s}_1 - m_2^2)\\
&- 2\left(m_1^2 - m_2^2\right)\hat{s}_-^2 - 9m_2^2\Sigma_{+-}^2 + \Delta^2\left(2\Sigma_{++} - m_2^2\right) + 2\hat{s}_1\left(2\Delta^2 + (m_1^2 - 5m_2^2)\Sigma_{+-}\right)\\
&+ \frac{\left(\Delta'^2 - 6Q^2\left(m_2^2 + \hat{s}_1\right)\right)\Sigma_{++}\left(\hat{s}_1 + \Sigma_{+-}\right)}{2(\hat{s}_- - m_2^2)} - \frac{2\Delta^2}{\hat{s}_1}\left(\Delta^2 + 2(2m_2^2 + \hat{s}_1)\Sigma_{+-}\right)\\
&+ \frac{(\hat{s}_- + m_2^2)}{\Delta'}\Big[\frac{-2}{\hat{s}_1}\Delta^2(\Delta^2 + 2\Sigma_{+-}\Sigma_{+-}) - 2\hat{s}_1(\Delta^2 - 6m_1^2Q^2)\\
&- (\Delta'^2 - 18m_1^2Q^2)\Sigma_{++} - 2\Delta^2\left(\Sigma_{-+} + 2\Sigma_{--}\right)\Big] L_{\xi'}\Big)\Big\}\\
\hat{f}_3^Q(\hat{s}_-) &= \frac{16}{\Delta'^2}\Big\{ -2\Delta^2 R_+ I_{\xi'} + 2m_1m_2R_-\left(1 - \frac{\Sigma_{+-}}{\hat{s}_1} - \frac{(\hat{s}_1 + m_2^2)(\hat{s}_1 + \Sigma_{+-})}{\Delta'\hat{s}_1} L_{\xi'}\right)\\
&+ R_+\Big(\Sigma_{-+} - 3\Sigma_{+-} - \frac{2}{\hat{s}_-}\left(\Delta^2 - 2m_2^2\Sigma_{+-}\right) - \frac{(\hat{s}_- - \Sigma_{-+})(\hat{s}_1 + \Sigma_{++})}{2(\hat{s}_1 + m_2^2)}\\
&+ \frac{\hat{s}_1 + m_2^2}{\Delta'\hat{s}_-}\left[-\hat{s}_1^2 + 4\left(m_1^2\Sigma_{-+} - \Delta^2\right) - 3\hat{s}_-\Sigma_{--}\right] L_{\xi'}\Big)\Big\}
\end{aligned}
$$

with

$$L_{\xi'} \equiv \ln\left(\frac{\Sigma_{++} + \hat{s}_1 - \Delta'}{\Sigma_{++} + \hat{s}_1 - \Delta'}\right)$$

and

$$I_{\xi'} = \left(\frac{\hat{s}_1 + 2m_2^2}{\hat{s}_1^2} + \frac{\hat{s}_1 - m_2^2}{\Delta'\hat{s}_1^2}\Sigma_{-+} L_{\xi'}\right).$$

S-ACOT

$$
\begin{aligned}
C_2^{(Vq)(1)} &= C_2(F)\frac{x}{2}\left[\frac{1 + x^2}{1 - x}\left(\ln\frac{(1 - x)}{x} - \frac{3}{4}\right) + \frac{1}{4}(9 + 5x)\right]_+ ,\\
C_1^{(Vq)(1)} &= \frac{1}{2x}C_2^{(Vq)(1)} - C_2(F)\frac{1}{2}x ,\\
C_3^{(Vq)(1)} &= \frac{1}{x}C_2^{(Vq)(1)} - C_2(F)(1 + x) .
\end{aligned}
$$

Coordinating Standard Model Parameters

Communication with Fred's code

```
COMMON /Ischeme/ ISCH, ISET, IFLG, IHAD
common /fred/ xmc,xmb,HMASS
common/fredew/ sinw2, xmw, xmz
```

Isch:	Scheme: ACOT, S-ACOT, S-ACOT-χ
Iset:	PDF set
Iflg:	*not yet used*
Ihad:	Hadron: proton, neutron …
Xmc:	Charm Mass
Xmb:	Bottom Mass
Hmass:	Proton mass
Sinw2:	Sin θ_W^2
Xmw:	M_W
Xmz:	M_Z

