Backgrounds for Heterotic Moduli Stabilization

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with M. Larfors And D. Lüst: arXiv:1205.6208

with L. Anderson, A. Lukas and B. Ovrut: arXiv: 1107.5076
1010.0255
and to appear.

SU(3) Structure Backgrounds:

- Consider compactification on a six manifold admitting an SU(3) structure.

Torsion classes:

$$\begin{aligned} dJ &= -\frac{3}{2}\mathrm{Im}(W_1\overline{\Omega}) + W_4 \wedge J + W_3 \\ d\Omega &= W_1 J \wedge J + W_2 \wedge J + \overline{W}_5 \wedge \Omega \end{aligned}$$

- SU(3) Holonomy: Calabi-Yau $W_i = 0 \; \forall \; i$
- SU(3) Structure $\mathcal{N} = 1$ vacuum: Strominger System

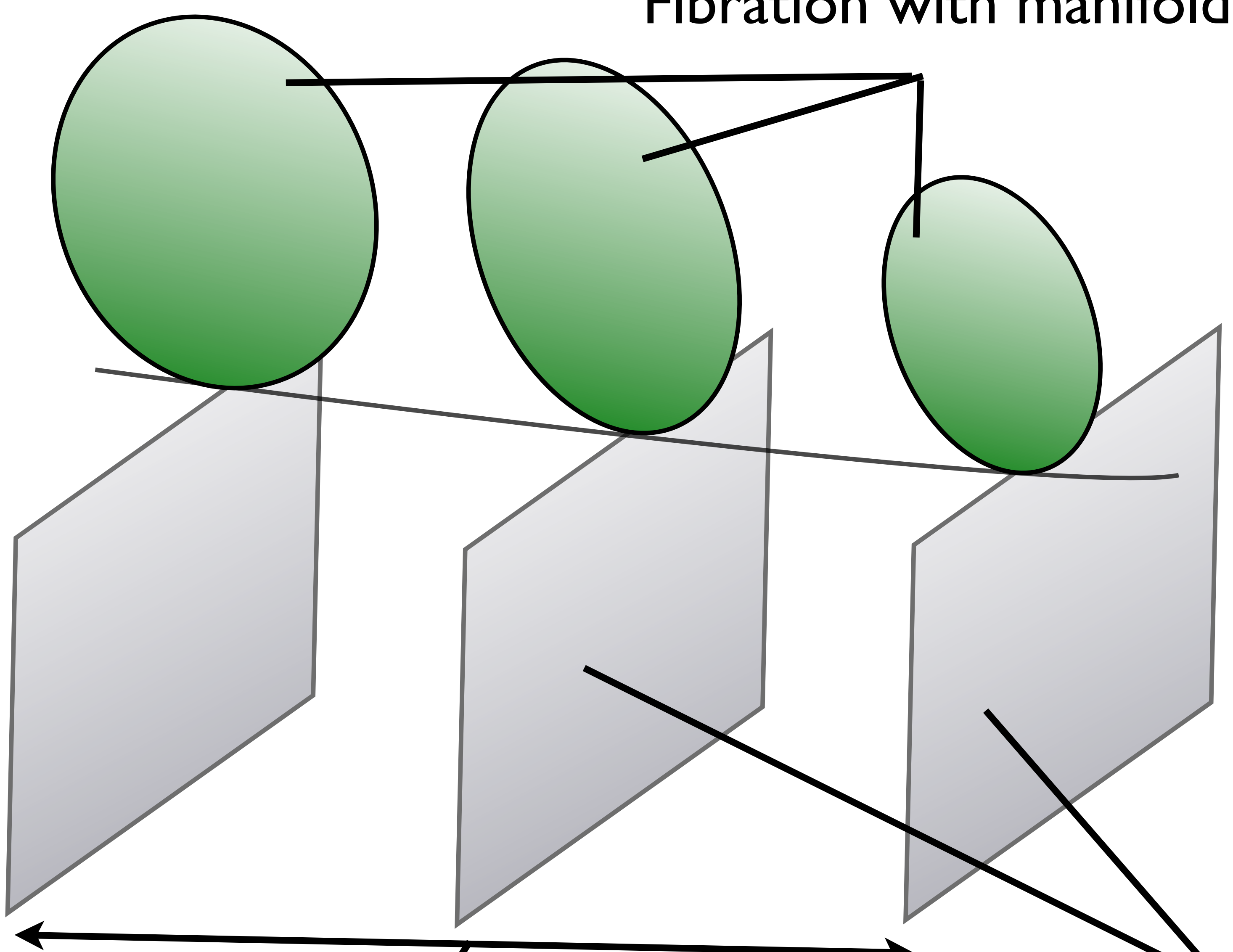
$W_1 = W_2 = 0 \qquad W_4 = \frac{1}{2}W_5 = d\hat{\phi}$ Lopes et al: hep-th/0211118

