Heterotic Line Bundle Models in Four Dimensions

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model building: arXiv:1106.4804, arXiv:1202.1757 with Lara Anderson, James Gray and Eran Palti

moduli stabilization: arXiv:0903.5088, arXiv:0905.1748, arXiv:1010.0255, arXiv:1102.0011, arXiv:1107.5076, with Lara Anderson, James Gray and Burt Ovrut

earlier work: hep-th/0702210, arXiv:0805.2875, arXiv:0911.1569, . . .
with Lara Anderson, James Gray, Yang-Hui He

Overview

- Introduction: Constructing Line Bundle Models
- Four-dimensional effective theory
- Phenomenological issues and examples
- Moduli stabilization
- Conclusion and outlook

Introduction

Data to define a heterotic line bundle model we need:

- A Calabi-Yau 3-fold X
- A line bundle sum $V = L_1 \oplus \cdots \oplus L_5$ on X,
  $c_1(V) = 0$, so structure group is $S(U(1)^5)$.
- vanishing slopes $\mu(L_a) \equiv c_1(L_a) \wedge J^2 \overset{!}{=} 0$
- Anomaly: $c_2(TX) - c_2(V) - c_2(\tilde{V}) = [C]$
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N=1, D=4 GUT with gauge group $SU(5) \times S(U(1)^5)$ and matter in $\mathbf{10}, \bar{\mathbf{10}}, \bar{\mathbf{5}}, \mathbf{5}, \mathbf{1}$

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- freely acting symmetry Γ on X, so $\hat{X} = X/\Gamma$ is smooth and non simply-connected
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standard-like model (hopefully) with gauge group $G_{\rm SM} \times S(U(1)^5)$

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10 multiplets: $$H^1(X,V) = \bigoplus_a H^1(X,L_a)$$

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- Additional U(1)s are usually Green-Schwarz anomalous, hence, associated vector bosons are massive.

- Abelian bundles still carries many of the properties of the generic non-Abelian bundles.

- But we have to remember they are usually part of a moduli space of non-Abelian bundles.

Recall: 4d gauge group is $SU(5) \times S(U(1)^5) \cong SU(5) \times U(1)^4$

Label $S(U(1)^5)$ repr. by integer vectors $\mathbf{q} = (q_1, \ldots, q_5)$ with identification $\mathbf{q} \sim \tilde{\mathbf{q}}$ iff $\mathbf{q} - \tilde{\mathbf{q}} \in \mathbb{Z}\,(1,1,1,1,1)$.

Describe Wilson line by two Γ representations W and $\tilde{W}$.

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$SU(5)$ repr.	$G_{\rm SM}$ repr.	name	cohomology
$\mathbf{10}_{\mathbf{e}_a}$	$(\mathbf{3},\mathbf{2})_1$ $(\bar{\mathbf{3}},\mathbf{1})_{-4}$ $(\mathbf{1},\mathbf{1})_6$	Q_a u_a e_a	$h^1(X, L_a, \chi_a \otimes W^* \otimes \tilde{W}^*)$ $h^1(X, L_a, \chi_a \otimes W^* \otimes W^*)$ $h^1(X, L_a, \chi_a \otimes \tilde{W}^* \otimes \tilde{W}^*)$
$\overline{\mathbf{10}}_{-\mathbf{e}_a}$	$(\bar{\mathbf{3}},\mathbf{2})_{-1}$ $(\mathbf{3},\mathbf{1})_4$ $(\mathbf{1},\mathbf{1})_{-6}$	$\bar{Q}_a$ $\bar{u}_a$ $\bar{e}_a$	$h^2(X, L_a, \chi_a \otimes W^* \otimes \tilde{W}^*)$ $h^2(X, L_a, \chi_a \otimes W^* \otimes W^*)$ $h^2(X, L_a, \chi_a \otimes \tilde{W}^* \otimes \tilde{W}^*)$
$\bar{\mathbf{5}}_{\mathbf{e}_a+\mathbf{e}_b}$	$(\bar{\mathbf{3}},\mathbf{1})_2$ $(\mathbf{1},\mathbf{2})_{-3}$	$d_{a,b},\ T_{a,b}$ $L_{a,b},\ H_{a,b}$	$h^1(L_a \otimes L_b, \chi_a \otimes \chi_b \otimes W)$ $h^1(L_a \otimes L_b, \chi_a \otimes \chi_b \otimes \tilde{W})$
$\mathbf{5}_{-\mathbf{e}_a-\mathbf{e}_b}$	$(\mathbf{3},\mathbf{1})_{-2}$ $(\mathbf{1},\mathbf{2})_3$	$\bar{T}_{a,b}$ $\bar{H}_{a,b}$	$h^2(L_a \otimes L_b, \chi_a \otimes \chi_b \otimes W)$ $h^2(L_a \otimes L_b, \chi_a \otimes \chi_b \otimes \tilde{W})$
$\mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}$	$(\mathbf{1},\mathbf{1})_0$	$S_{a,b}$	$h^1(L_a \otimes L_b^*, \chi_a \otimes \chi_b^*)$

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		$(\mathbf{1},\mathbf{2})_3$	$\bar{H}_{a,b}$	$h^2(L_a\otimes L_b,\chi_a\otimes\chi_b\otimes\tilde{W})$	> 0 (Higgs doublet)
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Search for promising models: on complete intersection CYs (CICYs)

for example: quintic $X \sim [\mathbb{P}^4|5]$ or bi-cubic $X \sim \left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array}\right]$

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Complete classification of about 8000 spaces

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Classification of freely-acting discrete symmetries

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Have scanned CICYs with freely-acting symmetries and $h^{1,1}(X) \leq 5$ (60 spaces) and line bundle sums $-9 + h^{1,1}(X) \leq k_a^i \leq 0 - h^{1,1}(X)$.

On CICYs with $h^{1,1}(X) = 4, 5$ we find ~ 2000 standard models.*

*standard model: SM gauge group times (anomalous) U(1)s, exact MSSM matter spectrum, one or more pairs of Higgs doublets, no exotics charged under standard model group.

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These models and their details are available at:

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There will be many more!

(L. Anderson, A. Constantin, J. Gray, AL, E. Palti, in progress)

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Four-dimensional effective theory

gauge group

$$G_{\text{SM}} \times S(U(1)^5)$$

charges $\mathbf{q} = (q_1, \ldots, q_5)$ with $\mathbf{q} \sim \tilde{\mathbf{q}} \Leftrightarrow \mathbf{q} - \tilde{\mathbf{q}} \in \mathbb{Z}(1,1,1,1,1)$

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matter

$$(\mathbf{10}^p) = (Q^p, u^p, e^p), \quad (\bar{\mathbf{5}}^p) = (d^p, L^p), \quad H, \quad \bar{H}, \quad S^\alpha \quad (p = 1, 2, 3)$$

$$\mathbf{Q}(\mathbf{10}^p) = \mathbf{e}_{a_p} \quad \mathbf{Q}(\bar{\mathbf{5}}^p) = \mathbf{e}_{b_p} + \mathbf{e}_{c_p}$$

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$\mathbf{Q}(S^\alpha) = \mathbf{e}_{d_\alpha} - \mathbf{e}_{f_\alpha}$ $\qquad S_{d_\alpha, f_\alpha}$

No exotics charged under SM group, no vector-like pairs etc.

moduli

$$\mathcal{S} = s + i\sigma, \quad T^i = t^i + 2i\chi^i, \quad Z, \quad \cdots$$

$$\delta_a \chi^i = -k_a^i \quad \delta_a \sigma = -k_a^i \beta_i$$

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U(1) masses

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D-terms

$$D_a = k_a^i t_i - \sum_\alpha Q_a(S^\alpha)|S^\alpha|^2$$

	4d U(1)s	bundle
all $S^\alpha = 0$:	$U(1)^4$	$S(U(1)^5)$

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$$> 4 - (\text{ number of } T^i)$$

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$$D_a = k^i_a t_i - \sum_\alpha Q_a(S^\alpha)|S^\alpha|^2$$

