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$SO(10)$ from F-theory

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Structure of the Talk

- Basic ingredients of F-theory model building
- Basic tools used in the spectral cover approach
- Implications of monodromies in model building
- Classification of emerging $SO(10)$ models
- Flux constraints
- Examples of $SO(10)$ constructions
- Implementation of matter parity

a brief road map to F-unification

▲ Candidates incorporating $\mathbf{G_{SM}}$ in a minimal context:

★ **SU(5)**: Chiral and Higgs Representations:

$$10 \rightarrow Q + u^c + e^c \; ; \; \bar{5} \rightarrow d^c + \ell$$

$$5 + \bar{5} \rightarrow (D + h_u) + (\overline{D} + h_d)$$

▲ Yukawa Couplings: $10 \cdot 10 \cdot 5 \rightarrow m_{top}, \; 10 \cdot \bar{5} \cdot \bar{5} \rightarrow m_b, m_\tau$

★ **SO(10)**: In addition, includes ν^c:

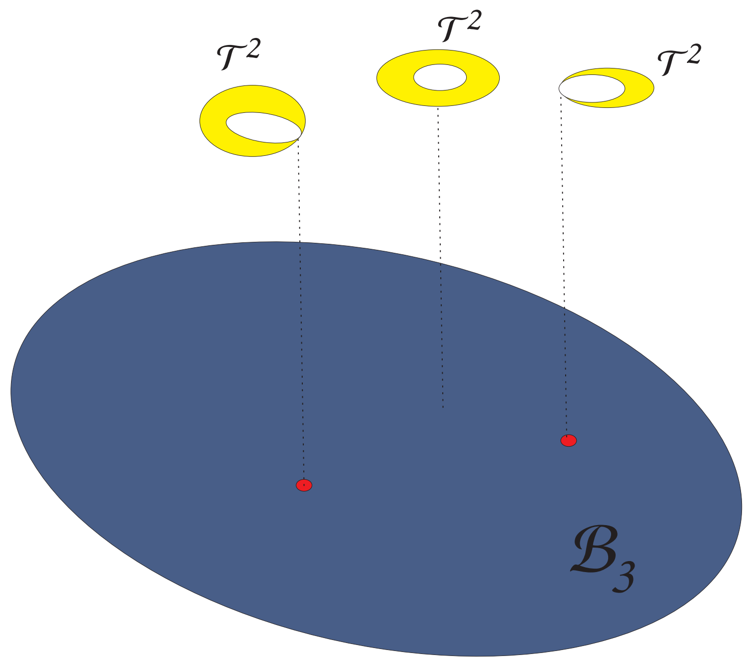
$$\mathbf{16} \rightarrow 10_{1/5} + \bar{5}_{-3/5} + 1_1, \; \mathbf{10} = \mathbf{5} + \bar{\mathbf{5}}$$

▲ Yukawa Coupling: $\mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} \rightarrow m_{top}, m_b, m_\tau, m_\nu$

★ F-Theory: $\mathcal{E}_8 \rightarrow$ **GUT**+ Internal Geometry

★ **F-theory** (*C. Vafa hep-th/9602022*)

- Defined on a background $\mathcal{R}^{3,1} \times \mathcal{X}$
- $\mathcal{X}$ elliptically fibered **CY** 4-fold over B_3
- B_3 complex 3-fold base.



CY 4-fold: Points of B_3 represented by torus $\tau = C_0 + \imath/g_s$. Red points: 7-branes, $\perp$ to B_3

Fibration is described by the $\mathcal{W}$eierstraß $\mathcal{E}$quation ($\mathcal{WE}$)

$$y^2 = x^3 + f(z)\,x + g(z) \tag{1}$$

x, y parameters of the fibration
$f(z)$, $g(z) \rightarrow 8\,\&\,12$ degree polynomials in z.

For each point of B_3, eq(9) describes a torus labeled by z

The fiber degenerates at the zeros of the discriminant

$$\Delta = 4\,f^3 + 27\,g^2 \tag{2}$$

$$\Downarrow$$

$\Delta = 0 \;\Rightarrow\;$ singularity of internal manifold

$$y^2 = x^3 + f\,x + g$$

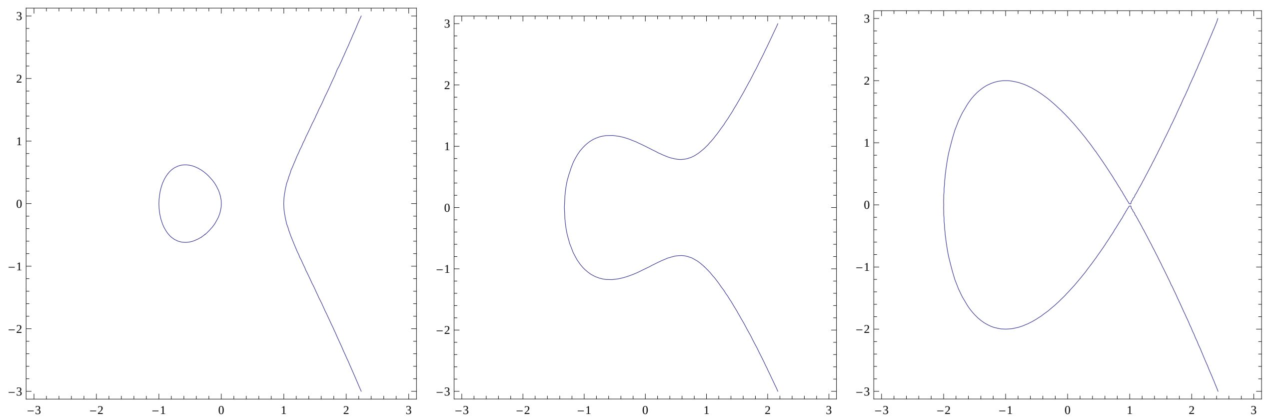


Figure 1: Fixing the values of the polynomials f, g to certain real numbers, elliptic fibrations reduce to elliptic curves for $\Delta \neq 0$. The three cases correspond to $\Delta > 0, \Delta < 0$ and $\Delta = 0$ respectively.

Interpretation of geometric singularities

(*Witten, hep-th/9507121, Bershadsky et al, hep-th/9510225;*)

- **Singularities** of Internal Manifold $\rightleftarrows$ **gauge symmetries**

  ... encoded in the structure of $f(z)$, $g(z)$

- Types of **singularities** : $\mathcal{ADE}$ (*Kodaira classif.*)

  ... they determine:

  A) **gauge symmetries**

$$\rightarrow \begin{cases} SU(n) \\ SO(m) \\ \mathcal{E}_n \end{cases}$$

  B) **matter content**

ord($f(z)$)	ord$g(z)$)	ord($\Delta(z)$)	fiber type	Singularity
0	0	n	I_n	A_{n-1}
≥ 1	1	2	II	none
1	≥ 2	3	III	A_1
≥ 2	2	4	IV	A_2
2	≥ 3	$n+6$	I_n^*	D_{n+4}
≥ 2	3	$n+6$	I_n^*	D_{n+4}
≥ 3	4	8	IV^*	$\mathcal{E}_6$
3	≥ 5	9	III^*	$\mathcal{E}_7$
≥ 4	5	10	II^*	$\mathcal{E}_8$

Table 1: Kodaira's classification of Elliptic Singularities with respect to the vanishing order of f, g, Δ.

Useful algorithm for local description: Tate's form

Procedure: (*see Katz et al 1106:3854*) Expand f, g

$$f(z) = \sum_n f_n z^n, \; g(z) = \sum_m g_m z^m$$

Then

$$\Delta = 4\left[f_0 + f_1 z + \cdots)\right]^3 + 27\left[g_0 + g_1 z + \cdots\right]^2$$

Demand $z/\Delta \;\Rightarrow$

$$f_0 = -\frac{1}{3}\, t^2, \;\; g_0 = \frac{2}{27}\, t^3$$

while $\mathcal{WE}$ obtains Tate's $\mathbf{I_1}$ form:

$$y^2 = x^3 + t\, x^2 + (f_1 + f_2 z + \cdots) z\, x + (\tilde{g}_1 + \tilde{g}_2 z + \cdots) z$$

Tate's Form

$$y^2 + a_1 x\, y + a_3 y = x^3 + a_2\, x^2 + a_4 x + a_6$$

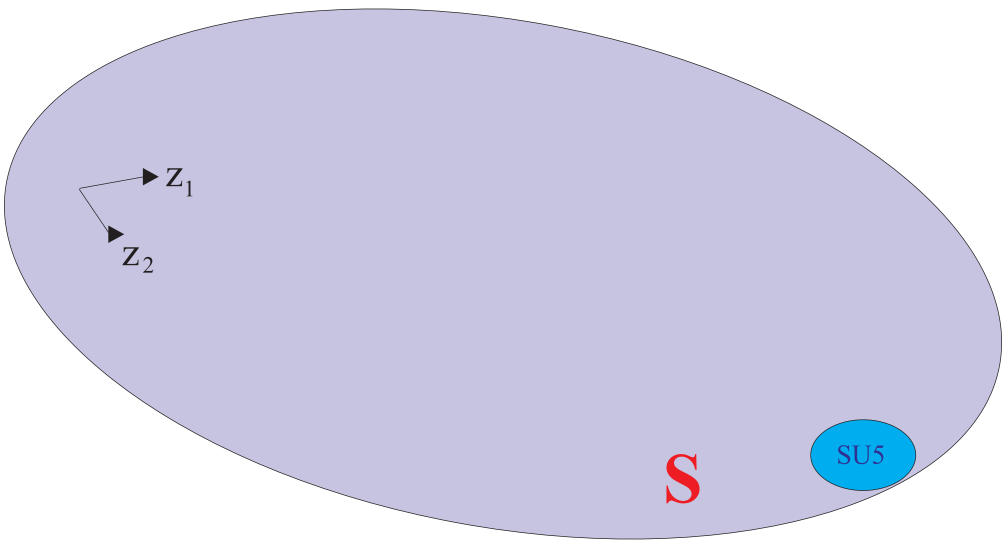
The algorithm (*Partial results*)

Group	a_1	a_2	a_3	a_4	a_6	Δ
$SU(2n)$	0	1	n	n	$2n$	$2n$
$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
$SO(10)$	1	1	2	3	5	7
$\mathcal{E}_6$	1	2	3	3	5	8
$\mathcal{E}_7$	1	2	3	3	5	9
$\mathcal{E}_8$	1	2	3	4	5	10

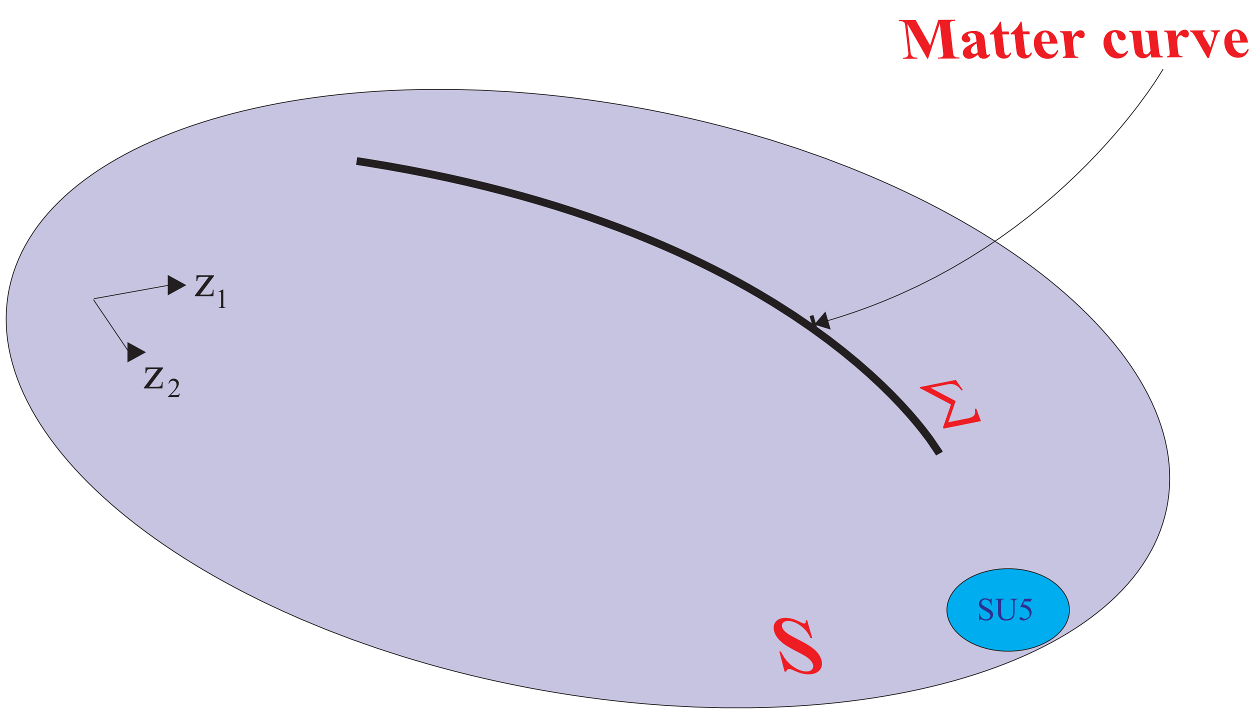
Model Building in **F-theory:**

Fundamental object: 7-branes

...wrap certain class of *'internal'* **2-complex dim.** surface **S** associated to gauge group G_S (*i.e.*, $SU(5)$ *or* $SO(10)$ *or...*)

