

Constraining General 2HDM by the Evolution of Yukawa Couplings

Jie Lu

IFIC, Valencia University

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Outline

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- ➌ The Evolution of Yukawas
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Johan Bijnens, Jie Lu, Johan Rathsman JHEP 1205 (2012) 118

Motivation

- The two Higgs doublet model (2HDM) is the minimal extension of Standard Model.
- Many beyond standard model theories contain two or more Higgs doublets.
Supersymmetry theories must contain even number of Higgs doublets.
- The general analysis of 2HDM can give hints or constraints for more fundamental beyond standard model theories.

Yukawa Couplings in Higgs Basis

- The most general Yukawa couplings in 2HDM

$$\begin{aligned} -\mathcal{L}_Y = & \overline{Q}_L \tilde{\Phi}_1 \eta_1^U U_R + \overline{Q}_L \Phi_1 \eta_1^D D_R + \overline{L}_L \Phi_1 \eta_1^L E_R \\ & + \overline{Q}_L \tilde{\Phi}_2 \eta_2^U U_R + \overline{Q}_L \Phi_2 \eta_2^D D_R + \overline{L}_L \Phi_2 \eta_2^L E_R + \text{h.c.} \end{aligned}$$

- The Higgs basis is convenient for studying Yukawa coupling

$$\begin{aligned} H_1 &= \begin{pmatrix} 0 \\ (\mathbf{v} + h)/\sqrt{2} \end{pmatrix} + \begin{pmatrix} G^+ \\ \frac{i}{\sqrt{2}} G^0 \end{pmatrix} \\ H_2 &= \begin{pmatrix} H^+ \\ (H + iA)/\sqrt{2} \end{pmatrix} \end{aligned}$$

- The Yukawa coupling in Higgs basis

$$\begin{aligned} -\mathcal{L}_Y = & \overline{Q}_L \tilde{H}_1 \kappa_0^U U_R + \overline{Q}_L H_1 \kappa_0^D D_R + \overline{L}_L H_1 \kappa_0^L E_R \\ & + \overline{Q}_L \tilde{H}_2 \rho_0^U U_R + \overline{Q}_L H_2 \rho_0^D D_R + \overline{L}_L H_2 \rho_0^L E_R + \text{h.c.} \end{aligned}$$

Yukawa Couplings and FCNC

- The Yukawa interaction in mass basis

$$\kappa^F = V_L^F \kappa_0^F V_R^{F\dagger} = \frac{\sqrt{2}}{v} \mathcal{M}_{ii}^F \quad \rho^F = V_L^F \rho_0^F V_R^{F\dagger}$$

$$\begin{aligned}
-\mathcal{L}_Y &= \frac{1}{\sqrt{2}} \bar{D} \left[\kappa^D s_{\beta-\alpha} + (\rho^D P_R + \rho^{D\dagger} P_L) c_{\beta-\alpha} \right] D h \\
&\quad + \frac{1}{\sqrt{2}} \bar{D} \left[\kappa^D c_{\beta-\alpha} - (\rho^D P_R + \rho^{D\dagger} P_L) s_{\beta-\alpha} \right] D H + \frac{i}{\sqrt{2}} \bar{D} (\rho^D P_R - \rho^{D\dagger} P_L) D A \\
&\quad + \frac{1}{\sqrt{2}} \bar{U} \left[\kappa^U s_{\beta-\alpha} + (\rho^U P_R + \rho^{U\dagger} P_L) c_{\beta-\alpha} \right] U h \\
&\quad + \frac{1}{\sqrt{2}} \bar{U} \left[\kappa^U c_{\beta-\alpha} - (\rho^U P_R + \rho^{U\dagger} P_L) s_{\beta-\alpha} \right] U H - \frac{i}{\sqrt{2}} \bar{U} (\rho^U P_R - \rho^{U\dagger} P_L) U A \\
&\quad + \dots
\end{aligned}$$

- Flavour changing neutral current (FCNC): $B_0 - \bar{B}_0$



Solutions for tree level FCNC

- Z_2 symmetry:

Type	U_R	D_R	L_R	ρ^U	ρ^D	ρ^L
I	+	+	+	$\kappa^U \cot \beta$	$\kappa^D \cot \beta$	$\kappa^L \cot \beta$
II	+	-	-	$\kappa^U \cot \beta$	$-\kappa^D \tan \beta$	$-\kappa^L \tan \beta$
III/Y	+	-	+	$\kappa^U \cot \beta$	$-\kappa^D \tan \beta$	$\kappa^L \cot \beta$
IV/X	+	+	-	$\kappa^U \cot \beta$	$\kappa^D \cot \beta$	$-\kappa^L \tan \beta$