- SU(3) Structure $\mathcal{N} = 1/2$ vacuum: Generalized half-flat

$W_{1-} = W_{2-} = 0 \quad W_4 = \frac{1}{2}W_5 = d\hat{\phi}$ Lukas et al: hep-th/1005.5302

We will add extra fluxes to the analysis, and provide solutions for the supergravity fields.

The setup:



Metric and associated field ansatzes

$$ds^2_{10} = e^{2A(x^m)}\left(ds^2_3 + e^{2\Delta(x^u)}dydy + g_{uv}(x^m)dx^u dx^v\right)$$

$$H_{\alpha\beta\gamma} = f\epsilon_{\alpha\beta\gamma} \qquad H_{\alpha mn} = H_{\alpha\beta n} = 0 \qquad \partial_\alpha\hat{\phi} = 0$$

- Three dimensional space is maximally symmetric.
- New fluxes: f and H_{yuv}
- Gravitino variation in x^α directions

$$\Longrightarrow A(x^m) = \text{constant}$$

- Define $\Theta = d\Delta$

The Killing spinor equations and Bianchi Identities become...

Consistency at fixed y

$$J \wedge dJ = J \wedge J \wedge d\hat{\phi} \;\;,\;\; d\Omega_- = 2d\hat{\phi} \wedge \Omega_- - e^{-\Delta} * H_y - \frac{1}{2} f J \wedge J \;\;,$$

$$0 = \frac{1}{2} * f - \Omega_+ \wedge H - \frac{1}{2} e^{-\Delta} H_y \wedge J \wedge J \;\;,\;\; e^{\Delta} * d\hat{\phi} = \frac{1}{2} H_y \wedge \Omega_- - \frac{1}{2} e^{\Delta} H \wedge J \;\;,$$

$$dH = 0 \;\;,\;\; d(*e^{-2\hat{\phi}-\Delta} H_y) = 0 \;\;,\;\; df = 0$$

Flow eqns

$$J \wedge J' = e^{\Delta} d\Omega_+ - \frac{1}{2} e^{\Delta} * (H \wedge \Omega_-) J \wedge J - 2e^{\Delta} d\hat{\phi} \wedge \Omega_+ - e^{\Delta} \Omega_+ \wedge \Theta$$

$$\Omega'_- = e^{\Delta} dJ - e^{\Delta} * (H \wedge \Omega_-) \Omega_- - 2e^{\Delta} d\hat{\phi} \wedge J + e^{\Delta} J \wedge \Theta - *He^{\Delta} - f e^{\Delta} \Omega_+$$

$$\hat{\phi}' = -\frac{1}{2} e^{\Delta} * (H \wedge \Omega_-)$$

$$H' = dH_y \;\;,\;\; (*e^{-2\hat{\phi}-\Delta} H_y)' = -d * (e^{-2\hat{\phi}+\Delta} H) \;\;,\;\; f' = 0$$

reduces correctly to previous cases.

