Fluxes and Chern-Simons Theories for F-theory

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Chirality + Flux: 1111.1232 [hep-th] with H. Hayashi

6d + Chiral tensors: 1111.1232 [hep-th] with F. Bonetti
1206.1600 [hep-th] with F. Bonetti, S. Hohenegger

Constraints: 1204.3092 [hep-th] with W. Taylor

CERN, July 2012

Introduction and Motivation

Systematics of compactifications

- Fundamental theory (String/M-/F-theory)

  $\xrightarrow[\text{low energies}]{}$ Wilsonian effective action: integrate out massive states which are heavier than a certain energy scale

- Classifying the core data of the effective actions by discrete information:

$$\vec{N} = \begin{pmatrix} \vec{N}_{\text{geom}} \\ \vec{N}_{\text{brane}} \\ \vec{N}_{\text{flux}} \end{pmatrix}$$

topological data of the compactifications manifold (Hodge-numbers, intersection numbers, Chern classes)

topological data of the brane configuration (number of branes, wrapping numbers, intersections...)

flux data, bulk + brane

- field dynamics is encoded by effective action
  effective potential, vacua, ... phenomenology

Looking for the unifying setup...

- ideally: look for unifying fundamental setup where Type IIA / IIB and the various branes are just different aspects

$\Rightarrow$ M-theory in 11d

key example: D6-branes admit geom. interpretation in M-theory $\Rightarrow$ unify $\vec{N}_{\text{geom}}$ and $\vec{N}_{\text{brane}}$

- major drawback: $\mathbb{M}^p \times X_{11-p}$ effective theory in even dim.

  $\Rightarrow$ internal manifold is odd dim.

F-theory provides an ideal setup for:

(1) unifying 7-brane and bulk physics $\vec{N}_{\text{brane}}$ & $\vec{N}_{\text{geom}}$ in complex geometries

(2) promising phenomenological scenarios (GUTs, moduli stabilization)

Obstacles in the study of F-theory

- contrast to string/M-theory: no 12-dimensional F-theory effective action
  also: fundamental formulation is poorly understood

- F-theory physics is often studied using limits and dualities:
  - weak coupling limit with D7-branes and O7-planes
  - F-theory / heterotic duality
  - local geometries

- Only known way to extract generic features of F-theory effective actions is via its formulation as a limit of M-theory.

- Remark: if objects like G_4 - flux and M5-branes are used in the context of F- theory this limit is always understood

Goals of this talk:

- Part I: Derive 4D chiral index for F-theory compactifications using M-theory G_4-flux in the F-theory limit
- Part II: Study corrections to the Chern-Simons terms due to Kaluza-Klein modes as predicted by M-theory
- Part III: Comment on constraints imposed by F-theory
- Message: - F-theory effective action can be reliably studied: bulk + 7-brane physics in a unified *N=1* framework
  - M-theory origin of various F-theory effects can be unexpected

F-theory via M-theory

F-theory compactifications

- Type IIB has non-perturbative $Sl(2,\mathbb{Z})$ symmetry rotating $\tau = C_0 + ie^{-\phi}$

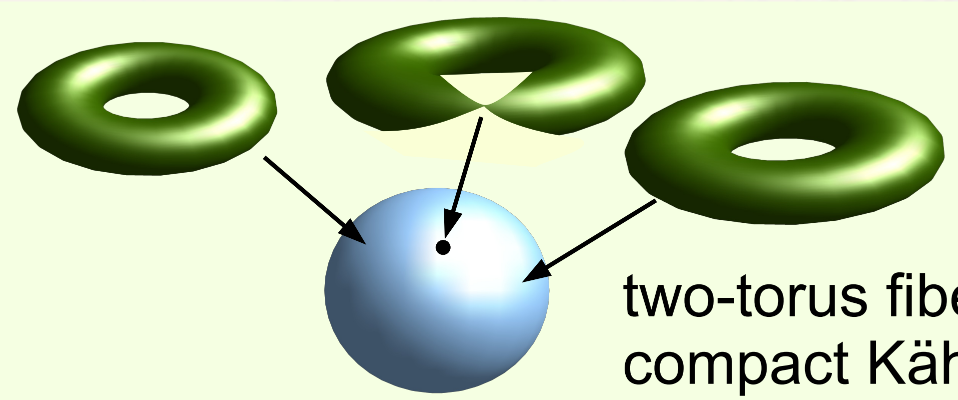
  $\Rightarrow$ interpret τ as complex structure of a two-torus (2 auxiliary dimensions)