	4d U(1)s	bundle
all $S^\alpha = 0$:	$U(1)^4$	$S(U(1)^5)$

moduli

$$\mathcal{S} = s + i\sigma, \quad T^i = t^i + 2i\chi^i, \quad Z, \quad \cdots$$

$$\delta_a \chi^i = -k_a^i \qquad \delta_a \sigma = -k_a^i \beta_i$$

U(1) masses

$$M_{ab} = \mathbf{k}_a^T G \mathbf{k}_b$$

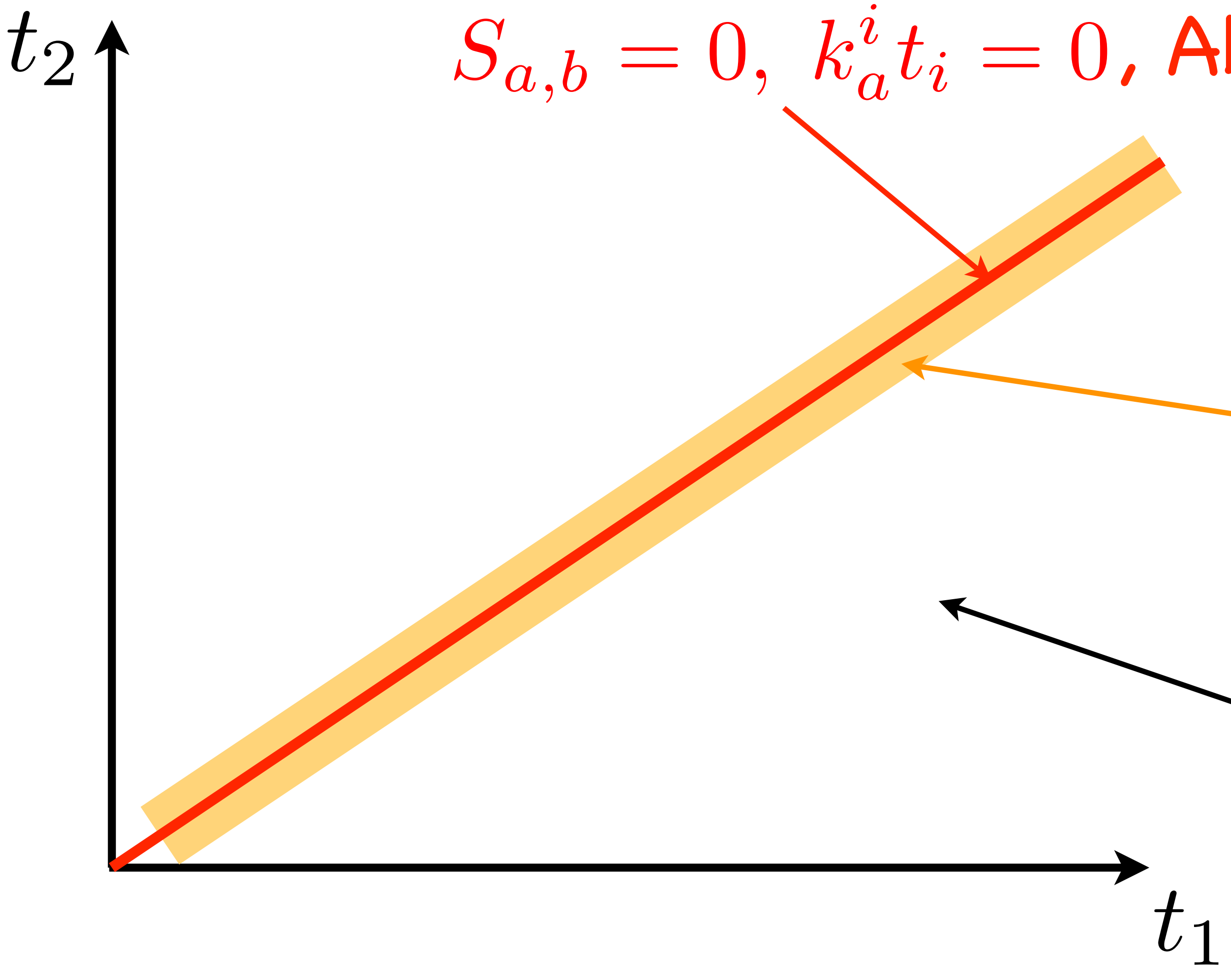
$$(\text{number of massless U(1)s}) = 4 - \text{rank}(k_a^i)$$

$$> 4 - (\text{ number of } T^i)$$

D-terms

$$D_a = k_a^i t_i - \sum_\alpha Q_a(S^\alpha)|S^\alpha|^2$$

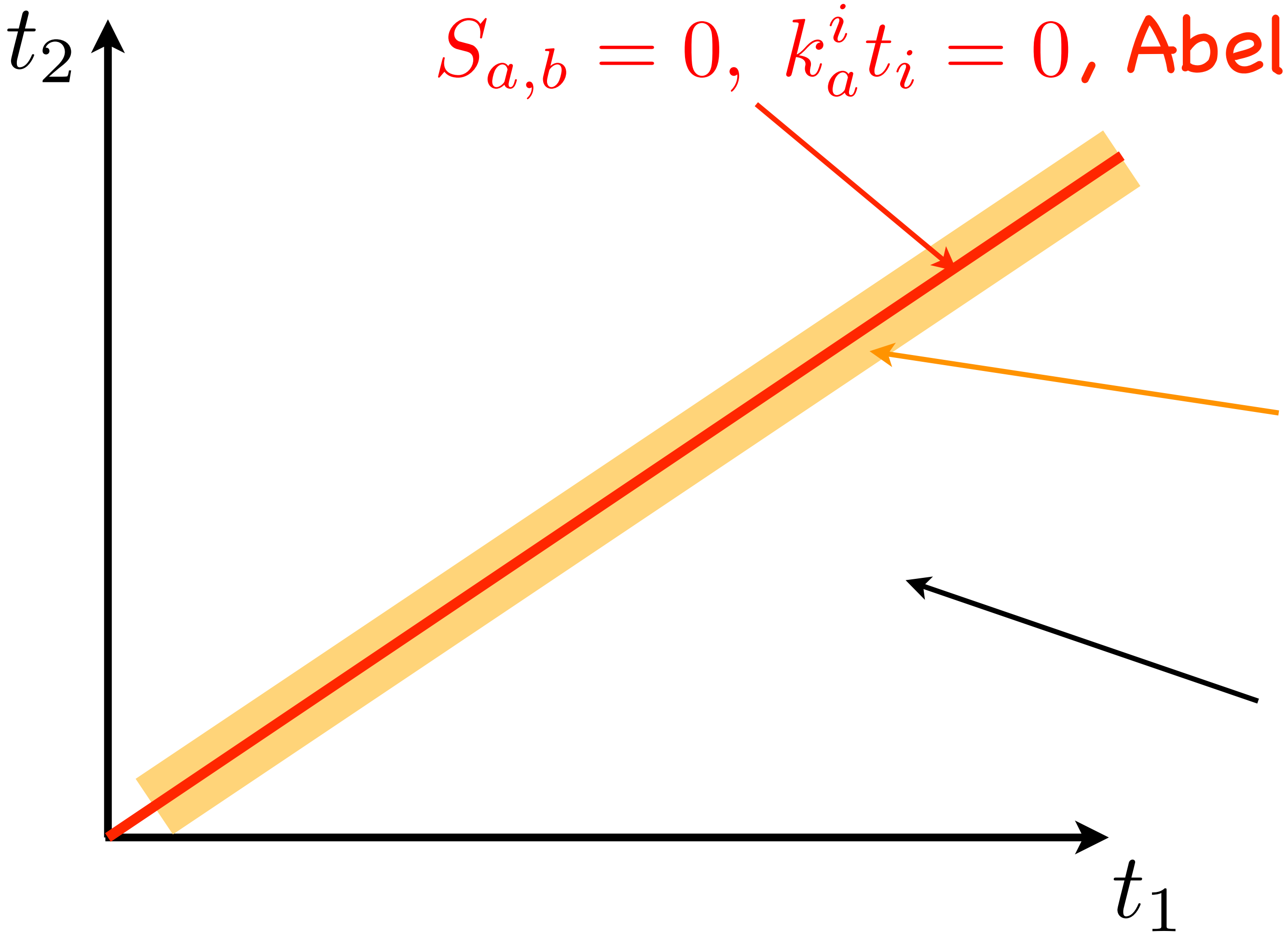
	4d U(1)s	bundle
all $S^\alpha = 0$:	$U(1)^4$	$S(U(1)^5)$
some $S^\alpha \neq 0$:	$U(1)^{f-1}$	$S(U(n_1) \times \cdots \times U(n_f))$



$S_{a,b} = 0,\ k_a^i t_i = 0$, Abelian

$S_{a,b}$ small, can be described in 4d theory

$S_{a,b}$ large, need to construct non-Abelian bundle



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Example: $S_{4,5} \neq 0$

4d U(1)s: $U(1)^4 \to U(1)^3$

structure group: $S(U(1)^5) \to S(U(1)^3 \times U(2))$

bundle: $V = L_1 \oplus \cdots \oplus L_5 \to V = L_1 \oplus L_2 \oplus L_3 \oplus U$

$$0 \to L_4 \to U \to L_5 \to 0\ , \quad \mathrm{Ext}^1(L_5, L_4) \cong H^1(X, L_4 \otimes L_5^*) \ni S_{4,5}$$

Constraints from $S(U(1)^5)$, e.g. superpotential:

$$W = \mu(S,Z)H\bar{H} + Y^{(d)}_{pq}(S,Z)H\bar{\mathbf{5}}^p\mathbf{10}^q + Y^{(u)}_{pq}(S,Z)\bar{H}\mathbf{10}^p\mathbf{10}^q$$

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$$\mu = \mu_0 + \mu_{1,\alpha}S^\alpha + \mu_{2,\alpha\beta}S^\alpha S^\beta + \ldots$$

0 ↗

↖ 0 for vector-like H, $\bar{H}$

↖ constrained by U(1)s

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Texture at each order in S^α

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$$Y = Y_0 + Y_{1,\alpha}S^\alpha + Y_{2,\alpha\beta}S^\alpha S^\beta + \ldots$$

Texture at each order in S^α

How to get fermion masses?

top Yukawa: $\mathrm{rk}(Y_0) > 0$ but only $\mathrm{rk}(Y_0) = 0, 2, 3$ possible:

$\bar{H}_{a,b}\mathbf{10}_a\mathbf{10}_b$ but $a \neq b$ since $\wedge^2 V = \sum_{a<b} L_a \otimes L_b$

Two possible ways out:

1) Non-trivial equivariant blocks of line bundles -> analogue of F-theory monodromy.