Rewrite fluxes and y derivatives

Helps with solving equations in a construction independent manner

$$
\begin{aligned}
H &= A_{1+}\Omega_+ + A_{1-}\Omega_- + A_{2+}\wedge J + A_{3+} \\
H_y &= B_1 J + B_2 + B_{3+}\ .
\end{aligned}
$$

such that

$$
\begin{aligned}
A_{3+}\wedge\Omega_\pm &= 0 \\
A_{3+}\wedge J &= 0 \\
B_2\wedge J\wedge J &= 0\ .
\end{aligned}
$$

and write:

$$
\begin{aligned}
J' &= \gamma_1 J + \gamma_{2+} + \gamma_3 \\
0 &= \gamma_{2+}\wedge J\wedge J = \gamma_3\wedge J\wedge J\ .
\end{aligned}
$$

$$
\begin{aligned}
\Omega'_- &= \alpha_{1+}\Omega_+ + \alpha_{1-}\Omega_- + \alpha_{2+}\wedge J + \alpha_3\ , \\
\Omega'_+ &= \beta_{1+}\Omega_+ + \beta_{1-}\Omega_- + \beta_{2+}\wedge J + \beta_3\ , \\
0 &= \Omega_\pm\wedge\alpha_3 = J\wedge\alpha_3\ , \\
0 &= \Omega_\pm\wedge\beta_3 = J\wedge\beta_3\ .
\end{aligned}
$$

- The quantities α, β and γ can easily be found in any given example (see paper for many worked cases).

Solving consistency conditions:

$$d\hat{\phi} = W_4$$
$$H_y = e^{\Delta}(-f - 2W_{1-})J - e^{\Delta}W_{2-} + \frac{1}{2}e^{\Delta}((2W_4 - W_5)\llcorner\overline{\Omega} + \text{c.c})$$

Also specifies some of the components of H

- Setting new fluxes to zero we recover the generalized half-flat conditions

$$W_{1-} = W_{2-} = 0 \quad W_4 = \frac{1}{2}W_5 = d\hat{\phi}$$

In general all but one of these conditions is relaxed.

Solving flow equations:

$$H = -\frac{1}{2}e^{-\Delta}\hat{\phi}'\Omega_+ + (\frac{7}{8} + \frac{3}{2}W_{1-})\Omega_-$$
$$+ * ((3W_4 - 2W_{5+}) \wedge J - W_3 + e^{-\Delta}\alpha_3)$$

- We also get equations for the flow itself.
  For example:

$$\gamma_3 = e^{\Delta} W_{2+} \quad \text{and} \quad \alpha_{1+} = -3e^{\Delta} W_{1-} - \frac{15}{8} e^{\Delta} f$$

- The explicit expressions for H allow us to check the Bianchi Identities and form field equations of motion trivially in any case.

- The equations for the flow yield the y dependence of the parameters in the SU(3) structure when used with any explicit construction.

Please see paper for egs:
- CY with flux
- Cosets
- Toric varieties (SCTV's)

Calabi-Yau Complex Structure and Bundles

- Gaugino variations tells us gauge bundle is poly-stable, slope zero and holomorphic:

$$g^{a\bar{b}} F_{a\bar{b}} = 0 \qquad\qquad F_{\bar{a}\bar{b}} = 0 = F_{ab}$$

- Ten dimensional action contains associated terms:

$$S = -\frac{1}{2\kappa_{10}^2}\alpha' \int_{\mathcal{M}_{10}} \sqrt{-g}\left(\frac{1}{2}\mathrm{tr}(g^{a\bar{b}}F_{a\bar{b}})^2 + \mathrm{tr}(g^{a\bar{a}}g^{b\bar{b}}F_{ab}F_{\bar{a}\bar{b}})\right)$$

- Assume we have such a bundle and perturb the complex structure, when can the connection adjust accordingly?

$$\delta\mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}}\delta A_{\bar{b}]} = 0$$

Algebraically this was worked out by Atiyah:

- Define a bundle Q:

$$0 \to V \otimes V^* \to Q \to TX \to 0$$

- Atiyah shows that the combined moduli of the holomorphic bundle are given by $H^1(Q)$, not $H^1(TX)$ and $H^1(V \otimes V^*)$.

