Progress on dS and FRW holography

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Based on works with X. Dong, B. Horn, S. Matsuura, E. Silverstein

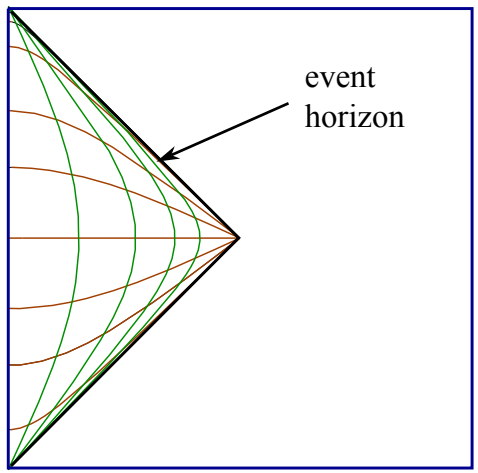
arXiv:1005.5403, 1108.5732, 1203.1680, and work in progress

String Phenomenology Institute, CERN, July 2012

The measurement of the late time acceleration of the universe is one of the fundamental discoveries of the 20th century.

Deep implications of $\Lambda_{cc} > 0$ and a de Sitter phase of exponential expansion.

★ Causal patch with event horizon:



$$R_{dS} \sim \frac{1}{H} \sim 10^{60} l_{Pl}$$

$$T_{dS} = \frac{1}{R_{dS}} \quad \text{[Hawking]}$$

$$S_{dS} = \frac{\text{Area}}{4G_N} \sim 10^{120}$$

$$QM: \quad e^{S_{dS}} = \begin{pmatrix} \text{microstates [macrosc. indistinguishable]} \\ \text{that build our causal patch} \end{pmatrix} \sim e^{10^{120}} \quad ?!$$

Basic questions in theoretical cosmology:

- What are the quantum mechanical degrees of freedom of our cosmology?
- QM role of horizon and regions outside our causal patch?
- Can we understand quantum gravity on cosmological spacetimes?

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Basic lesson from string theory: quantum gravity can be formulated as a holographic quantum mechanics theory. E.g. AdS/CFT.

$\rightsquigarrow$ **Our goal: Formulate cosmology holographically.**

In this talk we will present concrete steps towards a complete holographic duality for de Sitter and certain FRW cosmologies

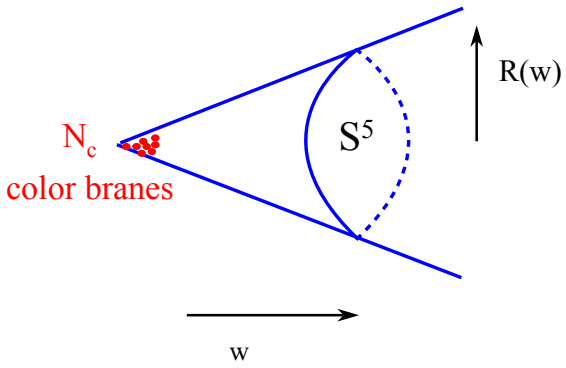
Road map for holographic formulation of cosmology:

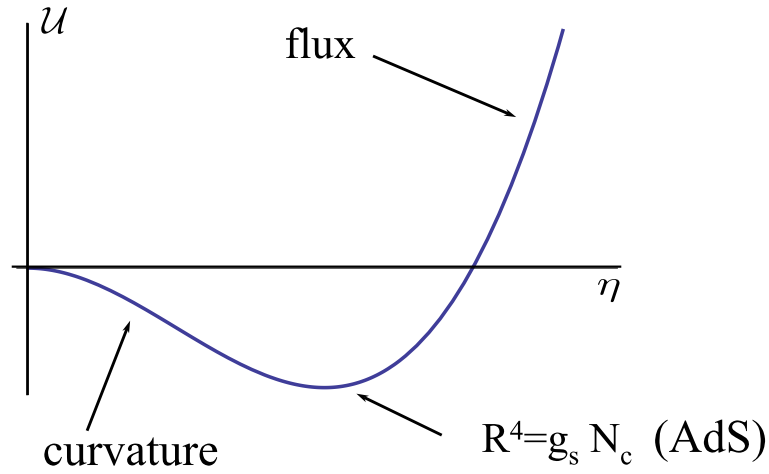
1. Uplifting AdS/CFT to de Sitter
2. Holography for FRW cosmologies
3. RG flow in time-dependent QFT