- Yukawa Alignment: the Yukawa coupling matrices κ^F and ρ^F are proportional to each other. So they can be diagonalized simultaneously.
[A. Pich and P. Tuzon, Phys. Rev. D 80 \(2009\)](#)
- Cheng-Sher Ansatz

$$\rho_{ij}^F = \lambda_{ij}^F \frac{\sqrt{2m_i m_j}}{v} \quad \lambda_{ij}^F \sim \mathcal{O}(1)$$

[T. P. Cheng and M. Sher, Phys. Rev. D 35 \(1987\) 3484](#)

Constraints on λ_{ij}^F

The constraints on λ_{ij}^F in general 2HDM from neutral meson mixing: $F^0 - \bar{F}^0$.

$$\begin{aligned}\Delta M_F^{\text{SM}} + \Delta M_F^{\text{2HDM}} &\leq \Delta M_F^{\text{expt}} + 2\sigma \\ F^0 &= \{K^0, D^0, B_d^0, B_s^0\}\end{aligned}$$

- $m_h = m_H = m_A = 120 \text{ GeV}$

$$\begin{aligned}\lambda_{uc} &\lesssim 0.13, \\ \lambda_{ds} &\lesssim 0.08, \quad \lambda_{db} \lesssim 0.03, \quad \lambda_{sb} \lesssim 0.05.\end{aligned}$$

- $m_h = m_H = m_A = 400 \text{ GeV}$

$$\begin{aligned}\lambda_{uc} &\lesssim 0.44, \\ \lambda_{ds} &\lesssim 0.27, \quad \lambda_{db} \lesssim 0.12, \quad \lambda_{sb} \lesssim 0.18.\end{aligned}$$

- $m_h = m_H = 120 \text{ GeV}, m_A = 400 \text{ GeV}$

$$\begin{aligned}\lambda_{uc} &\lesssim 0.30, \\ \lambda_{ds} &\lesssim 0.20, \quad \lambda_{db} \lesssim 0.08, \quad \lambda_{sb} \lesssim 0.12.\end{aligned}$$

Constraining the Yukawa couplings with two methods

Evolving the Yukawa couplings from EW scale to high energy scale.

- Detect the position of **Landau pole**: where the perturbative theory fails.
- Detect the energy scale where **large non-diagonal** $\lambda_{i \neq j} \geq 0.1$.
Answer the question that how much Z_2 symmetry breaking is allowed at EW scale.
If $\lambda_{i \neq j}$ grow quickly, the theory is either fine-tuned or incomplete in some way.

Renormalization Group Equations

The RGE for Yukawa couplings and correlated in general 2HDM

- The general 3×3 Yukawa matrices: $\kappa^U, \kappa^D, \kappa^L$ and ρ^U, ρ^D, ρ^L ;
- The gauge couplings: g_1, g_2, g_3 ;
- The Higgs parameters: vev, $\tan\beta, \theta$.

The total number of ODE equations is 114.

G. Cvetic, S. S. Hwang and C. S. Kim, Int. J. Mod. Phys. A 14, 769 (1999)

P. M. Ferreira, L. Lavoura and J. P. Silva, Phys. Lett. B 688 (2010) 341

G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, J. P. Silva Phys.Rept. 516 (2012) 1

The computing method:

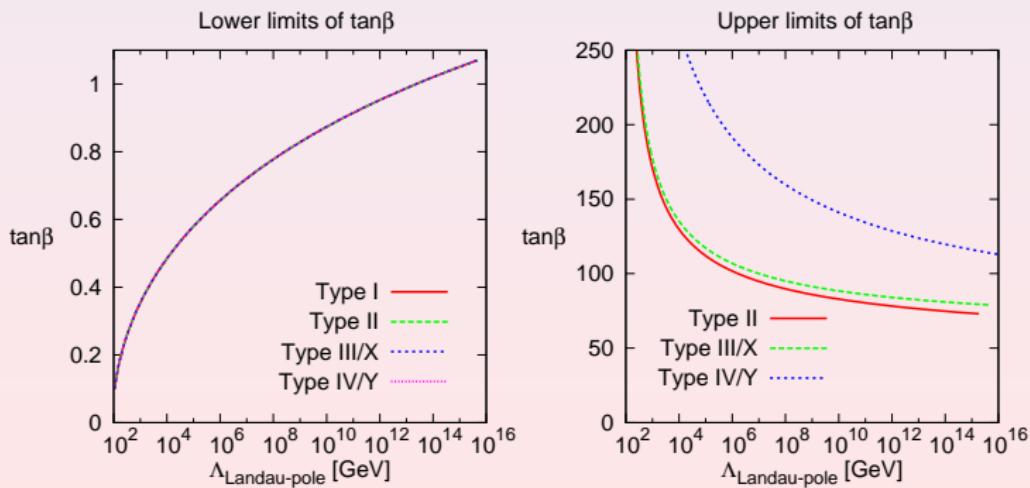
- C++ with *GSL* and *Eigen* (for matrix operations);
- explicit Runge-Kutta-Fehlberg(4,5) method in *GSL ODE-solver*.