  [Vafa] [Morrison,Vafa]

- minimally supersymmetric F-theory compactifications:
  - F-theory on torus fibered Calabi-Yau 4-fold Y_4

    $\Rightarrow$ 4 dim, N=1 supergravity theory

    $\Rightarrow$ base B_3 is a Kähler manifold



- singularities of the fibration are crucial to encode 7-brane physics

  $\Rightarrow$ pinching torus indicates presence of 7-branes magn. charged under τ

- brane and bulk physics encoded by <u>complex</u> geometry

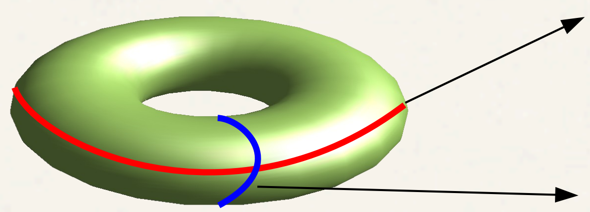
F-theory via M-theory

- F-theory viewed as auxiliary `12 dim.' theory (torus volume unphysical)

- F-theory effective actions has to be studied via M-theory

  Consider M-theory on space $T^2 \times M_9$

  τ is the complex structure modulus of the T^2, v volume of T^2



F-theory limit:

(1) A-cycle: if small than M-theory becomes Type IIA

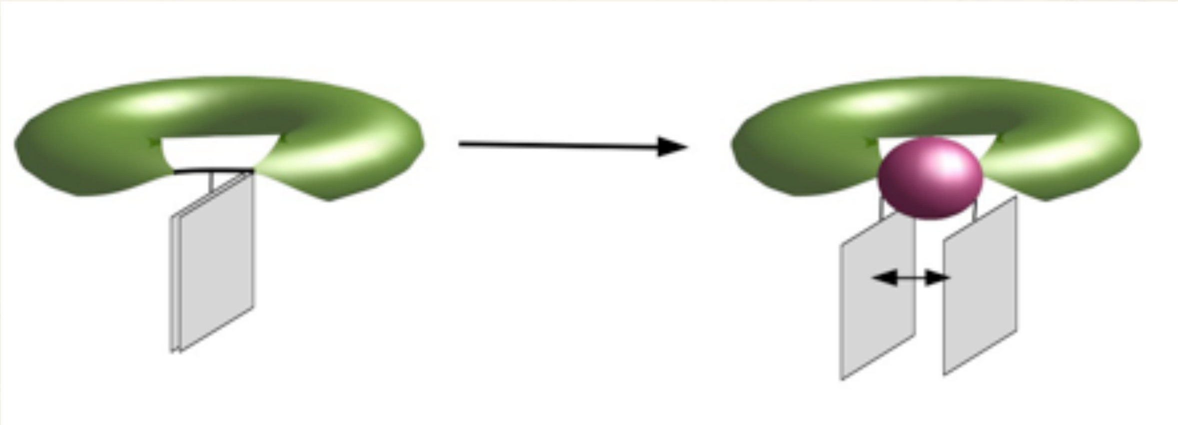
(2) B-cycle: T-duality $\Rightarrow$ Type IIA becomes Type IIB, τ is indeed dilaton-axion

(3) grow extra dimension: send $v \rightarrow 0$ than T-dual B-cycle becomes large

- can be generalized for singular T^2 fibrations: e.g. Taub-NUT → D6 → D7

F-theory / M-theory geometries

- F-theory geometries can be constructed and analyzed
  - singularities of elliptic fibration induce non-Abelian gauge symmetry
  - singularity resolution:
    (resolution at each codimension)



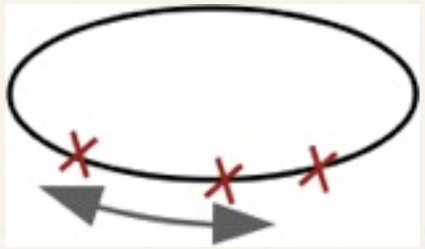
- Examples: compact, fully resolved Calabi-Yau three- / fourfolds
  $\Rightarrow$ toric geometry: numerous examples + various types of gauge groups

- Unification of $\vec{N}_{\text{geom}}$ and $\vec{N}_{\text{brane}}$ on resolved Calabi-Yau manifolds
  $\Rightarrow$ bulk and 7-brane geometries systematically classified by smooth higher-dimensional complex geometries!