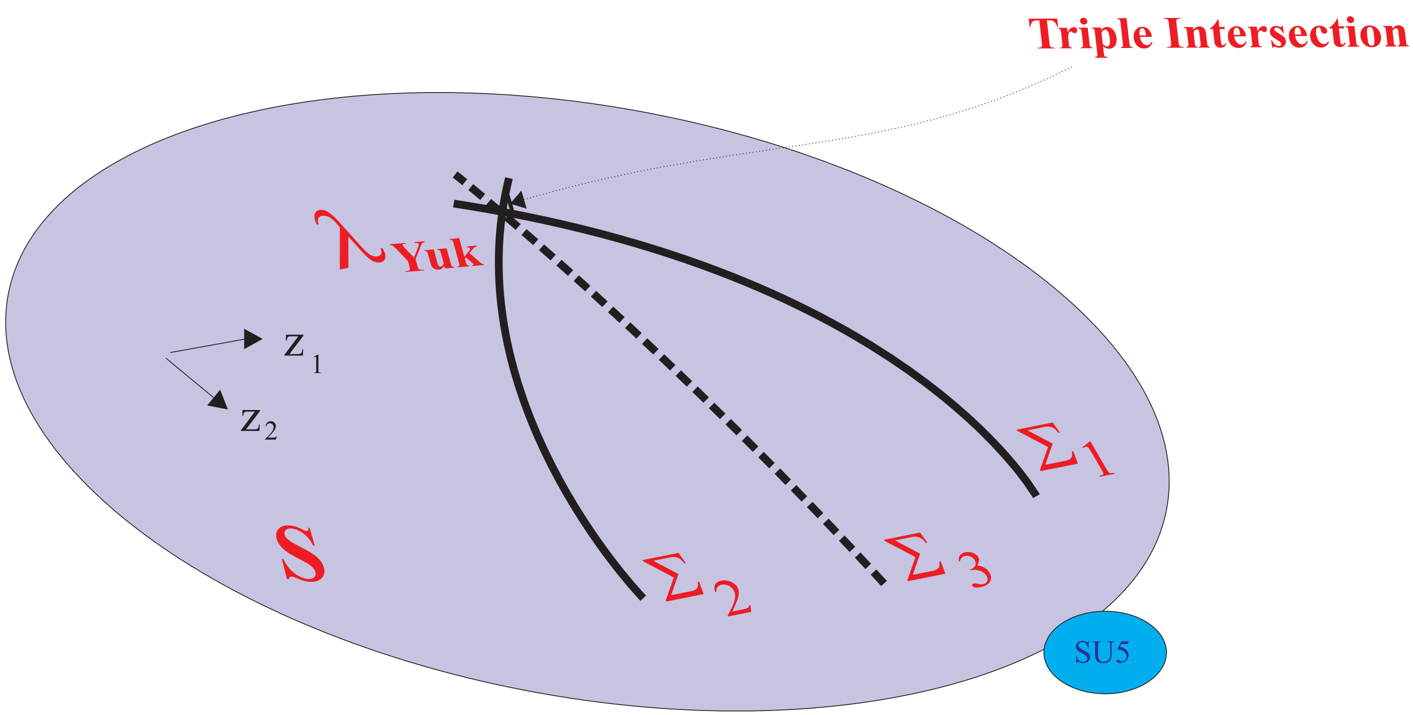


Matter resides along intersections with other 7-branes...



Along a matter curve Σ gauge symmetry is enhanced...

Yukawa couplings



gauge symmetry ... further ... enhanced!

★ Zero-modes Equations

Equs of motion from $d = 8$ effective action

⇓

D.E.s for **zero modes**:

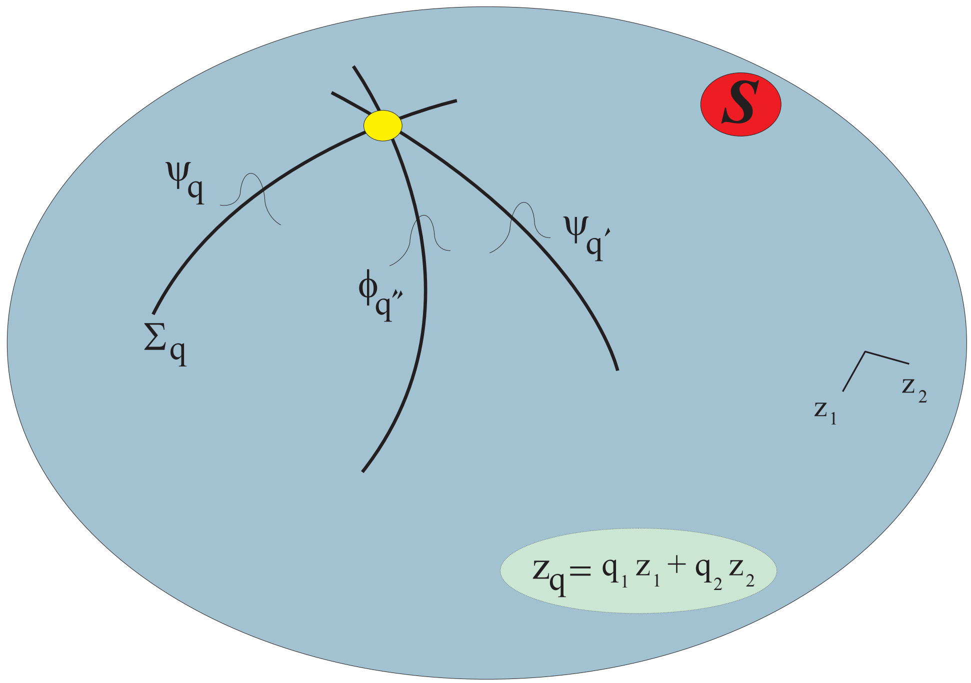
(*Font & Ibanez 2009, Heckman et al 2009* *)

- Solution: Gaussian profile for zero mode wavefunction:

$$\psi \propto e^{\{-m^2 q|\cos\theta z_1 + \sin\theta z_2|^2\}} \tag{3}$$

with $q = \sqrt{q_1^2 + q_2^2}$ *and* $\tan\theta = q_2/q_1$.

(* see also: *Aparicio, Font, Ibanez, Marchesano, 1104.2609*)



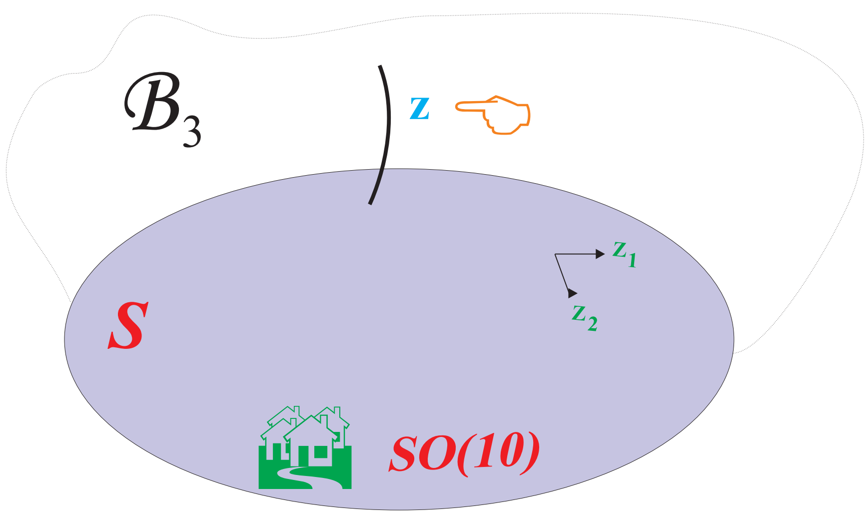
Depiction of the wavefuctions ψ_i and ϕ along the matter curves...

F-SO(10)

- **S** Internal complex manifold with $SO(10)$ singularity
- For $SO(10)$, we take $a_1 = -b_5\, z, a_3 = -b_3 z^2, \ldots$

$$y^2 = x^3 + b_5 xyz + b_4 x^2 z + b_3 y z^2 + b_2 x z^3 + b_0 z^5$$

- z the coordinate on normal bundle to S, in B_3
- S corresponds to $z = 0$ in B_3, $(B_3|_{z=0} \to S)$



▲ Lowest z-powers of f, g, Δ

$$f = -\frac{1}{3} b_4^2 \, z^2 + \cdots, \quad g = \frac{2b_4^3}{27} \, z^3 + \cdots, \quad \Delta = -(16 b_3^2 b_4^3) \, z^7$$

Comparing to Kodaira's classification: $\rightarrow D_5 = SO(10)$ singularity

▲ Symmetry enhancements

Order	Equation	Enhancement
1^{st}	$b_4 = 0$	E_6
2^{nd}	$b_4 = b_3 = 0$	E_7
1^{st}	$b_3 = 0$	$SO(12)$
2^{nd}	$b_3 = b_2^2 - 4b_0 b_4 = 0$	$SO(14)$

Triple Intersection: $E_7 : 133 \cdot 56 \cdot 56 \rightarrow \cdots \rightarrow 16_1 \cdot 16_1 \cdot 10_{-2}$

Spectral Cover approach

▲ Topological Properties and notation :

Chern classes associated to bundle structure

▲ $c_1 \to 1^{st}$ Chern class of the **Tangent** Bundle to S_{GUT}

▲ $-t \to 1^{st}$ Chern class of the **Normal** Bundle to S_{GUT}

Then:

$z \to [z] = -t$

If :$[x] = 2(c_1 - t)$; $[y] = 3(c_1 - t)$; $[b_k] = \eta - kc_1 = (6-k)c_1 - t$

$\mathcal{WE}$ transforms as: $\boxed{6\,(c_1 - t)}$.

For example:

$$[b_2 x z^3] \;=\; \{(6-2)c_1 - t\} + \{2(c_1 - t)\} - 3t \;=\; 6\,(c_1 - t)$$

★ Elliptic Fibration ⇒ $\mathcal{E}_8$ → *highest symmetry*

$$E_8 \supset SO(10) \times SU(4)_\perp$$

⇒ commutant $SU(4)_\perp$ → determines symmetry of **spectral cover**

★ **Correlating coefficients b_k of $SO(10)$ to $SU(4)$**

Local form of $\mathcal{WE}$ for the $SO(10)$

(*putting scaling dims* $(x, y, z) \sim (\frac{1}{3}, \frac{1}{2}, \frac{1}{5}) \to$ *dim*$(b_5 xyz > 1)$

$$y^2 = x^3 + b_4 x^2 z + b_3 y z^2 + b_2 x z^3 + b_0 z^5$$

Defining

$$z = s^{4/5}, \ x = s^{-2/5}, \ y = s^{-3/5}$$

$$\Rightarrow \boxed{0 = b_0 s^4 + b_2 s^2 + b_3 s + b_4 \equiv \sum_k b_k s^{4-k}}$$

... → with roots $t_i = SU(4)$ weights

★ Matter... ★

... descends from $\mathcal{E}_8$-adjoint... decomposed under

$$\mathcal{E}_8 \supset SO(10) \times SU(4)_\perp$$

$$248 \to (45,1) + (\overline{16}, \bar{4}_\perp) + \boxed{(16, 4_\perp) + (10, 6_\perp) + (1, 15_\perp)}$$

★ Turning on Fluxes

$$SO(10) \times SU(4)_\perp \to SO(10) \times U(1)^3$$

Geometric Interpetation

16's and **10**'s reside on matter curves Σ_n characterised by the $SU(4)_\perp$ weights t_i:

$$\Sigma_{16} \quad : \quad t_i$$

$$\Sigma_{10} \quad : \quad t_i + t_j$$

$$\Sigma_1 \quad : \quad \pm(t_i - t_j), \; i \neq j$$

★ Couplings... ★

... $SO(10)$ representations distinguished by $U(1)_i$'s

... t_i weights subject to $SU(4)_\perp$ constraint:

$$t_1 + t_2 + t_3 + t_4 = 0$$

... Yukawa coupling gauge invariance:

$$\mathbf{16}_{t_i}\ \mathbf{16}_{t_j}\ \mathbf{10}_{t_k+t_l}$$

...requires

$$t_i + t_j + t_k + t_l = 0$$

...implies that

$$i, j, k, l \Rightarrow \text{all differ!}$$

★ **not compatible with Rank-one tree-level Mass Matrix**

But!

$t_i \rightarrowtail$ roots of the $SU(4)_\perp$-spectral cover equation:

$$\sum_k^4 b_k s^{4-k} = 0 \quad \Rightarrow \quad b_0 \prod_{i=1}^4 (s - t_i) = 0$$

... t_i solutions require inversion of

$$b_k = b_k(t_i) \Rightarrow \; t_i = t_i(b_k)$$

implies **brunchcuts** $\Rightarrow$ monodromies among t_i (*Hayashi et al, 0901.4941, Heckmann et al, 0906.0581, Marsano et al 0906.4672*)

Monodromies for $U(1)^3 \in SU(4)$:

$$\mathcal{Z}_3 \quad : \quad \{t_1, t_2, t_3\}, \; \{t_4\}$$

$$\mathcal{Z}_2 \times \mathcal{Z}_2 \quad : \quad \{t_1, t_2\}, \; \{t_3, t_4\}$$

$$\mathcal{Z}_{\mathbf{2}} \quad : \quad \{t_1, t_2\}, \; \{t_3\}, \{t_4\}$$

$\mathcal{A}$: $\mathcal{Z}_2$ monodromy

factorization of the spectral cover equation is

$$\sum_k b_k s^{4-k} = \underbrace{(a_1 + a_2 s + a_3 s^2)}_{\mathcal{Z}_2}(a_4 + a_5 s)(a_6 + a_7 s)$$

$$\rightarrow \; b_k = \sum a_l a_m a_n, \quad \text{with} \;\; k + l + m + n = 15$$

Homologies $[a_n]$ of a_n:

$$[b_k] = \eta - k\,c_1 \;=\; [a_l] + [a_m] + [a_n], \tag{4}$$

solution of (4) in terms of two free parameters $\chi = \chi_5 + \chi_7$

a_1	a_2	a_3	a_4	a_5	a_6	a_7
$\eta - 2c_1 - \chi$	$\eta - c_1 - \chi$	$\eta - \chi$	$-c_1 + \chi_5$	χ_5	$-c_1 + \chi_7$	χ_7

$\Rightarrow$ *Important!* They tell us how flux restricts on matter curves

★ FLUX-restrictions

on matter curves described by the integers:

$$P_n = \mathcal{F}_x \cdot (\chi_n - c_1),\ M = \mathcal{F}_x \cdot (\eta - 3c_1),\ P = \mathcal{F}_x \cdot (\chi - c_1)$$

★ Defining equ.s of Matter Curves

$$P_{16}(s) = b_0 \prod_i (s - t_i) = 0,\ P_{10}(s) = b_0^2 \prod_{i<j} (s - t_i - t_j) = 0$$

$$\Sigma_{\mathbf{16}}: \quad P_{16}(0) = b_4 = 0 \rightarrow \quad a_1 a_4 a_6 = 0$$

$$\Sigma_{\mathbf{10}}: \quad P_{10}(0) = b_3^2 = 0 \rightarrow \quad ((a_1 - \lambda\, a_4 a_6)(a_5 a_6 + a_4 a_7))^2 = 0$$

Collecting all properties...

