An invitation to discrete gauge symmetries in string theory

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inspired on:
M.Berasaluce-González, L. Ibáñez, P. Soler, AU, arXiv:1106.4169
B. Schellekens, L.Ibáñez, AU, arXiv:1205:5364
M. Berasaluce-González, P. G. Cámara, F. Marchesano, AU, arXiv:1206.2383

String Phenomenology TH Institute, July 2012

Discrete symmetries in BSM and QG

- Discrete symmetries are a key ingredient in SM and a useful tool in BSM physics
cf. several talks

  R-parity or similar to prevent fast p-decay in MSSM

  Yukawas and flavour physics

- Violated in quantum gravity, unless they are gauge

Banks, Dixon '88

  • Microscopic arguments in string theory

  • General black hole arguments

  Key point is no global symmetry hair

  however, gauge charge can be measured at infinity

- Also true for discrete gauge symmetries (strings can lasso BH)

- Status of discrete symmetries in string theory?

  Focus on exact gauge discrete symmetries

  Preserved even by instantons or any non-pert. effect, QG,...

Prototype: Zn gauge symmetry

- Realize Z_n as U(1) Higgssed by field of charge n [many refs...]

  Lagrangian for gauge field and phase of scalar field

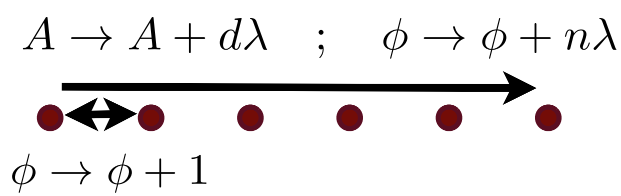
$$|\partial_\mu \phi - n A_\mu|^2$$

  Periodic axion $\phi \simeq \phi + 1$ defines lattice Γ

  Gauge transformation $A_\mu \to A_\mu + \partial_\mu \lambda \quad ; \quad \phi \to \phi + n\lambda$

  Embedding of U(1) S^1 into axion S^1 with winding n

  Defines lattice Γ'=nZ



Discrete gauge symmetry: field identifications not implemented by U(1) gauge transformations

$$\frac{\Gamma}{\Gamma'} = \mathbf{Z}_n$$

Generalizes to multiple U(1), non-abelian, etc

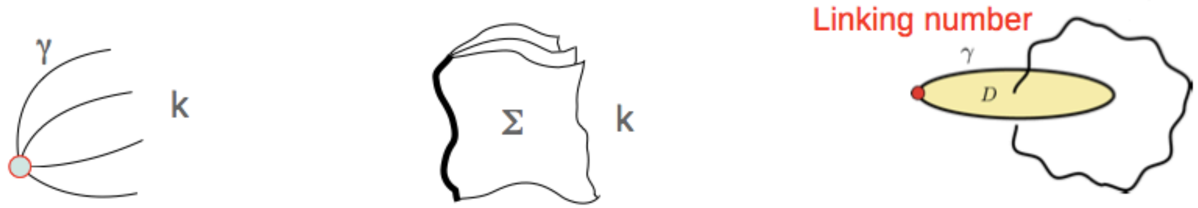
Understanding discrete gauge symmetries

- Charged particles and strings (conserved mod n)

  - Strings defined by holonomy given by scalar field shift (and coupling to dual 2-form B_2)

  - Particles defined by integral of H_3 on surrounding S^3 (and coupling to A_1)

$\mathcal{O}_{\text{particle}} \sim e^{2\pi i n \int_\gamma A}$ $\qquad$ $\mathcal{O}_{\text{string}} \sim e^{2\pi i m \int_\Sigma B_2}$ $\qquad$ from Pablo´s talk