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$$\wedge^2 U^* \neq 0 \;\Rightarrow\; \text{charge } -2\mathbf{e}_4$$

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How to generate other masses:

If some S^α small: Froggat-Nielsen mechanism

If all S^α large: non-perturbative effects

... or combination of both

Note: $e^{-n_i T^i} \times (\text{matter})$ constrained by U(1)s as well

Phenomenological issues and example

CY data:

- **Cicy 6777, Symmetry 1**

$$X = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

$\eta(X) = -64 \quad h^{1,1}(X) = 5 \quad h^{2,1}(X) = 37 \quad c_2(TX) = \{24, 24, 24, 24, 56\}$

$\kappa = 12\,t_1\,t_2\,t_3 + 12\,t_1\,t_2\,t_4 + 12\,t_1\,t_3\,t_4 + 12\,t_2\,t_3\,t_4 + 24\,t_1\,t_2\,t_5 + 24\,t_1\,t_3\,t_5 + 12\,t_2\,t_3\,t_5 + 24\,t_1\,t_4\,t_5 + 24\,t_2\,t_4\,t_5 + 24\,t_3\,t_4\,t_5 + 24\,t_1\,t_5^2 + 12\,t_2\,t_5^2 + 12\,t_3\,t_5^2 + 24\,t_4\,t_5^2 + 8\,t_5^3$

symmetry: 1   order: 2

Abelian: True   block diagonal: True   factors: {2}

Action on coordinates: $\left\{\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\right\}$

Action on polynomials: $\left\{\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\right\}$

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Phenomenological issues and example

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volume

symmetry: 1   order: 2

Abelian: True   block diagonal: True   factors: {2}

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Phenomenological issues and example

CY data:

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$$X = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

← definition of CY in $(\mathbb{P}^1)^{\times 4} \times \mathbb{P}^3$

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Action on polynomials: $\left\{\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\right\}$

bundle data:

Model number 13, Identifier {6777, 2, 1}

Basic properties

standard model? True massless U(1): 0 number of $5\,\overline{5}$ pairs: 1 $c_2(V) = \{8, 6, 6, 14, 24\}$

$$V:\ (k^i{}_a) = \begin{pmatrix} 1 & 1 & 0 & 0 & -2 \\ 0 & -1 & 1 & -1 & 1 \\ 0 & -2 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix}$$

Cohomology of V:

L_2	=	{1, −1, −2, 0, 1}	$h[L_2]$	=	{0, 4, 0, 0}	$h[L_2,R]$	=	{{0, 0}, {2, 2}, {0, 0}, {0, 0}}
L_5	=	{−2, 1, 0, 0, 0}	$h[L_5]$	=	{0, 2, 0, 0}	$h[L_5,R]$	=	{{0, 0}, {1, 1}, {0, 0}, {0, 0}}
$L_2 \times L_4$	=	{1, −2, −1, 0, 1}	$h[L_2 \times L_4]$	=	{0, 4, 0, 0}	$h[L_2 \times L_4,R]$	=	{{0, 0}, {2, 2}, {0, 0}, {0, 0}}
$L_2 \times L_5$	=	{−1, 0, −2, 0, 1}	$h[L_2 \times L_5]$	=	{0, 1, 1, 0}	$h[L_2 \times L_5,R]$	=	{{0, 0}, {1, 0}, {1, 0}, {0, 0}}
$L_4 \times L_5$	=	{−2, 0, 1, 0, 0}	$h[L_4 \times L_5]$	=	{0, 2, 0, 0}	$h[L_4 \times L_5,R]$	=	{{0, 0}, {1, 1}, {0, 0}, {0, 0}}
$L_1 \times L_2{}^*$	=	{0, 1, 2, −1, −1}	$h[L_1 \times L_2{}^*]$	=	{0, 0, 4, 0}	$h[L_1 \times L_2{}^*,R]$	=	{{0, 0}, {0, 0}, {2, 2}, {0, 0}}
$L_1 \times L_5{}^*$	=	{3, −1, 0, −1, 0}	$h[L_1 \times L_5{}^*]$	=	{0, 0, 8, 0}	$h[L_1 \times L_5{}^*,R]$	=	{{0, 0}, {0, 0}, {4, 4}, {0, 0}}
$L_2 \times L_3{}^*$	=	{1, −2, −3, −1, 2}	$h[L_2 \times L_3{}^*]$	=	{0, 4, 0, 0}	$h[L_2 \times L_3{}^*,R]$	=	{{0, 0}, {2, 2}, {0, 0}, {0, 0}}
$L_2 \times L_4{}^*$	=	{1, 0, −3, 0, 1}	$h[L_2 \times L_4{}^*]$	=	{0, 12, 0, 0}	$h[L_2 \times L_4{}^*,R]$	=	{{0, 0}, {6, 6}, {0, 0}, {0, 0}}
$L_2 \times L_5{}^*$	=	{3, −2, −2, 0, 1}	$h[L_2 \times L_5{}^*]$	=	{0, 11, 3, 0}	$h[L_2 \times L_5{}^*,R]$	=	{{0, 0}, {−2 + r1, 13 − r1}, {−6 + r1, 9 − r1}, {0, 0}}
$L_4 \times L_5{}^*$	=	{2, −2, 1, 0, 0}	$h[L_4 \times L_5{}^*]$	=	{0, 6, 0, 0}	$h[L_4 \times L_5{}^*,R]$	=	{{0, 0}, {3, 3}, {0, 0}, {0, 0}}

Wilson line: {{0}, {1}} Equivariant structure: {{0}, {0}, {0}, {0}, {1}} Higgs pairs: 1

Downstairs spectrum: $\left\{2\, 10_2,\ 10_5,\ 2\,\overline{5}_{2,4},\ \overline{5}_{4,5},\ H_{2,5},\ \overline{H}_{2,5},\ 2\, S_{2,1},\ 4\, S_{5,1},\ 2\, S_{2,3},\ 6\, S_{2,4},\ (13-r1)\, S_{2,5},\ (9-r1)\, S_{5,2},\ 3\, S_{4,5}\right\}$ Phys. Higgs: $\left\{H_{2,5},\ \overline{H}_{2,5}\right\}$

Legend: $\{r1 \to rk_1[H_2[N^* \times l2 \times l5*,\ 2],\ H_2[l2 \times l5*,\ 2]]\}$

Transfer format: {{6, 1, 1, 4, 6, 5, 5, 9, 8, 10, 12, 18, 1, 7, 13}, {6, 6, −1, −1, −1, −1}}

$rk(Y^{(u)}) = \{2, 3\}$ $rk(Y^{(d)})) = \{0, 0\}$ dim. 4 operators absent: {True, True} dim. 5 operators absent: {True, True}

bundle data:

- **Model number 13, Identifier {6777, 2, 1}**
- **Basic properties**

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$$V:\ (k^i{}_a) = \begin{pmatrix} 1 & 1 & 0 & 0 & -2 \\ 0 & -1 & 1 & -1 & 1 \\ 0 & -2 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix} \longleftarrow$$

integer matrix defining line bundle sum

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$L_2\times L_5$	=	{−1, 0, −2, 0, 1}	$h[L_2\times L_5]$	=	{0, 1, 1, 0}	$h[L_2\times L_5,R]$	=	{{0, 0}, {1, 0}, {1, 0}, {0, 0}}
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$L_1\times L_5{}^*$	=	{3, −1, 0, −1, 0}	$h[L_1\times L_5{}^*]$	=	{0, 0, 8, 0}	$h[L_1\times L_5{}^*,R]$	=	{{0, 0}, {0, 0}, {4, 4}, {0, 0}}
$L_2\times L_3{}^*$	=	{1, −2, −3, −1, 2}	$h[L_2\times L_3{}^*]$	=	{0, 4, 0, 0}	$h[L_2\times L_3{}^*,R]$	=	{{0, 0}, {2, 2}, {0, 0}, {0, 0}}
$L_2\times L_4{}^*$	=	{1, 0, −3, 0, 1}	$h[L_2\times L_4{}^*]$	=	{0, 12, 0, 0}	$h[L_2\times L_4{}^*,R]$	=	{{0, 0}, {6, 6}, {0, 0}, {0, 0}}
$L_2\times L_5{}^*$	=	{3, −2, −2, 0, 1}	$h[L_2\times L_5{}^*]$	=	{0, 11, 3, 0}	$h[L_2\times L_5{}^*,R]$	=	{{0, 0}, {−2 + r1, 13 − r1}, {−6 + r1, 9 − r1}, {0, 0}}
$L_4\times L_5{}^*$	=	{2, −2, 1, 0, 0}	$h[L_4\times L_5{}^*]$	=	{0, 6, 0, 0}	$h[L_4\times L_5{}^*,R]$	=	{{0, 0}, {3, 3}, {0, 0}, {0, 0}}

Wilson line: {{0}, {1}} Equivariant structure: {{0}, {0}, {0}, {0}, {1}} Higgs pairs: 1

Downstairs spectrum: $\left\{2\,10_2,\ 10_5,\ 2\,\overline{5}_{2,4},\ \overline{5}_{4,5},\ H_{2,5},\ \overline{H}_{2,5},\ 2\,S_{2,1},\ 4\,S_{5,1},\ 2\,S_{2,3},\ 6\,S_{2,4},\ (13-r1)\,S_{2,5},\ (9-r1)\,S_{5,2},\ 3\,S_{4,5}\right\}$ Phys. Higgs: $\left\{H_{2,5},\ \overline{H}_{2,5}\right\}$

Legend: $\{r1 \to rk_1[H_2[N^* \times l2 \times l5*,\ 2],\ H_2[l2 \times l5*,\ 2]]\}$

Transfer format: {{6, 1, 1, 4, 6, 5, 5, 9, 8, 10, 12, 18, 1, 7, 13}, {6, 6, −1, −1, −1, −1}}

$rk(Y^{(u)})$ = {2, 3} $rk(Y^{(d)}))$ = {0, 0} dim. 4 operators absent: {True, True} dim. 5 operators absent: {True, True}

bundle data:

- **Model number 13, Identifier {6777, 2, 1}**
- **Basic properties**

standard model? True massless U(1): 0 number of $5\,\overline{5}$ pairs: 1 $c_2(V) = \{8, 6, 6, 14, 24\}$

$$V:\ (k^i{}_a) = \begin{pmatrix} 1 & 1 & 0 & 0 & -2 \\ 0 & -1 & 1 & -1 & 1 \\ 0 & -2 & 1 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \end{pmatrix} \longleftarrow$$

integer matrix defining line bundle sum

Cohomology of V:

L_2	= {1, −1, −2, 0, 1}	$h[L_2]$	= {0, 4, 0, 0}	$h[L_2,R]$	= {{0, 0}, {2, 2}, {0, 0}, {0, 0}}
L_5	= {−2, 1, 0, 0, 0}	$h[L_5]$	= {0, 2, 0, 0}	$h[L_5,R]$	= {{0, 0}, {1, 1}, {0, 0}, {0, 0}}
$L_2{\times}L_4$	= {1, −2, −1, 0, 1}	$h[L_2{\times}L_4]$	= {0, 4, 0, 0}	$h[L_2{\times}L_4,R]$	= {{0, 0}, {2, 2}, {0, 0}, {0, 0}}
$L_2{\times}L_5$	= {−1, 0, −2, 0, 1}	$h[L_2{\times}L_5]$	= {0, 1, 1, 0}	$h[L_2{\times}L_5,R]$	= {{0, 0}, {1, 0}, {1, 0}, {0, 0}}
$L_4{\times}L_5$	= {−2, 0, 1, 0, 0}	$h[L_4{\times}L_5]$	= {0, 2, 0, 0}	$h[L_4{\times}L_5,R]$	= {{0, 0}, {1, 1}, {0, 0}, {0, 0}}
$L_1{\times}L_2{}^*$	= {0, 1, 2, −1, −1}	$h[L_1{\times}L_2{}^*]$	= {0, 0, 4, 0}	$h[L_1{\times}L_2{}^*,R]$	= {{0, 0}, {0, 0}, {2, 2}, {0, 0}}
$L_1{\times}L_5{}^*$	= {3, −1, 0, −1, 0}	$h[L_1{\times}L_5{}^*]$	= {0, 0, 8, 0}	$h[L_1{\times}L_5{}^*,R]$	= {{0, 0}, {0, 0}, {4, 4}, {0, 0}}
$L_2{\times}L_3{}^*$	= {1, −2, −3, −1, 2}	$h[L_2{\times}L_3{}^*]$	= {0, 4, 0, 0}	$h[L_2{\times}L_3{}^*,R]$	= {{0, 0}, {2, 2}, {0, 0}, {0, 0}}
$L_2{\times}L_4{}^*$	= {1, 0, −3, 0, 1}	$h[L_2{\times}L_4{}^*]$	= {0, 12, 0, 0}	$h[L_2{\times}L_4{}^*,R]$	= {{0, 0}, {6, 6}, {0, 0}, {0, 0}}
$L_2{\times}L_5{}^*$	= {3, −2, −2, 0, 1}	$h[L_2{\times}L_5{}^*]$	= {0, 11, 3, 0}	$h[L_2{\times}L_5{}^*,R]$	= {{0, 0}, {−2 + r1, 13 − r1}, {−6 + r1, 9 − r1}, {0, 0}}
$L_4{\times}L_5{}^*$	= {2, −2, 1, 0, 0}	$h[L_4{\times}L_5{}^*]$	= {0, 6, 0, 0}	$h[L_4{\times}L_5{}^*,R]$	= {{0, 0}, {3, 3}, {0, 0}, {0, 0}}

Wilson line: {{0}, {1}} Equivariant structure: {{0}, {0}, {0}, {0}, {1}} Higgs pairs: 1

Downstairs spectrum: $\left\{2\,10_2,\ 10_5,\ 2\,\overline{5}_{2,4},\ \overline{5}_{4,5},\ H_{2,5},\ \overline{H}_{2,5},\ 2\,S_{2,1},\ 4\,S_{5,1},\ 2\,S_{2,3},\ 6\,S_{2,4},\ (13-r1)\,S_{2,5},\ (9-r1)\,S_{5,2},\ 3\,S_{4,5}\right\}$ Phys. Higgs: $\left\{H_{2,5},\ \overline{H}_{2,5}\right\}$

Legend: $\{r1 \to rk_1[H_2[N^*{\times}l2{\times}l5{*},\ 2],\ H_2[l2{\times}l5{*},\ 2]]\}$

Transfer format: {{6, 1, 1, 4, 6, 5, 5, 9, 8, 10, 12, 18, 1, 7, 13}, {6, 6, −1, −1, −1, −1}}

$rk(Y^{(u)}) = \{2, 3\}$ $rk(Y^{(d)})) = \{0, 0\}$ dim. 4 operators absent: {True, True} dim. 5 operators absent: {True, True}

spectrum: $\mathbf{10}_2, \mathbf{10}_2, \mathbf{10}_5, \bar{\mathbf{5}}_{2,4}\bar{\mathbf{5}}_{2,4}, \bar{\mathbf{5}}_{4,5}, H_{2,5}, \bar{H}_{2,5},$

$S_{5,1}, S_{2,3}, S_{2,4}, S_{2,5}, S_{5,2}, S_{4,5}$

allowed operators (1):

Operators

basic superpotential terms:

$$\overline{H}10^p10^q\!:\ Y^{(u)} = \begin{pmatrix} \{S_{5,2}, S_{2,5}S_{5,2}^2, S_{2,4}S_{4,5}S_{5,2}^2, S_{2,5}^2S_{5,2}^3\} & \{S_{5,2}, S_{2,5}S_{5,2}^2, S_{2,4}S_{4,5}S_{5,2}^2, S_{2,5}^2S_{5,2}^3\} & \{1, S_{2,5}S_{5,2}, S_{2,4}S_{4,5}S_{5,2}, S_{2,5}^2S_{5,2}^2, S_{2,4}S_{2,5}S_{4,5}S_{5,2}^2\} \\ \{S_{5,2}, S_{2,5}S_{5,2}^2, S_{2,4}S_{4,5}S_{5,2}^2, S_{2,5}^2S_{5,2}^3\} & \{S_{5,2}, S_{2,5}S_{5,2}^2, S_{2,4}S_{4,5}S_{5,2}^2, S_{2,5}^2S_{5,2}^3\} & \{1, S_{2,5}S_{5,2}, S_{2,4}S_{4,5}S_{5,2}, S_{2,5}^2S_{5,2}^2, S_{2,4}S_{2,5}S_{4,5}S_{5,2}^2\} \\ \{1, S_{2,5}S_{5,2}, S_{2,4}S_{4,5}S_{5,2}, S_{2,5}^2S_{5,2}^2, S_{2,4}S_{2,5}S_{4,5}S_{5,2}^2\} & \{1, S_{2,5}S_{5,2}, S_{2,4}S_{4,5}S_{5,2}, S_{2,5}^2S_{5,2}^2, S_{2,4}S_{2,5}S_{4,5}S_{5,2}^2\} & \{S_{2,5}, S_{2,4}S_{4,5}, S_{2,5}^2S_{5,2}, S_{2,4}S_{2,5}S_{4,5}S_{5,2}, S_{2,4}^2S_{4,5}^2S_{5,2}, S_{2,5}^3S_{5,2}^2\} \end{pmatrix}$$

$$H\overline{5}^p10^q\!:\ Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$$

$$H\overline{H}\!:\ \mu = \{1, S_{2,5}S_{5,2}, S_{2,4}S_{4,5}S_{5,2}, S_{2,5}^2S_{5,2}^2, S_{2,4}S_{2,5}S_{4,5}S_{5,2}^2\}$$

$$W_{\text{sing}} = \{S_{2,4}S_{4,5}S_{5,2}, S_{2,5}^2S_{5,2}^2, S_{2,4}S_{2,5}S_{4,5}S_{5,2}^2\}$$

R−parity violating terms in superpotential:

$$\overline{H}L^p\!:\ \rho = \begin{pmatrix} \{S_{2,4}S_{5,2}, S_{2,4}S_{2,5}S_{5,2}^2, S_{2,4}^2S_{4,5}S_{5,2}^2\} \\ \{S_{2,4}S_{5,2}, S_{2,4}S_{2,5}S_{5,2}^2, S_{2,4}^2S_{4,5}S_{5,2}^2\} \\ \{S_{2,4}, S_{2,4}S_{2,5}S_{5,2}, S_{2,4}^2S_{4,5}S_{5,2}, S_{2,4}S_{2,5}^2S_{5,2}^2\} \end{pmatrix}$$

$$10^p\overline{5}^q\overline{5}^r\!:\ \lambda = \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\overline{5}^p10^q10^r10^s\!:\ \lambda' = \{\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$
$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$
$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}\}$$

D−terms:

$$\text{FI−terms: } k^i{}_a\kappa_i = \begin{pmatrix} -4t_1t_2 - 4t_1t_3 + 4t_2t_4 + 4t_3t_4 - 8t_1t_5 + 8t_4t_5 \\ 4t_1t_3 + 8t_2t_3 - 4t_1t_4 + 4t_2t_4 + 8t_3t_4 - 8t_1t_5 + 8t_2t_5 + 12t_3t_5 + 4t_5^2 \\ -4t_2t_4 - 4t_3t_4 + 8t_1t_5 + 4t_2t_5 + 4t_3t_5 + 8t_5^2 \\ 4t_1t_2 - 4t_1t_3 + 4t_2t_4 - 4t_3t_4 + 4t_2t_5 - 4t_3t_5 \\ 4t_1t_3 - 8t_2t_3 + 4t_1t_4 - 8t_2t_4 - 4t_3t_4 + 8t_1t_5 - 16t_2t_5 - 12t_3t_5 - 8t_4t_5 - 12t_5^2 \end{pmatrix}$$

$$\text{singlet D−terms: } q_{\alpha a}S^\alpha\overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1}S^\dagger{}_{2,1} - S_{5,1}S^\dagger{}_{5,1} \\ S_{2,1}S^\dagger{}_{2,1} + S_{2,3}S^\dagger{}_{2,3} + S_{2,4}S^\dagger{}_{2,4} + S_{2,5}S^\dagger{}_{2,5} - S_{5,2}S^\dagger{}_{5,2} \\ -S_{2,3}S^\dagger{}_{2,3} \\ -S_{2,4}S^\dagger{}_{2,4} + S_{4,5}S^\dagger{}_{4,5} \\ -S_{2,5}S^\dagger{}_{2,5} - S_{4,5}S^\dagger{}_{4,5} + S_{5,1}S^\dagger{}_{5,1} + S_{5,2}S^\dagger{}_{5,2} \end{pmatrix}$$

allowed operators (1):