... of $SO(10)$ representations

Matter	t_i charges	Section	Homology	$U(1)_X$
16	$t_{1,2}$	a_1	$\eta - 2c_1 - \chi$	$M - P$
16	t_3	a_4	$-c_1 + \chi_5$	P_5
16	t_4	a_6	$-c_1 + \chi_7$	P_7
10	$t_{1,2} + t_3$	$(a_1 - \lambda a_4 a_6)$	$\eta - 2c_1 - \chi$	$M - P$
10	$t_{1,2} + t_4$	$(a_1 - \lambda a_4 a_6)$	$\eta - 2c_1 - \chi$	$M - P$
10	$t_1 + t_2$	$(a_5 a_6 + a_4 a_7)$	$-c_1 + \chi$	P
10	$t_3 + t_4$	$(a_5 a_6 + a_4 a_7)$	$-c_1 + \chi$	P

$\mathcal{B}$: $\mathcal{Z}_2 \times \mathcal{Z}_2$ factorization

$$\sum_k b_k s^{4-k} = \underbrace{(a_1 + a_2 s + a_3 s^2)}_{\mathcal{Z}_2}\underbrace{(a_4 + a_5 s + a_6 s^2)}_{\mathcal{Z}_2}$$

Properties of $SO(10)$ representations

Matter	Section	Homology	t_i charges	$U(1)_X$
$\mathbf{16}_a$	a_1	$\eta - 2c_1 - \chi$	$t_{1,2}$	$M - P$
$\mathbf{16}_b$	a_4	χ	$t_{3,4}$	$P - C$
$\mathbf{10}_a$	$a_1 - \lambda a_4$	$\eta - 2c_1 - \chi$	$t_{1,2} + t_{3,4}$	$M - P$
$\mathbf{10}_b$	a_5	$\chi - c_1$	$t_3 + t_4/t_1 + t_2$	P

$\mathcal{C}$: $\mathcal{Z}_3$ factorization

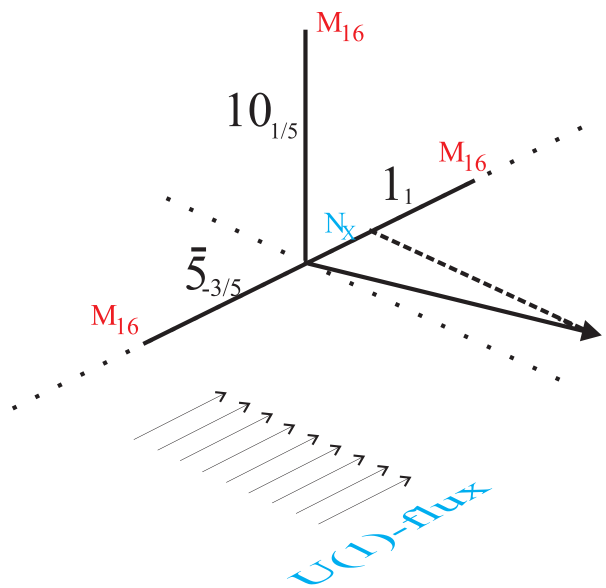
$$\sum_k b_k s^{4-k} = \underbrace{(a_1 + a_2 s + a_3 s^2 + a_4 s^3)}_{\mathcal{Z}_3}(a_5 + a_6 s)$$

Properties of $SO(10)$ representations, $\mathcal{Z}_3$ case

Matter	Section	Homology	t_i charges	$U(1)_X$
16	a_1	$\eta - 3c_1 - \chi$	t_i	$M - P + C$
16	a_5	$\chi - c_1$	t_4	P
10	$a_1 + \lambda a_2$	$\eta - 3c_1 - \chi$	$2t_i$	$M - P + C$
10	a_6	χ	$t_i + t_4$	$P - C$

$SO(10) \to SU(5)$ breaking by $U(1)_X$ flux

$$\mathbf{16} \Rightarrow \mathbf{10}_{1/5} + \mathbf{5}_{-3/5} + \mathbf{1}_{1}$$



$$\# \, 10_{1/5} = M_{16}$$
$$\# \, \bar{5}_{-3/5} = M_{16} - N_X$$
$$\# \, 1_{1} = M_{16} + N_X$$

Emergent $SO(10) \rightarrow SU(5)$ content for the $\mathcal{Z}_3$ case

... repeating procedure to determine $SU(5)$ properties...
(*Dudas-Palti: 1007:1297*)

$SO(10) \supset SU(5)$	$U(1)_Y$-flux	$U(1)_X$
$16_{t_i} \supset 10^{(1)}$	$-N_x - N_y$	M_{16}^1
$16_{t_4} \supset 10^{(2)}$	N_x	M_{16}^2
$10_{t_i+t_4} \supset 5^{(0)}$	0	$M_{10} + n\,M$
$10_{t_i+t_j} \supset 5^{(1)}$	$-N_y$	$-M_{10} + n\,M$
$16_{t_i} \supset 5^{(2)}$	$-N_x$	$-M_{16}^1 + (M - P + C)$
$16_{t_4} \supset 5^{(3)}$	$N_x + N_y$	$-M_{16}^2 + P$
$45 \supset 10^{(3)}$	N_y	M_{10}'

$U(1)_Y$ **flux controls the SM matter content and more...**

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

$SU(5)$ irreps... **ABSENCE** OF $U(1)_Y$ flux:

... multiplicities of the resulting **SM** spectrum:

$$\mathbf{10} \in SU(5) \Rightarrow \begin{cases} n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} & = & M_{10} \\ n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} & = & M_{10} \\ n_{(1,1)_1} - n_{(1,1)_{-1}} & = & M_{10} \end{cases}$$

and

$$\mathbf{5} \in SU(5) \Rightarrow \begin{cases} n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} & = & M_5 \\ n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} & = & M_5 \end{cases}$$

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

$U(1)_Y$ flux **splits** $SU(5)$ irreps...

... multiplicities of the resulting **SM** spectrum:

$$\mathbf{10} \in SU(5) \Rightarrow \begin{cases} n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} &= M_{10} \\ n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} &= M_{10} - N_Y \\ n_{(1,1)_1} - n_{(1,1)_{-1}} &= M_{10} + N_Y \end{cases} \quad (5)$$

and

$$\mathbf{5} \in SU(5) \Rightarrow \begin{cases} n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} &= M_5 \\ n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} &= M_5 + N_Y \end{cases} \quad (6)$$

Model Building Recipe

▲ Choose the desired Gauge Group G_S
(spectral cover symmetry G_{SC} automatically specified!)

$$G_S \times G_{SC} \subset \mathcal{E}_8$$

▲ Choose Monodromy : $\mathcal{Z}_2$, $\mathcal{Z}_3$, ...

▲ Pick up Flux parameters (M_n, N_X, N_Y)

▲ Make judicious choice of matter curves to accommodate spectrum

▲ Define matter parity

A $\mathcal{Z}_3$ example

Choice of $U(1)_X, U(1)_Y$ **flux** parameters:

$$N_x = -N_y = 1, M_{16}^1 = 3, M_{10} = 1, P = -3, M = C = 0$$

$SU(5)$	N_Y	M_X	Matter
$10_{t_i}^{(1)} = 10_M$	0	3	$3(Q + u^c + e^c)$
$5_{-2t_i}^{(0)} = 5_{h_u}$	0	+1	$h_u + \mathbf{D}$
$\bar{5}_{t_i+t_4}^{(1)} = \bar{5}_{\bar{D}}$	+1	−1	$\overline{\mathbf{D}}$
$\bar{5}_{t_i+t_5}^{(2)} = \bar{5}_{h_d}$	−1	0	h_d
$\bar{5}_{t_4+t_5}^{(3)} = \bar{5}_M$	0	−3	$3(d^c, \ell)$
$\theta_{ij} = 1_{t_i-t_j}$			singlets

and *extraneous matter...*

$$X\, 10^{(a)}\overline{10}^{(b)} \cdots \to \langle X\rangle\, (\bar{e}^c e^c + \bar{u}^c u^c)$$

Mass terms for charged fermions and Higgs triplets

$$Matter\,curves \quad : \quad 10^1 10^1 5^0 + 10^1 \bar{5}^3 \bar{5}^2 \theta_{15} + 5^0 \bar{5}^1 \theta_{14}$$

$$\to\ couplings \qquad 10_M 10_M 5_{h_u} + 10_M \bar{5}_M \bar{5}_{h_d} \langle\theta_{15}\rangle + \langle\theta_{14}\rangle\, 5_{h_u} \bar{5}_{\bar{D}}$$

Dangerous Terms

$$10_M \bar{5}_M \bar{5}_M \theta_{14} \theta_{15},\ 10_M 10_M 10_M \bar{5}_M,\ 5^0 \bar{5}^2\, \theta_{15}$$

$$\Downarrow$$

Eliminate them using **Matter Parity**

Matter Parity from Geometry?

topological properties are encoded in b_k coefficients

Consider the phase transformation (*Hayashi et al, 0910:2762*)

$$s \to s\, e^{i\phi},\; b_k \to b_k\, e^{i(\xi-(6-k)\phi)}$$

Let's apply to $SU(5)$ case:
...spectral cover equation picks up an overall phase

$$\mathcal{C}_5 :\; \sum_k b_k s^{5-k} \to e^{i(\xi-\phi)} \sum_k b_k s^{5-k}$$

★ Z_2-parity: $\phi = \pi$:

$$s \to -s,\; b_k \to (-1)^k e^{i\xi} b_k$$

★ Communicating Matter Parity to Matter Curves ★

Example: *Consider relations in $\mathcal{Z}_2$ monodromy:*

$$b_k = \sum a_l a_m a_n, \quad l + m + n = N - k,\ N = 17 \tag{7}$$

Choose a_n to transform as

$$a_n \to a_n\, e^{i(\zeta - n\,\phi)}$$

$$\to\ b_k \propto a_l a_m a_n \to a_l a_m a_n\, e^{3\zeta-(N-k)\phi} \tag{8}$$

(7) & (8) consistent for $\xi = \phi = \pi, \zeta = 0$,

$$\boxed{a_n \to (-1)^n\, a_n}$$

★ Implications on matter curves:

$$5^{(0)} \sim a_6 a_7 \to (-1)^{(6+7)} = (-)$$

... associate this to matter parity!

★ Application: The case of the $\mathcal{Z}_3$ model ★

$$\boxed{a_n \to (-1)^{n+1}\, a_n}$$

Defining equations and induced matter parity

$SU(5)$	Equation	Matter	R-parity
$10^{(1)}_{t_i} = 10_M$	a_1	$3(Q + u^c + e^c)$	$+$
$5^{(0)}_{-2t_i} = 5_{h_u}$	$a_1 a_6 a_8 + \cdots$	$h_u + D$	$+$
$\bar{5}^{(1)}_{t_i+t_4} = \bar{5}_{\bar{D}}$	$a_1 a_6 + \cdots$	$\overline{D}$	$+$
$\bar{5}^{(2)}_{t_i+t_5} = \bar{5}_{h_d}$	$a_1 a_8 + \cdots$	h_d	$+$
$\bar{5}^{(3)}_{t_4+t_5} = \bar{5}_M$	$a_6 a_7 + \cdots$	$3(d^c + \ell)$	$-$
θ_{14}, θ_{15}	$\cdots$	$\cdots$	$-, +$

Terms Forbidden by *R*-parity: Higgs mixing $\cancel{5^0 \bar{5}^2 \theta_{15}}$ and

$$\cancel{10_M \bar{5}_M \bar{5}_M \theta_{14} \theta_{15}}, \; \cancel{10_M 10_M 10_M \bar{5}_M}$$

A few remarks

⋆ **Geometric** Nature of **F-Theory** Models:

Manifold Singularity ⇆ **Gauge Symmetry** G_{GUT}

⋆ **Elliptic Fibration:**

$$\mathcal{E}_8 \to \mathbf{G_{GUT}} \times \mathcal{C}_{\text{spectral cover}}$$

⋆ **Spectral Cover** ⇉ useful local properties of G_{GUT}:

$U(1)$-symmetries, Monodromies, Matter Parity ...