1. Uplifting AdS/CFT to de Sitter

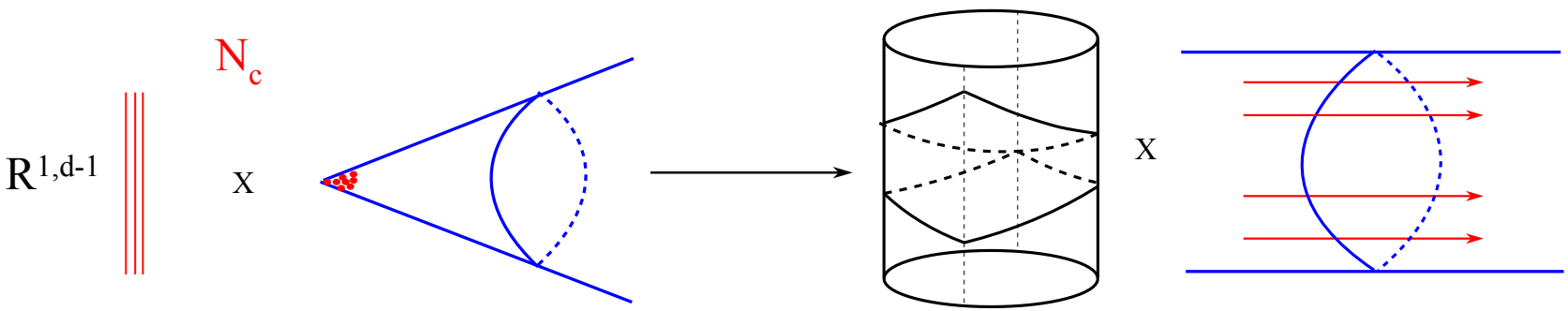
Recall basics of AdS/CFT: N_c D3-branes in ten dimensions

$$ds_6^2 = dw^2 + R(w)^2 ds_B^2$$

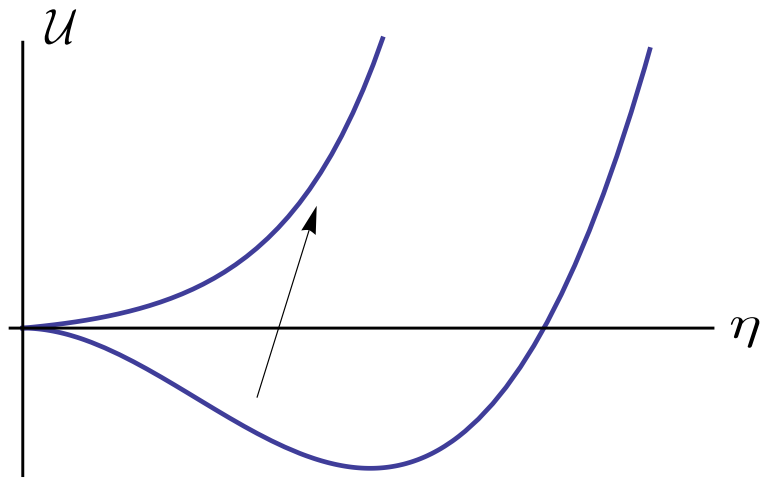




$$\frac{(dR/dw)^2}{R^2} = +\frac{1}{R^2} - g_s^2 \left(\frac{N_c}{R^5}\right)^2 \Rightarrow R^4 = g_s N_c$$



Cosmological sols from known AdS_d/CFT_{d-1} pairs by uplifting:



Need energy sources that compete w/ curvature, and such that they have a QFT interpretation in the $d-1$ holographic dual.

