Shift Symmetries (Part II)
M5-instantons in F-theory and their Type IIB duals

- 1204.2551 (JHEP) with **A. Hebecker, A. Knochel**
- 1109.3454 (NPB) with **S. Krause, C. Mayrhofer**
- 1202.3138 (submitted to JHEP) with **S. Krause, C. Mayrhofer**
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Outline

Part I

Stringy origin of shift symmetries in Kähler potential of bulk matter

`see talk by Arthur Hebecker`

Part II

M5-brane instantons, G_4-fluxes and $U(1)$s in F-theory compactifications

Shift symmetries in het. orbifolds

Shift symmetry $\leftrightarrow$ **Higher-dimensional gauge symmetry**

$\Rightarrow$ pertinent to matter arising from components of higher dimensional gauge field (**bulk matter**)

Best studied example: Heterotic orbifolds $[T^2 \times T^4]/\mathbb{Z}_N$

- **Shift symmetry for matter propagating on entire T^2**

  $\leftrightarrow$ does **not** hold for **twisted states** localised at orbifold fixed points

  [Lopes,Lüst,Mohaupt'94] [Antoniadis,Gava,Narain,Taylor'95]

  [Brigonole,Ibanez,Munoz,Scheich'95]

- for B, C: vectorlike pair of states on T^2

$$K = -\ln[(T + \bar{T})(U + \bar{U}) - (B + \bar{C})(\bar{B} + C)]$$

- characteristic for recent class of heterotic orbifolds

  [Buchmüller,Hamaguchi,Lebedev,Ratz'05]

  [Lebedev,Nilles,Raby,Samos-Sanchez,Ratz,Vaudrevange,Wingerter'05], ...

CFT argument for shift symmetry

Stringy reason for shift symmetry:
form of vertex operator for flat Wilson lines [Dine,Seiberg,Wen,Witten'86]

Similar structure expected for bulk matter in Type II with D-branes descending from brane Wilson line moduli

Cleanest version of argument: 1 single **Type IIA D6-brane on sLag Σ**

[Kachru,Katz,Lawrence,McGreevy'99]

- Chiral superfield: $\Phi^{(i)} = \varphi^{(i)} + ia^{(i)}$ $\qquad i = 1, \ldots, b_1(\Sigma)$
  $\varphi^{(i)}$: normal deformation, $\qquad a^{(i)}$: Wilson line
- No contact terms of $a^{(i)}$ perturbatively in α', tree-level in g_s
- Such contact terms would involve zero-momentum vertex operator

$$V_{a^{(i)}}|_{k=0} = \int_{\partial D} a_\mu^{(i)}(X)\partial X^\mu \qquad D : \text{worldsheet disk}$$

$V_{a^{(i)}}|_{k=0}$ vanishes for top. trivial D for flat Wilson lines

Type IIA D6-branes

At leading order: $\mathbf{K}(\mathbf{\Phi}, \bar{\mathbf{\Phi}}) = \mathbf{K}(\mathbf{\Phi} + \bar{\mathbf{\Phi}})$

Sources of corrections to $K = K(\Phi + \bar{\Phi})$:

- non-pert. in α' $\leftrightarrow$ worldsheet instantons
- higher order in g_s
- couplings to boundary changing operators of type $\Phi^{(i)}\Phi_{ab}\Phi_{ba}$

Generalisation to multiple D6-branes:

- $\Phi^{(i)}$ in adjoint of $U(N)$
- $K = \mathrm{tr}(\Phi + \bar{\Phi})^2 f(S, \bar{S}) + \mathcal{O}(g_s, e^{-\Phi/\alpha'}, \Phi^3, \Phi\Phi_{ab}\Phi_{ba})$
- Structure persists upon breaking $U(N) \to G \times H$

$$N^2 \to \textstyle\sum_i (R_i, Q_i)$$

similarly to heterotic (by Wilson lines or orbifolds)