⇓

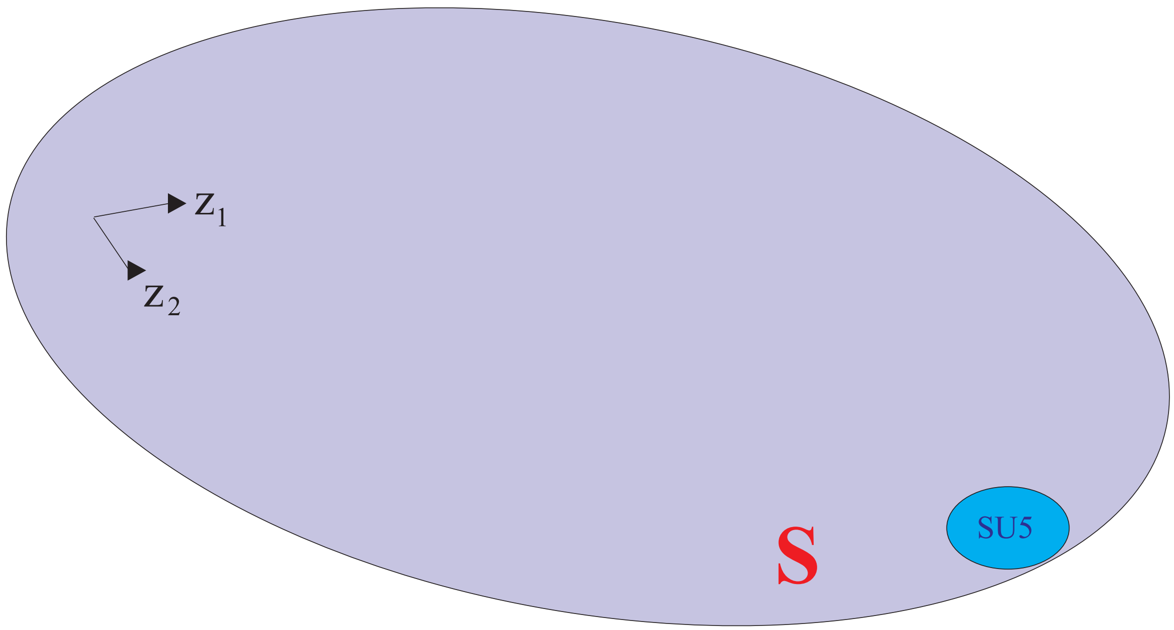
Additional Structure beyond GUTs

Possible Signatures:

KK-massive modes ↣ relevance to Cosmology?

Exotic Matter ↣ Experimental detection?

- S certain class of *‘internal’* **2-complex dim.** surface ($S \in B_3$)
- 7-branes wrap **S**
- S has a singularity associated to gauge group G_S



SU(5)

Write Weierstrass equ. in 'expanded' form

$$y^2 + a_1 x\, y + a_3 y \quad = \quad x^3 + a_2\, x^2 + a_4 x + a_6 \tag{9}$$

z the coordinate on normal bundle to S, in B_3

S corresponds to $z = 0$ in B_3, $(B_3|_{z=0} \to S)$

▲ The order of vanishing of $a_i = b_i\, z^{n_i}$ characterizes the type of singularity

i.e, the **gauge group** supported by S.

Choice: $a_1 = -b_5, a_2 = b_4 z, a_3 = -b_3 z^2, a_4 = b_2 z^3, a_6 = z^5 b_0$

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 z^2 y + b_4 x^2 z + b_5 xy$$

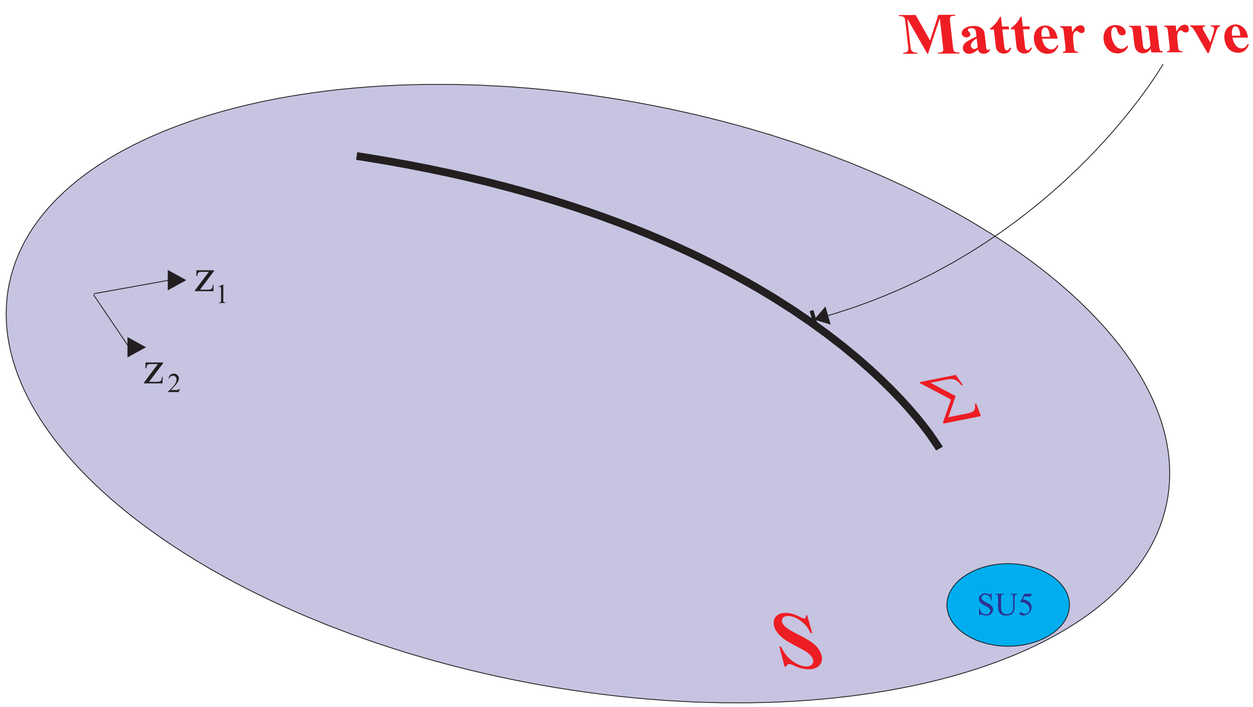
$\Rightarrow$ **SU(5)** Singularity.

Tate's Algorithm

Group	a_1	a_2	a_3	a_4	a_6	Δ
...	...	...	...	...	...	...
$SU(2n)$	0	1	n	n	$2n$	$2n$
$SU(2n+1)$	0	1	n	$n+1$	$2n+1$	$2n+1$
$SO(10)$	1	1	2	3	5	7
E_6	1	2	3	3	5	8
E_7	1	2	3	3	5	9
E_8	1	2	3	4	5	10

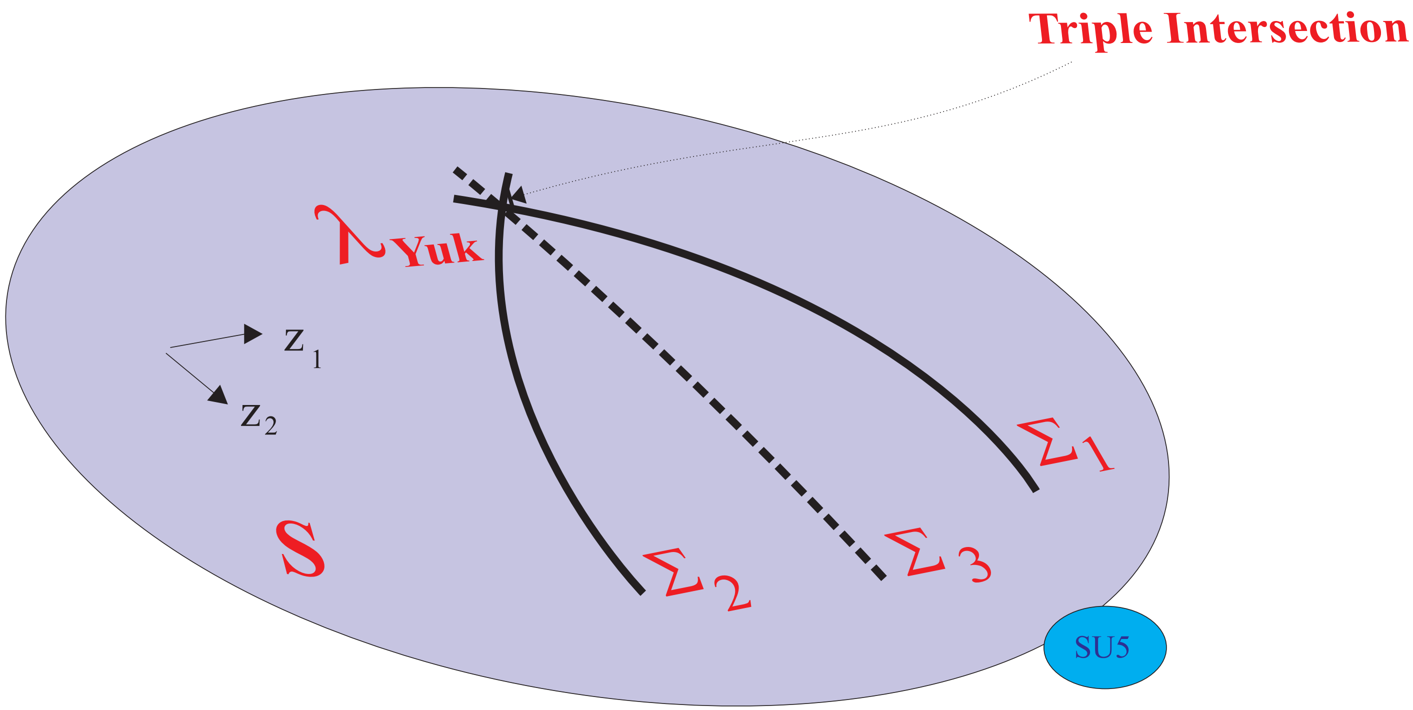
Table 2: (*Tate, 1975, Bershadsky et.al. hep-th/9605200.*) The order of vanishing of the coefficients $a_i \sim z^{n_i}$ and the corresponding gauge group. The highest singularity allowed in the elliptic fibration is E_8.

Matter resides along intersections with other 7-branes...

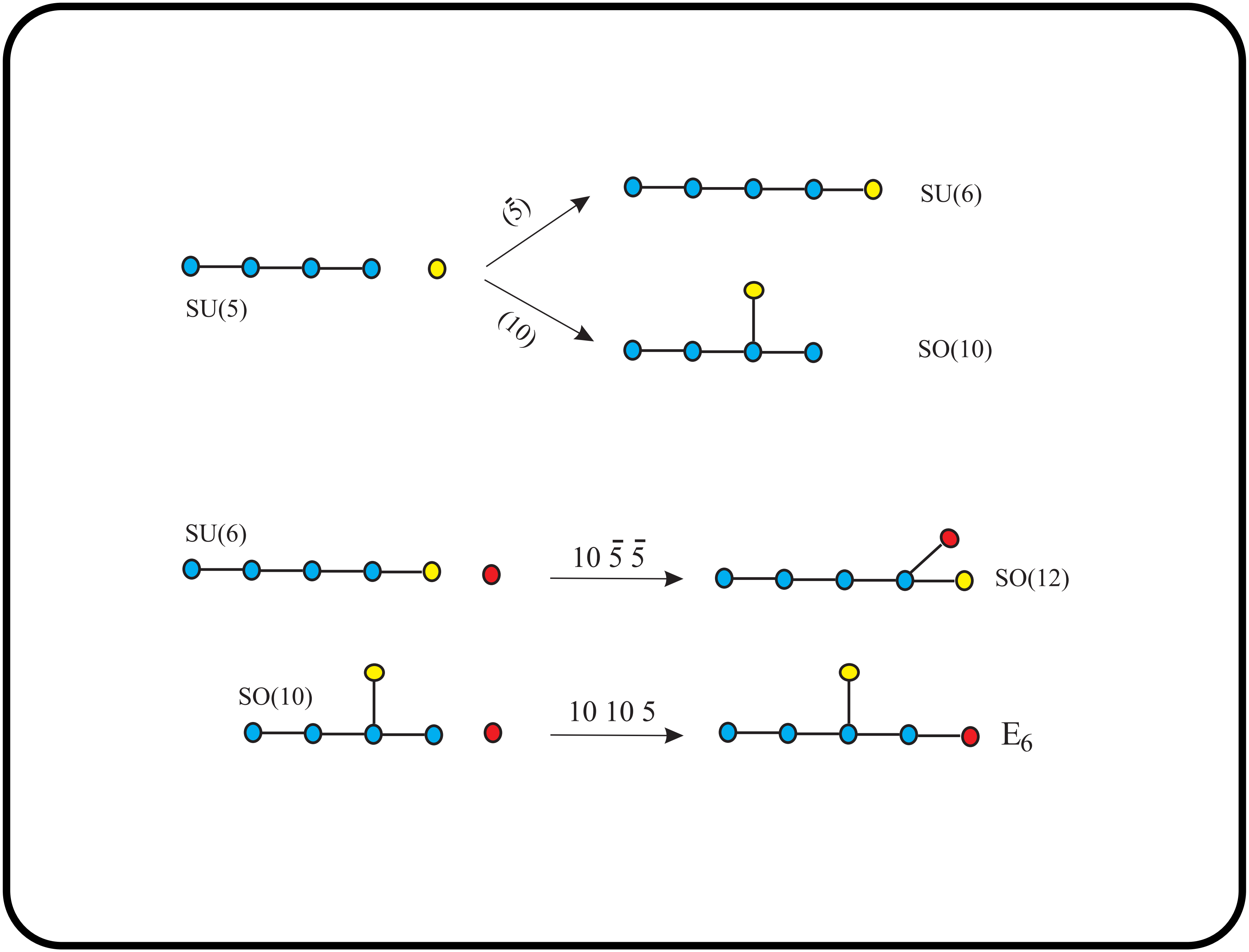


Along a matter curve Σ gauge symmetry is enhanced...