Examples:

- cosmic strings in 4d
- magnetic monopole as Hopf fibration of S^1 over S^2

$\Rightarrow$ magnetic flavors in dual QFT

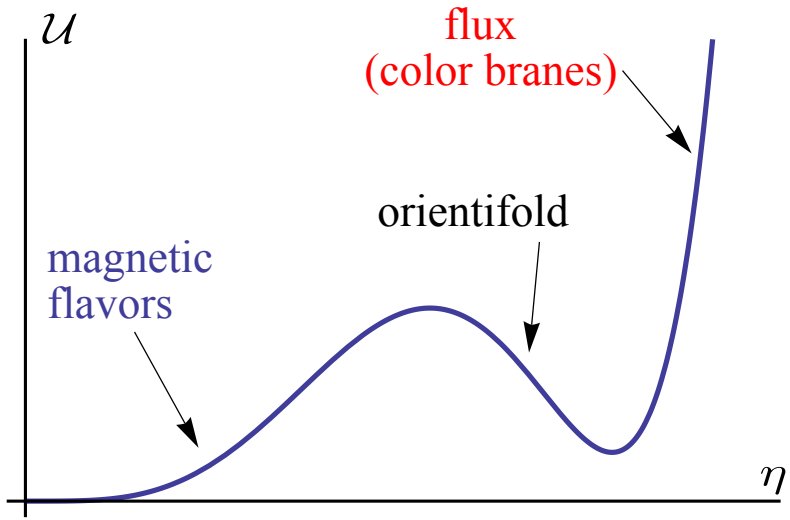
Effective theory of de Sitter

[Silverstein, ...]

String theory has three classes of energy sources:

- $\mathcal{U} \propto g_s^0$: curvature, "geometric" branes
- $\mathcal{U} \propto \pm g_s$: D-branes/orientifold planes
- $\mathcal{U} \propto g_s^2$: fluxes

A metastable de Sitter vacuum with only classical sources requires



Ten-dimensional perspective

- 1) *Background geometry* for color-branes:

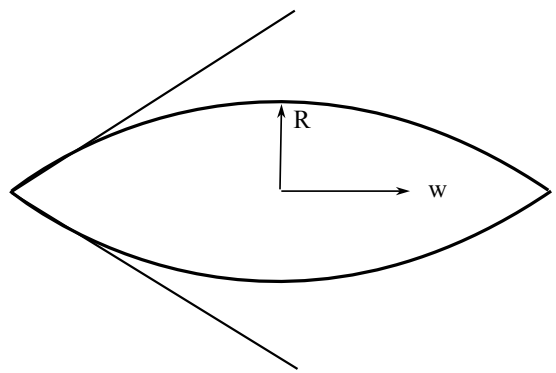
uplifted curvature + orientifold

$$ds^2 = dw^2 + R(w)^2 ds_B^2 \quad \Rightarrow \quad \frac{R'(w)^2}{R^2} \sim -\frac{1}{R^2} + \frac{g_s}{R^{n_O}}$$

with $n_O > 2$ related to the codimension of the orientifold

$\rightsquigarrow$ $R(w)$ first grows, reaches a maximum size, and then decreases

The cone of AdS/CFT has become a compact space!