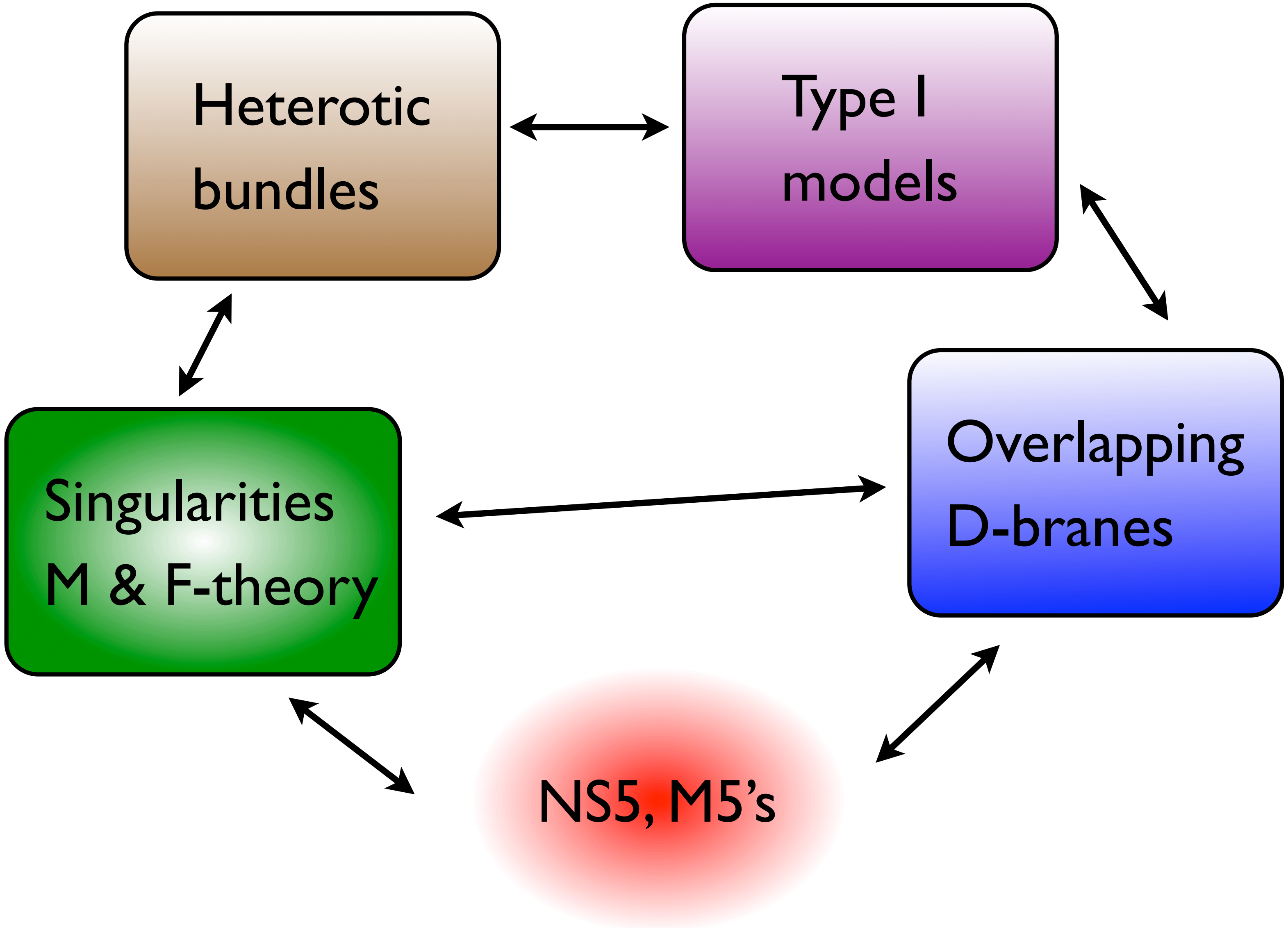
$e^{-2\pi i \phi} e^{2\pi i k \int_\gamma A}$ $\qquad$ $e^{-2\pi i \int_{\partial\Sigma} A} e^{2\pi i k \int_\Sigma B_2}$ $\qquad$ $\exp\left[2\pi i \frac{nm}{k} L(\Sigma,\gamma)\right]$

- Selection rules for couplings: Non-perturbative, Yukawas,...

- Structure of the underlying gauging: who are the actors?

Overview (partial) understandings in several realizations

Road map of (continuous) gauge symmetries



Analogous global view for discrete gauge symmetries?

Higgsing continuous symmetries by vevs

Perhaps simplest, perhaps not...

- Often done in heterotic upon taking D-flat directions

  Field theoretical: Lacks glamour?

