Non-Abelian discrete gauge symmetries in String Theory

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arXiv:**1203.2686** with M. Berasaluce-Gonzalez, F. Marchesano, D. Regalado and A. Uranga
arXiv:**1106.0060** with L. E. Ibañez and F. Marchesano

CERN TH Institute on String Phenomenology. 6[Th] July 2012

Discrete gauge symmetries

- Discrete symmetries are a key ingredient in BSM model building

- For instance, in the MSSM dim 4 operators can induce fast proton decay:

  - *Matter-parity*

  - *Baryon triality* [Ibáñez, Ross '92]

Experimental signatures (at the LHC) depend on the symmetry!

- Non-Abelian discrete flavour symmetries might also explain textures of quark and lepton masses and mixings

- Discrete quantum symmetries of quiver theories, condensed matter, etc

Accidental or exact symmetries at the fundamental level?

Discrete gauge symmetries

- Diverse arguments strongly suggest that global symmetries are violated by quantum gravitational effects: [e.g. Banks, Seiberg '11 for a recent discussion]

  - Microscopic arguments in string theory [Banks, Dixon '88]

  - General black hole arguments (charged black holes evaporate thermally into uncharged vacuum)

- Hence, fundamental discrete symmetries should have a gauge nature in quantum theories of gravity, and in particular in string theory

- This has important implications: powerful selection rules, valid also in a complete non-perturbative formulation of the theory

Discrete gauge symmetries

Previous works on discrete gauge symmetries in string theory:

Heterotic orbifolds: Forste, Ko, Kobayashi, Nilles, Park, Ploger, Raby, Ramos-Sanchez, Ratz, Vaudrevange... '04 - '12

Magnetized branes: Abe, Choi, Kobayashi, Ohki, Sakai... '09 - '10

Intersecting branes: Berasaluce, Ibanez, Soler, Uranga... '11

Gepner models: Ibanez, Schellekens, Uranga '12

Outlook

- Review of Abelian discrete gauge symmetries in 4d QFT

[Banks, Seiberg '11]

- General formalism for non-Abelian 4d discrete gauge symmetries and axions

- Non-Abelian discrete symmetries from torsion homology

[Gukov, Rangamani, Witten '98]

- Non-Abelian discrete flavour symmetries in magnetized / intersecting branes

- Conclusions

Discrete gauge symmetries in 4d QFT

Abelian discrete gauge symmetries

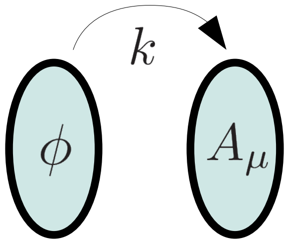
[Banks, Seiberg '11]

- The basic Lagrangian for a $\mathbf{Z}_k$ discrete gauge symmetry is:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi - k A_\mu) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad \phi \to \phi + 1$$

- It represents the gauging of continuous shift symmetry by a U(1):

$$A_\mu \to A_\mu + \partial_\mu \lambda \ , \qquad \phi \to \phi + k\lambda$$



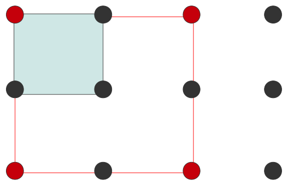
k acts as a winding number
in the map between the two $\mathbf{S}^1$

- The discrete scalar equivalence therefore corresponds to fractional 1/k U(1) gauge transformations

Abelian discrete gauge symmetries

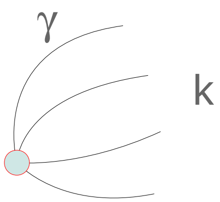
- This picture can be easily extended to the multiple Abelian case:

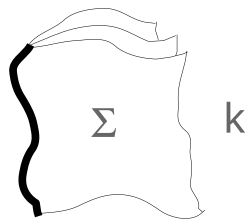
$$\mathbf{P} = \frac{\Gamma}{\hat{\Gamma}} = \mathbf{Z}_{k_1} \times \mathbf{Z}_{k_2} \times \ldots$$

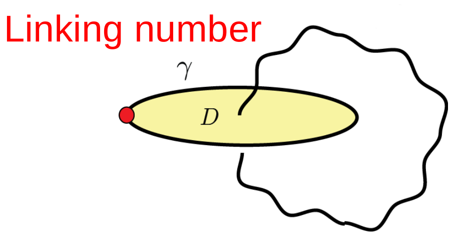


- Theories with discrete gauge symmetries have sets of charged Aharonov-Bohm particle and string states, with charge conserved modulo k: [Alford, Krauss, Preskill, Wilczek '89]

$$\mathcal{O}_{\text{particle}} \sim e^{2\pi i n \int_\gamma A} \qquad \mathcal{O}_{\text{string}} \sim e^{2\pi i m \int_\Sigma B_2}$$







$$e^{-2\pi i \phi} e^{2\pi i k \int_\gamma A} \qquad e^{-2\pi i \int_{\partial\Sigma} A} e^{2\pi i k \int_\Sigma B_2} \qquad \exp\left[2\pi i \frac{nm}{k} L(\Sigma, \gamma)\right]$$