M-theory on resolved CY manifolds

- physical interpretation of resolution only possible in M-theory
  - moving branes apart on the B-circle:

    Coulomb branch of the lower-dimensional gauge theory: $G \rightarrow U(1)^{\mathrm{rank}G}$



- Massive states from M2 branes on geometric 2-cycles:
  - M2-branes on resolution $\mathbb{P}^1$'s over generic points of S $\Rightarrow$ massive `W-bosons' of G-breaking
  - M2-branes on resolution $\mathbb{P}^1$'s over intersection $\Rightarrow$ massive matter multiplets
  - M2-branes on the elliptic fiber $\Rightarrow$ massive Kaluza-Klein modes

- All massive states have to be integrated out to determine Wilsonian effective action $\Rightarrow$ in circle compactification also KK-modes are crucial!!

4D F-theory effective actions via M-theory

- effective actions can be computed via M-theory / 11-dimensional supergravity on the resolved Calabi-Yau fourfolds

F-theory on singular CY_4
'Gauge bundle'

4d, N=1 effective theory with non-Abelian gauge symmetry G

circle compactification

3d, N=2 effective theory pushed to 3d Coulomb branch: $U(1)^{\mathrm{rk}G}$

M-theory on resolved $\tilde{CY}_4$
G_4- flux

3d, N=2 effective theory with only abelian gauge symmetries

compare

4d / 3d [TG] [TG,Kerstan,Palti,Weigand] [TG, Savelli] [TG, Hayashi] [TG,Klevers,Poretschkin]

- explicit: N=1 characteristic data determining the action

Part I: Chiral index from flux

Finally facing F-theory fluxes for fibered fourfolds

- specification of flux bundle in F-theory is hard
  spectral cover-type methods, link to weak coupling

- M-theory dual model on fully resolved $\widetilde{\mathrm{CY}}_4$:
  non-Abelian groups: new resolution two-forms (dual to resolution divisors)

  $\omega_i \quad i = 1, \ldots, \mathrm{rank}(G)$ fully resolved geometry has new intersection numbers (compute explicitly): $\int \omega_i \wedge \omega_j \wedge \omega_k \wedge \omega_l$

  extra U(1)'s: new resolution two-forms $\tilde{\omega}_m \quad m = 1, \ldots, n_{\mathrm{U}(1)}$

- M-theory fluxes we consider: $$G_4 = m^{\Sigma\Lambda} \omega_\Sigma \wedge \omega_\Lambda$$
  - $m^{\Sigma\Lambda}$ subject to vanishing conditions to lift to F-theory
  - $m^{\Sigma\Lambda}$ subject to quantization conditions (Freed-Witten) [Collinucci,Savelli]

F-theory chiral spectrum via M-theory

- determination of charged chiral spectrum is much harder:
  - chirality induced by fluxes on 7-branes, but in M-theory on resolved 4-fold there are no chiral fields
  - Chirality formulas for M/F-theory setups?

$$\chi(\mathbf{R}) = \int_{S_{\mathbf{R}}} G_4$$

$G_4 = \langle dC_3 \rangle$ flux on resolved fourfold

[Braun,Collinucci,Valandro]
[Marsano,Schäfer-Nameki]
[Krause,Mayrhofer,Weigand]
[TG,Hayashi]
[Intriligator,Jockers,Mayr, Morrison,Plesser]
[Küntzler,Schäfer-Nameki]

  - 3D one-loop Chern-Simons terms linked to 4D chiral index $\chi(\mathbf{R})$
  - 3D M-theory Chern-Simons terms computes anomaly free chiral spectrum

[TG,Hayashi]
[TG,Klevers] in progress

3D Chern-Simons vs. 4D chiral matter

[TG,Hayashi]