D7-branes on divisor D

[Jockers,Louis '04]

- $h^1(D)$ Wilson line moduli expected to exhibit similar behaviour
- $h^{2,0}(D)$ deformation moduli from polarisation normal to D
  $\Rightarrow$ no general CFT argument available,
  but specific examples $(K3 \times T^2/\mathbb{Z}_2)$ indicates possibility of leading order flat direction

Extra ingredient for bulk matter model building: Gauge flux

L line bundle breaks $U(N) \rightarrow G \times U(1) \Rightarrow$ bulk matter states in (R_G, q)

- Wilson line type: $H^1(D, L^q) \oplus H^1(D, L^{-q})$
- deformation type: $H^2(D, L^{-q}) \oplus H^2(D, L^q)$

Caveat:

These do not descend from universal (gauge) adjoint multiplet in 8D
$\Rightarrow$ shift symmetry structure not obvious

Summary - Shift Symmetry

✓Leading order shift symmetry $K(\Phi + \bar{\Phi})$ for bulk matter in Type IIA

✓In certain regions of moduli space expected also for Type IIB

✓Applications to bulk Higgs model builiding and brane inflation

`see talk by Arthur Hebecker`

M5-instantons

Non-perturbative superpotentials from M5-branes play important roles in F-theory

- Kähler moduli stabilisation
- generation of matter couplings including neutrino masses, μ-terms, SUSY breaking F-terms, corrections to Yukawas, ...
- interesting from formal perspective e.g. relation to U(1)s and G_4-fluxes

Recent investigations of D3/M5-instantons in context of F-theory model building include [Blumenhagen,Collinucci,Jurke'10] [Cvetič,Etxebarria,Halversson'10-12] [Donagi,Wijnholt'10] [Marchesano,Martucci'10] [Marsano,Saulina,Nameki'11] [Bianchi,Collinucci,Martucci'11] [Grimm,Savelli'11] ...

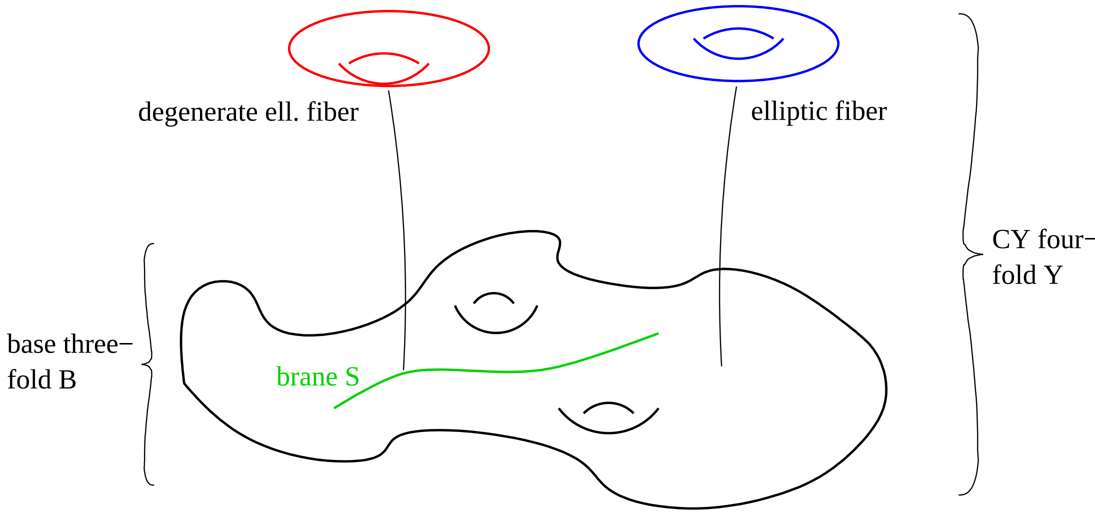
[Kerstan,TW'12] analyses

1) M5-partition function in M/F-theory and D3-instanton limit

2) M5-instantons in presence of gauge flux

M5-instantons - Setup

Elliptic fibration $Y_4 : T^2 \to B$



M5-instanton on vertical divisor $D_M = \pi^{-1} D_M^{\mathrm{b}} \subset Y_4$

Generic fiber of D_M smooth except at intersections with discriminant (7-branes)