Non-Abelian discrete gauge symmetries

- Take a set of axion-like scalars with non-commuting shift symmetries:

$$\phi^b \to \phi^b + \epsilon^A X_A^b \qquad\qquad [X_A, X_B] = f_{AB}{}^C X_C$$

- The effective action can be described in terms of a non-linear σ-model
- For an axionic scalar manifold M the number of independent shift symmetries equals the dim. of M ➡ group manifold
- The effective action is given in terms of the right-invariant 1-forms of M

$$(dgg^{-1})(\vec{\phi}) = \eta^a(\vec{\phi})t_a \qquad\qquad \int d^4x\, \mathcal{K}_{ab}\eta^a_\mu \eta^{b,\mu} = \int d^4x\, G_{ab}(\vec{\phi})\partial^\mu \phi^a \partial_\mu \phi^b$$

- In addition there is a set of discrete identifications, so that the actual compact axionic moduli space is M/Γ, with Γ a lattice in M
- Discrete non-Abelian gauge symmetries described by gauging the above σ-model action

$$\partial_\mu \phi^a \;\; \to \;\; \partial_\mu \phi^a - k_\alpha{}^a A_\mu^\alpha$$

Non-Abelian discrete gauge symmetries

- Same logic than in the Abelian case: the discrete gauge symmetry is the group of field identifications in the scalar manifold modulo those already accounted by the gauging

$$\mathbf{P} = \frac{\Gamma}{\hat{\Gamma}}$$

- Example: Heisenberg group

$$[t_1, t_2] = t_3$$

Right-invariant 1-forms:

$$\eta^1_\mu = \partial_\mu \phi^1 \qquad \eta^2_\mu = \partial_\mu \phi^2 \qquad \eta^3_\mu = \partial_\mu \phi^3 + \frac{1}{2}(\phi^1 \partial_\mu \phi^2 - \phi^2 \partial_\mu \phi^1)$$

$$\Gamma: \quad \begin{cases} \phi^1 \to \phi^1 + 1 \,, \quad \phi^3 \to \phi^3 - \dfrac{\phi^2}{2} \\ \phi^2 \to \phi^2 + 1 \,, \quad \phi^3 \to \phi^3 + \dfrac{\phi^1}{2} \\ \phi^3 \to \phi^3 + 1 \,, \end{cases}$$

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Gauged right-invariant 1-forms:

$$\eta^1_\mu = \partial_\mu \phi^1 \underline{- kA^1_\mu} \qquad \eta^2_\mu = \partial_\mu \phi^2 \underline{- kA^2_\mu} \qquad \eta^3_\mu = \partial_\mu \phi^3 \underline{- kA^3_\mu} + \frac{1}{2}[\phi^1(\partial_\mu \phi^2 \underline{- kA^2_\mu}) - \phi^2(\partial_\mu \phi^1 \underline{- kA^1_\mu})]$$

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Gauged right-invariant 1-forms:

$$\eta^1_\mu = \partial_\mu \phi^1 - kA^1_\mu \qquad \eta^2_\mu = \partial_\mu \phi^2 - kA^2_\mu \qquad \eta^3_\mu = \partial_\mu \phi^3 - kA^3_\mu + \frac{1}{2}[\phi^1(\partial_\mu \phi^2 - kA^2_\mu) - \phi^2(\partial_\mu \phi^1 - kA^1_\mu)]$$

$$A^1_\mu \to A^1_\mu + \partial_\mu \lambda^1 \ , \quad A^2_\mu \to A^2_\mu + \partial_\mu \lambda^2$$

$$A^3_\mu \to A^3_\mu + \partial_\mu \lambda^3 + \frac{k}{2}(\lambda^2 A^1_\mu - \lambda^1 A^2_\mu) + \frac{1}{2}(\phi^1 \partial_\mu \lambda^2 - \phi^2 \partial_\mu \lambda^1)$$

$$\phi^1 \to \phi^1 + k\lambda^1 \ , \quad \phi^2 \to \phi^2 + k\lambda^2 \ , \quad \phi^3 \to \phi^3 + \frac{k}{2}(\phi^1 \lambda^2 - \phi^2 \lambda^1) + k\lambda^3$$

Non-Abelian discrete gauge symmetries

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$$\mathbf{P} = \frac{\Gamma}{\hat{\Gamma}}$$

- Example: Heisenberg group

$$\mathbf{P} = \frac{\Gamma}{\hat{\Gamma}} = (\mathbf{Z}_k \times \mathbf{Z}_k) \rtimes \mathbf{Z}_k$$

with generators $T_1^k = T_2^k = T_3^k = 1 \ , \quad T_1 T_2 = T_3 T_2 T_1$

E.g., for k = 2, $\mathbf{P} = \mathrm{Dih}_4$

k = 3, $\mathbf{P} = \Delta(27)$

k = ...