Z_2 -symmetric models

- The diagonal λ_{ii}^F in 2HDM models with Z_2 symmetry.

Type	λ_{ii}^U	λ_{ii}^D	λ_{ii}^L
I	$1/\tan\beta$	$1/\tan\beta$	$1/\tan\beta$
II	$1/\tan\beta$	$-\tan\beta$	$-\tan\beta$
III/Y	$1/\tan\beta$	$-\tan\beta$	$1/\tan\beta$
IV/X	$1/\tan\beta$	$1/\tan\beta$	$-\tan\beta$

- The position of the Landau pole as a function of the input $\tan\beta$ at EW scale.



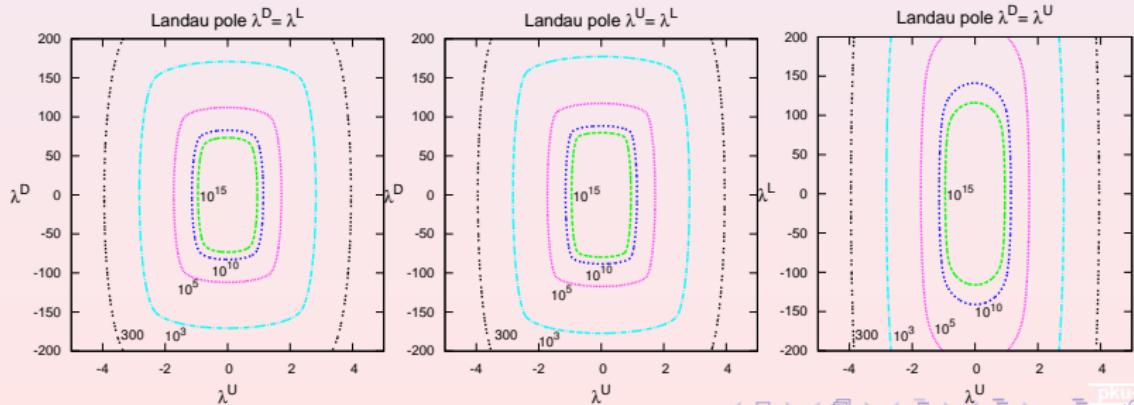
Aligned models

- Yukawa alignment in general 2HDM:

$$\rho^U = s_u^* \kappa^U, \quad \rho^D = s_d \kappa^D, \quad \rho^L = s_l \kappa^L \quad \rightarrow \quad \text{diagonal matrices}$$

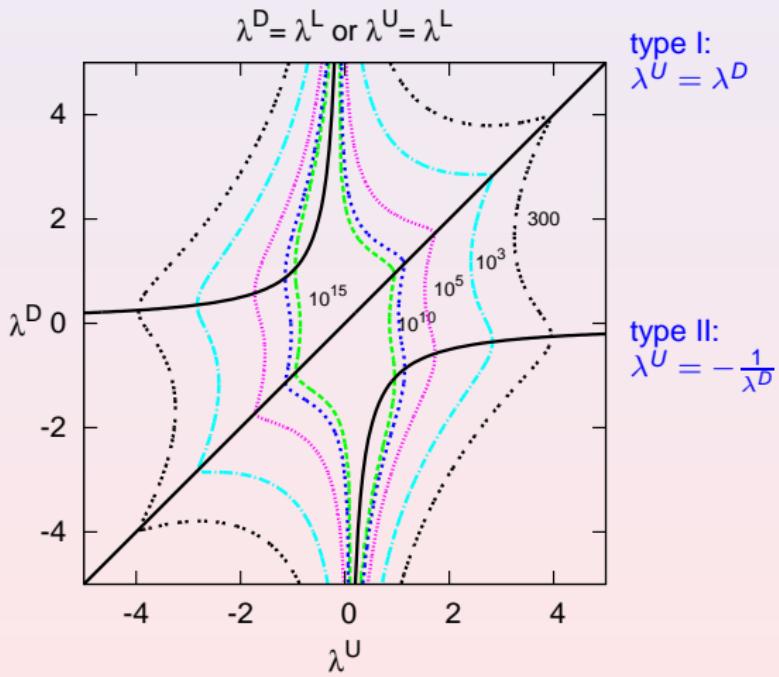
- Aligned I/II: $\lambda_{ii}^U, \quad \lambda_{ii}^D = \lambda_{ii}^L$
- Aligned III: $\lambda_{ii}^D, \quad \lambda_{ii}^U = \lambda_{ii}^L$
- Aligned IV: $\lambda_{ii}^L, \quad \lambda_{ii}^U = \lambda_{ii}^D$