Technical complication:

M5 hosts 2-form $\mathcal{B}$ with self-dual 3-form field strength $*\mathcal{H} = i\mathcal{H}$

see [Pasti,Sorokin,Tonin '96&'97] for first explicit action

We work with Witten's auxiliary action [Witten'96]

- $S_{M5} = 2\pi(\mathrm{Vol}_{D_M} + i \int C_6) + S_{\mathcal{B}}, \quad S_{\mathcal{B}} = -2\pi \int \left[\mathcal{H} \wedge * \mathcal{H} - 2i\, \mathcal{H} \wedge C_3^- + \frac{1}{4} C_3 \wedge * C_3 \right]$
- $*\mathcal{H} = i\mathcal{H}$ only imposed later by extracting holomorphic piece in W

Partition function - Overview

For superpotential need holomorphic piece of the partition function

$$Z_{M5} = e^{-2\pi(\mathrm{Vol}_{D_M} + i\int C_6)} \int \mathcal{D}\mathcal{B}\ e^{-S_{\mathcal{B}}}$$

- This is merely part of the partition function corresponding in IIB language to fluctuations of gauge potential along instanton
- to be supplemented by fermionic partners and deformation moduli(ni)
- assume first absence of bulk fluxes (isolated instanton)

Partition function includes classical + quantum piece (Pfaffian):

$$Z_{M5} = \sum_{\mathcal{H}_0} e^{-S_{M5}[\mathcal{H}_0]} \int \mathcal{D}\delta\mathcal{B}\ e^{-S'_{M5}[d\delta\mathcal{B},\mathcal{H}_0]}$$

Quantum piece (Pfaffian) $\leftrightarrow$ non-zero modes $\delta\mathcal{B}$

$\rightsquigarrow$ non-trivial info on bulk moduli

$\rightsquigarrow$ zeroes expected whenever instanton hits other branes   IIB: [Baumann et al'06],...

$\rightsquigarrow$ hard to compute explicitly in SUGRA/closed string channel

(cf. Type II: open string channel very helpful!)

Classical partition function

Formal evaluation of classical sum over $\mathcal{H}_0$ possible

- Expansion of $\mathcal{H}_0, C_3$ along integer symplectic basis (E_M, F^M) of $H^3(D_M)$ and summation yields [Henningson,Nilsson,Salomonson'99]

$$Z_{M5} = \sum_{\alpha,\beta=0,\frac{1}{2}} e^{-2\pi(\mathrm{Vol}_{M5}+i\int C_6)} \mathcal{Z}_-\left[{\alpha \atop \beta}\right] \mathcal{Z}_+\left[{\alpha \atop \beta}\right]$$

- $\mathcal{Z}_-\left[{\alpha \atop \beta}\right] = e^{\frac{\pi}{2} b^2 C_-^M Z_{MN}(C_-^N - C_+^N)} \times$

$$\sum_{k_M} e^{i\pi\left((k+\alpha)_M \overline{Z}^{MN}(k+\alpha)_N + 2(k+\alpha)_M(\beta^M - ibC_-^M)\right)}$$

- $\int_{D_M} E_M \wedge F^N = \delta_M^N, \quad F^N = X^{MN} E_N + Y^{MN}(*E_N),$
  $Z^{MN} = X^{MN} + iY^{MN}$

Witten ('96): The final superpotential corresponds to choice of correct spin structure α_c, β_c:

$$W_{\mathrm{cl.}} = e^{-2\pi(\mathrm{Vol}_{M5}+i\int C_6)} \mathcal{Z}_-\left[{\alpha_c \atop \beta_c}\right]$$

Finding α_c, β_c in M-theory is in general hard!