Yukawa couplings

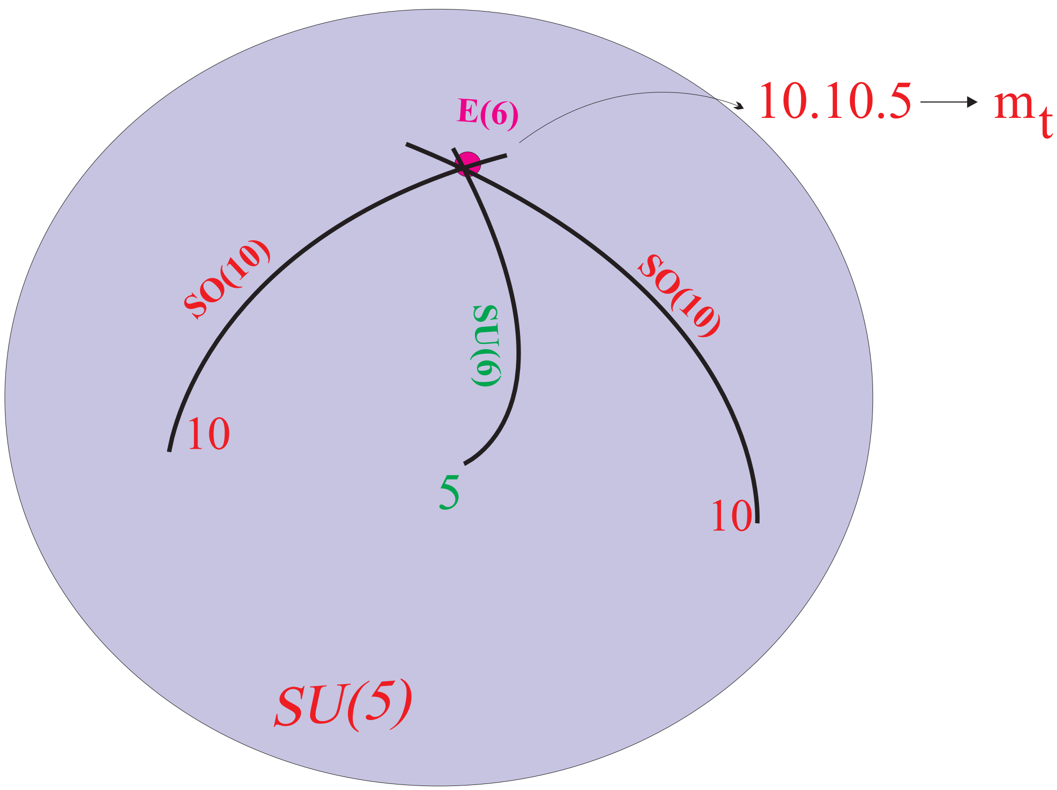


gauge symmetry ... further ... enhanced!



$(\bar{5})$
SU(6)
SU(5)
(10)
SO(10)
SU(6)
$10\ \bar{5}\ \bar{5}$
SO(12)
SO(10)
10 10 5
E_6

top Yukawa enhancement ...



Along a matter curve Σ gauge symmetry is enhanced...

★ Twisted YM and F-Spectrum

10-d Super YM theory :

$$\begin{cases} 10dim \text{ Gauge Field } A \\ \text{Adjoint fermions in } 16_+ \text{ of } SO(9,1) \end{cases}$$

Under Reduction $SO(9,1) \to SO(7,1) \times U(1)_R$ fields decompose to

$$\begin{cases} 8dim \text{ Gauge Field } A \\ \text{scalars } \varphi, \bar{\varphi} = A_8 \pm i\, A_9 \\ \text{fermions } \Psi_\pm = (S_\pm, \pm\frac{1}{2}) \end{cases}$$

F-theory described by 8-d YM Compactified on $R^{7,1} = R^{3,1} \times S$.

$$SO(7,1) \times U(1)_R \to SO(3,1) \times SO(4) \times U(1)_R$$

The 8-d spinor Ψ_+ decomposes $(O(4) \sim SU(2) \times SU(2))$

$$\left(S_+, \frac{1}{2}\right) \to \left((2,1), (2,1), \frac{1}{2}\right) \oplus \left((1,2), (1,2), -\frac{1}{2}\right)$$

$\Rightarrow$ globally, NOT well defined!

TWIST:

$J \sim U(1) \in U(2)$, $J_R \sim U(1)_R \to J_\pm = J \pm 2J_R$

$\Rightarrow$

$$\left(S_+, \frac{1}{2}\right) \to \{(2,1) \otimes 2_1\} \oplus \{(1,2) \otimes (1_2 \oplus 1_0)\}$$

preserving $\mathcal{N} = 1$ SUSY.

(*Beasley, Heckmann, Vafa, 0802.3391*)

- Under twisting, scalars & fermions become **forms:**

$$\text{scalars} : \varphi \quad = \quad \varphi_{mn}\, dz^m \wedge dz^n$$

$$\text{fermions} : \quad = \quad \begin{cases} \eta_\alpha & (0,0) \\ \psi_{\dot\alpha} = \psi_{\dot\alpha m}\, dz^m & (1,0) \\ \chi_\alpha = \chi_{\dot\alpha mn}\, dz^m \wedge dz^n & (2,0) \end{cases}$$

The above can be organised in $\mathcal{N}=1$ multiplets

$$(\mathbf{A}_\mu, \eta),\ (\mathbf{A}_{\bar{\mathbf{m}}}, \psi_{\bar{\mathbf{m}}}),\ (\phi_{\mathbf{12}}, \chi_{\mathbf{12}})$$

Spectral Cover Approach...

• E_8 Highest symmetry in **Elliptic Fibration**

F-$SU(5)^{\star}$ from... E_8-breaking chain...

$$E_8 \supset SU(5) \times U(5)_{\perp} \supset SU(5) \times \mathbf{U(1)^4}$$

All matter in the adjoint of E_8,

$$248 \quad \rightarrow \quad (24,1) + (1,24) + (10,5) + (\overline{5},10) + (5,\overline{10}) + (\overline{10},\overline{5})$$

Recall Weierstrass' equation for the $SU(5)$ singularity

$$y^2 = x^3 + b_0 z^5 + b_2 x z^3 + b_3 y z^2 + b_4 x^2 z + b_5 x y$$

$\rightarrow$ spectral cover obtained by defining homogeneous coordinates

$$z \rightarrow U,\ x \rightarrow V^2,\ y \rightarrow V^3,\ s = U/V$$

so Weierstrass becomes

$$0 \quad = \quad b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5$$

roots $\sum_i s_i = 0$ identified with $SU(5)_\perp$ Cartan subalgebra:

$$Q_t = \text{diag}\{t_1, t_2, t_3, t_4, t_5\}$$

★ *Matter curves* characterised by t_i's

Polynomial coefficients depend on t_i

$$b_k = b_k(t_i)$$

Inversion implies **brunchcuts**! ⇒ ..Simplest monodromy Z_2 : :

$$a_1 + a_2 s + a_3 s^2 = 0 \rightarrow s_{1,2} = \frac{-a_2 \pm \sqrt{w}}{2a_3}$$

Under $\theta \rightarrow \theta + 2\pi \rightarrow \sqrt{w} \rightarrow -\sqrt{w}$ branes interchange locations

$$s_1 \leftrightarrow s_2 \text{ or } t_1 \leftrightarrow t_2$$

2 $\mathbf{U(1)}$'s related by monodromies ... gauge symmetry reduces to:

$$SU(5) \times U(1)^4 \rightarrow \mathbf{SU(5)} \times \mathbf{U(1)^3}$$

Field	$SU(5) \times SU(5)_\perp$ **Rep.**	t_i direction	R-parity
Q_3, U_3^c, l_3^c	$(10, 5)$	$t_{1,2}$	$-$
Q_2, U_2^c, l_2^c	$(10, 5)$	t_4	$-$
Q_1, U_1^c, l_1^c	$(10, 5)$	t_3	$-$
D_3^c, L_3	$(\overline{5}, 10)$	$t_{1,2} + t_4$	$-$
D_2^c, L_2	$(\overline{5}, 10)$	$t_{1,2} + t_3$	$-$
D_1^c, L_1	$(\overline{5}, 10)$	$t_3 + t_4$	$-$
H_u	$(5, \overline{10})$	$-t_1 - t_2$	$+$
H_d	$(\overline{5}, 10)$	$t_3 + t_5$	$+$
θ_{ij}	$(1, 24)$	$t_i - t_j$	$+$
θ'_{ij}	$(1, 24)$	$t_i - t_j$	$-$

Fermion Masses

Two ways to obtain Fermion Mass Hierarchy in F-theory

- All families on the same curve(s) ($\Sigma_{10}, \Sigma_{\bar{5}}$)

  Flux corrections $\Rightarrow$ Hierarchy...

- Families assigned on different matter curves ($\Sigma_{10}^{1,2,3}, \Sigma_{\bar{5}}^{1,2,3}$)

  Monodromy $\rightarrow$ $\mathbf{10_{t_1}} = \mathbf{10_{t_2}} \equiv \mathbf{10_3}$: Rank one mass matrices:

$$\lambda_t \, 10_3 \cdot 10_3 \cdot 5_H$$

Hierarchy organised by $U(1)$**'s** from underlying E_8 via:
Singlet vevs $\langle \theta_{ij} \rangle$

Choice: $\langle\theta_{14}\rangle \cdot \langle\theta_{43}\rangle \neq 0$

- **Rank one** Quark mass matrices (*Dudas-Palti; GKL, Ross*)

$$M_d = \begin{pmatrix} \lambda^d_{11}\theta^2_{14}\theta^2_{43} & \lambda^d_{12}\theta_{14}\theta^2_{43} & \lambda^d_{13}\theta_{14}\theta_{43} \\ \lambda^d_{21}\theta^2_{14}\theta_{43} & \lambda^d_{22}\theta_{14}\theta_{43} & \lambda^d_{23}\theta_{14} \\ \lambda^d_{31}\theta_{14}\theta_{43} & \lambda^d_{32}\theta_{43} & 1\times\lambda^d_{33} \end{pmatrix} v_b, \qquad (10)$$

$$M^u = \begin{pmatrix} \lambda^u_{11}\theta^2_{14}\theta^2_{43} & \lambda^u_{12}\theta^2_{14}\theta_{43} & \lambda^u_{13}\theta_{14}\theta_{43} \\ \lambda^u_{21}\theta^2_{14}\theta_{43} & \lambda^u_{22}\theta^2_{14} & \lambda^u_{23}\theta_{14} \\ \lambda^u_{31}\theta_{14}\theta_{43} & \lambda^u_{32}\theta_{14} & 1\times\lambda^u_{33} \end{pmatrix} v_u \qquad (11)$$

- λ_{ij} computed from overlapping integrals ... *expected of* $\mathcal{O}(1)$.

Are λ_{ij} really $\sim \mathcal{O}(1)$???

★ Zero-modes Equations

Recall that **S** Kähler spanned by $z_{1,2}$, with form given by

$$\omega = \frac{\imath}{2}\left(dz^1 \wedge d\bar{z}^1 + dz^2 \wedge d\bar{z}^2\right)$$

Assume a background for the adjoint scalar φ

$$\langle\varphi\rangle = m^2(z_1 Q_1 + z_2 Q_2)$$

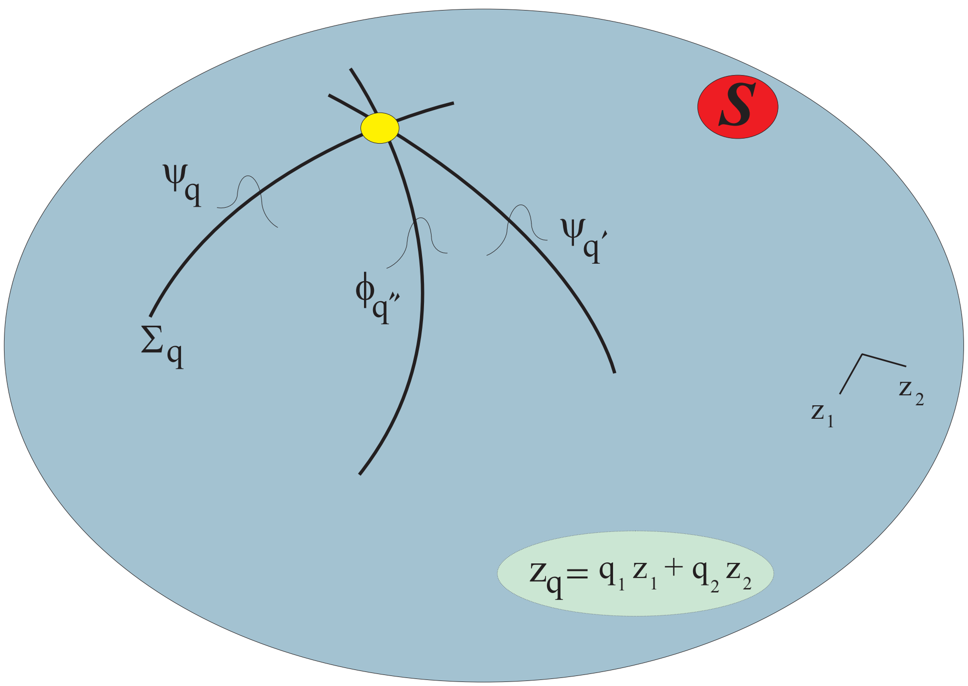
Equs of motion → D.E.s for **zero modes**:

$$\begin{aligned}
\partial_1\psi_1 + \partial_2\psi_2 - m^2(q_1\bar{z}_1 + q_2\bar{z}_2)\chi &= 0 \\
\bar{\partial}_1\chi - m^2(q_1\, z_1 + q_2\, z_2)\psi_1 &= 0 \\
\bar{\partial}_2\chi - m^2(q_1\, z_1 + q_2\, z_2)\psi_2 &= 0
\end{aligned}$$

- Solution: Gaussian profile for zero mode wavefunction:

$$\psi \propto e^{\{-m^2 q|\cos\theta z_1 + \sin\theta z_2|^2\}} \tag{12}$$

with $q = \sqrt{q_1^2 + q_2^2}$ *and* $\tan\theta = q_2/q_1$.