- 3d Chern-Simons terms: $S_{\rm CS}^{(3)} = \int \Theta_{ij} A^i \wedge F^j$ $\qquad U(1)^{{\rm rk} G}$
  - No such terms from classical circle reduction of 4d, N=1 theory $\Rightarrow$ generated at one loop by massive fermions

$$\Theta_{ij}^{1-{\rm loop}} = \frac{1}{2} \sum_{\rm rep} n_{\rm rep} \sum_{\lambda \in W({\rm rep})} \lambda_i \lambda_j \; {\rm sign}(w_k \xi^i)$$

[Aharony,Hanany,Intriligator Seiberg,Strassler]

  - M-theory on resolved $\widetilde{\rm CY}_4$ has classical Chern-Simons term: $\Theta_{ij}^{\rm flux} = \int_{\widetilde{\rm CY}_4} G_4 \wedge \omega_i \wedge \omega_j$
  - G_4 fluxes on $\widetilde{\rm CY}_4$ count 4D chiral matter spectrum massive in the 3D Coulomb branch $\Rightarrow$ determine chiral index $\Theta_{ij}^{1-{\rm loop}} = \Theta_{ij}^{\rm flux}$
  - 4D anomaly induced by chiral matter is captured by 3D Chern-Simons term at one loop in the Coulomb branch (F-theory vs. M-theory)

Examples: with or without U(1)

[TG,Hayashi]

- M-theory fluxes which do not break SU(5) gauge group

  consider e.g. base $\mathbb{P}^2 \times \mathbb{P}^1$

  (1) toric construction of fully resolved CY_4

  (2) computation of intersection numbers: fluxes of form $G_4 = m^{ij}\omega_i \wedge \omega_j$

  (3) computation of Mori cone to evaluate: $\mathrm{sign}(\int_\Sigma J)$

  [Marsano,Schäfer-Nameki]

  $\Rightarrow$ compute chirality by using one-loop equation

- U(1) restricted Tate model: simple way to globally obtain geometrically massless U(1) for a fourfold

  [TG,Weigand]

  $\Rightarrow$ same program can be applied for fluxes of the form $G_4 = F_{U(1)} \wedge \omega_{U(1)}$

- Algorithmic implementation and model searches are possible!

  $\Rightarrow$ scanning over Kreuzer-Skarke list for fourfolds....

Part II: Kaluza-Klein modes in M- to F-theory duality

Extending M-theory to F-theory limit

- Comparing geometrically large M-theory compactification with circle compactification requires to integrate out Kaluza-Klein modes

- one-loop corrections to Chern-Simons terms do not depend on mass of the modes in the loop:

key example: Kaluza-Klein theory of S^1 – compactification from D to D-1 dimensions (D even)

$$k_{\rm KK} \int_{M_{D-1}} A^0 \wedge F^0 \wedge \ldots \wedge F^0$$

KK-vector

$$ds^2_D = r^2(dy + A^0)^2 + ds^2_{D-1}$$

$k_{\rm KK}$ only arise at one loop due to integrated out KK-modes

- match of 5D M-theory compactifications and 6D F-theory reveals physics of self-dual tensors in 6D

[Bonetti,TG]
[Bonetti,TG,Hohenegger]

Tool: anomaly cancellation in 6D, N=1 Sugra

- anomalies cancel among the fields: e.g. $\mathrm{tr}(R^4)$ - gravitational anomaly

$$H - V = 273 - 29T$$

hypermultiplets (H), vector multiplets (V), tensor multiplets (T)

- Green-Schwarz mechanism to cancel residual anomaly [Sagnotti]

$$S_{\rm GS} = \int B_2^\beta\, \Omega_{\alpha\beta} \wedge \left[a^\alpha\, \mathrm{tr}(R\wedge R) + b^\alpha\, \mathrm{Tr}(F \wedge F)\right]$$

a^α, b^α: constant anomaly vectors for T+1 tensors B_2^β

e.g. $\mathrm{tr}(R^2)^2$- gravitational anomaly

$$a^\alpha \Omega_{\alpha\beta} a^\beta = 9 - T$$

Derivation for F-theory via M-theory

[Bonetti,TG]

- start with topological terms of 11D sugra:

$$S^{(11)}_{\rm CS} = \int C_3 \wedge G_4 \wedge G_4 + \int C_3 \wedge I_8(R)$$

fourth order polynomial in the 11d Riemann tensor

- expand M-theory three-form along $H^2(\tilde{CY}_3)$

$$C_3 = A^0 \wedge \omega_{B_2} + A^\alpha \wedge \omega_\alpha + A^i \wedge \mathrm{w}_i$$

ω_α pulled back from the base $\rightarrow A^\alpha$ 6D tensors

ω_{B_2} base $\rightarrow A^0$ is KK vector in going from 6D to 5D

w_i resolution of gauge sing. $\rightarrow A^i$ 6D vectors (Cartans)

- insert KK-ansatz into 11D topological terms

$\rightarrow$ read off: e.g.