- **Operators**

rank 2 for $\langle S^\alpha \rangle = 0$

basic superpotential terms:

$$\overline{\mathrm{H}}10^p10^q:\ Y^{(u)} = \begin{pmatrix} \{S_{5,2},\ S_{2,5}\,S_{5,2}^2,\ S_{2,4}\,S_{4,5}\,S_{5,2}^2,\ S_{2,5}^2\,S_{5,2}^3\} & \{S_{5,2},\ S_{2,5}\,S_{5,2}^2,\ S_{2,4}\,S_{4,5}\,S_{5,2}^2,\ S_{2,5}^2\,S_{5,2}^3\} & \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\} \\ \{S_{5,2},\ S_{2,5}\,S_{5,2}^2,\ S_{2,4}\,S_{4,5}\,S_{5,2}^2,\ S_{2,5}^2\,S_{5,2}^3\} & \{S_{5,2},\ S_{2,5}\,S_{5,2}^2,\ S_{2,4}\,S_{4,5}\,S_{5,2}^2,\ S_{2,5}^2\,S_{5,2}^3\} & \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\} \\ \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\} & \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\} & \{S_{2,5},\ S_{2,4}\,S_{4,5},\ S_{2,5}^2\,S_{5,2},\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2},\ S_{2,4}^2\,S_{4,5}^2\,S_{5,2},\ S_{2,5}^3\,S_{5,2}^2\} \end{pmatrix}$$

$$\mathrm{H}\overline{5}^p10^q:\ Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$$

$$\mathrm{H}\overline{\mathrm{H}}:\ \mu = \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\}$$

$$W_{\mathrm{sing}} = \{S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\}$$

R−parity violating terms in superpotential:

$$\overline{\mathrm{H}}\mathrm{L}^p:\ \rho = \begin{pmatrix} \{S_{2,4}\,S_{5,2},\ S_{2,4}\,S_{2,5}\,S_{5,2}^2,\ S_{2,4}^2\,S_{4,5}\,S_{5,2}^2\} \\ \{S_{2,4}\,S_{5,2},\ S_{2,4}\,S_{2,5}\,S_{5,2}^2,\ S_{2,4}^2\,S_{4,5}\,S_{5,2}^2\} \\ \{S_{2,4},\ S_{2,4}\,S_{2,5}\,S_{5,2},\ S_{2,4}^2\,S_{4,5}\,S_{5,2},\ S_{2,4}\,S_{2,5}^2\,S_{5,2}^2\} \end{pmatrix}$$

$$10^p\overline{5}^q\overline{5}^r:\ \lambda = \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\overline{5}^p10^q10^r10^s:\ \lambda' = \{\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$
$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$
$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}\}$$

D−terms:

$$\text{FI−terms:}\ k^i{}_a\kappa_i = \begin{pmatrix} -4\,t_1\,t_2 - 4\,t_1\,t_3 + 4\,t_2\,t_4 + 4\,t_3\,t_4 - 8\,t_1\,t_5 + 8\,t_4\,t_5 \\ 4\,t_1\,t_3 + 8\,t_2\,t_3 - 4\,t_1\,t_4 + 4\,t_2\,t_4 + 8\,t_3\,t_4 - 8\,t_1\,t_5 + 8\,t_2\,t_5 + 12\,t_3\,t_5 + 4\,t_5^2 \\ -4\,t_2\,t_4 - 4\,t_3\,t_4 + 8\,t_1\,t_5 + 4\,t_2\,t_5 + 4\,t_3\,t_5 + 8\,t_5^2 \\ 4\,t_1\,t_2 - 4\,t_1\,t_3 + 4\,t_2\,t_4 - 4\,t_3\,t_4 + 4\,t_2\,t_5 - 4\,t_3\,t_5 \\ 4\,t_1\,t_3 - 8\,t_2\,t_3 + 4\,t_1\,t_4 - 8\,t_2\,t_4 - 4\,t_3\,t_4 + 8\,t_1\,t_5 - 16\,t_2\,t_5 - 12\,t_3\,t_5 - 8\,t_4\,t_5 - 12\,t_5^2 \end{pmatrix}$$

$$\text{singlet D−terms:}\ q_{\alpha a}S^\alpha\overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1}\,S^\dagger{}_{2,1} - S_{5,1}\,S^\dagger{}_{5,1} \\ S_{2,1}\,S^\dagger{}_{2,1} + S_{2,3}\,S^\dagger{}_{2,3} + S_{2,4}\,S^\dagger{}_{2,4} + S_{2,5}\,S^\dagger{}_{2,5} - S_{5,2}\,S^\dagger{}_{5,2} \\ -S_{2,3}\,S^\dagger{}_{2,3} \\ -S_{2,4}\,S^\dagger{}_{2,4} + S_{4,5}\,S^\dagger{}_{4,5} \\ -S_{2,5}\,S^\dagger{}_{2,5} - S_{4,5}\,S^\dagger{}_{4,5} + S_{5,1}\,S^\dagger{}_{5,1} + S_{5,2}\,S^\dagger{}_{5,2} \end{pmatrix}$$

allowed operators (1):

- **Operators**

basic superpotential terms:

rank 2 for $\langle S^{\alpha} \rangle = 0$

$$\overline{H}10^p10^q:\ Y^{(u)} = \begin{pmatrix} \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} \\ \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} \\ \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} & \{S_{2,5}, S_{2,4} S_{4,5}, S_{2,5}^2 S_{5,2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}, S_{2,4}^2 S_{4,5}^2 S_{5,2}, S_{2,5}^3 S_{5,2}^2\} \end{pmatrix}$$

$$H\overline{5}^p10^q:\ Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix} \longleftarrow \text{rank 0}$$

$$H\overline{H}:\ \mu = \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\}$$

$$W_{\text{sing}} = \{S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\}$$

R−parity violating terms in superpotential:

$$\overline{H}L^p:\ \rho = \begin{pmatrix} \{S_{2,4} S_{5,2}, S_{2,4} S_{2,5} S_{5,2}^2, S_{2,4}^2 S_{4,5} S_{5,2}^2\} \\ \{S_{2,4} S_{5,2}, S_{2,4} S_{2,5} S_{5,2}^2, S_{2,4}^2 S_{4,5} S_{5,2}^2\} \\ \{S_{2,4}, S_{2,4} S_{2,5} S_{5,2}, S_{2,4}^2 S_{4,5} S_{5,2}, S_{2,4} S_{2,5}^2 S_{5,2}^2\} \end{pmatrix}$$

$$10^p\overline{5}^q\overline{5}^r:\ \lambda = \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\overline{5}^p10^q10^r10^s:\ \lambda' = \{\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$

$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$

$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}\}$$

D−terms:

$$\text{FI−terms: } k^i{}_a \kappa_i = \begin{pmatrix} -4 t_1 t_2 - 4 t_1 t_3 + 4 t_2 t_4 + 4 t_3 t_4 - 8 t_1 t_5 + 8 t_4 t_5 \\ 4 t_1 t_3 + 8 t_2 t_3 - 4 t_1 t_4 + 4 t_2 t_4 + 8 t_3 t_4 - 8 t_1 t_5 + 8 t_2 t_5 + 12 t_3 t_5 + 4 t_5^2 \\ -4 t_2 t_4 - 4 t_3 t_4 + 8 t_1 t_5 + 4 t_2 t_5 + 4 t_3 t_5 + 8 t_5^2 \\ 4 t_1 t_2 - 4 t_1 t_3 + 4 t_2 t_4 - 4 t_3 t_4 + 4 t_2 t_5 - 4 t_3 t_5 \\ 4 t_1 t_3 - 8 t_2 t_3 + 4 t_1 t_4 - 8 t_2 t_4 - 4 t_3 t_4 + 8 t_1 t_5 - 16 t_2 t_5 - 12 t_3 t_5 - 8 t_4 t_5 - 12 t_5^2 \end{pmatrix}$$

$$\text{singlet D−terms: } q_{\alpha a} S^{\alpha} \overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1} S^{\dagger}{}_{2,1} - S_{5,1} S^{\dagger}{}_{5,1} \\ S_{2,1} S^{\dagger}{}_{2,1} + S_{2,3} S^{\dagger}{}_{2,3} + S_{2,4} S^{\dagger}{}_{2,4} + S_{2,5} S^{\dagger}{}_{2,5} - S_{5,2} S^{\dagger}{}_{5,2} \\ -S_{2,3} S^{\dagger}{}_{2,3} \\ -S_{2,4} S^{\dagger}{}_{2,4} + S_{4,5} S^{\dagger}{}_{4,5} \\ -S_{2,5} S^{\dagger}{}_{2,5} - S_{4,5} S^{\dagger}{}_{4,5} + S_{5,1} S^{\dagger}{}_{5,1} + S_{5,2} S^{\dagger}{}_{5,2} \end{pmatrix}$$

allowed operators (1):