Trilinear coupling of two fermion fields ψ_i and a Higgs ϕ

Trilinear Yukawa coupling Integral:

- Computation in terms of overlapping wavefunction integrals

$$\lambda_{ij} = \frac{M_*^4}{(2\pi)^2} \int_S \psi_i \cdot \psi_j \cdot \phi \, dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2$$

- ψ normalization : (*Font,Ibanez 2009, Heckman et al 2009*)

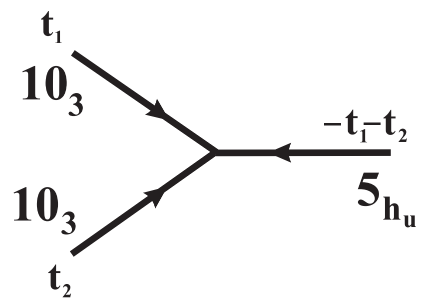
$$\mathcal{C} = M_*^4 \int_S |\psi|^2 dz \wedge d\bar{z} = \frac{\pi}{q} \frac{M_*^4}{m^2} R^2 = \frac{\pi}{q} \frac{1}{\sqrt{a_G}}$$

$(assuming\ M_* \sim m\ and\ M_*^2 R^2 \sim a_G,\ R \to\ curvature)$

$$\lambda_{ij} = \frac{4\sqrt{\pi\, a_G}^{3/2}}{q + q' + q''} \frac{(qq'q'')^{3/2}}{(q_1 q_2' - q_1' q_2)^2}.$$

Computing $\lambda_{33} \equiv \lambda_{top}$: ($GKL$, *GG Ross* JHEP 1102:108,2011.)

Define 'vector basis': $|t_i>_j = \delta_{ij}$ ($i.e.$ $< t_1| = \{1,0,0,0,0\}$, etc)



Define Locally the operators Q_i: ($U(1)_i \in SU(5)_\perp$)

$$Q_1 = \frac{1}{\sqrt{30}}\{3,3,-2,-2,-2\}$$

$$Q_2 = \frac{1}{\sqrt{2}}\{1,-1,0,0,0\},$$

$$Q_3 = \frac{1}{\sqrt{2}}\{0,0,1,0,-1\}$$

$$Q_4 = \frac{1}{2}\{0,0,1,-2,1\}$$

Vertex states $|t_{1,2}>, |-t_1-t_2>$ of top coupling are annihilated by:

$$Q_{3,4}\,|t_{1,2}> = 0$$

while, acting by $Q_{1,2}$:

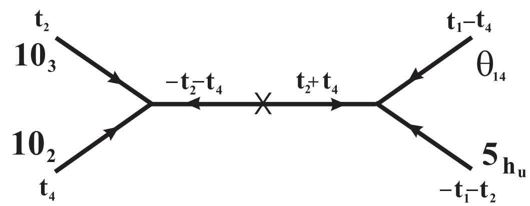
$$\{q_1, q_2\} = \left\{\sqrt{\frac{3}{10}}, \frac{1}{\sqrt{2}}\right\},\ \{q_1', q_2'\} = \left\{\sqrt{\frac{3}{10}}, -\frac{1}{\sqrt{2}}\right\}$$

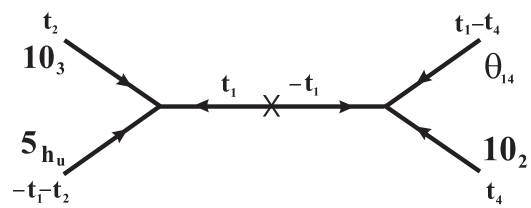
Substitution to the overlapping integral:

$$\lambda_{top} = 0.31 \times \left(\frac{a_G}{a_{G_0}}\right)^{\frac{3}{4}}, \quad a_{G_0} = \frac{1}{24}$$

(*RG analysis indicates a better fit ....see GKL, N.D. Tracas, G. Tsamis, arXiv:1102.5244*)

Computing higher dimensional couplings: Example: λ^u_{23}.





$\lambda^u_{23} \rightarrow 10_3\, 10_2\, 5_{h_u} \theta_{14}$ with exchange of massive messenger states

- KK-mode wavefunction: Obeys modified DE

$$(\partial_1 - \imath A_1)\psi_1 - m^2 q_1 \bar{z}_1 \chi = 0$$

Solution: $\psi_1 \sim e^{-q_1 m^2 \xi\, |z_1|^2}$, $\xi < 1$.

Left vertex of U_{23}-graphs:

$$10_i 10_j 5_{_{KK}} \;:\; \{t_i\}_{0-mode} + \{t_j\}_{0-mode} \rightarrow \{-t_i - t_j\}_{KK}$$

Calculation of the overlapping integral:

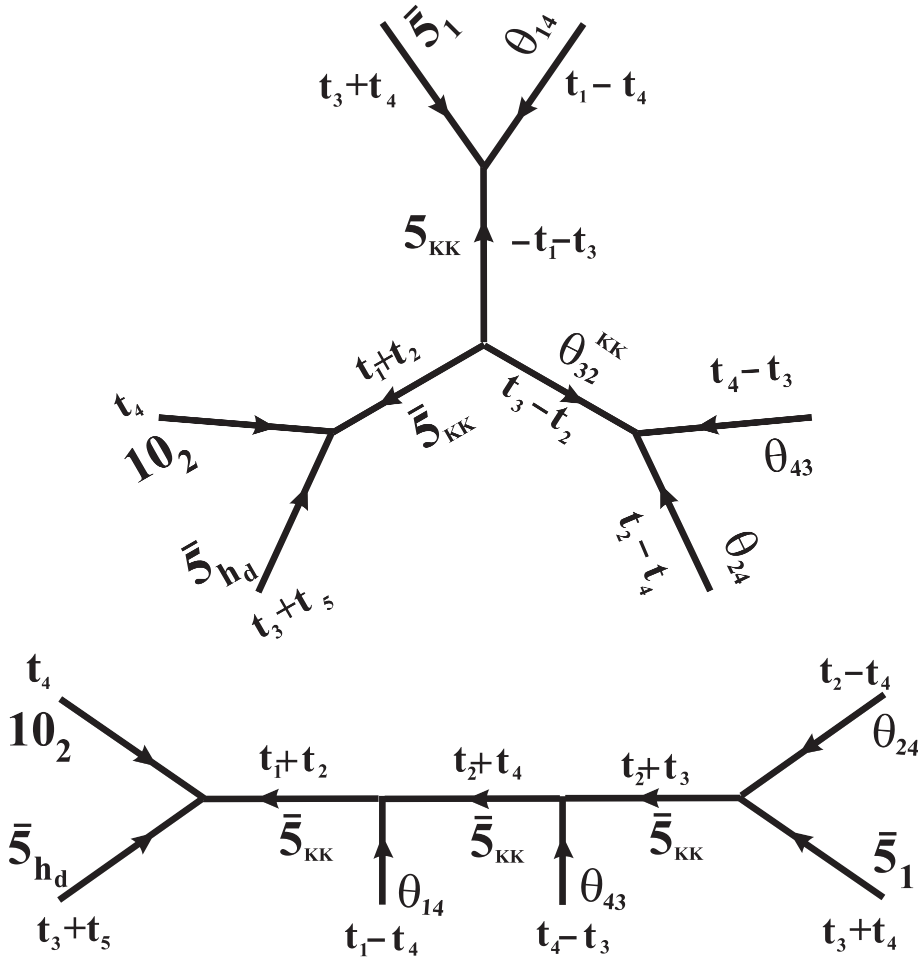
$$I_a(\xi) = \frac{8\sqrt{2\pi\,\xi}}{3 \cdot 5^{3/4}\left(4\xi + \sqrt{6}\right)}$$

Right vertices ($\overline{10} \cdot 10 \cdot 1$ and $\overline{5} \cdot 5 \cdot 1$) :

$$I_x(\xi) = \frac{8\sqrt{10\pi\,\xi}}{3 \cdot 3^{3/4}\left(4\sqrt{5}\xi + 5\sqrt{2}\right)},\; I_y = \frac{2 \cdot 3^{3/4}\sqrt{10\pi}\sqrt{\xi}}{7\left(\left(5 + \sqrt{15}\right)\xi + \sqrt{15}\right)}$$

U_{23} - **Yukawa Coupling:**

$$\lambda^u_{23}(\xi) = I_a(\xi) \cdot \left(I_x(\xi) + I_y(\xi)\right)$$



Representative graphs for λ^b_{21} Yukawa coupling.

Results: (simplified case $\xi = 1$)

$$M_d = \begin{pmatrix} 0.12\,\theta_{14}^2\theta_{43}^2 & 0.11\,\theta_{14}\theta_{43}^2 & 0.18\,\theta_{14}\,\theta_{43} \\ 0.14\,\theta_{14}^2\theta_{43} & 0.16\,\theta_{14}\theta_{43} & 0.20\,\theta_{14} \\ 0.09\,\theta_{14}\theta_{43} & 0.17\,\theta_{43} & 0.29 \end{pmatrix}$$

$$M_u = \begin{pmatrix} 0.09\,\theta_{14}^2\theta_{43}^2 & 0.22\,\theta_{14}^2\theta_{43} & 0.16\,\theta_{14}\theta_{43} \\ 0.22\,\theta_{14}^2\theta_{43} & 0.18\,\theta_{14}^2 & 0.22\,\theta_{14} \\ 0.16\,\theta_{14}\theta_{43} & 0.22\,\theta_{14} & 0.31 \end{pmatrix}$$

For $\langle\theta_{43}\rangle \sim \frac{1}{2}$, $\langle\theta_{14}\rangle \sim \frac{1}{10} \rightarrow$

Reasonable hierarchy and CKM-mixing

▲▼ $\mathcal{C}$harged $\mathcal{L}$eptons ▲▼

Major problem in **SU(5)**: Wrong mass relations:

$$m_s = m_\mu \;\&\; m_d = m_e \quad \text{at } M_{GUT}$$

SOLUTION:

▲▼**Splitting of $SU(5)$-reps** via the **FLUX mechanism**

Two types of fluxes:

▲M_{10}, M_5: connected to $U(1)$'s$\in SU(5)_\perp$: determine the chirality of complete $10, 5 \in SU(5)$.

▲N_Y: related to Cartan generators of $SU(5)_{GUT}$. They are taken along $U(1)_Y \in SU(5)_{GUT}$ and **split** $SU(5)$-reps.

$U(1)_\perp$–Flux on SM reps $\in$ **10**'s:

$$n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} = M_{10} \tag{13}$$

$$n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} = M_{10}$$

$$n_{(1,1)_1} - n_{(1,1)_{-1}} = M_{10} \tag{14}$$

$U(1)_\perp$– Flux on SM reps $\in$ **5**'s:

$$n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} = M_5 \tag{15}$$

$$n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5$$

(*...subject to:* $\sum_i M_{10}^i + \sum_j M_5^j = 0$)

$U(1)_Y-$**Flux**-splitting of **10**'s:

$$\begin{aligned} n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} &= M_{10} \\ n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} &= M_{10} - \mathbf{N_Y} \\ n_{(1,1)_1} - n_{(1,1)_{-1}} &= M_{10} + \mathbf{N_Y} \end{aligned}$$

$U(1)_Y-$ **Flux**-splitting of **5**'s:

$$\begin{aligned} n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{\frac{1}{3}}} &= M_5 \\ n_{(1,2)_{\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} &= M_5 + \mathbf{N_Y} \end{aligned}$$

Application: Choice of M_{10_i}, M_{5_j} so that:

$$matter\,curve\,10^{(3)}\;:\quad\rightarrow\quad Q_2+u_2^c\equiv\begin{pmatrix}c\\s\end{pmatrix}+c^c$$

$$matter\,curve\,10^{(4)}\;:\quad\rightarrow\quad e_2^c\equiv\mu^c$$

(*...plus extra vector-like* $u^c+\bar{u}^c$)

$$\Rightarrow M_{\ell epton}\neq M_{down}$$

$$M_\ell \quad=\quad \begin{pmatrix}\theta_{14}^2\theta_{43}^2 & \theta_{15}\theta_{14}\theta_{43} & \theta_{14}\,\theta_{43}\\ \theta_{14}^2\theta_{43} & \theta_{15}\theta_{43} & \theta_{14}\\ \theta_{14}\theta_{43} & \theta_{15} & 0.295\end{pmatrix}$$

$\rightarrow$ $m_\mu\sim 3\cdot m_s$!, $m_d\sim m_e$?...

Neutrinos

Z_2-monodromy $\theta'_{12} \leftrightarrow \theta'_{21} \Rightarrow$

$$\theta'_{12} = \textbf{Right-Handed Majorana Neutrino}$$

(*Bouchard et al, 0904.1419*)

Assume also:

$$\theta'_{14},\ \theta'_{41},\ \theta'_{14},\ \theta'_{41}$$

States are

$$\nu_{@} \approx \frac{1}{\sqrt{2}}\left[\nu_3 - \nu_2 + \sqrt{2}\langle\theta_{14}\rangle(\langle\theta_{43}\rangle - 1))\nu_1\right]$$

$$\nu_{\odot} \approx \frac{1}{N}\left[\nu_3 + \nu_2 + \sqrt{2}\langle\theta_{14}\rangle(1 + \langle\theta_{43}\rangle)\nu_1\right]$$

Parameters can be adjusted to give bi-tri maximal mixing.