- The constraints from the presence of Landau pole in three different Aligned models.

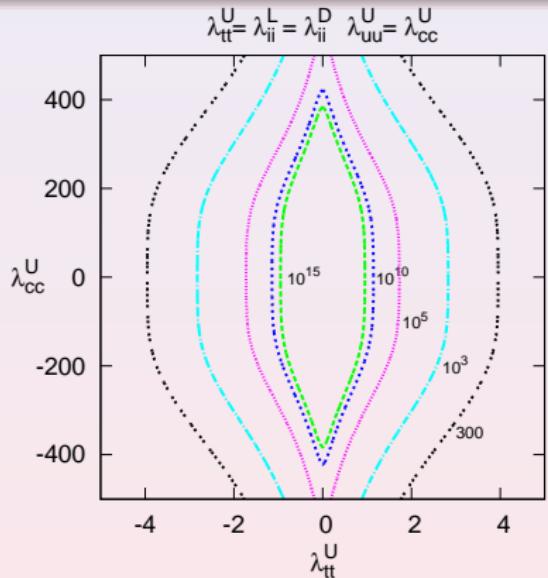


Aligned models

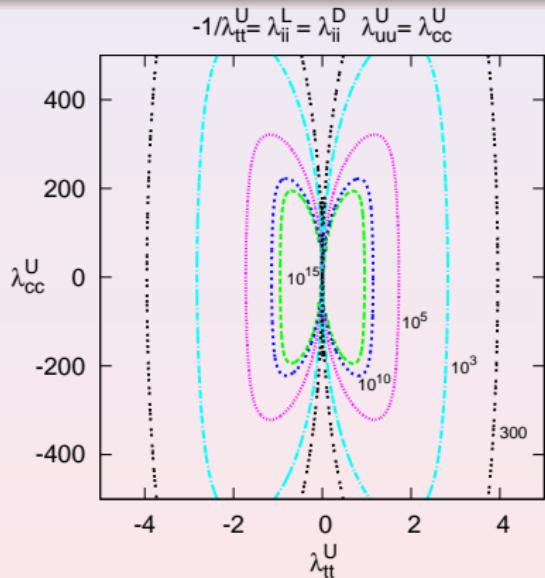
Combination of constraints from Landau pole and large non-diagonal $\lambda_{i \neq j}^F$.