Recap

- The effective Lagrangian for discrete gauge symmetries is given by gauged non-linear σ-models for a set of axion-like scalars

- The order of each gauge generator is determined by the gauging, whereas the non-Abelian structure is determined by the non-commutativity of the shift symmetries

- States in 4d electrically (magnetically) charged under discrete gauge symmetries are Aharonov-Bohm particles (strings)

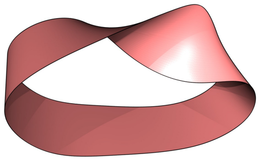
- Aharonov-Bohm strings and particles induce fractional holonomies on each other

Discrete gauge symmetries from torsion homology in string theory

NADGS from torsion homology

- A simple way to obtain Aharonov-Bohm strings and particles in string theory is to consider D-branes and/or NS-branes wrapped on torsion cycles of the compactification manifold

$$H_p(X_D, \mathbf{Z}) = (\mathbf{Z} \oplus \ldots \oplus \mathbf{Z}) \oplus (\mathbf{Z}_{k_1} \oplus \ldots \oplus \mathbf{Z}_{k_n})$$



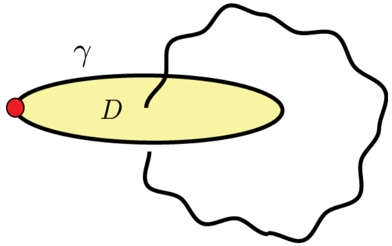
$$\partial S_{p+1} = k\pi_p^{\text{tor}}$$

p-brane on torsion p-cycle → 4d Aharonov-Bohm particle

(D-p)-brane on torsion (D-p-1)-cycle → 4d Aharonov-Bohm string

UCT + Poincaré duality: $\text{Tor } H_p(X_D, \mathbf{Z}) \simeq \text{Tor } H_{D-p-1}(X_D, \mathbf{Z})$

- They satisfy $\mathbf{Z}_k$ holonomies:



$$\frac{1}{2\pi i}\log[\text{hol}(\Sigma,\gamma)] = \frac{1}{k}\int_{D\times k\pi_p^{\text{tor}}} F_{p+2} = \frac{1}{k}\int_{D\times S_{p+1}} \delta_{p+3} = \frac{1}{k}L(\Sigma,\gamma)$$

$$L([\pi_p^{\text{tor}}],[\pi_{D-p-1}^{\text{tor}}]) = \frac{1}{k}$$

NADGS from torsion homology

- A-B strings and particles are the smoking gun of massive U(1)'s Higgsed down to discrete $\mathbf{Z}_k$ gauge symmetries via the Stuckelberg mechanism

- We can see this more explicitly from dimensional reduction. For that we introduce the set of eigenforms of the Laplacian that correspond to the generators of $\text{Tor}\ H^{p+1}(X_D) \simeq \text{Tor}\ H_p(X_D)$ and $\text{Tor}\ H^{D-p}(X_D) \simeq \text{Tor}\ H_{D-p-1}(X_D)$

$$d\omega = k\beta\ , \quad d\alpha = (-1)^{D-p}k\tilde{\omega} \qquad \int_{X_D} \alpha \wedge \beta = \int_{X_D} \tilde{\omega} \wedge \omega = 1$$

- Expanding in these forms,

$$A_{p+1} = \phi(x^\mu) \wedge \beta + A(x^\mu) \wedge \omega \quad \Rightarrow \quad dA_{p+1} = (d\phi - kA) \wedge \beta + dA \wedge \omega$$