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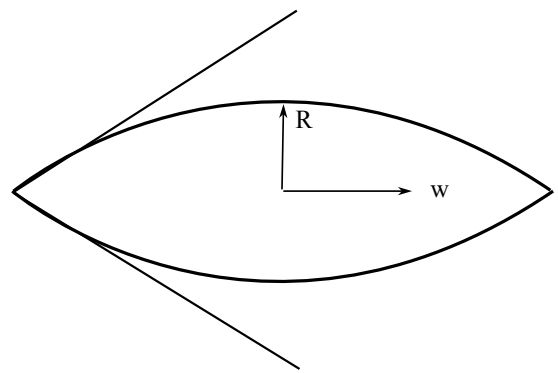
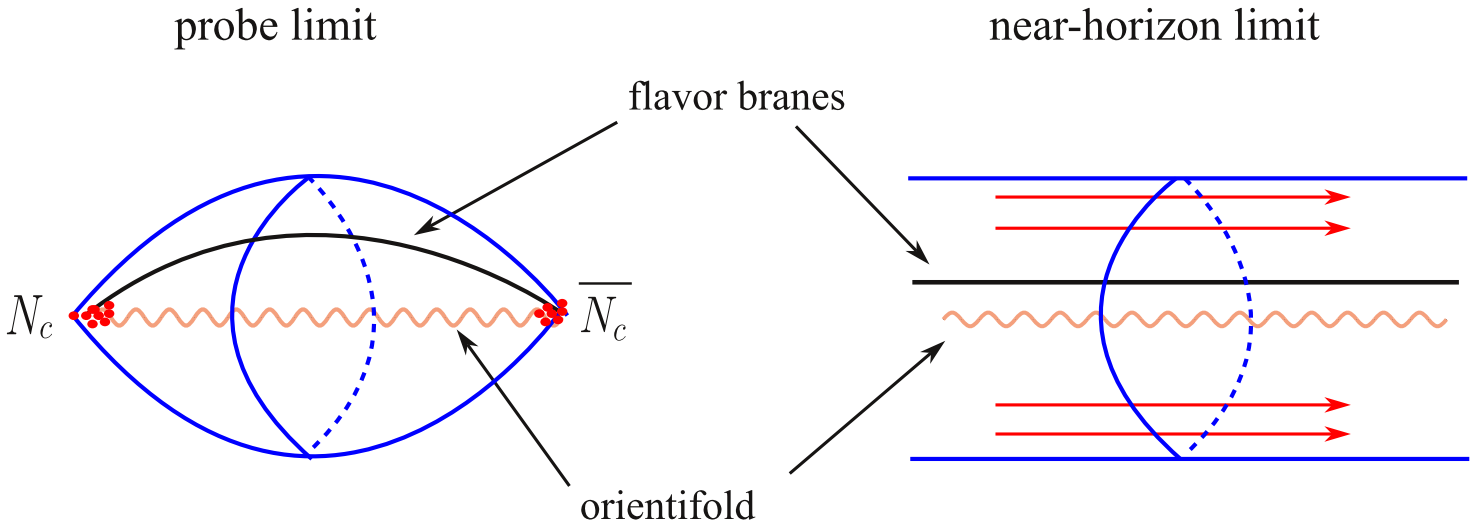


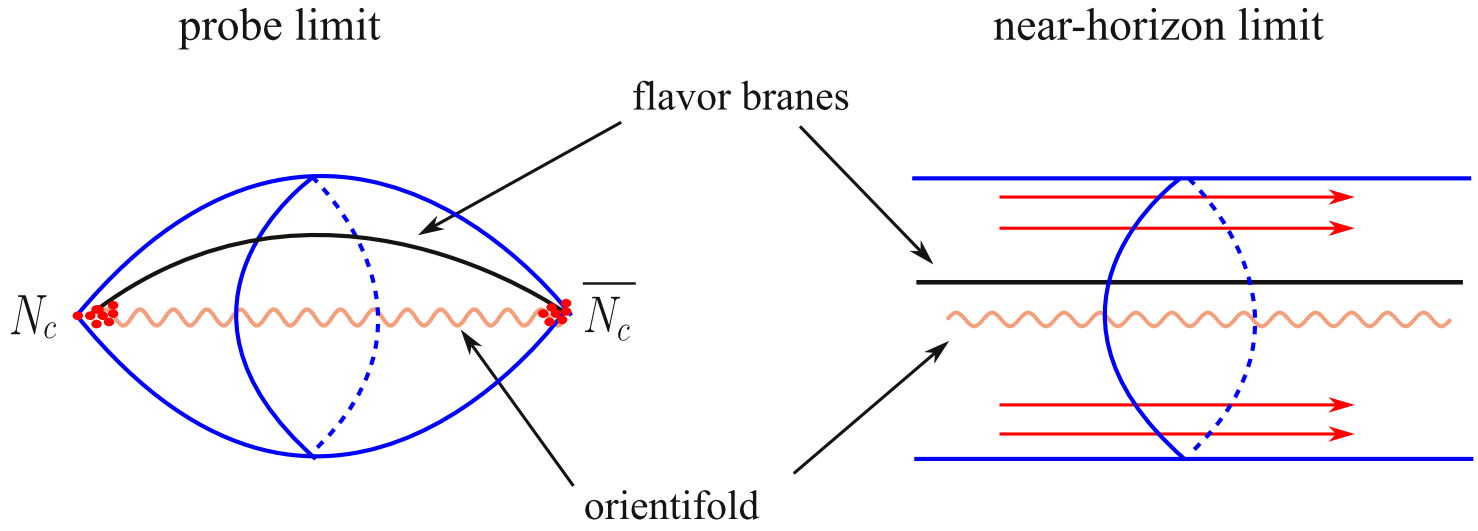
- 2) Charge conservation requires color branes at one tip and anti-branes at the other tip.