- Actually occurs in pretty glamorous setups

  Type I tachyon condensation — Sen; Witten

  Z_2 symmetry with particles/"strings" being non-BPS D0's D7's

  Remnant of gauge symmetry on brane-antibranes Higgsed by tachyon (analogous to Higgsing SU(2) by a triplet, leaves Z_2 charged doublets)

  Brane positions

  U(n) on coincident branes broken by adjoint vevs has a Weyl group remnant, which makes branes indistinguishable

  Gluing morphism — Donagi, Wijnholt

- Could be useful to think field theory effects in stringy terms

  However we take a different route

U(1)'s from p-forms

Compactifications of p-forms on spaces with torsion homology

- Ex: M-theory on 7d space (e.g. G2) X with $H_2(X,Z)=H_4(X,Z)=Z_k$

  M2's on 2-cycles are Z_k particles

  M5's on 4-cycles are Z_k strings

- Gauging manifest using non-harmonic forms (massive U(1))

  Ex: $d\omega_2 = k\beta_3$ — Cámara, Ibáñez, Marchesano

  KK reduction
  $$C_3 = A_1(x^\mu) \wedge \omega_2(y^m) + \phi(x^\mu)\, \beta_3(y^m)$$
  $$G_4 = (d\phi - kA_1) \wedge \omega_2 + \ldots$$

- Non-abelian if torsion classes have relations — cf. Pablo's talk

  E.g. Δ_{27} in $AdS_5 \times S^5/Z_3$ — Gukov, Rangamani, Witten '98

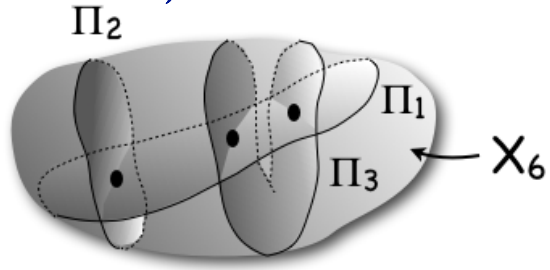
  H_k in $AdS_5 \times Y_5/Z_k$ — Burrington, Liu, Pando-Zayas '06

U(1)'s from D-branes

- The Zk lagrangian can be dualized to a coupling $kB_2 \wedge F_2$
- BF couplings for D-branes from Chern-Simons action,

e.g for D6´s on 3-cycles

$$B_2^k = \int_{[\alpha_k]} C_5 \quad , \quad [\Pi_A] = \sum_k r_A^k [\alpha_k]$$



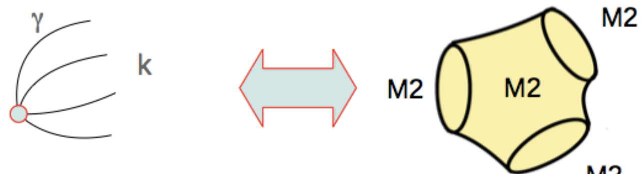
For $Q = \sum_A c_A Q_A \Rightarrow \left(\sum_A c_A N_A r_A^k\right) B_k \wedge F$ cf. Schelleken's talk

Zn gauge symmetry iff $\sum_A c_A N_A r_A^k = 0 \bmod n, \text{ for all } k$

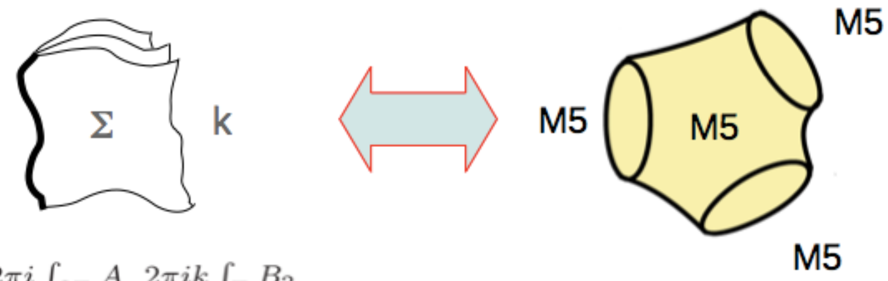
Zk charged particles are open strings at intersections

- Actually, lift to M-theory on G2 with torsion classes

instantons are M2 on 3-chains, emit torsion M2's (Zk particles)



$e^{-2\pi i\phi} e^{2\pi i k \int_\gamma A}$



$e^{-2\pi i \int_{\partial\Sigma} A} e^{2\pi i k \int_\Sigma B_2}$

Discrete isometries: CYs and orbifolds

- CYs have no continuous isometries, but admit discrete ones

  e.g. freely acting discrete isometries used for non-trivial Π_1

  charged particles are momenta, charged strings are Taub NUTs (!)