NADGS from torsion homology

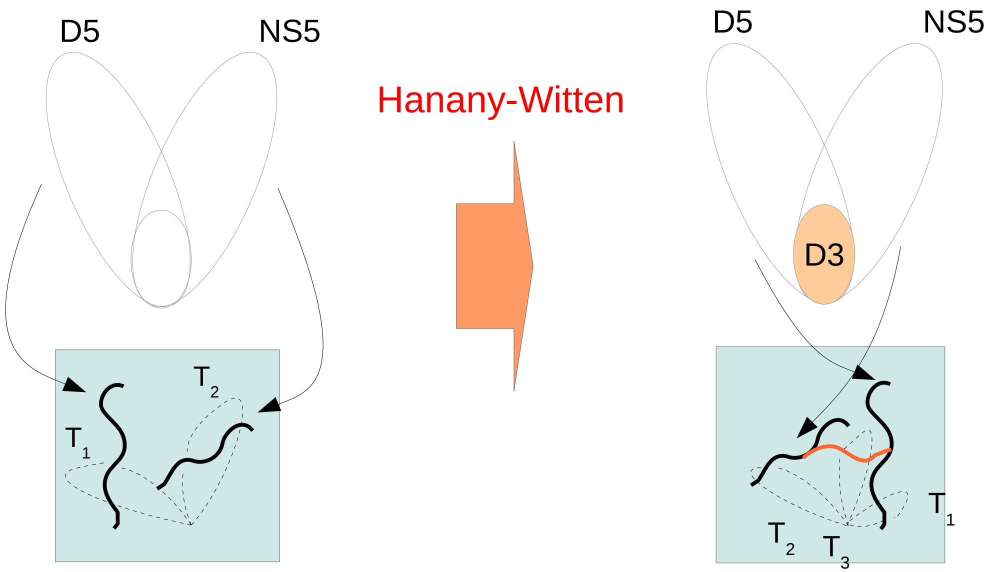
- Non-Abelianity arises in this context from non-trivial relations between torsion homology classes

- To be more specific, consider type IIB compactifications to 4d

$$\text{Tor } H_1(X_6, \mathbf{Z}) = \text{Tor } H_4(X_6, \mathbf{Z}) = \mathbf{Z}_k \ , \qquad \text{Tor } H_2(X_6, \mathbf{Z}) = \text{Tor } H_3(X_6, \mathbf{Z}) = \mathbf{Z}_{k'}$$

F1, D1   NS5, D5   D3   D3

$\mathbf{Z}_k \times \mathbf{Z}_k$ $\mathbf{Z}_{k'}$



$$T_1^k = T_2^k = T_3^{k'} = 1$$
$$T_1 T_2 = T_3 T_2 T_1$$

NADGS from torsion homology

- Macroscopic counterpart in terms of torsion forms with relations

$$d\gamma_1 = k\rho_2 \ , \quad d\tilde{\rho}_4 = k\zeta_5$$
$$d\alpha_3 = k'\tilde{\omega}_4 \ , \quad d\omega_2 = k'\beta_3$$

$$\rho_2 \wedge \rho_2 = M\tilde{\omega}_4$$

NADGS from torsion homology

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$$d\gamma_1 = k\rho_2 \ , \quad d\tilde{\rho}_4 = k\zeta_5$$
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$$\rho_2 \wedge \rho_2 = M\tilde{\omega}_4$$

- Dimensionally reducing 10d type IIB sugra action on these forms:

$$S_{10\text{d}} = \frac{-1}{4\kappa_{10}^2} \int d^{10}x \left[(-G)^{1/2} \left(e^{-2\phi}(H_3)^2 + (F_3)^2 + \frac{1}{2}(F_5)^2 \right) - dC_4 \wedge B_2 \wedge dC_2 \right] + \ldots$$

$$B_2 = \phi^1 \rho_2 + A^1 \wedge \gamma_1$$
$$C_2 = \phi^2 \rho_2 + A^2 \wedge \gamma_1$$
$$C_4 = \phi^3 \tilde{\omega}_4 + A^3 \wedge \alpha_3 + V^3 \wedge \beta_3 + c_2 \wedge \omega_2$$

$$S_{4d} = \frac{1}{4} \int d^4x [(-g)^{1/2}(-\mathcal{M}_{ij}\mathcal{T}\eta^i \cdot \eta^j - \mathcal{G}^{-1}(\eta^3)^2 - \mathcal{M}_{ij}\mathcal{N} dA^i \cdot dA^j + \mathcal{S}^{-1}(F_2^3)^2) + \mathcal{Q}\mathcal{S}^{-1} F_2^3 \wedge F_2^3]$$

$$\eta^1 = d\phi^1 - kA^1$$
$$\eta^2 = d\phi^2 - kA^2$$
$$\eta^3 = d\phi^3 - k'A^3 - M\phi^2\eta^1$$

$$k'F_2^3 = d\eta^3 - \frac{\epsilon_{ij}}{2} M\eta^i \wedge \eta^j$$

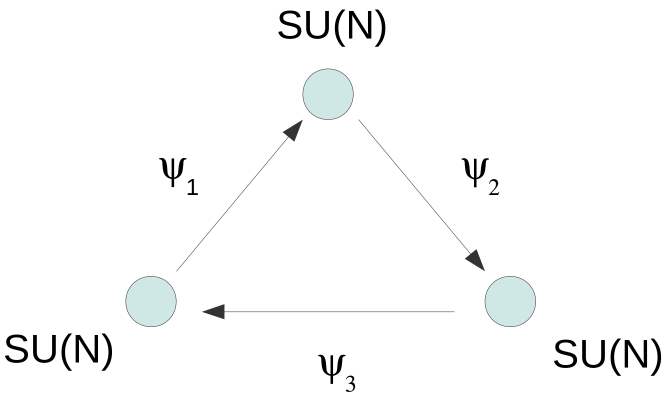
$$[t_1, t_2] = Mt_3$$

$$T_1^k = T_2^k = T_3^{k'} = 1$$
$$T_1 T_2 = T_3 T_2 T_1$$

NADGS from torsion homology

- Example: N fractional D3-branes at a $\mathbf{C}^3/\mathbf{Z}_3$ singularity

[Gukov, Rangamani, Witten '98]