Comparison with Type IIB

[Kerstan,TW'12]

✓Quantitative match with fluxed $O(1)$ E3-instantons in Type IIB as described in [Grimm,Kerstan,Palti,TW'11]

✓sum over $\mathcal{H}$-flux $\leftrightarrow$ sum over E_3-flux $\mathcal{F}_E$

$$W_{E3}^{cl.} = \sum_{\mathcal{F}_E} e^{-S_E[\mathcal{F}_E]},$$

$$W_{E3}^{cl.} = \exp\left[-\pi\left(\frac{1}{2}C_E^\alpha \mathcal{K}_{\alpha\beta\gamma}v^\beta v^\gamma + iC_E^\alpha(c_\alpha - \frac{1}{2}\mathcal{K}_{\alpha ab}c^a b^b)\right)\right]$$
$$\times \exp\left[-\frac{i\pi}{\tau-\overline{\tau}}\delta_{MN}G^M(G^N - \overline{G}^N)\right] \sum_{\mathcal{F}^M \in \mathbb{Z}} e^{-i\pi\left(2\delta_{MN}G^M\mathcal{F}^N + \tau\delta_{MN}\mathcal{F}^M\mathcal{F}^N\right)}$$

✓allows us to fix spin structure of M5-instanton: $\alpha_M = 0 = \beta^M$

$$\mathcal{F}^M \leftrightarrow \delta^{MN}k_N \ ; \quad G^M \leftrightarrow C_-^M \ ; \quad \overline{G}^M \leftrightarrow C_+^M \ ; \quad \overline{Z}^{MN} = -\tau\delta^{MN}.$$

G_4-Fluxes - Overview

Consider next M5-instanton in presence of gauge flux

Gauge fluxes described by $\mathbf{G_4 \in H^{2,2}(Y_4)}$ with **'1 leg along fiber'**

a) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge D_b = 0$ b) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge Z = 0 \quad \forall D_i \in H^2(B)$, Z: fibre

Construction requires detailed knowledge of geometry of 4-fold Y_4

$$\mathbf{H^{2,2}(Y_4) = H^{2,2}_{\mathrm{vert}}(Y_4) \oplus H^{2,2}_{\mathrm{hor}}(Y_4)}$$

- $\mathbf{H^{2,2}_{\mathrm{vert}}(Y_4)}$ generated by elements of $H^{1,1} \wedge H^{1,1}$: factorisable fluxes

  $\Longleftrightarrow$ extra 2-forms obtained by **resolution of singularities**

  - fluxes associated with massless U(1)s [Grimm,TW '10], [Braun,Collinucci,Valandro '11], [Krause,Mayrhofer,TW'11],[Grimm,Hayashi'11]
  - extra special fluxes, e.g. 'spectral cover' fluxes [Marsano,Nameki'11][Küntzler,Nameki'12][Tatar,Walters'12]]

- $\mathbf{H^{2,2}_{\mathrm{hor}}(Y_4)}$: non-factorisable fluxes [Braun,Collinucci,Valandro '11], [Krause,Mayrhofer,TW'11]

M5-instantons and gauge flux

Flux dependence via $S_{\mathcal{B},G_4} = S_{\mathcal{B}} - 2\pi i \int \mathcal{B} \wedge \iota^* G_4$

- breaks large gauge trafos on $\mathcal{B}$ to $H^2(D_M, \mathbb{R}) \to H^2(D_M, \mathbb{Z})$
- extra integration over $H^2(D_M, \mathbb{R}) / H^2(D_M, \mathbb{Z})$ gives zero:

$$\int d\mathcal{B}\, e^{2\pi i \int \mathcal{B} \wedge \iota^* G_4} \simeq \delta(G_4)$$ [Donagi,Wijnholt'10]