- **Operators**

basic superpotential terms:

rank 2 for $\langle S^{\alpha} \rangle = 0$

$$\overline{H}10^{p}10^{q}:\ Y^{(u)} = \begin{pmatrix} \{S_{5,2}, S_{2,5} S_{5,2}^{2}, S_{2,4} S_{4,5} S_{5,2}^{2}, S_{2,5}^{2} S_{5,2}^{3}\} & \{S_{5,2}, S_{2,5} S_{5,2}^{2}, S_{2,4} S_{4,5} S_{5,2}^{2}, S_{2,5}^{2} S_{5,2}^{3}\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^{2} S_{5,2}^{2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^{2}\} \\ \{S_{5,2}, S_{2,5} S_{5,2}^{2}, S_{2,4} S_{4,5} S_{5,2}^{2}, S_{2,5}^{2} S_{5,2}^{3}\} & \{S_{5,2}, S_{2,5} S_{5,2}^{2}, S_{2,4} S_{4,5} S_{5,2}^{2}, S_{2,5}^{2} S_{5,2}^{3}\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^{2} S_{5,2}^{2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^{2}\} \\ \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^{2} S_{5,2}^{2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^{2}\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^{2} S_{5,2}^{2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^{2}\} & \{S_{2,5}, S_{2,4} S_{4,5}, S_{2,5}^{2} S_{5,2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}, S_{2,4}^{2} S_{4,5}^{2} S_{5,2}, S_{2,5}^{3} S_{5,2}^{2}\} \end{pmatrix}$$

$$H\overline{5}^{p}10^{q}:\ Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix} \longleftarrow \text{rank 0}$$

$$H\overline{H}:\ \mu = \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^{2} S_{5,2}^{2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^{2}\} \longleftarrow \mu\text{-term}$$

$$W_{\text{sing}} = \{S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^{2} S_{5,2}^{2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^{2}\}$$

R−parity violating terms in superpotential:

$$\overline{H}L^{p}:\ \rho = \begin{pmatrix} \{S_{2,4} S_{5,2}, S_{2,4} S_{2,5} S_{5,2}^{2}, S_{2,4}^{2} S_{4,5} S_{5,2}^{2}\} \\ \{S_{2,4} S_{5,2}, S_{2,4} S_{2,5} S_{5,2}^{2}, S_{2,4}^{2} S_{4,5} S_{5,2}^{2}\} \\ \{S_{2,4}, S_{2,4} S_{2,5} S_{5,2}, S_{2,4}^{2} S_{4,5} S_{5,2}, S_{2,4} S_{2,5}^{2} S_{5,2}^{2}\} \end{pmatrix}$$

$$10^{p}\overline{5}^{q}\overline{5}^{r}:\ \lambda = \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\begin{aligned} \overline{5}^{p}10^{q}10^{r}10^{s}:\ \lambda' = \{&\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}, \\ &\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}, \\ &\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}\} \end{aligned}$$

D−terms:

$$\text{FI−terms:}\ k^{i}{}_{a}\kappa_{i} = \begin{pmatrix} -4 t_1 t_2 - 4 t_1 t_3 + 4 t_2 t_4 + 4 t_3 t_4 - 8 t_1 t_5 + 8 t_4 t_5 \\ 4 t_1 t_3 + 8 t_2 t_3 - 4 t_1 t_4 + 4 t_2 t_4 + 8 t_3 t_4 - 8 t_1 t_5 + 8 t_2 t_5 + 12 t_3 t_5 + 4 t_5^{2} \\ -4 t_2 t_4 - 4 t_3 t_4 + 8 t_1 t_5 + 4 t_2 t_5 + 4 t_3 t_5 + 8 t_5^{2} \\ 4 t_1 t_2 - 4 t_1 t_3 + 4 t_2 t_4 - 4 t_3 t_4 + 4 t_2 t_5 - 4 t_3 t_5 \\ 4 t_1 t_3 - 8 t_2 t_3 + 4 t_1 t_4 - 8 t_2 t_4 - 4 t_3 t_4 + 8 t_1 t_5 - 16 t_2 t_5 - 12 t_3 t_5 - 8 t_4 t_5 - 12 t_5^{2} \end{pmatrix}$$

$$\text{singlet D−terms:}\ q_{\alpha a} S^{\alpha} \overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1} S^{\dagger}{}_{2,1} - S_{5,1} S^{\dagger}{}_{5,1} \\ S_{2,1} S^{\dagger}{}_{2,1} + S_{2,3} S^{\dagger}{}_{2,3} + S_{2,4} S^{\dagger}{}_{2,4} + S_{2,5} S^{\dagger}{}_{2,5} - S_{5,2} S^{\dagger}{}_{5,2} \\ -S_{2,3} S^{\dagger}{}_{2,3} \\ -S_{2,4} S^{\dagger}{}_{2,4} + S_{4,5} S^{\dagger}{}_{4,5} \\ -S_{2,5} S^{\dagger}{}_{2,5} - S_{4,5} S^{\dagger}{}_{4,5} + S_{5,1} S^{\dagger}{}_{5,1} + S_{5,2} S^{\dagger}{}_{5,2} \end{pmatrix}$$

allowed operators (1):

■ Operators

basic superpotential terms:

rank 2 for $\langle S^\alpha \rangle = 0$

$$\overline{H}10^p10^q:\ Y^{(u)} = \begin{pmatrix} \{S_{5,2},\ S_{2,5}\,S_{5,2}^2,\ S_{2,4}\,S_{4,5}\,S_{5,2}^2,\ S_{2,5}^2\,S_{5,2}^3\} & \{S_{5,2},\ S_{2,5}\,S_{5,2}^2,\ S_{2,4}\,S_{4,5}\,S_{5,2}^2,\ S_{2,5}^2\,S_{5,2}^3\} & \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\} \\ \{S_{5,2},\ S_{2,5}\,S_{5,2}^2,\ S_{2,4}\,S_{4,5}\,S_{5,2}^2,\ S_{2,5}^2\,S_{5,2}^3\} & \{S_{5,2},\ S_{2,5}\,S_{5,2}^2,\ S_{2,4}\,S_{4,5}\,S_{5,2}^2,\ S_{2,5}^2\,S_{5,2}^3\} & \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\} \\ \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\} & \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\} & \{S_{2,5},\ S_{2,4}\,S_{4,5},\ S_{2,5}^2\,S_{5,2},\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2},\ S_{2,4}^2\,S_{4,5}^2\,S_{5,2},\ S_{2,5}^3\,S_{5,2}^2\} \end{pmatrix}$$

$$H\overline{5}^p10^q:\ Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$$

rank 0

$$H\overline{H}:\ \mu = \{1,\ S_{2,5}\,S_{5,2},\ S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\}$$

μ-term

$$W_{\text{sing}} = \{S_{2,4}\,S_{4,5}\,S_{5,2},\ S_{2,5}^2\,S_{5,2}^2,\ S_{2,4}\,S_{2,5}\,S_{4,5}\,S_{5,2}^2\}$$

Majorana masses for RH neutrinos

R–parity violating terms in superpotential:

$$\overline{H}L^p:\ \rho = \begin{pmatrix} \{S_{2,4}\,S_{5,2},\ S_{2,4}\,S_{2,5}\,S_{5,2}^2,\ S_{2,4}^2\,S_{4,5}\,S_{5,2}^2\} \\ \{S_{2,4}\,S_{5,2},\ S_{2,4}\,S_{2,5}\,S_{5,2}^2,\ S_{2,4}^2\,S_{4,5}\,S_{5,2}^2\} \\ \{S_{2,4},\ S_{2,4}\,S_{2,5}\,S_{5,2},\ S_{2,4}^2\,S_{4,5}\,S_{5,2},\ S_{2,4}\,S_{2,5}^2\,S_{5,2}^2\} \end{pmatrix}$$

$$10^p\overline{5}^q\overline{5}^r:\ \lambda = \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\overline{5}^p10^q10^r10^s:\ \lambda' = \{\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$

$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$

$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}\}$$

D–terms:

$$\text{FI–terms:}\ k^i{}_a\kappa_i = \begin{pmatrix} -4\,t_1\,t_2 - 4\,t_1\,t_3 + 4\,t_2\,t_4 + 4\,t_3\,t_4 - 8\,t_1\,t_5 + 8\,t_4\,t_5 \\ 4\,t_1\,t_3 + 8\,t_2\,t_3 - 4\,t_1\,t_4 + 4\,t_2\,t_4 + 8\,t_3\,t_4 - 8\,t_1\,t_5 + 8\,t_2\,t_5 + 12\,t_3\,t_5 + 4\,t_5^2 \\ -4\,t_2\,t_4 - 4\,t_3\,t_4 + 8\,t_1\,t_5 + 4\,t_2\,t_5 + 4\,t_3\,t_5 + 8\,t_5^2 \\ 4\,t_1\,t_2 - 4\,t_1\,t_3 + 4\,t_2\,t_4 - 4\,t_3\,t_4 + 4\,t_2\,t_5 - 4\,t_3\,t_5 \\ 4\,t_1\,t_3 - 8\,t_2\,t_3 + 4\,t_1\,t_4 - 8\,t_2\,t_4 - 4\,t_3\,t_4 + 8\,t_1\,t_5 - 16\,t_2\,t_5 - 12\,t_3\,t_5 - 8\,t_4\,t_5 - 12\,t_5^2 \end{pmatrix}$$

$$\text{singlet D–terms:}\ q_{\alpha a}S^\alpha\overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1}\,S^\dagger{}_{2,1} - S_{5,1}\,S^\dagger{}_{5,1} \\ S_{2,1}\,S^\dagger{}_{2,1} + S_{2,3}\,S^\dagger{}_{2,3} + S_{2,4}\,S^\dagger{}_{2,4} + S_{2,5}\,S^\dagger{}_{2,5} - S_{5,2}\,S^\dagger{}_{5,2} \\ -S_{2,3}\,S^\dagger{}_{2,3} \\ -S_{2,4}\,S^\dagger{}_{2,4} + S_{4,5}\,S^\dagger{}_{4,5} \\ -S_{2,5}\,S^\dagger{}_{2,5} - S_{4,5}\,S^\dagger{}_{4,5} + S_{5,1}\,S^\dagger{}_{5,1} + S_{5,2}\,S^\dagger{}_{5,2} \end{pmatrix}$$

allowed operators (1):