- From the associated long exact sequence:

$$0 \to H^1(V \otimes V^*) \to H^1(Q) \to H^1(TX) \xrightarrow{\alpha} H^2(V \otimes V^*)$$

where $\alpha = [F]$

- Thus: $H^1(Q) = H^1(V \otimes V^*) \oplus \mathrm{Ker}(\alpha)$

- This should be compared to the differential expression on the previous slide.

Problem: All this analysis requires that you know a starting point to fluctuate around!

- We would like instead a way of describing the moduli space, and its properties, globally. Which loci are we restricted to in complex structure moduli space?
- To make things more explicit we move to an example:

Manifold: $$X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array} \right] / \mathbb{Z}_2 \times \mathbb{Z}_4$$

(10 complex struct.)

Bundle: $$0 \to \mathcal{L} \to V \to \mathcal{L}^* \to 0$$

where $\mathcal{L} = \mathcal{O}(-2, -2, 1, 1)$

- Bundle is controlled by $H^1(X, \mathcal{L}^2)$
  - This vanishes generically
  - But can jump to a non-zero value on special loci in complex structure moduli space.
  - Loci in complex structure where bundle support jumps is where you get stabilized to.

Complex structure dependence of the controlling cohomology:

Koszul: $$0 \to \mathcal{N}^* \otimes \mathcal{L}^2 \to \mathcal{L}^2 \to \mathcal{L}^2|_X \to 0$$

where: $\mathcal{N} = \mathcal{O}(2,2,2,2)$

Tells us:

$$0 \to H^1(\mathcal{L}^2|_X) \to H^2(\mathcal{N}^* \otimes \mathcal{L}^2) \xrightarrow{p} H^2(\mathcal{L}^2) \to H^2(\mathcal{L}^2|_X) \to 0$$

- Source and target spaces are described in terms of polynomials in ambient space coordinates.
- Map p is complex structure dependent degree (2,2,2,2) polynomial - the defining relation!
- Procedure:
  - Take a general element of the source: $\sum_i b_i S^i$
  - and a general defining relation: $\sum_a c_a p^a$
  - Ask that the product of the two vanishes in the target polynomial space:

$$\sum_{a,i} \lambda^{ia} b_i c_a = 0$$

→ Algebraic variety for vacuum space.

- We want to know the stabilized loci in complex structure moduli space:
  - Primary decompose to obtain one equation for each locus in combined complex structure "bundle" modulus space.
  - Perform elimination (projection) to the complex structure moduli space for each piece.

25 distinct interesting loci:

Dim.	Num.
7	2
5	2
4	3
3	4
2	6
1	5
0	3

We must also check the smoothness of the CY on each locus.

- In this case only one of the loci is smooth.
- Many have point like singularities on the CY - may ask if they can be resolved
- Answer is definitely yes, at least for some of them.

Dim.	Num.	Sing.
7	2	0
5	2	0
4	2	0
4	1	-1
3	2	0
3	2	1
2	5	0
2	1	2
1	3	0
1	2	2
0	3	2

Complete description of stable loci in c.s. moduli space

Summary

- SU(3) structure backgrounds:
  - Showed how to generalise the torsion classes giving rise to a good heterotic background.
  - Gave explicit solutions for supergravity fields: especially important for solving Bianchi Identities.
- Calabi-Yau complex structure stabilization:
  - Reviewed the basic mechanism.
  - Described the problem of knowing where to start the standard fluctuation analysis for stability of the vacuum.
  - Showed how to algorithmically map out the vacua in complex structure moduli space.