Fluxes from color-branes $\Rightarrow$ $\dfrac{R'(w)^2}{R^2} \sim -\dfrac{1}{R^2} + \dfrac{g_s}{R^{n_O}} - g_s^2 \left(\dfrac{N_c}{R^n}\right)^2$

Fluxes dominate at small R ($2n > n_O$) preventing the singularities at the tips.

- Concrete example: $dS_3 \times Y_7$ from uplifting D1-D5.
  Y_7: T^4 fibration over S^3.



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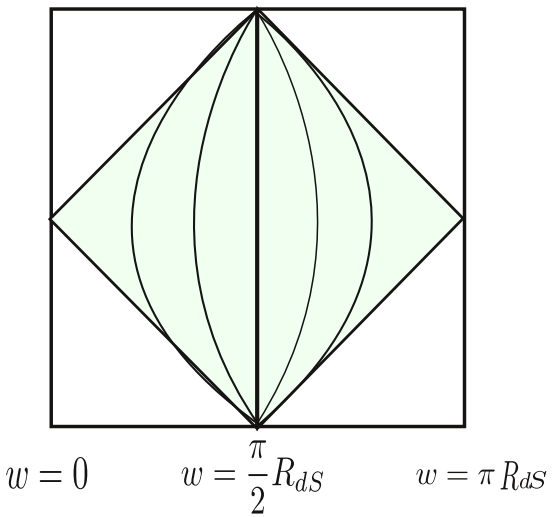
$\Rightarrow$ dS_d as two throat compactification. Microscopic d.o.f.:

Two gauge theories with magnetic flavors, cutoff $1/R_{dS}$, and dynamical gravity in $d-1$ dims.

- Realizes the static patch of dS_d sliced by dS_{d-1}

$$ds^2_{dS_d} = dw^2 + \sin^2(w/R_{dS})\, ds^2_{dS_{d-1}}$$



Two dS_{d-1} throats glued at $w = \frac{\pi}{2} R_{dS}$

$\Rightarrow$ dS/dS correspondence of
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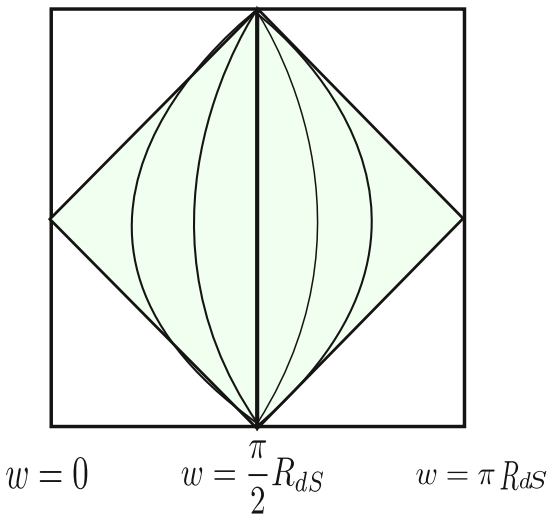