- **Operators**

basic superpotential terms:

rank 2 for $\langle S^\alpha \rangle = 0$

$$\overline{H}10^p10^q\text{:}\ \ Y^{(u)} = \begin{pmatrix} \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} \\ \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} \\ \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} & \{S_{2,5}, S_{2,4} S_{4,5}, S_{2,5}^2 S_{5,2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}, S_{2,4}^2 S_{4,5}^2 S_{5,2}, S_{2,5}^3 S_{5,2}^2\} \end{pmatrix}$$

$$H\overline{5}^p10^q\text{:}\ \ Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$$

rank 0

$$H\overline{H}\text{:}\ \ \mu = \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\}$$

μ-term

$$W_{\text{sing}} = \{S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\}$$

Majorana masses for RH neutrinos

R−parity violating terms in superpotential:

$$\overline{H}L^p\text{:}\ \ \rho = \begin{pmatrix} \{S_{2,4} S_{5,2}, S_{2,4} S_{2,5} S_{5,2}^2, S_{2,4}^2 S_{4,5} S_{5,2}^2\} \\ \{S_{2,4} S_{5,2}, S_{2,4} S_{2,5} S_{5,2}^2, S_{2,4}^2 S_{4,5} S_{5,2}^2\} \\ \{S_{2,4}, S_{2,4} S_{2,5} S_{5,2}, S_{2,4}^2 S_{4,5} S_{5,2}, S_{2,4} S_{2,5}^2 S_{5,2}^2\} \end{pmatrix}$$

Dirac neutrino masses

$$10^p\overline{5}^q\overline{5}^r\text{:}\ \ \lambda = \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\overline{5}^p10^q10^r10^s\text{:}\ \ \lambda' = \{\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$
$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$
$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}\}$$

D−terms:

$$\text{FI−terms:}\ \ k^i{}_a\kappa_i = \begin{pmatrix} -4t_1t_2 - 4t_1t_3 + 4t_2t_4 + 4t_3t_4 - 8t_1t_5 + 8t_4t_5 \\ 4t_1t_3 + 8t_2t_3 - 4t_1t_4 + 4t_2t_4 + 8t_3t_4 - 8t_1t_5 + 8t_2t_5 + 12t_3t_5 + 4t_5^2 \\ -4t_2t_4 - 4t_3t_4 + 8t_1t_5 + 4t_2t_5 + 4t_3t_5 + 8t_5^2 \\ 4t_1t_2 - 4t_1t_3 + 4t_2t_4 - 4t_3t_4 + 4t_2t_5 - 4t_3t_5 \\ 4t_1t_3 - 8t_2t_3 + 4t_1t_4 - 8t_2t_4 - 4t_3t_4 + 8t_1t_5 - 16t_2t_5 - 12t_3t_5 - 8t_4t_5 - 12t_5^2 \end{pmatrix}$$

$$\text{singlet D−terms:}\ \ q_{\alpha a}S^\alpha\overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1}S^\dagger{}_{2,1} - S_{5,1}S^\dagger{}_{5,1} \\ S_{2,1}S^\dagger{}_{2,1} + S_{2,3}S^\dagger{}_{2,3} + S_{2,4}S^\dagger{}_{2,4} + S_{2,5}S^\dagger{}_{2,5} - S_{5,2}S^\dagger{}_{5,2} \\ -S_{2,3}S^\dagger{}_{2,3} \\ -S_{2,4}S^\dagger{}_{2,4} + S_{4,5}S^\dagger{}_{4,5} \\ -S_{2,5}S^\dagger{}_{2,5} - S_{4,5}S^\dagger{}_{4,5} + S_{5,1}S^\dagger{}_{5,1} + S_{5,2}S^\dagger{}_{5,2} \end{pmatrix}$$

allowed operators (1):

- **Operators**

basic superpotential terms:

rank 2 for $\langle S^{\alpha} \rangle = 0$

$$\overline{\mathrm{H}}10^{p}10^{q}:\; Y^{(u)} = \begin{pmatrix} \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} \\ \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{S_{5,2}, S_{2,5} S_{5,2}^2, S_{2,4} S_{4,5} S_{5,2}^2, S_{2,5}^2 S_{5,2}^3\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} \\ \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} & \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} & \{S_{2,5}, S_{2,4} S_{4,5}, S_{2,5}^2 S_{5,2}, S_{2,4} S_{2,5} S_{4,5} S_{5,2}, S_{2,4}^2 S_{4,5}^2 S_{5,2}, S_{2,5}^3 S_{5,2}^2\} \end{pmatrix}$$

$$\mathrm{H}\overline{5}^{p}10^{q}:\; Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix} \longleftarrow \text{rank } 0$$

$$\mathrm{H}\overline{\mathrm{H}}:\; \mu = \{1, S_{2,5} S_{5,2}, S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\} \longleftarrow \mu\text{-term}$$

$$W_{\mathrm{sing}} = \{S_{2,4} S_{4,5} S_{5,2}, S_{2,5}^2 S_{5,2}^2, S_{2,4} S_{2,5} S_{4,5} S_{5,2}^2\}$$

Majorana masses for RH neutrinos

R–parity violating terms in superpotential:

$$\overline{\mathrm{H}}L^{p}:\; \rho = \begin{pmatrix} \{S_{2,4} S_{5,2}, S_{2,4} S_{2,5} S_{5,2}^2, S_{2,4}^2 S_{4,5} S_{5,2}^2\} \\ \{S_{2,4} S_{5,2}, S_{2,4} S_{2,5} S_{5,2}^2, S_{2,4}^2 S_{4,5} S_{5,2}^2\} \\ \{S_{2,4}, S_{2,4} S_{2,5} S_{5,2}, S_{2,4}^2 S_{4,5} S_{5,2}, S_{2,4} S_{2,5}^2 S_{5,2}^2\} \end{pmatrix} \longleftarrow \text{Dirac neutrino masses}$$

$$10^{p}\overline{5}^{q}\overline{5}^{r}:\; \lambda = \{\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\overline{5}^{p}10^{q}10^{r}10^{s}:\; \lambda' = \{\{\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\}\},$$
$$\{\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\}\},$$
$$\{\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\}\}\}$$

proton stable

D–terms:

$$\text{FI–terms: } k^{i}{}_{a}\kappa_{i} = \begin{pmatrix} -4 t_1 t_2 - 4 t_1 t_3 + 4 t_2 t_4 + 4 t_3 t_4 - 8 t_1 t_5 + 8 t_4 t_5 \\ 4 t_1 t_3 + 8 t_2 t_3 - 4 t_1 t_4 + 4 t_2 t_4 + 8 t_3 t_4 - 8 t_1 t_5 + 8 t_2 t_5 + 12 t_3 t_5 + 4 t_5^2 \\ -4 t_2 t_4 - 4 t_3 t_4 + 8 t_1 t_5 + 4 t_2 t_5 + 4 t_3 t_5 + 8 t_5^2 \\ 4 t_1 t_2 - 4 t_1 t_3 + 4 t_2 t_4 - 4 t_3 t_4 + 4 t_2 t_5 - 4 t_3 t_5 \\ 4 t_1 t_3 - 8 t_2 t_3 + 4 t_1 t_4 - 8 t_2 t_4 - 4 t_3 t_4 + 8 t_1 t_5 - 16 t_2 t_5 - 12 t_3 t_5 - 8 t_4 t_5 - 12 t_5^2 \end{pmatrix}$$

$$\text{singlet D–terms: } q_{\alpha a} S^{\alpha} \overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1} S^{\dagger}{}_{2,1} - S_{5,1} S^{\dagger}{}_{5,1} \\ S_{2,1} S^{\dagger}{}_{2,1} + S_{2,3} S^{\dagger}{}_{2,3} + S_{2,4} S^{\dagger}{}_{2,4} + S_{2,5} S^{\dagger}{}_{2,5} - S_{5,2} S^{\dagger}{}_{5,2} \\ -S_{2,3} S^{\dagger}{}_{2,3} \\ -S_{2,4} S^{\dagger}{}_{2,4} + S_{4,5} S^{\dagger}{}_{4,5} \\ -S_{2,5} S^{\dagger}{}_{2,5} - S_{4,5} S^{\dagger}{}_{4,5} + S_{5,1} S^{\dagger}{}_{5,1} + S_{5,2} S^{\dagger}{}_{5,2} \end{pmatrix}$$

allowed operators (2):

Kinetic terms:

GM term: $\tilde{\mu} = \{S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\}$