SUMMARY

1. F-theory offers new insights in **Yukawa** textures
2. **Matter** localised on "curves" in **Internal Geometry**

   Chirality related to topology and to the internal flux

3. Wave functions localised along these "matter curves"
4. Yukawa couplings, at **triple intersections**
5. **Fermion mass textures** completely determined from

   a) $\lambda(q, a_G)$ and,

   b) familon vevs $\langle\theta_{ij}\rangle$

6. Flux mechanism:

   i) $m_\mu - m_s$ splitting at M_{GUT}

   ii) Doublet-Triplet splitting (*suppressing Proton Decay...*)

THRESHOLDS

1. $U(1)_Y$ Flux Thresholds
2. KK-massive Modes
3. D_3 brane probes
4. SUSY

I. Flux thresholds

Correction terms through CS-action $S_{CS} \sim \int C_0 \wedge F^4$:

$$\frac{1}{a_3(M_G)} = \frac{1}{a_G} - y, \; \frac{1}{a_2(M_G)} = \frac{1}{a_G} - y + x, \; \frac{1}{a_1(M_G)} = \frac{1}{a_G} - y + \frac{3}{5}x$$

(*Blumenhagen, Phys.Rev.Lett.102:071601,2009.*)

$$x = -\frac{1}{2} S \int c_1^2(\mathcal{L}_Y), \quad y = \frac{1}{2} S \int c_1^2(\mathcal{L}_a)$$

$S = e^{-\phi} + \imath C_0 \;\rightarrow$ axion-dilaton field.

x: *not completely arbitrary*:

$$24 \rightarrow (8,1)_0 + (1,1)_0 + (3,2)_{-5/6} + (\bar{3},2)_{5/6}$$

elimination of exotics $(3,2), (\bar{3},2) \;\Rightarrow$

$$\chi(S, \mathcal{L}_{\mathcal{Y}}) = 0 \;\Rightarrow \int c_1^2(\mathcal{L}_Y) = -2$$

At M_{GUT} :

$$\frac{5}{3}\frac{1}{a_1(M_G)} = \frac{1}{a_2(M_G)} + \frac{2}{3}\frac{1}{a_3(M_G)} \tag{16}$$

★ $a_i(M_G)$ relation weaker than unification $a_i(M_G) = a_G$.
Let M_X : decoupling scale of extra matter

$$[5(b_1^x - b_1) - 2(b_3^x - b_3)] \ln\left(\frac{M_G}{M_X}\right) = 0 \tag{17}$$

a: **Minimal case:** If extra matter **only** *color* triplets, $[\cdots] \equiv 0 \Rightarrow$

$$\forall\, x \quad \exists\ M_X$$

$\Rightarrow M_G \sim 2 \times 10^{16}$GeV is found as the scale where (16) holds.
(GKL, N.D.Tracas arXiv:0912.1557)

II: Threshold effects from heavy KK-modes

A: The gauge multiplet

One Loop running of gauge couplings:

$$\frac{16\pi^2}{g_a^2(\mu)} = \frac{16\pi^2 k_a}{g_s^2} + b_a \log\frac{\Lambda^2}{\mu^2} + \mathcal{S}_a^{(g)}, \quad a = 3, 2, Y \cdot \tag{18}$$

Λ : Cutoff scale.

KK-Thresholds:

$$\mathcal{S}_a^{(g)} = 2\sum_i \mathrm{Tr}_{R_i} Q_a^2 \;\; \mathrm{Str}_{M\neq 0}\left(\frac{1}{12} - \chi^2\right)\log\left(\frac{\Lambda^2}{M^2}\right)$$

χ: helicity operator

R_i: representations from the adjoint decomposition $24 = \bigoplus R_i$

$$24 \to (8,1)_0 + (1,3)_0 + (1,1)_0 + (3,2)_{-5/6} + (\bar{3},2)_{5/6}$$

M: KK-mass scale

Laplacian $\Delta = \bar\partial\bar\partial^\dagger + \bar\partial^\dagger\bar\partial$ on Riemann surface acting on k-form

$$\Delta_{k,R}\,\psi_n^{(k)} = \lambda_n^{(k)}\,\psi_n^{(k)}$$

In 4-d, eigenvalues $\rightarrow$ mass squared: $\lambda_n^{(k)} \rightarrow M^2$

Thresholds written as (*Friedmann, Witten, hep-th/0211269*)

$$\mathcal{S}_a^{(g)} = 2\sum_i \mathrm{Tr}_{R_i}(Q_a^2)\mathcal{K}_i$$

$\mathcal{K}_i$ equal twice the Ray-Singer analytic torsion

$$\mathcal{T}_{R_i} = \frac{1}{2}\sum_{k=0}^{2}(-1)^{k+1}\log\det{}'\frac{\Delta_{k,R_i}}{\Lambda^2}$$

Ray-Singer Theorem:

Torsion independent of the metric of the Manifold and $\Lambda \rightarrow$

$$\mathcal{T}_R = \mathit{Topological\ Invariant}$$

Thresholds can be cast to the form

$$\mathcal{S}'^{(g)}_a = 20\, k_a \mathcal{T}_{5/6} + \frac{4}{3} b_a{}^{(g)} \left(\mathcal{T}_{5/6} - \mathcal{T}_0\right) \tag{19}$$

(*with* $b_a^{(g)} = (0, -6, -9)$)

Gauge coupling running becomes:

$$\frac{16\pi^2}{g_a^2(\mu)} = \underbrace{k_a \left(\frac{16\pi^2}{g_s^2} + 20\mathcal{T}_{5/6}\right)} + \underbrace{b_a^{(g)} \log \frac{\exp\left[4/3\left(\mathcal{T}_{5/6} - \mathcal{T}_0\right)\right]}{\mu^2 V^{1/2}}}$$

$$= k_a \frac{16\pi^2}{g_{GUT}^2} + b_a^{(g)} \log \frac{M_{GUT}^2}{\mu^2} \tag{20}$$

with the GUT scale defined as: $(M_C = V^{-1/4})$

$$M_{GUT} = e^{2/3\left(\mathcal{T}_{5/6} - \mathcal{T}_0\right)} M_C$$

B: The Chiral and Higgs Fields

1) Bulk fields $(3,2)_{-5/6}+(\bar{3},2)_{5/6}$, eliminated...

2) $10,\overline{10}\in\Sigma_{10}$, $5,\bar{5}\in\Sigma_5$

2a) KK-massive modes:

Thresholds	$SU(3)$	$SU(2)$	$U(1)$
$S_a^{\bar{5}}$	$\mathcal{K}_{1/3}$	$\mathcal{K}_{-1/2}$	$\mathcal{K}_{-1/2}+2/3\,\mathcal{K}_{1/3}$
S_a^{10}	$2\mathcal{K}_{1/6}+\mathcal{K}_{-2/3}$	$3\mathcal{K}_{1/6}$	$\mathcal{K}_{1/6}+2\mathcal{K}_1+8/3\mathcal{K}_{-2/3}$

★ : ∃ examples of line-bundles with the **only** effect:

SHIFT of gauge coupling value at M_{GUT}

$$k_a\,\frac{16\pi^2}{g_{GUT}^2}\to k_a\left(\frac{16\pi^2}{g_{GUT}^2}+\sum_i c_i\mathcal{T}_i\right)$$

2b)Chiral and Higgs massless spectrum:

Complete $SU(5)$ **multiplets:** $b_a \propto k_a \to (b_3, b_2, b_Y) \propto (1:1:\frac{5}{3})$

replace :

$$b_a \ \log\left(\frac{\Lambda'^2}{\mu^2}\right) \to b_a \ \log\left(\frac{M_{GUT}^2}{\mu^2}\right)$$

However:

i) Color triplets should be eliminated

$$5_H \to \not{T} + h_u \ \bar{5}_H \to \not{\bar{T}} + h_d$$

ii)$U(1)_Y$ **Flux** (breaking $SU(5)$) splits $10, 5$ multiplets

$\Rightarrow$ **Restrictions on spectrum of low energy models.**

(*GKL*, *N.D. Tracas*, *G. Tsamis*, *arXiv:1102.5244*)

An $F - SU(5)$ fulfilling the requirements (GKL, GG Ross JHEP 1102:108,2011.)

Chiral Matter											
	M	N	Q	u^c	e^c		M	N	d^c	L
$10^{(1)}\ (F_3)$	1	0	1	1	1	$5^{(4)}\ (\bar{f}_1)$	-1	0	-1	-1
$10^{(2)}\ (F_{2,1})$	1	-1	1	2	0	$5^{(1)}\ (\bar{f}_2)$	-1	0	-1	-1
$10^{(3)}\ (F_{1,2})$	1	1	1	0	2	$5^{(2)}\ (\bar{f}_3)$	-1	0	-1	-1
$10^{(4)}\ (-)$	0	0	0	0	0	$5^{(3)}\ (-)$	0	0	0	0

Higgs and Colour Triplets				
	M	N	T	$h_{u,d}$
$5^{(0)}\ (h_u, T)$	1	0	1	1
$5^{(5)}\ (h_d)$	0	-1	0	-1
$5^{(6)}\ (\bar{T})$	-1	1	-1	0

OUTCOME

1)KK-thresholds $\propto$ Analytic Tortion (*topologically invariant*)

2) GUT-scale intimately related to KK-massive modes scale

3) cutoff scale independence $\rightarrow$ constraints on spectrum

4) scales' ratios topologically invariant

$$\frac{M_{GUT}}{M_C} = e^{\frac{2}{3}(\mathcal{T}_{5/6}-\mathcal{T}_0)}$$

5) Yukawas

$$\lambda_{top} = e^{2(\mathcal{T}_{5/6}-\mathcal{T}_0)} \frac{4\sqrt{\pi}}{q+q'+q''} \frac{(qq'q'')^{3/2}}{(q_1 q_2' - q_1' q_2)^2} \sim \mathcal{O}(1)$$

ADDITIONAL MATERIAL

★ Gauge and Matter Fields in F-theory:

Gravity: 10-d space

Gauge Fields: 8d space on **seven-branes** (S) supporting gauge group G_S

Matter Fields: 6d, on Riemann surfaces

$$\mathbf{\Sigma_i} = S \cap S_i, \quad i = 1, 2, \ldots$$

(i.e., on the intersections of the compact surface S with other surfaces S_i supporting some group G_i usually taken to be $G_i = U(1)_i$.)

Interactions: 4d, triple intersections

$$\mathbf{W_Y} \; : \; S \cap S_i \cap S_j \rightarrow \text{point}$$

At the intersections symmetry is enhanced!

$$G_{\Sigma_i} \supset G_S \times G_{S_i}$$

$G_S = SU(5)$: Singularity enhancement:

- Matter curves accommodating $\bar{\mathbf{5}}$ are associated with $SU(6)$

$$\Sigma_{\bar{5}} = S \cap S_{\bar{5}} \quad \Rightarrow \quad SU(5) \to SU(6)$$

$$\mathrm{ad}_{SU_6} = 35 \quad \to \quad 24_0 + 1_0 + 5_6 + \bar{5}_{-6}$$

- Matter curves accommodating $\mathbf{10}$ are associated with $SO(10)$

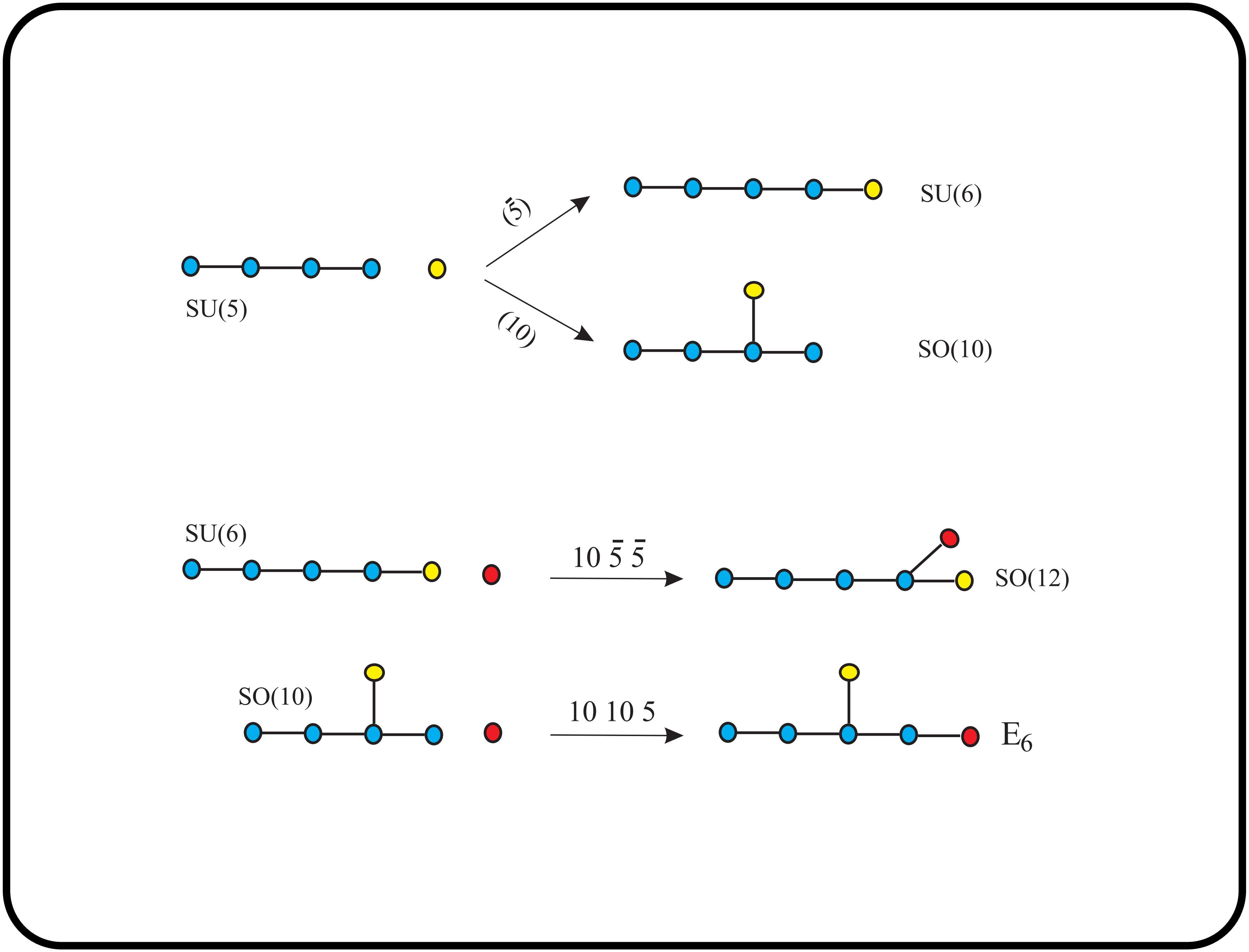
$$\Sigma_{10} = S \cap S_{10} \quad \Rightarrow \quad SU(5) \to SO(10)$$

$$\mathrm{ad}_{SO_{10}} = 45 \quad \to \quad 24_0 + 1_0 + 10_4 + \overline{10}_{-4}$$

- Further enhancement in triple intersections→ **Yukawas**:

$$SO(10) \equiv E_5 \quad \Rrightarrow \quad E_6 \to \mathbf{10} \cdot \mathbf{10} \cdot \mathbf{5}$$

$$SU(6) \quad \Rrightarrow \quad SO(12) \to \mathbf{10} \cdot \bar{\mathbf{5}} \cdot \bar{\mathbf{5}}$$



SU(5)
($\bar{5}$)
(10)
SU(6)
SO(10)
SU(6)
10 $\bar{5}$ $\bar{5}$
SO(12)
SO(10)
10 10 5
E_6

MONODROMIES

Recall coefficients b_i from $E_8 \to SU(5)$ deformed singularity.