$a^\alpha = K^\alpha$ $\quad [K] = K^\alpha \omega_\alpha$ canonical class of base B_2

New Chern-Simons terms at one loop

- M-theory on $\widetilde{\text{CY}}_3$: extra Chern-Simons terms with <u>no classical</u> KK analog, e.g.

① $\int \mathcal{K}_{ijk} A^i \wedge F^j \wedge F^k$

<u>three exceptional indices:</u> singularity structure
→ one loop: massive W-bosons + charged matter

gauge theory comp: [Witten],[Intriligator,Morrison,Seiberg]

② $\int \Omega_{\alpha\beta} K^\alpha K^\beta A^0 \wedge F^0 \wedge F^0$

<u>three times base:</u> use for ell. fibrations $\omega_0^2 = [K] \wedge \omega_0$
→ one loop: massive KK modes for all 6D fermions and 6D tensors
→ eff. theory: $\Omega_{\alpha\beta} K^\alpha K^\beta = 9 - T$

[Bonetti,TG,Hohenegger]

③ $\int (c_2)_0 \; A^0 \wedge \text{tr}(R \wedge R)$

<u>M-theory prediction:</u> $(c_2)_0 = 48 - 4T$

eff. theory derivation is not known

- 6D anomaly cancellation → relations among 5D CS-coefficients & spectrum

6D Self-dual tensors and their KK-action

- $A^0 \wedge F^0 \wedge F^0$ coupling cannot only be generated by KK-fermions: also KK-modes of self-dual tensors need to run in loop

- Kaluza-Klein inspired action: KK-ansatz $\hat{B} = \sum_{n\in\mathbb{Z}} e^{iny}\left[B_n + A_n \wedge (dy + A^0)\right]$

$$S = \int -\tfrac{1}{2} r^{-1} \mathcal{F} \wedge *\mathcal{F} + \tfrac{1}{2} c\, A^0 \wedge \mathcal{F} \wedge \mathcal{F}$$
$$+ \sum_{n=1}^{\infty} \int -r^{-1} \bar{\mathcal{F}}_n \wedge *\mathcal{F}_n + \tfrac{i}{n} c\, \bar{\mathcal{F}}_n \wedge \mathcal{D}\mathcal{F}_n$$

$$\mathcal{F}_n = \mathcal{D}A_n + inB_n$$

$$\mathcal{D} = d - inA^0$$

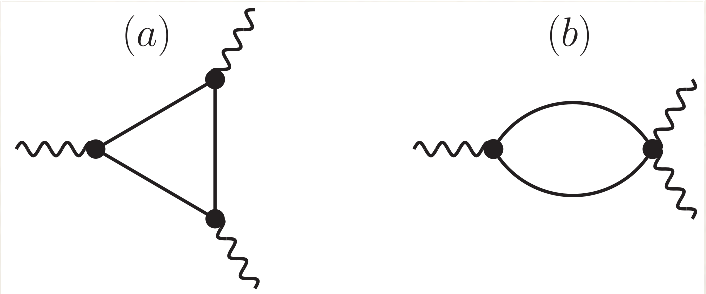
[Bonetti,TG,Hohenegger]

self-dual tensors have in 5D KK-tensor modes with Chern-Simons kinetic terms

[Townsend etal.]