AdS/CFT

$AdS_5 \times S^5/Z_3$

$$W = \mathrm{Tr}(\psi_1\psi_2\psi_3)$$

$$\xi = e^{2\pi i/3N}$$

Non-Abelian discrete quantum symmetry:

$$\begin{cases} T_1 & : \quad (\psi_1,\psi_2,\psi_3) \to (\psi_1,\xi\psi_2,\xi^2\psi_3) \\ T_2 & : \quad (\psi_1,\psi_2,\psi_3) \to (\psi_2,\psi_3,\psi_1) \\ T_3 & : \quad (\psi_1,\psi_2,\psi_3) \to (\xi\psi_1,\xi\psi_2,\xi\psi_3) \end{cases}$$

$$\mathrm{Tor}\ H_1(\mathbf{S^5}/\mathbf{Z}_3,\mathbf{Z}) = \mathbf{Z}_3$$

$$\mathrm{Tor}\ H_3(\mathbf{S^5}/\mathbf{Z}_3,\mathbf{Z}) = \mathbf{Z}_3$$

$$d\gamma_1 = 3\rho_2$$

$$d\alpha_3 = 3\tilde{\omega}_4$$

$$\rho_2 \wedge \rho_2 = \tilde{\omega}_4$$

dim. reduction

$$T_1^3 = T_2^3 = T_3^3 = 1$$

$$T_1T_2 = T_3T_2T_1$$

Discrete flavour symmetries in magnetized/intersecting brane models

Discrete flavour symmetries

- Non-Abelian discrete flavour symmetries arise in systems of magnetized or intersecting branes due to the interplay between discrete isometries and massive D-brane U(1)'s

- Consider a T^2 with a U(1) gauge field background

$$F_2 = 2\pi M dx \wedge dy \quad \Rightarrow \quad A = \pi M (x dy - y dx)$$

- Magnetization breaks translational symmetries

$$A(x+\lambda_x, y) = A(x,y) + \pi M \lambda_x dy$$
$$A(x, y+\lambda_y) = A(x,y) - \pi M \lambda_y dx$$

and need to be compensated with a U(1) gauge trasformation

$$\psi(x,y) \to e^{-i\pi q M \lambda_x y}\psi(x+\lambda_x, y) = e^{q\lambda_x X}\psi(x,y)$$
$$\psi(x,y) \to e^{i\pi q M \lambda_y x}\psi(x, y+\lambda_y) = e^{q\lambda_y Y}\psi(x,y)$$

$$X = \partial_x - i\pi M y \ , \quad Y = \partial_y + i\pi M x \qquad [X, Y] = MQ$$

Discrete flavour symmetries

- Compatibility with the T^2 identifications implies that only a discrete subgroup survives

$$\lambda_x q M \in \mathbf{Z} \qquad \lambda_y q M \in \mathbf{Z}$$

- Discrete isometries act as flavour symmetries on matter fields (degenerate Landau levels)

$$\begin{aligned} T_x &: \quad \psi_j \to e^{2\pi i (j-1)/M} \psi_j \\ T_y &: \quad \psi_j \to \psi_{j+1} \quad \text{with } \psi_{M+1} \equiv \psi_1 \\ T_q &: \quad \psi_j \to e^{2\pi i q} \psi_j \end{aligned}$$

- Lead to selection rules e.g. in Yukawa couplings

$$\lambda_{ijk} \psi_i^{ab} \psi_j^{bc} \psi_k^{ca} \qquad \begin{aligned} \lambda_{ijk} &= 0 \ \text{ if } i+j+k \neq 0 \bmod M \\ \lambda_{ijk} &= \lambda_{i+1,j+1,k+1} \end{aligned}$$

[Cremades et al. '03; Abe et al. '09]

- Underlaying continuous symmetry preserved perturbatively but violated by non-perturbative effects. Discrete subgroup exact in the full theory.