$\Longleftrightarrow$ integration over charged zero modes in IIB language

$$\int \mathcal{D}\lambda_a \mathcal{D}\tilde{\lambda}_b \sum_{\mathcal{F}_E} e^{-S_E[\mathcal{F}_E]} = 0$$

- equivalently: $\iota^* G_4 \neq 0$ implies Freed-Witten anomaly

e.g. [Marsano,Saulina,Schäfer-Nameki'11; Grimm,Savelli'11]

Superpotential without 'charged matter operators' $\Rightarrow \iota^* G_4 \overset{!}{=} 0$

Quantitative criterion: **Compute** $\int_{C_{(4)}} G_4$ **with** $C_{(4)} \in H_4(D_M)$

[Kerstan,TW'12]

M5-instantons and gauge flux

[Kerstan,TW'12]

Test for $C_{(4)} \in H_4(D_M)$ with $C_{(4)}$ neither inside base nor wrapping fiber

1st type of $C_{(4)} \leftrightarrow U(1)_X$

- $U(1)_X$ from $C_3 = A_X \wedge \mathsf{w}_X \qquad \mathsf{w}_X \in H^{1,1}(\hat{Y}_4)$
- Associated with $U(1)_X$: **Gauge flux G_4** [Grimm,TW' 10]

  [Braun,Collinucci,Valandro'11] [Krause,Mayrhofer,TW '11] [Grimm,Hayashi'11]

  $$C_3 = A_X \wedge \mathsf{w}_X \Longrightarrow \mathbf{G_4 = F_X \wedge \mathsf{w}_X}, \qquad F_X \in H^{1,1}(\hat{Y}_4) \cap H^2(B_3)$$

- $[C^X_{(4)}] = -D_M \wedge \mathsf{w}_X \in H^{2,2}_{\text{vert.}}(\hat{Y}_4)$
- $q_X = \int_{C^X_{(4)}} \iota^* G_4 = -\int_{\hat{Y}_4} D_M \wedge \mathsf{w}_X \wedge G_4 \leftrightarrow U(1)_X$ shift of C_6
- This is **sensitive to linear combination of zero modes** with both brane stacks!

Example: $SU(5) \times U(1)_X$

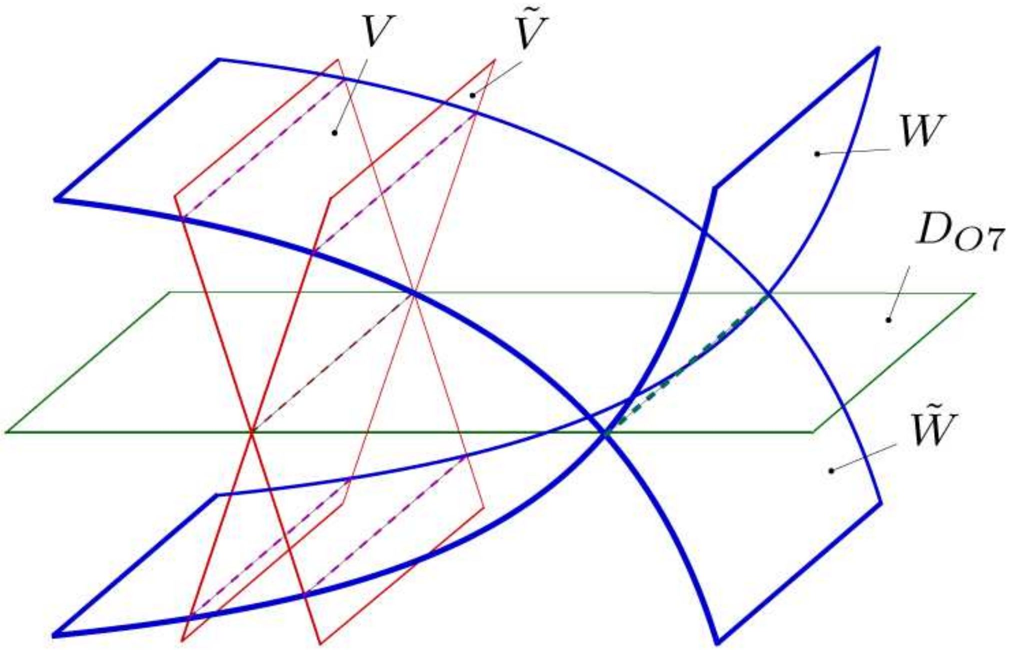
Explicit resolution of singularity yields [Krause,Mayrhofer,TW '11]