- Computed contribution of all KK-modes of N=1, 6D sugra:

e.g. tensors



Proof:

$$k_{\rm KK} \propto 9 - T$$

Part III: Topological terms and constraints

Basic idea:

[TG,Taylor]

- in 6D F-theory compactifications on CY_3 :
  - effective theory: strong constraints from anomalies (gauge+gravitational...)
  - F-theory geometry: topological properties of resolved elliptic fibrations can be matched with the anomaly constraints

  $\Rightarrow$ relating terms in the effective action and spectrum (Green-Schwarz)

- in 4D F-theory compactifications on CY_4 :
  - F-theory geometry: various topological properties genuine to consistent compactification

  $\Rightarrow$ Can one relate terms in the effective action and spectrum despite the absence of certain anomalies? Are there universal constraints?

4D topological terms

[TG,Taylor]

- to formulate constraints we also need topological terms - axion couplings
  - topological terms with 7-brane gauge field

    $\int b^\alpha \chi_\alpha \mathrm{Tr}(F \wedge F)$ $\longrightarrow$ $[S] = b^\alpha \omega_\alpha$ location of 7-branes in B_3

  - topological terms involving curvature

    $\int a^\alpha \chi_\alpha \mathrm{tr}(R \wedge R)$ $\longrightarrow$ $[K] = a^\alpha \omega_\alpha$ canonical class of B_3

    $\rightarrow$ our derivation using orientifold limit (likely also via M-theory as in 6D/5D)

  - higher curvature terms are crucial to determine 7-brane configuration: discriminant:

$$[\Delta] = \mathrm{rank}(G)[S] + [\Delta']$$

$\longleftarrow$ I_1 locus only visible via higher curvature terms in effective action

Constraints

[TG,Taylor]

- Relating the moduli spectrum to couplings: (no anomaly interpretation)

define $\langle\langle x, y, z\rangle\rangle := \mathcal{K}_{\alpha\beta\gamma} x^\alpha y^\beta z^\gamma$

- no gauge group

complex str. → C_{cs}; Kähler + axion → C_{sa}

$$39 - 60\langle\langle a, a, a\rangle\rangle = C_{\text{cs}} + C_{\text{sa}} - C_{21}$$ ← axion+axion

derivation of Euler number of fourfold [Sethi,Vafa,Witten] [Andreas,Curio]

- including gauge group, but no charged matter

$$39 - 60\langle\langle a, a, a\rangle\rangle = C_{\text{cs}} + C_{\text{sa}} - C_{21} + r_G + \frac{1}{6} r_G (c_G + 1)\langle\langle a + b, a + b, b\rangle\rangle$$

- complications from a general 4D theory:
  - distinguish the moduli fields (all chiral multiplets in 4D), as in 5D
  - scalar potential e.g. due to fluxes → focus on light fields
  - corrections and additional axion-(curvature)^2 , e.g. dilaton-axion at weak string coupling, additional het. axion

Conclusions

- M-theory to F-theory limit is powerful (double-dimensional reduction)
  - importance of Chern-Simons terms: study theory and formulate constraints
    loop corrections ↔ classical M-theory results
  - importance of Kaluza-Klein modes in circle reduction:
    6D self-dual tensors → Kaluza-Klein modes: interesting action, induce new couplings
  - key properties of F-theory compactifications encoded by topological terms
    → might lead to interesting constraints beyond anomaly conditions
- Many open questions:
  - M5-branes instantons behave non-trivially in the M-theory to F-theory limit
    [Cvetič,TG,Halverson,Klevers] in progress
  - extend constraint analysis in 4D, including fluxes and potentials
  - constraining continuos parameters

The End.
Thank you!