Thanks to

Voica Radescu

QCDNUM Package

QCDNUM ACOT Package

$$F_{123L} = \int dz \int dy \int dx \; C_{123L}(x,y,z) f(x)$$

Change order of integrations

$$F_{123L} = \int dx \; f(x) \left[\underbrace{\int dy \int dz \; C_{123L}(x,y,z)}_{\text{Weight tables}} \right]$$

Weight tables

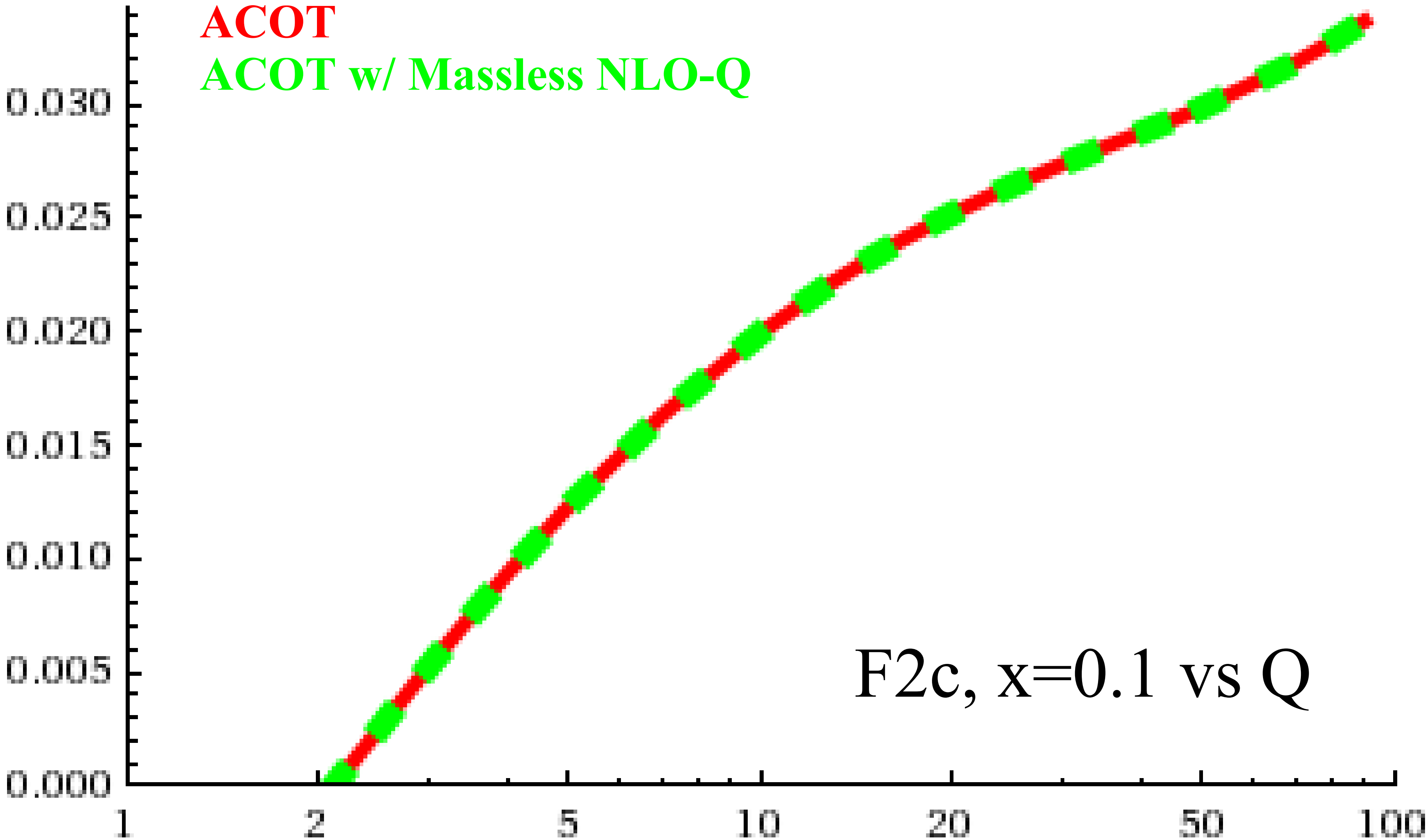
Need tables for:

{q,c,b} ⊗ {F_2,F_L} ⊗ { LO, NLO, SUB, ~~NLO-Q~~, ~~SUB-Q~~}

Thanks to
Michiel Botje

QCDNUM ACOT Package

Comparison:
ACOT
ACOT w/ Massless NLO-Q



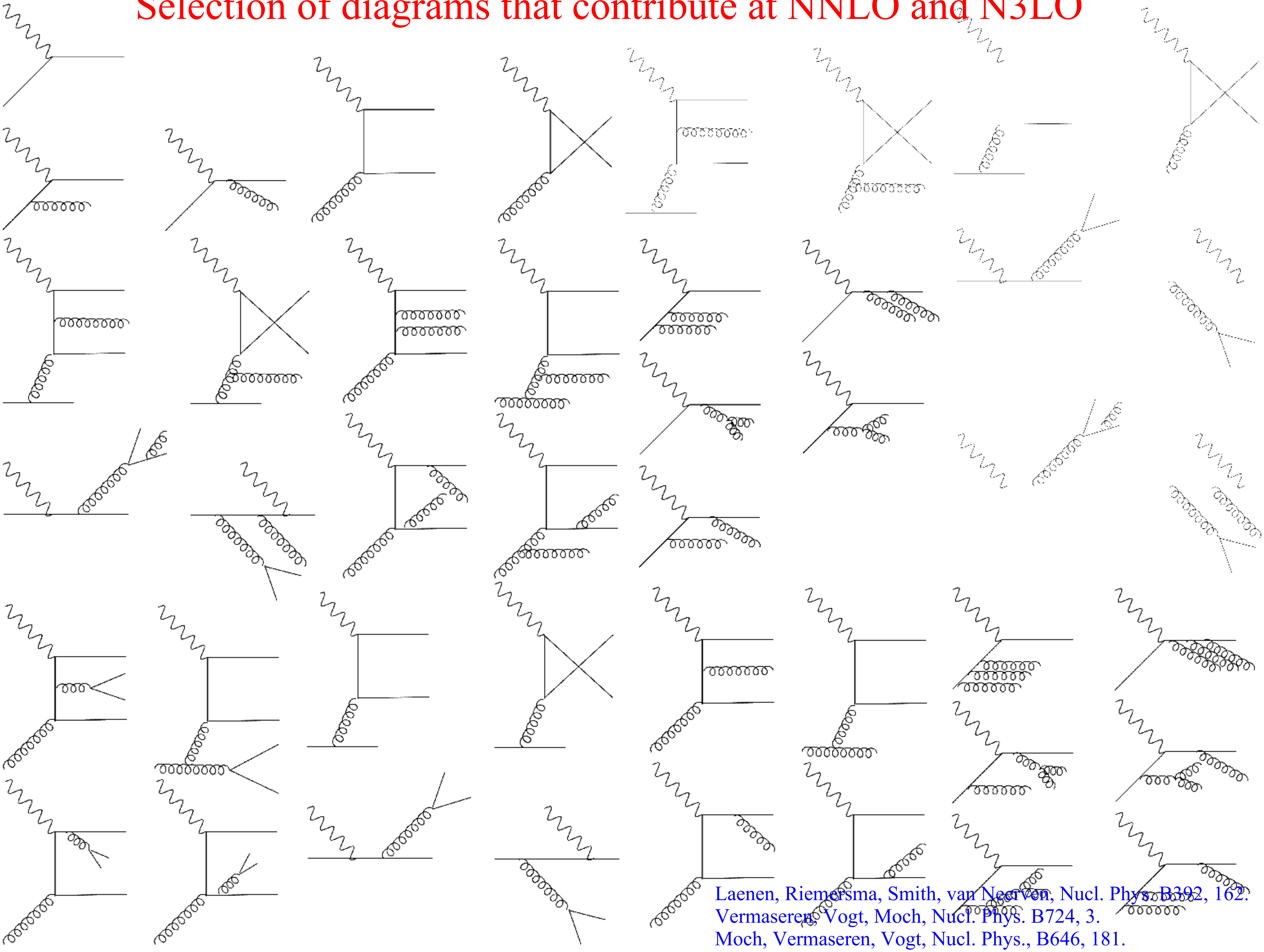
NNLO

+

N^3LO

T.P. Stavreva, I Schienbein,

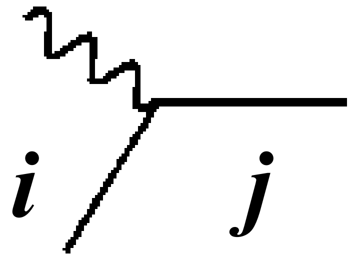
Selection of diagrams that contribute at NNLO and N3LO



Laenen, Riemersma, Smith, van Neerven, Nucl. Phys. B392, 162.
Vermaseren, Vogt, Moch, Nucl. Phys. B724, 3.
Moch, Vermaseren, Vogt, Nucl. Phys., B646, 181.
Moch, Vermaseren, Vogt, Phys. Lett., B606, 123.