$E_i, i = 1,2,3,4 \leftrightarrow U(1)_i \subset SU(5)$ $\quad$ $\mathrm{w}_X = (S - Z - \bar{K}) + \sum_i t_i E_i \leftrightarrow U(1)_X$

Type IIB limit:

[Krause,Mayrhofer,TW '12]

- $U(5)_a \times U(1)_b$ model
- $U(1)_a$ and $U(1)_b$ massive
- massless combination: $U(1)_X = \frac{1}{5}(U(1)_a - 5U(1)_b)$
- $G_4^X = \mathcal{F} \wedge \mathrm{w}_X \leftrightarrow$ $F_a = \mathcal{F}, \quad F_b = -5\mathcal{F}$



$$q_X = \int_{C^X_{(4)}} \iota^* G_4^X \Longleftrightarrow \int_{D_E \cap D_a} \mathcal{F}_a - 5 \int_{D_E \cap D_b} \mathcal{F}_b$$

In IIB this counts weighted sum of instanton zero modes!

Fluxes in F-theory vs. Type IIB

What about further fluxes in IIB?

- D5-tadpole generically allows only 1 more flux:
  F_λ: $F_a = \frac{\lambda}{5} D_{O7}, \quad F_b = 0 \qquad \leftrightarrow \qquad$ massive $U(1)_a$
- In F-theory: global version of universal spectral cover fluxes
  $G_4^\lambda = \quad \lambda \left(E_2 \wedge E_4 + \frac{1}{5}(2, -1, 1, -2)_i E_i \wedge \mathcal{K} \right)$ [Marsano,S-Nameki '11]
- G_4^λ and G_4^X are the only possible factorisable gauge fluxes.
  These exhaust the generically possible $U(1)$ fluxes in Type IIB

Back to selection rules for M5-instanton:

We expect as many constraints as individual brane stacks

2) Proposal

- If $D_M \cap \mathcal{W} \neq 0$ **extra surfaces** exist which are **not in** $H^{2,2}_{\text{vert}}(\hat{Y}_4)$
- These are algebraic only for special complex structure.
  in different contexts: [Braun,Collinucci,Valandro'11;Collinucci,Savelli'12]

Fluxes in F-theory vs. Type IIB

Evidence:

- Suppose $D_M \cap \mathcal{W} \subset C_{10}$ (10-matter surface) $\Longrightarrow$ take surface $C_{10}|_{D_M}$
  - ✓ $\int_{C_{10}|_{D_M}} \iota^* G_4 \neq 0$ yields correct $U(1)_a$ charge in IIB limit!
  - ✓ measures effect of intersection with $SU(5)$ stack individually
- If $D_M \cap \mathcal{W}$ not in C_{10}, one can deform intersection locus, possibly on auxiliary space defined in [Kerstan,TW'12]

Conclusion:

Consistent with intuition about zero modes even in absence of IIB dual

Summary

✓ Structure of M5-instanton and fluxed E3-instanton partition function match

✓ Selection rules in presence of G_4-flux match IIB zero mode intuition

Many open questions, including

- Evaluation of Pfaffian directly in M5-picture
- Vertex operators/microscopic picture for M5-instantons
- M5-instantons with chirality inducing $\mathcal{H}$ flux?