Diagonal models: Z_2 -breaking in the up-sector



(a) Diagonal model I

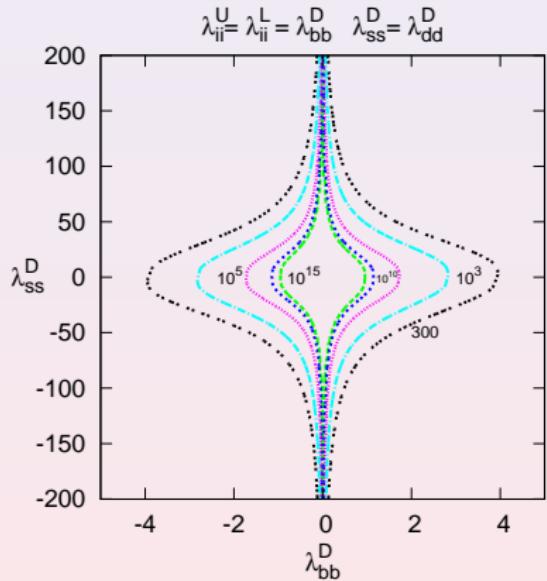


(b) Diagonal model II

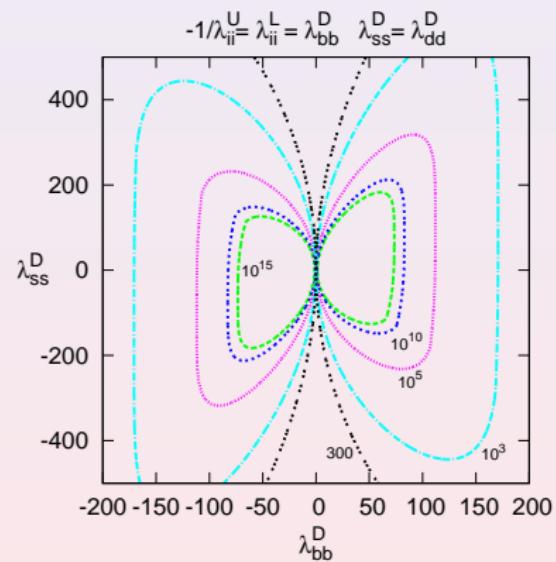
$$\lambda^U = \begin{pmatrix} \lambda_{uu} & & \\ & \lambda_{cc} & \\ & & \lambda_{tt} \end{pmatrix} \quad \lambda^D = \begin{pmatrix} \lambda_{dd} & & \\ & \lambda_{ss} & \\ & & \lambda_{hh} \end{pmatrix} \quad \lambda^L = \begin{pmatrix} \lambda_{ee} & & \\ & \lambda_{\mu\mu} & \\ & & \lambda_{\tau\tau} \end{pmatrix}$$

F. Mahmoudi, O. Stål, Phys. Rev. D81 (2010) 035016

Diagonal models: Z_2 -breaking in the down-sector



(c) Diagonal model I

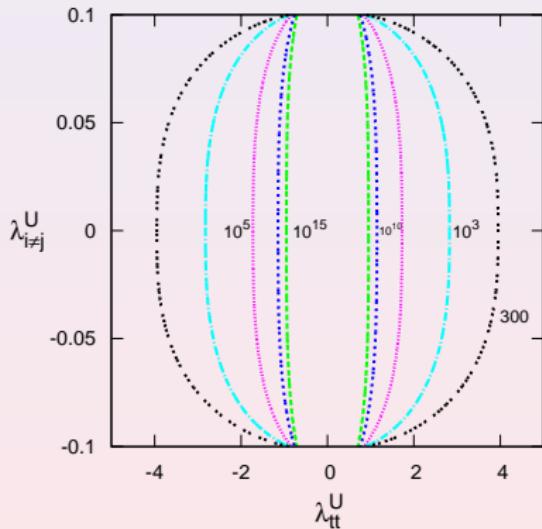


(d) Diagonal model II

$$\lambda^U = \begin{pmatrix} \lambda_{uu} & & \\ & \lambda_{cc} & \\ & & \lambda_{tt} \end{pmatrix} \quad \lambda^D = \begin{pmatrix} \color{orange}\lambda_{dd} & & \\ & \color{brown}\lambda_{ss} & \\ & & \color{red}\lambda_{bb} \end{pmatrix} \quad \lambda^L = \begin{pmatrix} \lambda_{ee} & & \\ & \lambda_{\mu\mu} & \\ & & \lambda_{\tau\tau} \end{pmatrix}$$

Non-diagonal models: Z_2 symmetry breaking at the up sector

$$\lambda_{ii}^U = \lambda_{ii}^L = \lambda_{ii}^D \quad \lambda_{i \neq i}^U \neq 0$$



(e) Non-diagonal model I

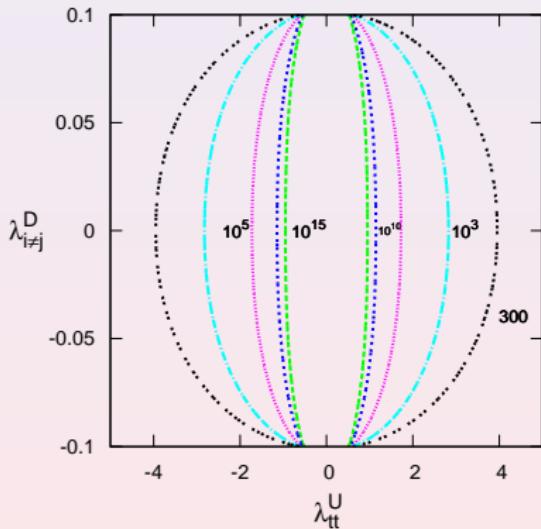
$$\lambda_{ii}^U = \lambda_{ii}^D = \lambda_{ii}^L; \quad \lambda_{i \neq j}^D = 0.$$

$$\lambda_{ii}^D = \lambda_{ii}^L = -\frac{1}{\lambda_{ii}^U}; \quad \lambda_{i \neq j}^D = 0$$