  Hard to interpret as gauging unless underlying U(1) is displayed

  Try some simplified setups

- Toroidal orbifolds Scrutinized in heterotic setup Nilles, Raby, Ratz, ...

  Discrete symmetries from worldsheet selection rules

  Point group Non-geometric "quantum" symmetry
  p-th twisted sector has charge p under Z_N
  (relates to NSNS field in Gukov-Rangamani-Witten)

  H-momentum Rotations, R-symmetry: interesting but hard
  (but underlying gauge bosons?)

  Space group Discrete translations (plus point group)

Discrete isometries: twisted tori

Go beyond CYs: Twisted tori

- Twisted torus: S^1 fibered over T^2 with Chern class N

- Isometries along T^2 break to Z_N and commute to fiber U(1) transf.
- Construct as a coset of a continuous Heisenberg group by a discrete subgroup

Resulting discrete isometry group $H_N=Z_N$ "x" Z_N "x" U(1)

- Twisted torus: T^3 with non-trivial geometric flux

U(1) transf. corresponding to T^2 translations gauge some of the toroidal moduli — Kaloper, Myers

Realize the discrete gauge symmetry as subgroup of U(1)^3 in language of gauging

- Interesting to generalize to other fluxes (all are gaugings)

Magnetized branes below are another particular example

Discrete isometries and D-branes (I)

Or even simpler: tori with defects

- **Circle compactification with localized O-plane**

  "Gauged U(1)" is massive KK replica of KK gauge boson

  Momentum conserved mod 2: Z_2 symmetry

- **Circle compactification with localized D-brane**

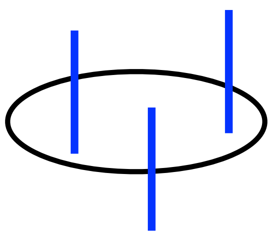
  Worldvolume scalar is brane position.

  Shifts upon translation in circle (gauge transf. of KK gauge boson)

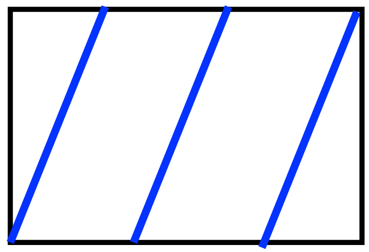
  Gauging description of the breaking of traslational invariance

  (T-dual to wrapped brane: gauging of Wilson line by B-field)

Z_k symmetries obtained by symmetric distribution of k branes
(effectively reduces the period of the worldvolume scalar)



Not so strange... ⇒



Discrete isometries and D-branes (II)

- **Most interesting: magnetized branes on tori**
  Translations are broken to ZN, by eating up brane Wilson lines, and commute to Dbrane U(1) gauge transformation (which may itself be broken by BF couplings) cf Pablo's talk

- Consider a T^2 with a U(1) gauge field background

$$F_2 = 2\pi M dx \wedge dy \quad \Rightarrow \quad A = \pi M(xdy - ydx)$$

- Magnetization breaks translational symmetries

$$A(x+\lambda_x, y) = A(x,y) + \pi M \lambda_x dy$$
$$A(x, y+\lambda_y) = A(x,y) - \pi M \lambda_y dx$$

and need to be compensated with a U(1) gauge trasformation

$$\psi(x,y) \to e^{-i\pi q M \lambda_x y}\psi(x+\lambda_x, y) = e^{q\lambda_x X}\psi(x,y)$$
$$\psi(x,y) \to e^{i\pi q M \lambda_y x}\psi(x, y+\lambda_y) = e^{q\lambda_y Y}\psi(x,y)$$