A parameter s can be defined locally on **S**

$$f(s) = b_0 s^5 + b_2 s^3 + b_3 s^2 + b_4 s + b_5 = 0$$

whose roots are t_i. Thus $b_j = b_j(t_i)$. $\quad (b_1 = \sum_i t_i = 0)$

Factorization of $f(s)$ implies symmetries (monodromies for t_i

Monodromies for $SU(5) \to U(1)^4$, $(t_1 + t_2 + t_3 + t_4 + t_5 = 0)$:

$$\begin{array}{rcl} \mathcal{S}_4 & : & \{t_1, t_2, t_3, t_4\},\ \{t_5\} \\ \mathcal{Z}_2 \times \mathcal{Z}_2 & : & \{t_1, t_2\},\ \{t_3, t_4\}, \{t_5\} \\ \mathcal{Z}_2 & : & \{t_1, t_2\},\ \{t_3\}, \{t_4\}, \{t_5\} \\ \cdots & & \cdots \end{array} \tag{21}$$

Gauge Couplings and the GUT scale

In F-theory gauge coupling unification is distorted by

I: Y-Flux breaking mechanism

&

II: Threshold effects from heavy modes

$SU(5)$ Fields expected to contribute:

$$\begin{aligned}
\Sigma_{5/\bar{5}} &\rightarrow D + \ell \quad + c.c. \\
\Sigma_{10,\overline{10}} &\rightarrow Q + u^c + e^c \ + c.c. \\
24_{(Bulk)} &\rightarrow (8,1)_0 + (1,3)_0 + (1,1)_0 + \\
& \quad\ Q' = (3,\bar{2})_{-5} + \bar{Q}' = (\bar{3},2)_5
\end{aligned}$$

$\cdots + \cdots$ states decoupling at M_{CFT} (see *Vafa*, 2010)

b: General case:

Extra matter (in vector like pairs) n_u, n_L, n_Q, n_{e^c}.

Assume decoupling at scale M_X.

Inequalities (**??**) for $a_i(M_U)$ are converted to extra matter constraints

$$(n_d - n_L - 2n_e - n_u + 3n_Q)\ \ln\frac{M_U}{M_X} > 32\ \ln\frac{M_U}{M_Z} - 4\pi\left(\frac{\cos^2\theta_W}{a_{em}} - \frac{5}{3a_3}\right)$$

$$(n_d - n_L + 3n_e + 4n_u - 7n_Q)\ \ln\frac{M_U}{M_X} > -28\ \ln\frac{M_U}{M_Z} + 2\pi\frac{3 - 8\sin^2\theta_W}{a_{em}}$$

Constraints can be converted to $M_X - M_U$ plots for values of

$$\beta_x = b_Y^x - b_2^x - (2/3)b_3^x$$

(For example $\beta_x = 10$ for $\{2Q, 2u\}$, $\{4Q,\ 6u\}$, or $\{6Q,\ 8u,\ 2e\}, \ldots$ etc.)

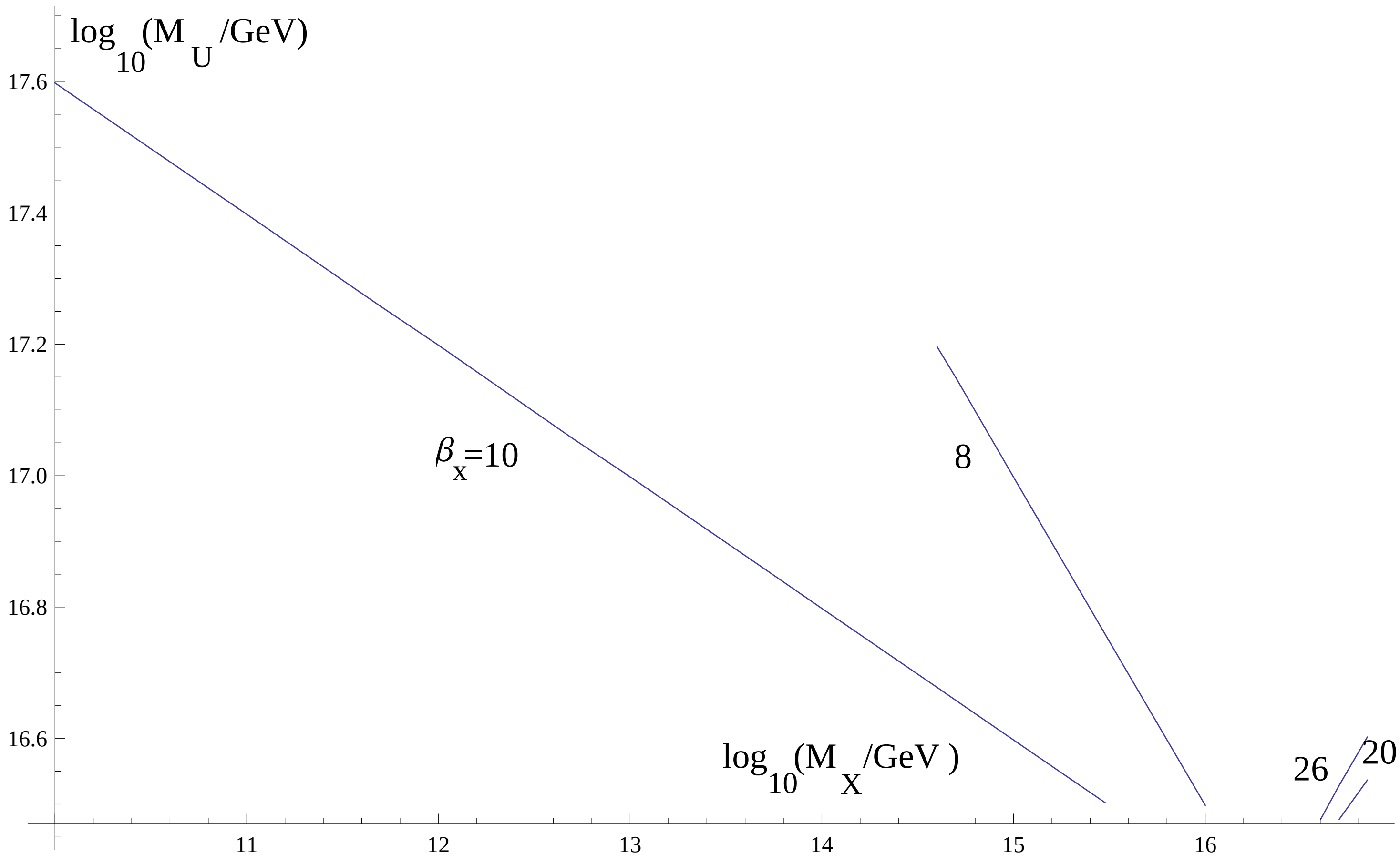


Figure 2: M_U as a function of the decoupling scale M_X for extra matter context $\beta_x = 10, 8$, and $\beta_x = 20, 26$.

M_X(GeV)	M_U(GeV)	$a_3(M_U)$	$(5/3)a_1(M_U)$	$a_2(M_U)$	x
10^{11}	6.23×10^{17}	0.1568	0.1510	0.1473	0.4103
10^{12}	2.90×10^{17}	0.0993	0.0973	0.0961	0.3335
10^{13}	1.51×10^{17}	0.0743	0.0735	0.0730	0.2308
10^{14}	8.22×10^{16}	0.0599	0.0596	0.5947	0.1154

Table 3: Two-loop results for the $SU(5)$ GUT scale M_U and the 'shifted' gauge coupling values $a_i(M_U)$ in the case of two vector-like $Q + \bar{Q}$ quark pairs and three $u^c + \bar{u}^c$ pairs. The corresponding decoupling scale M_X is shown in the first column.

Gaugino masses

Departure from Standard gaugino scenario due to gauge coupling splitting at M_{GUT} ⋆:

$$\mathbf{2\,\frac{M_3}{a_3} + 3\,\frac{M_2}{a_2} - 5\,\frac{M_1}{a_1} = 0} \tag{22}$$

GKL & N.D. Tracas, arXiv:0912.1557, E.P.J.C67:489-498,2010;

Sum rule (22) holds irrespectively of the initial $M_i(M_G)$ (universal or not) and leads to interesting phenomenological implications. (see also *T. Li et al, arXiv:1002.1031*)

⋆ *The same sum rule has been obtained in the context of no-scale SUGRA. See J. Ellis et al, Phys.Lett.B155:381,1985.*

Additional Material

Example... Let S is a Hirzebruch surface F_n. This is generated by the effective classes f, σ with intersections

$$f \cdot f = 0,\ f \cdot \sigma = 1,\ \sigma \cdot \sigma = -n \tag{23}$$

with first Chern class $c_1(F_n) = (n+2)f + 2\sigma$ and $K_S = -c_1(F_n)$. We choose $n = 1$ and then $F_n \to F_1$. Now chiral zero modes 16_{-3} and $\overline{16}_3$ belong to the cohomology groups

$$\begin{aligned} 16_{-3} &\in H^1_{\bar{\partial}}(F_1, \mathcal{L}^{-3}) \\ \overline{16}_{+3} &\in H^1_{\bar{\partial}}(F_1, \mathcal{L}^{+3}) \end{aligned}$$

Now, a line bundle $\mathcal{L}$ is supersymmetric if there exist integers a, b so that $a\,b < 0$ and $\mathcal{L}$ given by

$$\mathcal{L} = \mathcal{O}_{F_1}(af + b\sigma) \tag{24}$$

Then, the net number of chiral fermion reps is

$$
\begin{aligned}
n_{16} - n_{\overline{16}} &= -\int_{F_1} c_1(\mathcal{T}) c_1(F_1) \\
&\rightarrow -(-3)\left\{(af + b\sigma)\cdot(3f + 2\sigma)\right\} \\
&= 3\left(3af\cdot f + 3bf\cdot\sigma + 2af\cdot\sigma + 2b\sigma\cdot\sigma\right) \\
&= 3(3a\cdot 0 + 3b\cdot 1 + 2a\cdot 1 - 2b\cdot n) \\
&= 3(2a + b) \qquad (25)
\end{aligned}
$$

where $n = 1$ for F_1 was used. We can set $a = -b = 1$ to obtain three generations and satisfy $ab < 0$.

Blow Up: To get a feeling of the blow-up procedure, consider the following cubic curve

$$y^2 = x^3 + x \tag{26}$$

which intersects itself at the origin. To blow-up the curve and resolve the singularity, we consider two homogeneous coordinates u, v. Then the blowing up is defined by the equation $xu = yv$. We can set $v = 1$ and take u as affine parameter, then we have the equations $y = u(u^2 - 1)$ and $x = u^2 - 1$. We show the result in figure 3.

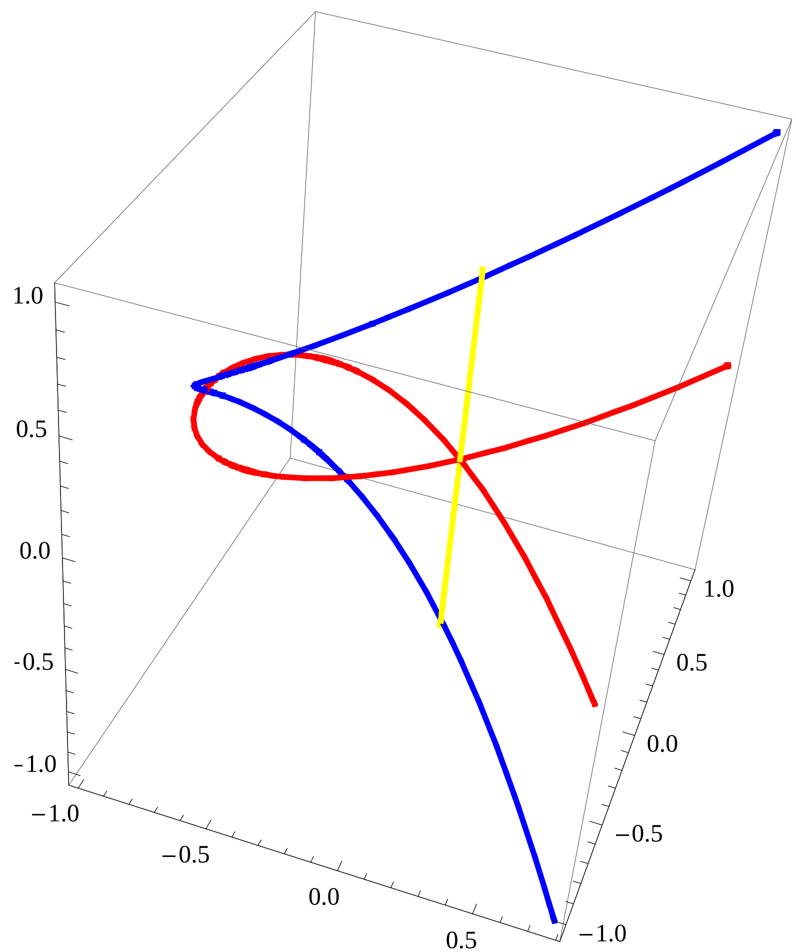


Figure 3: Blow-up of a singularity....