Must decompose into individual flavor components

$$F = \sum_{i,j}^{5,6} F^{ij}$$



$$F^c = \sum_{i}^{5} F^{i4}$$

$$x^{-1} F_a = q_{ns} \otimes C^{\mathrm{ns}}_{a,q} + < e^2 > (q_{\mathrm{s}} \otimes C^{\mathrm{s}}_{a,q} + g \otimes C_{a,g}), \quad (a = 2, L)$$

$$q_{\mathrm{ns}} = \sum_{i=1}^{n_f} (e_i^2 - < e^2 >) q_i^+$$

$$q_{\mathrm{s}} = \sum_{i=1}^{n_f} q_i^+, \quad q_i^+ = q_i + \bar{q}_i$$

$$< e^2 > = < e^2 >^{(n_f)} = \frac{1}{n_f} \sum_{i=1}^{n_f} e_i^2$$

The Goal: Convert from {s, ns, ps} to {q,g, ...}

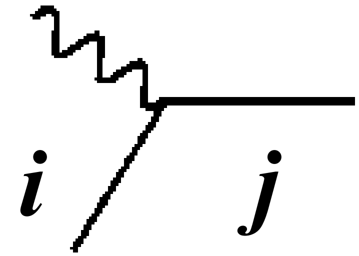
$$x^{-1} F_{a,q_i} = q_i^+ \otimes \left[e_i^2 \, C^{\mathrm{ns}}_{a,q} + < e^2 > C^{\mathrm{ps}}_{a,q} \right]$$

$$C^{\mathrm{ps}}_{a,q} = C^{\mathrm{ns}}_{a,q} - C^{\mathrm{s}}_{a,q}$$

$$< e^2 >^{(n_f)} \rightarrow a^+(n_f) = \frac{1}{n_f} \sum_{i=1}^{n_f} a^+_{q_i}$$

Master formula for decomposing the flavor components

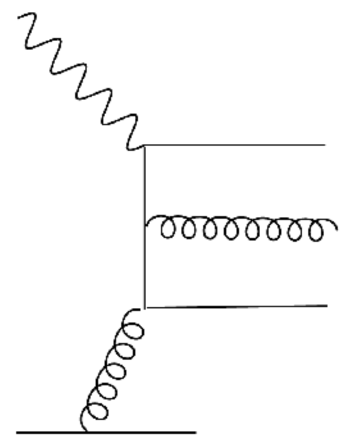
$$x^{-1} F_a^{ij} = q_i^+ \otimes \Big\{ e_i^2 \, \Big[C_{a,q}^{\rm ns}(n_f = 0)\ \delta_{ij} + C_{a,q}^{\rm ns}(j) - C_{a,q}^{\rm ns}(j-1) \Big] + < e^2 >^{(j)}\ C_{a,q}^{\rm ps}(j) - < e^2 >^{(j-1)}\ C_{a,q}^{\rm ps}(j-1) \Big\}$$



Issues: Flavor separation:

New diagrams at this order

- c,b, goes down beam pipe
- both c & b in final state



Conclusion

Conclusions

Heavy Quarks & Higher Order Corrections:

- Essential to properly incorporate mass effects for required precision
- Improved measurements of F^2, F^{cc}, F^{bb}, and F_L:
- Improved precision for LHC where heavy flavors play a prominent role

Theoretically, we can now compute full dynamic mass range [10^{-150},10^{+150}]

- ACOT natural massive extension of MS-bar
- Separate roles of dynamic and kinematic masses illustrated
- Mass effects are essential:

N2LO and N3LO Correction Implemented:

- Flavor Decomposition Essential
- Stable results for F2 and FL
- Important reference point

Interface to HERA Fitter framework:

Thanks to: A. Kusina, T.P. Stavreva I Schienbein, J.-Y. Yu, K. Kovarik,
P. Nadolsky, M. Guzzi
J. Owens, J. Morfin, C. Keppel, D. Soper …
& the HERA-PDF Working Group