Discrete flavour symmetries

- We can get further insight from dimensional reduction. Consider a stack of magnetized D9-branes on a $T^6 = (T^2)_1 \times (T^2)_2 \times (T^2)_3$

$$F_2 = \sum_{r=1}^{3} \frac{\pi i}{\mathrm{Im}\, U^r} \begin{pmatrix} \frac{m_a^r}{n_a^r}\mathbb{I}_{n_a^r} & & & \\ & \frac{m_b^r}{n_b^r}\mathbb{I}_{n_b^r} & & \\ & & \frac{m_c^r}{n_c^r}\mathbb{I}_{n_c^r} & \\ & & & \ddots \end{pmatrix} dz^r \wedge d\bar{z}^r$$

$$\begin{array}{lll} \text{D9} & c_\alpha^0 = n_\alpha^1 n_\alpha^2 n_\alpha^3 \,, \quad d_\alpha^0 = m_\alpha^1 m_\alpha^2 m_\alpha^3 \,, & \text{D3}/\overline{\text{D3}} \\ & c_\alpha^1 = n_\alpha^1 m_\alpha^2 m_\alpha^3 \,, \quad d_\alpha^1 = m_\alpha^1 n_\alpha^2 n_\alpha^3 \,, & \\ \text{D5} & c_\alpha^2 = m_\alpha^1 n_\alpha^2 m_\alpha^3 \,, \quad d_\alpha^2 = n_\alpha^1 m_\alpha^2 n_\alpha^3 \,, & \text{D7}/\overline{\text{D7}} \\ & c_\alpha^3 = m_\alpha^1 m_\alpha^2 n_\alpha^3 \,, \quad d_\alpha^3 = n_\alpha^1 n_\alpha^2 m_\alpha^3 & \end{array}$$

10d type I sugra action

dim. reduction

4d non-linear σ-model $\int d^4x\, \mathcal{K}_{ab} \eta_\mu^a \eta^{b,\mu}$

$$\eta_\mu^{\phi^p} = \partial_\mu \phi^p + \frac{1}{2} \sum_\alpha \left(-2 d_\alpha^p A_\mu^\alpha + c_\alpha^0 \xi_{x,\alpha}^p \eta_\mu^{\xi_{y,\alpha}^p} - c_\alpha^0 \xi_{y,\alpha}^p \eta_\mu^{\xi_{x,\alpha}^p} \right)$$

$$\eta_\mu^{\phi^0} = \partial_\mu \phi^0 + \frac{1}{2} \sum_\alpha \left[2 d_\alpha^0 A_\mu^\alpha - \sum_{p=1}^{3} \left(c_\alpha^p \xi_{x,\alpha}^p \eta_\mu^{\xi_{y,\alpha}^p} - c_\alpha^p \xi_{y,\alpha}^p \eta_\mu^{\xi_{x,\alpha}^p} \right) \right]$$

$$\eta_\mu^{\xi_{x,\alpha}^p} = \partial_\mu \xi_{x,\alpha}^p + \frac{m_\alpha^p}{n_\alpha^p} V_\mu^{y,p}$$

$$\eta_\mu^{\xi_{y,\alpha}^p} = \partial_\mu \xi_{y,\alpha}^p - \frac{m_\alpha^p}{n_\alpha^p} V_\mu^{x,p}$$

Kahler metrics agree with CFT computation of

[Lust, Mayr, Richter, Stieberger '09]

Discrete flavour symmetries

- Linear combinations of D-brane U(1) gauge symmetries

$$Q^P = \sum_\alpha d_\alpha^P Q^\alpha \ , \quad P = 0, 1, 2, 3$$

are spontaneously broken to (flavour universal) discrete gauge symmetries by 'eating' RR scalars [Berasaluce, Ibanez, Soler, Uranga '11]

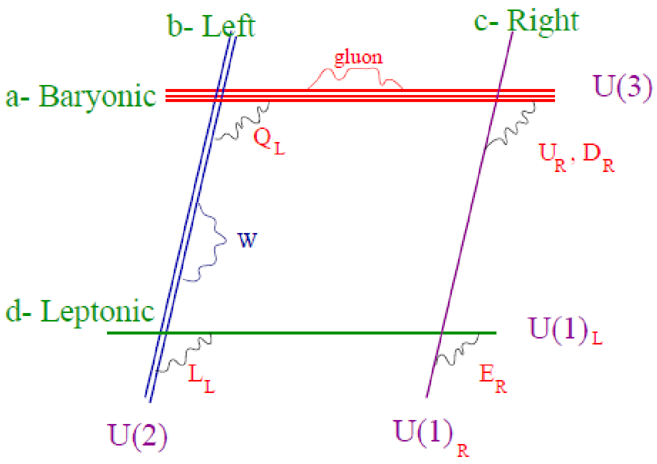
- Translational isometries of the torus are spontaneously broken to a discrete flavour gauge symmetries by 'eating' some D-brane Wilson line scalars

- These symmetries span a non-Abelian alegebra of the form

$$[X^p, Y^p] = -\frac{m_\alpha^p}{n_\alpha^p} Q^\alpha$$

Discrete flavour symmetries

- Let us consider an example:



N_α	(n_α^1, m_α^1)	(n_α^2, m_α^2)	(n_α^3, m_α^3)
$N_a = 3$	$(1, 0)$	$(3, 1)$	$(3, -1)$
$N_b = 1$	$(0, 1)$	$(1, 0)$	$(0, -1)$
$N_c = 1$	$(0, 1)$	$(0, -1)$	$(1, 0)$
$N_d = 1$	$(1, 0)$	$(3, -1)$	$(3, 1)$

[Cremades et al. '03]
[Marchesano,Shiu '04]

$$d_a^2 = d_d^3 = 3 \; , \quad d_a^3 = d_d^2 = -3$$

$$SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \times \mathbf{Z}_3$$

$$Q_Y = \frac{1}{6}(Q_a - 3Q_c + 3Q_d)$$

$$Q_{B-L} = \frac{Q_a}{3} + Q_d$$

$$Q_{\mathbf{Z}_3} = 3Q_a - Q_d$$

⇨ baryon triality
(non-pert. exact!)

[Berasaluce, Ibanez, Soler, Uranga '11]

Sector	Field	$SU(3) \times SU(2)_L$	Q_Y	Q_{B-L}	$Q_{\mathbf{Z}_3}$
ab	Q_L	$3(\mathbf{3}, \mathbf{2})$	1/6	1/3	3
ac	U_R	$3(\bar{\mathbf{3}}, \mathbf{2})$	-2/3	-1/3	-3
ac^*	D_R	$3(\bar{\mathbf{3}}, \mathbf{2})$	1/3	-1/3	-3
db	L	$3(\mathbf{1}, \mathbf{2})$	-1/2	-1	1
dc	N_R	$3(\mathbf{1}, \mathbf{1})$	0	1	-1
dc^*	E_R	$3(\mathbf{1}, \mathbf{1})$	1	1	-1
bc	H_u	$(\mathbf{1}, \mathbf{2})$	1/2	0	0
bc	H_d	$(\mathbf{1}, \bar{\mathbf{2}})$	-1/2	0	0

Discrete flavour symmetries

- Baryon triality in this model is the center of a $\Delta(27)_L \bowtie \Delta(27)_R$ discrete flavour symmetry, generated by the four $\mathbf{Z}_3$ discrete isometries of the 2nd and 3rd tori

$$[X^2_{\mathbf{Z}_3}, Y^2_{\mathbf{Z}_3}] = -[X^3_{\mathbf{Z}_3}, Y^3_{\mathbf{Z}_3}] = -\frac{Q_{\mathbf{Z}_3}}{3}$$

- The four flavour symmetry generators act on the MSSM fields as

$$\begin{array}{rcl} e^{X^2_{\mathbf{Z}_3}} & : & \psi^k_R \to e^{-\frac{2\pi i k}{3}} \psi^k_R \\ e^{X^3_{\mathbf{Z}_3}} & : & \psi^k_L \to e^{\frac{2\pi i k}{3}} \psi^k_L \\ e^{Y^2_{\mathbf{Z}_3}} & : & (\psi^1_R, \psi^2_R, \psi^3_R) \to (\psi^2_R, \psi^3_R, \psi^1_R) \\ e^{Y^3_{\mathbf{Z}_3}} & : & (\psi^1_L, \psi^2_L, \psi^3_L) \to (\psi^3_L, \psi^1_L, \psi^2_L) \end{array}$$

and imply exact relations between Yukawa couplings

$$\frac{Y_{11}}{Y_{21}} = \frac{Y_{12}}{Y_{22}} = \frac{Y_{13}}{Y_{23}}, \quad \frac{Y_{21}}{Y_{31}} = \frac{Y_{22}}{Y_{32}} = \frac{Y_{23}}{Y_{33}}, \quad \frac{Y_{31}}{Y_{11}} = \frac{Y_{32}}{Y_{12}} = \frac{Y_{33}}{Y_{13}}$$

$$\frac{Y_{11}}{Y_{12}} = \frac{Y_{21}}{Y_{22}} = \frac{Y_{31}}{Y_{32}}, \quad \frac{Y_{12}}{Y_{13}} = \frac{Y_{22}}{Y_{23}} = \frac{Y_{32}}{Y_{33}}, \quad \frac{Y_{13}}{Y_{11}} = \frac{Y_{23}}{Y_{21}} = \frac{Y_{33}}{Y_{31}}$$

⇒ rank 1 Yukawa couplings (non-pert. exact!)

From D-brane discrete symmetries back to torsion cycles

From D-branes back to torsion cycles

- D-brane discrete gauge symmetries can also be understood in terms of torsion homology in M-theory.