$\bar{5}^p \bar{5}^{\bar{q}\dagger}$: $K^{(\bar{5})} =$

$$\{\{\{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\},$$

$$\{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\}, \{S_{2,5}, S^\dagger_{5,2}, S_{2,1} S^\dagger_{5,1}, S_{2,4} S_{4,5}, S^2_{2,5} S_{5,2}, S^\dagger_{2,5} (S^\dagger)^2_{5,2}\}\},$$

$$\{\{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\},$$

$$\{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\}, \{S_{2,5}, S^\dagger_{5,2}, S_{2,1} S^\dagger_{5,1}, S_{2,4} S_{4,5}, S^2_{2,5} S_{5,2}, S^\dagger_{2,5} (S^\dagger)^2_{5,2}\}\},$$

$$\{\{S^\dagger_{2,5}, S_{5,2}, S_{5,1} S^\dagger_{2,1}, S^\dagger_{2,4} S^\dagger_{4,5}, (S^\dagger)^2_{2,5} S^\dagger_{5,2}, S_{2,5} S^2_{5,2}\}, \{S^\dagger_{2,5}, S_{5,2}, S_{5,1} S^\dagger_{2,1}, S^\dagger_{2,4} S^\dagger_{4,5}, (S^\dagger)^2_{2,5} S^\dagger_{5,2}, S_{2,5} S^2_{5,2}\},$$

$$\{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\}\}\}$$

$10^p 10^{\bar{q}\dagger}$: $K^{(10)} =$

$$\{\{\{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\},$$

$$\{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\}, \{S^\dagger_{2,5}, S_{5,2}, S_{5,1} S^\dagger_{2,1}, S^\dagger_{2,4} S^\dagger_{4,5}, (S^\dagger)^2_{2,5} S^\dagger_{5,2}, S_{2,5} S^2_{5,2}\}\},$$

$$\{\{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\}, \{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1},$$

$$S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\}, \{S^\dagger_{2,5}, S_{5,2}, S_{5,1} S^\dagger_{2,1}, S^\dagger_{2,4} S^\dagger_{4,5}, (S^\dagger)^2_{2,5} S^\dagger_{5,2}, S_{2,5} S^2_{5,2}\}\}, \{\{S_{2,5}, S^\dagger_{5,2}, S_{2,1} S^\dagger_{5,1}, S_{2,4} S_{4,5}, S^2_{2,5} S_{5,2}, S^\dagger_{2,5} (S^\dagger)^2_{5,2}\},$$

$$\{S_{2,5}, S^\dagger_{5,2}, S_{2,1} S^\dagger_{5,1}, S_{2,4} S_{4,5}, S^2_{2,5} S_{5,2}, S^\dagger_{2,5} (S^\dagger)^2_{5,2}\}, \{1, S_{2,5} S_{5,2}, S^\dagger_{2,5} S^\dagger_{5,2}, S_{2,5} S_{5,1} S^\dagger_{2,1}, S_{2,1} S^\dagger_{2,5} S^\dagger_{5,1}, S_{2,1} S_{5,2} S^\dagger_{5,1}, S_{5,1} S^\dagger_{2,1} S^\dagger_{5,2}, S_{2,4} S_{4,5} S^\dagger_{2,5}, S_{2,5} S^\dagger_{2,4} S^\dagger_{4,5}, S_{2,4} S_{4,5} S_{5,2}, S^\dagger_{2,4} S^\dagger_{4,5} S^\dagger_{5,2}\}\}\}$$

$$L^p H^\dagger: \ \hat{\rho} = \begin{pmatrix} \{S_{2,4} S^\dagger_{2,5}, S_{2,4} S_{5,2}, S_{2,4} S_{5,1} S^\dagger_{2,1}, S_{2,5} S_{5,2} S^\dagger_{4,5}\} \\ \{S_{2,4} S^\dagger_{2,5}, S_{2,4} S_{5,2}, S_{2,4} S_{5,1} S^\dagger_{2,1}, S_{2,5} S_{5,2} S^\dagger_{4,5}\} \\ \{S_{2,4}, S_{2,5} S^\dagger_{4,5}, S_{2,1} S^\dagger_{4,5} S^\dagger_{5,1}, S_{2,4} S_{2,5} S_{5,2}, S_{2,4} S^\dagger_{2,5} S^\dagger_{5,2}\} \end{pmatrix}$$

allowed operators (2):

Kinetic terms:

GM term: $\tilde{\mu} = \{S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\}$

$\overline{5}^{p}\overline{5}^{\overline{q}\,\dagger}$: $K^{(\overline{5})} = \{\{\{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\},$

$\{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\},\ \{S_{2,5},\ S^{\dagger}_{5,2},\ S_{2,1}\, S^{\dagger}_{5,1},\ S_{2,4}\, S_{4,5},\ S^{2}_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, (S^{\dagger})^{2}_{5,2}\}\},$

$\{\{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\},$

$\{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\},\ \{S_{2,5},\ S^{\dagger}_{5,2},\ S_{2,1}\, S^{\dagger}_{5,1},\ S_{2,4}\, S_{4,5},\ S^{2}_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, (S^{\dagger})^{2}_{5,2}\}\},$

$\{\{S^{\dagger}_{2,5},\ S_{5,2},\ S_{5,1}\, S^{\dagger}_{2,1},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ (S^{\dagger})^{2}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S^{2}_{5,2}\},\ \{S^{\dagger}_{2,5},\ S_{5,2},\ S_{5,1}\, S^{\dagger}_{2,1},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ (S^{\dagger})^{2}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S^{2}_{5,2}\},$

$\{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\}\}\}$

$10^{p}10^{\overline{q}\,\dagger}$: $K^{(10)} = \{\{\{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\},$

$\{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\},\ \{S^{\dagger}_{2,5},\ S_{5,2},\ S_{5,1}\, S^{\dagger}_{2,1},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ (S^{\dagger})^{2}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S^{2}_{5,2}\}\},$

$\{\{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\},\ \{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},$

$S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\},\ \{S^{\dagger}_{2,5},\ S_{5,2},\ S_{5,1}\, S^{\dagger}_{2,1},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ (S^{\dagger})^{2}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S^{2}_{5,2}\}\},\ \{\{S_{2,5},\ S^{\dagger}_{5,2},\ S_{2,1}\, S^{\dagger}_{5,1},\ S_{2,4}\, S_{4,5},\ S^{2}_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, (S^{\dagger})^{2}_{5,2}\},$

$\{S_{2,5},\ S^{\dagger}_{5,2},\ S_{2,1}\, S^{\dagger}_{5,1},\ S_{2,4}\, S_{4,5},\ S^{2}_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, (S^{\dagger})^{2}_{5,2}\},\ \{1,\ S_{2,5}\, S_{5,2},\ S^{\dagger}_{2,5}\, S^{\dagger}_{5,2},\ S_{2,5}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,1}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,1},\ S_{2,1}\, S_{5,2}\, S^{\dagger}_{5,1},\ S_{5,1}\, S^{\dagger}_{2,1}\, S^{\dagger}_{5,2},\ S_{2,4}\, S_{4,5}\, S^{\dagger}_{2,5},\ S_{2,5}\, S^{\dagger}_{2,4}\, S^{\dagger}_{4,5},\ S_{2,4}\, S_{4,5}\, S_{5,2},\ S^{\dagger}_{2,4}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,2}\}\}\}$

$$L^{p}H^{\dagger}:\ \hat{\rho} = \begin{pmatrix} \{S_{2,4}\, S^{\dagger}_{2,5},\ S_{2,4}\, S_{5,2},\ S_{2,4}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,5}\, S_{5,2}\, S^{\dagger}_{4,5}\} \\ \{S_{2,4}\, S^{\dagger}_{2,5},\ S_{2,4}\, S_{5,2},\ S_{2,4}\, S_{5,1}\, S^{\dagger}_{2,1},\ S_{2,5}\, S_{5,2}\, S^{\dagger}_{4,5}\} \\ \{S_{2,4},\ S_{2,5}\, S^{\dagger}_{4,5},\ S_{2,1}\, S^{\dagger}_{4,5}\, S^{\dagger}_{5,1},\ S_{2,4}\, S_{2,5}\, S_{5,2},\ S_{2,4}\, S^{\dagger}_{2,5}\, S^{\dagger}_{5,2}\} \end{pmatrix}$$

$$K^{\mathbf{\bar{5}}} \sim K^{\mathbf{10}} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Overview of phenomenological properties:

standard models	no massless $U(1)$	1 Higgs pair	2 Higgs pairs	3 Higgs pairs	$\mathrm{rk}(Y^{(u)}) > 0$	no proton decay, $\lambda = \lambda' = 0$	1 Higgs, $\mathrm{rk}(Y^{(u)}) > 0$, $\lambda = \lambda' = 0$, $U(1)$s massive
407	237	262	77	63	45	198	13

Moduli stabilization

- dilaton $\mathcal{S}$
- $h^{1,1}(X)$ Kahler moduli T^i
- $h^{2,1}(X)$ complex structure moduli Z^a
- $h^1(X, V \otimes V^*)$ bundle moduli S^α

} typically sum up to O(10) - O(100)

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Usually $c_2(V) < c_2(TX)$, so there is room for a hidden bundle $\tilde{V}$

Stabilize moduli by hidden E_8 flux

- Hidden line bundles $L \subset \tilde{V}$ can stabilize (up to) all but one of $(\mathcal{S}, T^i)$ through FI terms.

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Probably need to combine D-terms, matter superpotential for S^α and non-perturbative effects to stabilize all moduli and break Susy.

(B. Dundee, S. Raby, A. Westphal 2010;
S. Parameswaran, I. Zavala, S. Ramos-Sanchez, 2011)

Conclusion and outlook

- We can "mass-produce" heterotic CY standard models. Many more than the current 2000 models will be found.
- The additional U(1) symmetries of these models constrain the 4d theory significantly.
- Calculation of superpotential terms feasible but matter field Kahler metric remains a problem.
- Finding discrete symmetries... (M. Klaput, C. Mishra, AL, in progress)
- More detailed model building (proton stability, fermion masses) is now possible.
- Moduli stabilization via hidden bundle looks promising. Remaining moduli have to be stabilized non-perturbatively, with Susy broken -> soft breaking parameters.

Thanks