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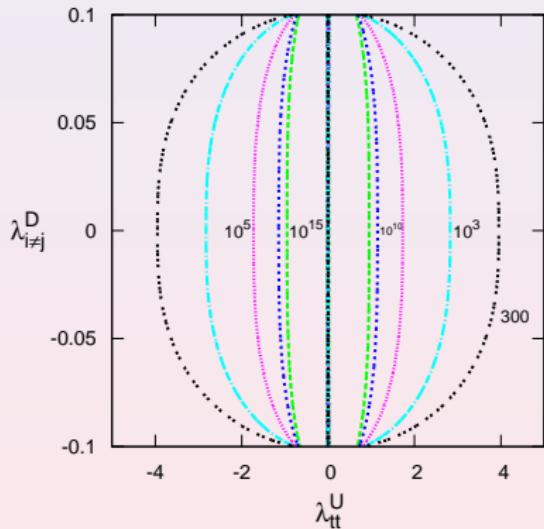
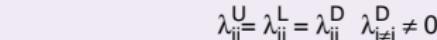
Non-diagonal models: Z_2 symmetry breaking at the down sector

$$\lambda_{ii}^U = \lambda_{ii}^L = \lambda_{ii}^D \quad \lambda_{i \neq i}^D \neq 0$$



(g) Non-diagonal model I

$$\lambda_{ii}^U = \lambda_{ii}^D = \lambda_{ii}^L; \quad \lambda_{i \neq j}^U = 0$$



(h) Non-diagonal model II

$$\lambda_{ii}^D = \lambda_{ii}^L = -\frac{1}{\lambda_{ii}^U}; \quad \lambda_{i \neq j}^U = 0$$

Conclusions

- **Z_2 -symmetric models:** constraints from avoiding a Landau-pole are very similar;
- **Aligned models:**
 - $\lambda^D/\lambda^U \lesssim 10$,
 - $-\lambda^D\lambda^U \lesssim 10$,
 - $\lambda^D \lesssim 2$ and $\lambda^U \lesssim 2$;
- For **diagonal models**, the constraints from the large nondiagonal $\lambda_{i \neq j}^F$ is stronger in the case of breaking in the **down**-sector than the **up**-sector;
- For **non-diagonal models**, the constraints are generally weak and $\lambda_{i \neq j}^D$ are much more sensitive compare to $\lambda_{i \neq j}^U$.

Backup Slides

$$\mathcal{D} \equiv 16\pi^2 \frac{d}{d(\ln \mu)}$$

$$\mathcal{D}(g_1) = \left(\frac{1}{3} + \frac{10}{9} n_q \right) g_1^3,$$

$$\mathcal{D}(g_2) = - \left(7 - \frac{2}{3} n_q \right) g_2^3,$$

$$\mathcal{D}(g_3) = - \frac{1}{3} (11N_c - 2n_q) g_3^3$$

$$\mathcal{D}(v^2) = -2\text{Tr} \left[N_c \left(\kappa_0^U \kappa_0^{U\dagger} + \kappa_0^D \kappa_0^{D\dagger} \right) + \kappa_0^L \kappa_0^{L\dagger} \right] v^2 + \left[\frac{3}{2} g_1^2 + \frac{9}{2} g_2^2 \right] v^2,$$

$$\begin{aligned} \mathcal{D}(\tan \beta) = & - \frac{1}{2 \cos^2 \beta} \text{Tr} \left[N_c \left(\rho_0^U \kappa_0^{U\dagger} + \kappa_0^U \rho_0^{U\dagger} + \kappa_0^D \rho_0^{D\dagger} + \rho_0^D \kappa_0^{D\dagger} \right) \right. \\ & \left. + \kappa_0^L \rho_0^{L\dagger} + \rho_0^L \kappa_0^{L\dagger} \right] I \end{aligned}$$

$$\begin{aligned} \mathcal{D}(\theta) = & \frac{1}{i \sin(2\beta)} \text{Tr} \left[N_c \left(\kappa_0^U \rho_0^{U\dagger} - \rho_0^U \kappa_0^{U\dagger} \right) - N_c \left(\kappa_0^D \rho_0^{D\dagger} - \rho_0^D \kappa_0^{D\dagger} \right) \right. \\ & \left. - \left(\kappa_0^L \rho_0^{L\dagger} - \rho_0^L \kappa_0^{L\dagger} \right) \right]. \end{aligned}$$