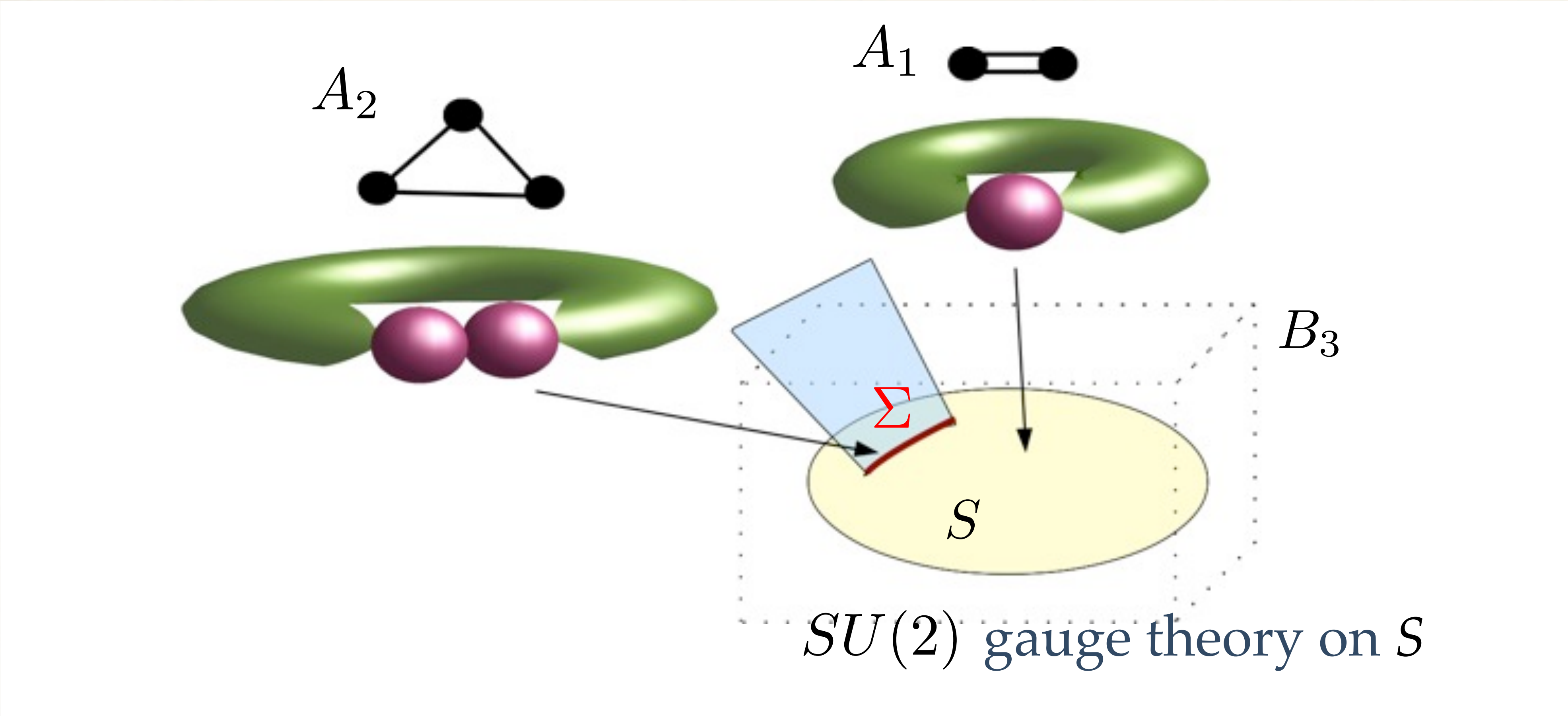
Gauge theory and singularities

- a closer look at the resolution space: e.g. single stack of branes

4D: - gauge theory on surface 4-cycle $S \subset B_3$

- further enhancement along intersection curve Σ (2-cycle) of two 7-branes

$\Rightarrow$ matter representations $\boldsymbol{R}$



Classical 5D Chern-Simons terms

- 5D Chern-Simons actions:

$$\int b^\alpha \Omega_{\alpha\beta} A^\beta \wedge (C_{ij} F^i \wedge F^j) \longrightarrow \quad [S] = b^\alpha \omega_\alpha \quad \text{location of 7-branes in } B_2$$

$$\int a^\alpha \Omega_{\alpha\beta} A^\beta \wedge \mathrm{tr}(R \wedge R) \longrightarrow \quad [K] = a^\alpha \omega_\alpha \quad \text{canonical class of } B_2$$

  - topological $\mathrm{tr}(R^2)$ - terms are given by second Chern class of $\tilde{CY}_3$
    [Antoniadis,Ferrara,Minasian,Narain]

  - have to use non-trivial geometric facts for Chern-classes of elliptic fibrations: (e.g. for smooth elliptic fibrations)

$$c_2(\tilde{CY}_3) = c_2(B_2) + 11c_1^2(B_2) + 12c_1(B_2) \wedge \omega_{B_2}$$

- 5D terms lift to be part of 6D Green-Schwarz term → M-theory deriv. of (b^α, a^α)
  compare with [Kumar,Morrison,Taylor] [Park,Taylor]