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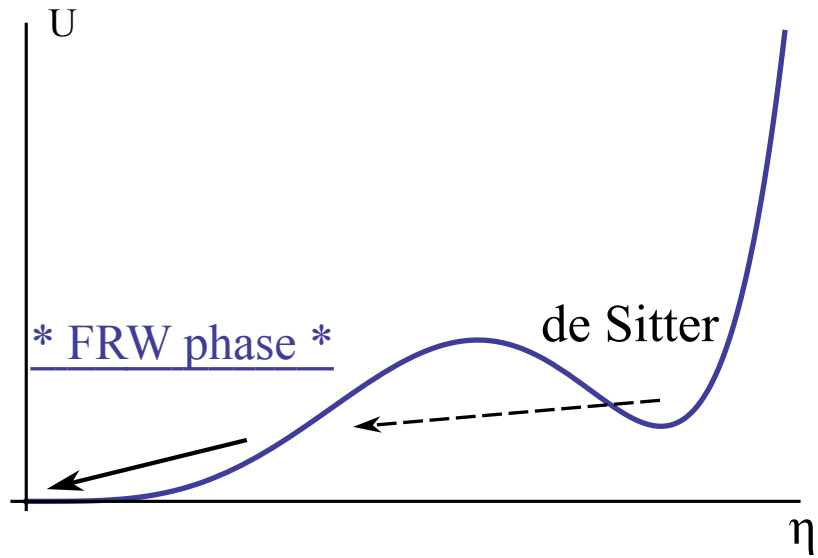
- Explanation of dS entropy in terms of gauge theory microstates:

$$\mathcal{S}_{dS} = (M_{Pl} R_{dS})^{d-2} \sim N_{dof} \text{ of dual QFT}$$

2. Holography for FRW cosmologies

Now focus on the FRW phase, where the 'uplifting' branes give the dominant contribution:



For concreteness, consider $AdS_5 \times S^5$ case.

$\rightsquigarrow$ **(p,q) 7-branes** play role of uplifting ingredient.

View S^5 as S^1_f fibered over $\mathbb{P}^2$. 7-branes wrap $AdS_5 \times S^1_f \times \Sigma_2 (\subset \mathbb{P}^2)$

♦ (p,q) 7-branes compete with curvature: they are codimension 2 and have tension $T_7 \sim 1/g_s^2$.

E.g. 24 7-branes exactly cancel the curvature of $\mathbb{P}^1$.

For n 7-branes,

$$\Delta n \equiv n - n_* \quad , \quad \mathcal{U}_R \propto \frac{\Delta n}{R^2}$$

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♦ D3-(p,q)7 gauge theory contains electric and magnetic matter from (3-7) strings [Sen; Banks, Douglas, Seiberg; Argyres, Douglas; ...]

- $\Delta n < 0$: AdS/CFT sols. are known
  [Aharony, Fayazzudin, Maldacena; Polchinski, Silverstein]
- $\Delta n \geq 0$: no static sol in the D3-(p,q)7 theory

  $\mathrm{Dim}\,\mathcal{O} <$ unitarity bound; e.g. $\mathrm{Dim}(u) = \frac{12}{12-n}$ in SW

However: consistent time-dependent cosmologies for $\Delta n > 0$!

Cosmological 10d solution

Late time solution for $\Delta n > 0$, sourced by magnetic flavor branes (color flux subdominant), in string frame:

$$ds_s^2 = -dt_s^2 + \frac{t_s^2}{c^2} dH_4^2 + \frac{t_s^2}{c^2} dB_4^2 + dx_f^2 \quad , \quad c^2 = \frac{7}{3}$$

with B_4 is a compact 4-dim hyperbolic space.

- Internal space: S^5+ 7-branes $\Rightarrow$ $\frac{t_s^2}{c^2} dB_4^2 + dx_f^2$
- 5d spacetime: open FRW (instead of AdS_5)

More general set of FRW solutions:

$$ds_s^2 = -dt_s^2 + \frac{t_s^2}{c^2} dH_{d-1}^2 + \frac{t_s^2}{\hat{c}^2} dB_{2m}^2 + dx_f^2$$

Goal: using this concrete solution, set up the holographic dictionary for FRW.