$$\begin{aligned}
\mathcal{D}(\kappa_0^U) &= -A_U \kappa_0^U + \text{Tr} \left[N_C \left(\kappa_0^U \kappa_0^{U\dagger} + \kappa_0^D \kappa_0^{D\dagger} \right) + \kappa_0^{L\dagger} \kappa_0^L \right] \kappa_0^U \\
&\quad - \frac{1}{2} \tan \beta \text{Tr} \left\{ N_C \left(\kappa_0^U \rho_0^{U\dagger} - \rho_0^U \kappa_0^{U\dagger} \right) - N_C \left(\kappa_0^D \rho_0^{D\dagger} - \rho_0^D \kappa_0^{D\dagger} \right) - \left(\kappa_0^L \rho_0^{L\dagger} - \rho_0^L \kappa_0^{L\dagger} \right) \right\} \kappa_0^U \\
&\quad + \left\{ \frac{1}{2} \left[\rho_0^U \rho_0^{U\dagger} + \rho_0^D \rho_0^{D\dagger} + \kappa_0^U \kappa_0^{U\dagger} + \kappa_0^D \kappa_0^{D\dagger} \right] \kappa_0^U + \kappa_0^U \left[\rho_0^{U\dagger} \rho_0^U + \kappa_0^{U\dagger} \kappa_0^U \right] \right. \\
&\quad \left. - 2\rho_0^D \kappa_0^{D\dagger} \rho_0^U - 2\kappa_0^D \kappa_0^{D\dagger} \kappa_0^U \right\}, \\
\mathcal{D}(\kappa_0^D) &= -A_D \kappa_0^D + \text{Tr} \left[N_C \left(\kappa_0^U \kappa_0^{U\dagger} + \kappa_0^D \kappa_0^{D\dagger} \right) + \kappa_0^L \kappa_0^{L\dagger} \right] \kappa_0^D \\
&\quad + \frac{1}{2} \tan \beta \text{Tr} \left\{ N_C \left(\kappa_0^U \rho_0^{U\dagger} - \rho_0^U \kappa_0^{U\dagger} \right) - N_C \left(\kappa_0^D \rho_0^{D\dagger} - \rho_0^D \kappa_0^{D\dagger} \right) - \left(\kappa_0^L \rho_0^{L\dagger} - \rho_0^L \kappa_0^{L\dagger} \right) \right\} \kappa_0^D \\
&\quad + \left\{ \frac{1}{2} \left[\rho_0^U \rho_0^{U\dagger} + \rho_0^D \rho_0^{D\dagger} + \kappa_0^U \kappa_0^{U\dagger} + \kappa_0^D \kappa_0^{D\dagger} \right] \kappa_0^D + \kappa_0^D \left[\rho_0^{D\dagger} \rho_0^D + \kappa_0^{D\dagger} \kappa_0^D \right] \right. \\
&\quad \left. - 2\rho_0^U \kappa_0^{U\dagger} \rho_0^D - 2\kappa_0^U \kappa_0^{U\dagger} \kappa_0^D \right\}, \\
\mathcal{D}(\kappa_0^L) &= -A_L \kappa_0^L + \text{Tr} \left\{ N_C \left(\kappa_0^{U\dagger} \kappa_0^U + \kappa_0^D \kappa_0^{D\dagger} \right) + \kappa_0^{L\dagger} \kappa_0^L \right\} \kappa_0^L \\
&\quad + \frac{1}{2} \tan \beta \text{Tr} \left\{ N_C \left(\kappa_0^U \rho_0^{U\dagger} - \rho_0^U \kappa_0^{U\dagger} \right) - N_C \left(\kappa_0^D \rho_0^{D\dagger} - \rho_0^D \kappa_0^{D\dagger} \right) - \left(\kappa_0^L \rho_0^{L\dagger} - \rho_0^L \kappa_0^{L\dagger} \right) \right\} \kappa_0^L \\
&\quad + \frac{1}{2} \left(\rho_0^L \rho_0^{L\dagger} + \kappa_0^L \kappa_0^{L\dagger} \right) \kappa_0^L + \kappa_0^L \left(\rho_0^{L\dagger} \rho_0^L + \kappa_0^{L\dagger} \kappa_0^L \right),
\end{aligned}$$