- Consider M-theory on a G_2 manifold admitting at least one perturbative type IIA CY_3 orientifold limit

$$\hat{\mathcal{M}}_7 \to (\mathcal{M}_6 \times \mathbf{S}^1)/\hat{\sigma} \qquad \hat{\sigma} = (\sigma, -1)$$

b_2 massless U(1)'s and b_3 massless complex scalars

$$A_3 = \phi^a(x^\mu) \wedge \beta_a + A_\alpha(x^\mu) \wedge \omega^\alpha \qquad \begin{array}{l} a = 1 \ldots b_3 \\ \alpha = 1 \ldots b_2 \end{array}$$

- In the perturbative limit some of them become massless D6-brane U(1)'s and moduli

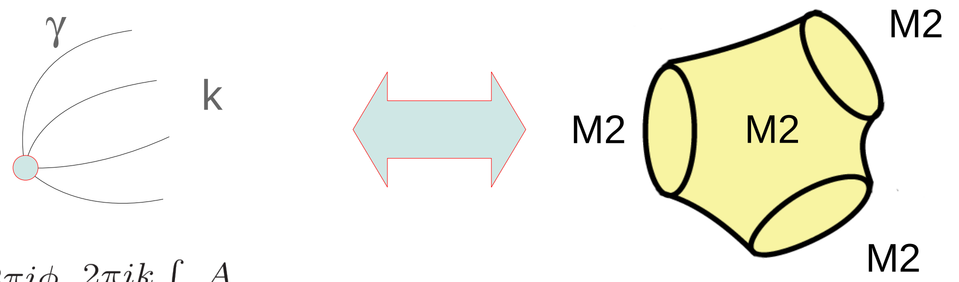
- In addition massive U(1) gauge symmetries spontaneously broken to discrete gauge symmetries arise from $\mathrm{Tor}\ H_2(\hat{\mathcal{M}}_7, \mathbf{Z}) = \mathrm{Tor}\ H_4(\hat{\mathcal{M}}_7, \mathbf{Z})$

$$dA_3 = (d\phi^a - k^{a\alpha} A_\alpha) \wedge \beta_a + dA_\alpha \wedge \omega^\alpha$$

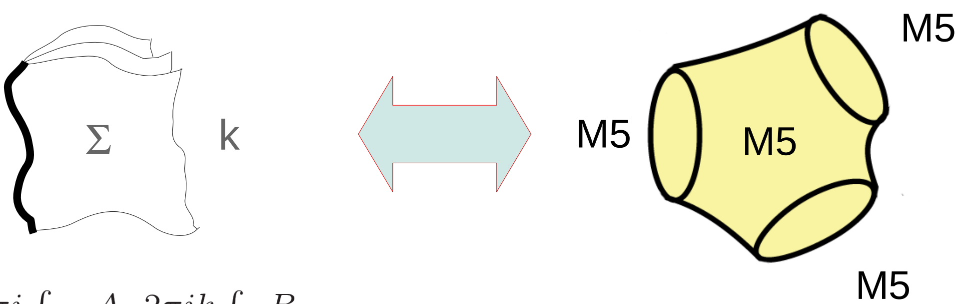
From D-branes back to torsion cycles

- In the perturbative limit some of these become D6-brane discrete gauge symmetries arising from the D6-brane Stuckelberg couplings

M2-brane on torsion 2-cycle → 4d Aharonov-Bohm particle
M5-brane on torsion 4-cycle → 4d Aharonov-Bohm string



$$e^{-2\pi i\phi}e^{2\pi ik\int_\gamma A}$$



$$e^{-2\pi i\int_{\partial\Sigma}A}e^{2\pi ik\int_\Sigma B_2}$$

Conclusions

Conclusions

- Discrete gauge symmetries are described in 4d dimensions in terms of gauged non-linear σ-models for axion-like scalars.
- They arise naturally in string theory compactifications from massive gauge symmetries (isometries, D-brane / RR gauge symmetries, etc).
- Symmetries arising from torsion homology (e.g. branes at singularities) can be suitably accounted for in dimensional reduction by considering torsion forms. Non-Abelianity arises from intersection of torsion forms via the Hanany-Witten effect.
- Non-Abelian interplay between discrete isometries (flavour symmetries) and discrete D-brane symmetries (matter parity, baryon triality, etc)
- Discrete symmetries are non-perturbatively exact; underlying continous symmetry only perturbatively exact. Powerful selection rules: Yukawa textures, proton decay, etc. Sometimes too restrictive.

[Abel, Goodsell '06]
[Blumenhagen, Cvetic, Lust, Richter, Weigand '07]
[Marchesano, Martucci '09]

Future directions

- R-symmetries
- Application to F-theory
- Systematic classification of flavour symmetries
- Models of lepton flavour mixing
- Discrete symmetries in heterotic line bundle models
- ...