Warped solution

Basic requirement for holographic dual: warped region that redshifts energies. [Maldacena]

Focus on d-dim part and go to Einstein frame metric + change vars ...

$$ds_d^2 = c^2 \left(T^{2/c} - w^2\right)^{c-1} dw^2 + \left(1 - \frac{w^2}{T^{2/c}}\right)^{c-1} \left(-dT^2 + c^2 T^2 \, dH_{d-2}^2\right)$$

- $(d-1)$ dual lives on $ds_{d-1}^2 = -dT^2 + c^2 T^2 \, dH_{d-2}^2$
- UV slice $w = 0$, two IR regions $w \to \pm T^{1/c}$:

$$E(w, T) = \left(1 - \frac{w^2}{T^{2/c}}\right)^{\frac{c-1}{2}} E_{pr} \ll M_{Pl}$$

- Dual QFT has time-dep couplings and nontrivial RG.

Properties of the holographic dual

- 1) $(d-1)$-dimensional gravity: $M_{Pl}^{d-1} \sim T$

At finite times, dual has propagating gravity, but as $T \to \infty$ gravity decouples!
Suggests a precise QFT description of FRW physics at late times.

- 2) Field-theoretic degrees of freedom: $\Lambda_c \sim 1/T$, $N_{\text{dof}} \sim T^{d-2}$

$\rightsquigarrow$ System has finite cutoff and accumulates d.o.f. per lattice point.
Agrees with parametric counting of magnetic flavors.

- 3) Correlation functions: semicl, $\langle \mathcal{O}(x)\mathcal{O}(x') \rangle \approx \exp\left[-S(x,x')\right]$

Geodesics become shorter by moving along the radial direction.

$\rightsquigarrow$ Power-law correlator for KK modes in time-dependent QFT

3. RG flow in time dependent QFT

In our FRW construction, time-dependence allows for $n_{\text{mag}} - n_* > 0$ and leads to new scale invariant regimes.

[Recall $n_{\text{mag}} - n_* > 0 \;\Rightarrow \Delta_{\mathcal{O}} < \Delta_{\text{unitary}}$ in static theory]

QFT question of more general relevance:

How do time-dependent couplings affect the IR dynamics of QFTs?

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Tractable t-dep QFT: large-N CFT plus scalar field

- $L = L_{CFT} - \frac{1}{2}\left((\partial\phi)^2 + m^2\phi^2\right) + g(t)\phi\mathcal{O}_{CFT}$

$\Rightarrow \quad L = L_{CFT} + \frac{g(t)^2}{2m^2}\mathcal{O}^2_{CFT}$ for $E \ll m$

Choose dim. of $\mathcal{O}$ at $g = 0$ fixed pt. by $\Delta_{\mathcal{O}} = d/2 + \nu$, $(\nu > 1)$.

- **Static case, $g = g_0$**
  - requiring that $g_0 \phi \mathcal{O}$ be marginal in the IR sets

$$\Delta_\phi = d - \left(\frac{d}{2} + \nu\right) = \frac{d}{2} - \nu \quad \Rightarrow \quad \text{violates unitarity!}$$

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- **t-dep case, $g \to g_0 t^\alpha$**
  - no effect for $E \gg \partial g / g$
  - for $\alpha > \nu$, $g(t)$ dominates the IR, reversing direction of flow
  - two-point functions can be calculated exactly at large-N

$$\langle \phi(x)\phi(x') \rangle \to \frac{\text{const}}{t^\alpha |x - x'|^{d-2\nu} t'^\alpha} \quad \Rightarrow \quad \Delta_{IR}(\phi) = \frac{d}{2} - \nu + \alpha$$

4. Conclusions and future directions

- We have constructed a holographic duality for de Sitter and certain FRW cosmologies.
- Holographic description in terms of two coupled QFTs with magnetic flavors. Explains entropy of cosmological sols.
- Time-dependent couplings can strongly affect the IR physics and induce novel scale invariant regimes

- Central role played by magnetic flavors. More general relevance for cosmological sols?
- Develop further the holographic description, with time-dependent and running couplings.
- Construct realistic dS4 solutions.