$$\begin{aligned}
\mathcal{D}(\rho_0^U) &= -A_U \rho_0^U + 2\text{Tr} \left[N_C \left(\rho_0^U \kappa_0^U U^\dagger + \kappa_0^D \rho_0^D D^\dagger \right) + \kappa_0^L \rho_0^L L^\dagger \right] \kappa_0^U \\
&\quad + \text{Tr} \left[N_C \left(\rho_0^U \rho_0^U U^\dagger + \rho_0^D \rho_0^D D^\dagger \right) + \rho_0^L \rho_0^L L^\dagger \right] \rho_0^U \\
&\quad - \frac{1}{2} \cot \beta \text{Tr} \left\{ N_C \left(\kappa_0^U \rho_0^U U^\dagger - \rho_0^U \kappa_0^U U^\dagger \right) - N_C \left(\kappa_0^D \rho_0^D D^\dagger - \rho_0^D \kappa_0^D D^\dagger \right) - \left(\kappa_0^L \rho_0^L L^\dagger - \rho_0^L \kappa_0^L L^\dagger \right) \right\} \rho_0^U \\
&\quad + \left\{ \frac{1}{2} \left[\rho_0^U \rho_0^U U^\dagger + \rho_0^D \rho_0^D D^\dagger + \kappa_0^U \kappa_0^U U^\dagger + \kappa_0^D \kappa_0^D D^\dagger \right] \rho_0^U + \rho_0^U \left[\rho_0^U \rho_0^U + \kappa_0^U \kappa_0^U \right] - 2\rho_0^D \rho_0^D \rho_0^U - 2\kappa_0^D \rho_0^D \kappa_0^U \right\}, \\
\mathcal{D}(\rho_0^D) &= -A_D \rho_0^D + 2\text{Tr} \left[N_C \left(\kappa_0^U \rho_0^U U^\dagger + \rho_0^D \kappa_0^D D^\dagger \right) + \rho_0^L \kappa_0^L L^\dagger \right] \kappa_0^D \\
&\quad + \text{Tr} \left[N_C \left(\rho_0^U \rho_0^U U^\dagger + \rho_0^D \rho_0^D D^\dagger + \rho_0^L \rho_0^L L^\dagger \right) \right] \rho_0^D \\
&\quad + \frac{1}{2} \cot \beta \text{Tr} \left\{ N_C \left(\kappa_0^U \rho_0^U U^\dagger - \rho_0^U \kappa_0^U U^\dagger \right) - N_C \left(\kappa_0^D \rho_0^D D^\dagger - \rho_0^D \kappa_0^D D^\dagger \right) - \left(\kappa_0^L \rho_0^L L^\dagger - \rho_0^L \kappa_0^L L^\dagger \right) \right\} \rho_0^D \\
&\quad + \left\{ \frac{1}{2} \left[\rho_0^U \rho_0^U U^\dagger + \rho_0^D \rho_0^D D^\dagger + \kappa_0^U \kappa_0^U U^\dagger + \kappa_0^D \kappa_0^D D^\dagger \right] \rho_0^D + \rho_0^D \left[\rho_0^D \rho_0^D + \kappa_0^D \kappa_0^D \right] - 2\rho_0^U \rho_0^U \rho_0^D - 2\kappa_0^U \rho_0^U \kappa_0^D \right\}, \\
\mathcal{D}\rho_0^L &= -A_L \rho_0^L + 2\text{Tr} \left\{ N_C \left(\kappa_0^U \rho_0^U U^\dagger + \rho_0^D \kappa_0^D D^\dagger \right) + \rho_0^L \kappa_0^L L^\dagger \right\} \kappa_0^L \\
&\quad + \text{Tr} \left\{ N_C \left(\rho_0^U \rho_0^U U^\dagger + \rho_0^D \rho_0^D D^\dagger \right) + \rho_0^L \rho_0^L L^\dagger \right\} \rho_0^L \\
&\quad + \frac{1}{2} \cot \beta \text{Tr} \left\{ N_C \left(\kappa_0^U \rho_0^U U^\dagger - \rho_0^U \kappa_0^U U^\dagger \right) - N_C \left(\kappa_0^D \rho_0^D D^\dagger - \rho_0^D \kappa_0^D D^\dagger \right) - \left(\kappa_0^L \rho_0^L L^\dagger - \rho_0^L \kappa_0^L L^\dagger \right) \right\} \rho_0^L \\
&\quad + \frac{1}{2} \left(\rho_0^L \rho_0^L L^\dagger + \kappa_0^L \kappa_0^L L^\dagger \right) \rho_0^L + \rho_0^L \left(\rho_0^L \rho_0^L + \kappa_0^L \kappa_0^L \right).
\end{aligned}$$