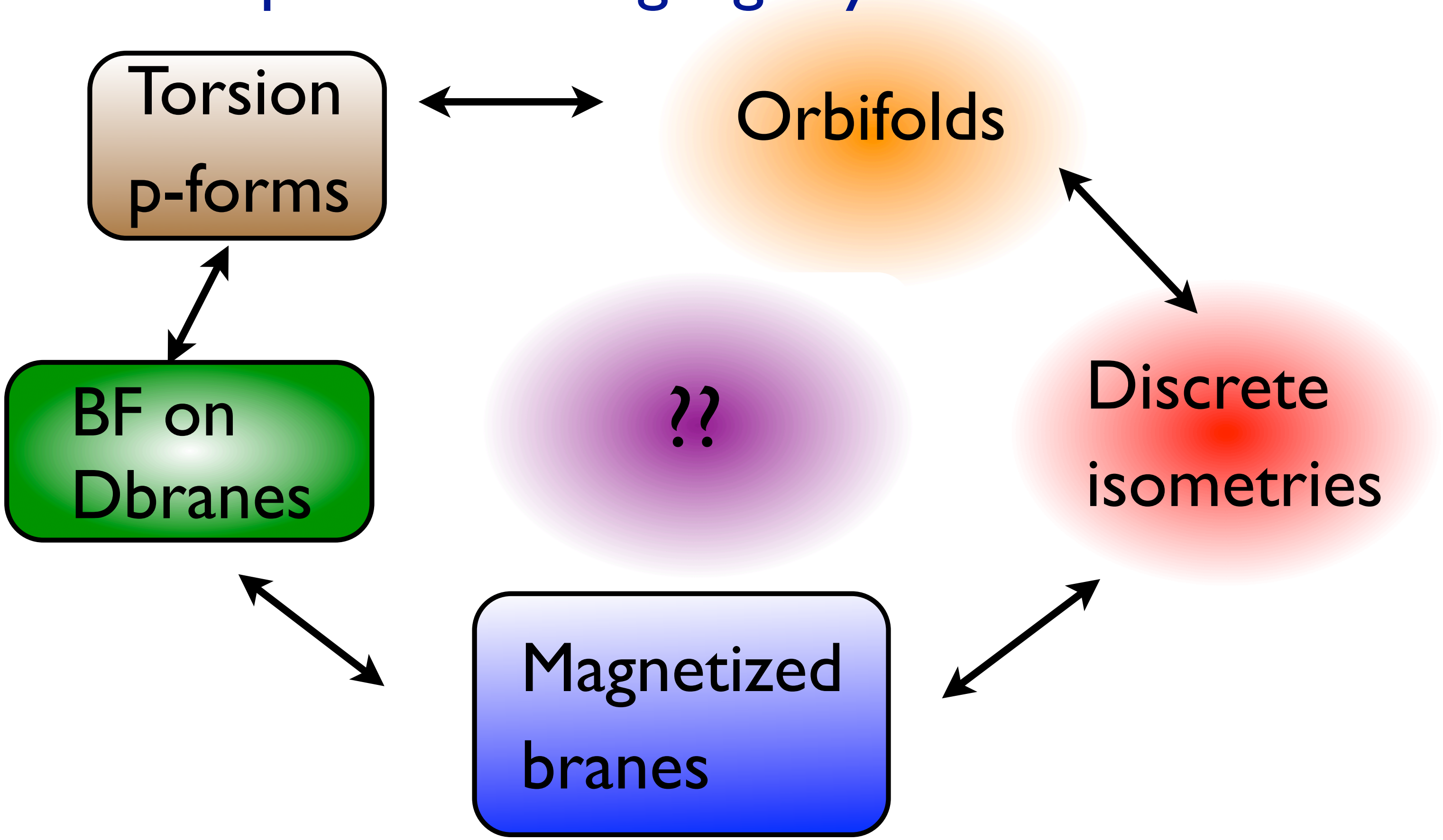
$$X = \partial_x - i\pi M y\ , \quad Y = \partial_y + i\pi M x \qquad [X,Y] = MQ$$

Discrete Heisenberg groups

explaining empirical obs. by Cremades, Ibáñez, Marchesano; Abe, Choi, Kobayashi, Ohki

Conclusions

- Partial road map of discrete gauge symmetries



- Focused on geometric side, rather than CFT

  (e.g. orbifolds or Gepners)

- Things becoming clearer, yet much work remains...