FLUXES & WARPING IN F-THEORY

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Based on: Work in progress; T.W. Grimm, M. Poretschkin, D.K.: **arXiv:1202.0285 [hep-th];**
T.W. Grimm, T.-W. Ha, A. Klemm, DK: **arXiv:0912.3250 [hep-th], arXiv:0909.2025 [hep-th]**.

MOTIVATION

Why 4D F-theory compactifications?

- F-theory describes a broad class of 4D N=1 Type II string compactifications with all necessary ingredients for constructing phenomenologically interesting models.
- Different complicated objects (7-branes, varying coupling constant) of Type IIB string theory mapped to F-theory geometry of elliptically fibered Calabi-Yau fourfold.

MOTIVATION

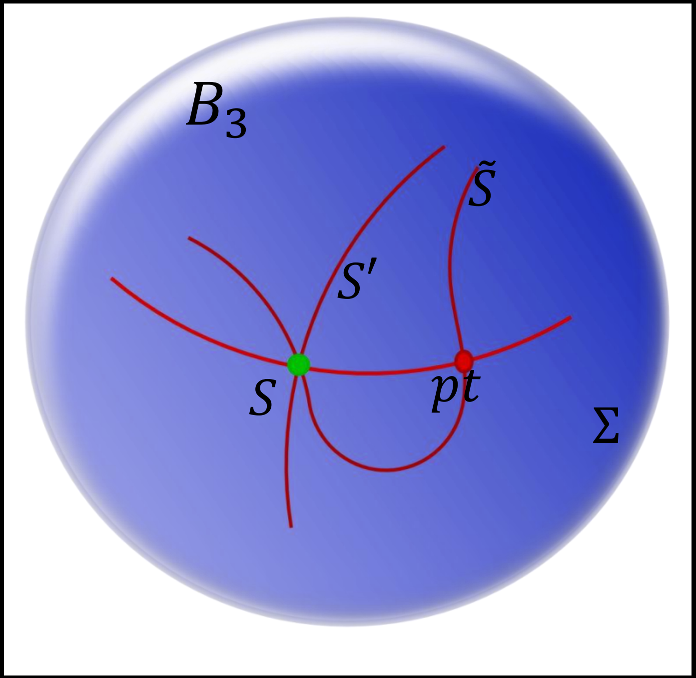
- F-theory is a geometric, non-perturbative formulation of Type IIB with non-perturbative structure encoded in the fibration of two-torus T^2 over base B_3.

Vafa '96

- Singularities in fibration= strong coupling regions of $\tau = C_0 + i g_s^{-1}$.

Complicated setup of 7-branes on B_3 mapped to elliptically fibered CY X_4.

- Gauge theory (ADE…) in 8d
  S in B_3
- Matter in 6d
  Σ in B_3
- flux F_2 on Σ (G_4-flux)
  4d matter
- Yukawas in 4d
  pt in B_3



- Recent advances in realistic GUT model building in F-theory based on this structure.

Donagi,Wijnholt '08; Beasley,Heckman,Vafa '08, Hayashi,Kawano,Tatar,Watari'09;Dudas,Palti'09,…
Global models: Marsano,Saulina,Schäfer-Nameki '09; Blumenhagen,Grimm,Jurke,Weigand '09,..

MOTIVATION

Requirement of G-fluxes in F-theory

- Extra discrete degrees of freedom to specify vacuum.
- Required by consistency of compactification: D3-tadpole cancellation.

Goal of this talk: How do G-fluxes enter the 4D N=1 effective action of F-theory?

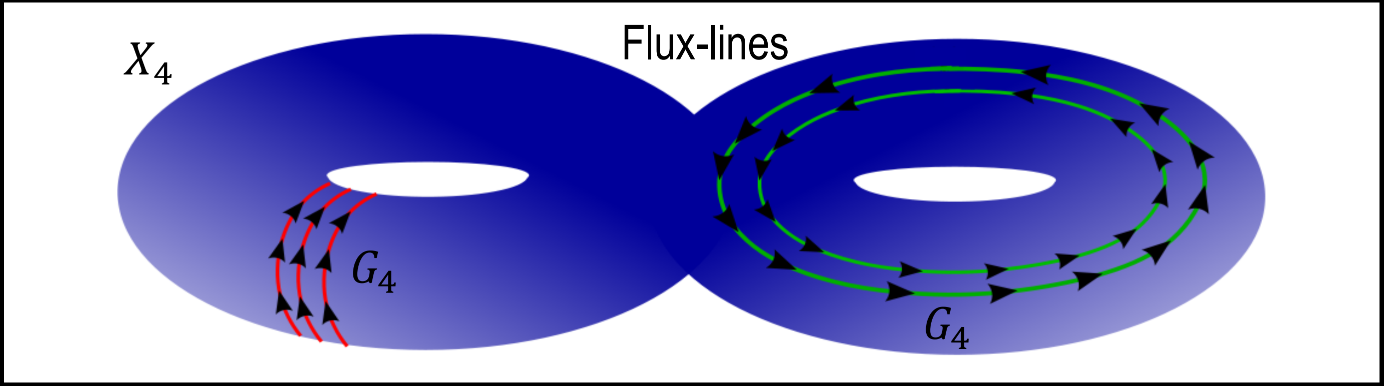
- Induce superpotential W: stabilization of complex structure moduli.
- Generation of 4D anomaly-free chiral spectrum by appropriate fluxes.
- More instanton-effects: gauged instantons with charged matter prefactor.
- Induce warping in F-theory: full understanding of 4D effective physics requires back-reaction of fluxes ⤇ derivation of 7-brane gauge coupling.

Two types of fluxes

F-THEORY WITH FLUXES

PROBE-FLUXES IN F-THEORY

- Additional discrete degrees of freedom have to be added: G-flux G_4
  - Quantized G-flux: $G_4 \in H^4(X_4, \mathbb{Z}/2).$ — Witten '96
  - D3-tadpole: $\frac{\chi(X_4)}{24} = n_{D3} + \frac{1}{2}\int_{X_4} G_4 \wedge G_4$ — Sethi, Vafa, Witten '96



- There are two qualitatively different fluxes on X_4 — Greene, Morrison, Plesser '94
  - Horizontal fluxes $H^4_H(X_4, \mathbb{Z})$ superpotential, moduli stablization.
  - Vertical fluxes $H^4_V(X_4, \mathbb{Z})$ D-term potential, 4d chirality, warping.

Goal: Understand the effect of the two types of fluxes on F-theory geometry and the effective action of F-theory.

The flux superpotential

HORIZONTAL FLUXES

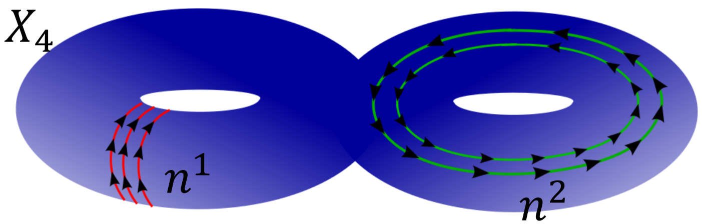
THE FLUXSUPERPOTENTIAL

- The horizontal flux G_4 enters the superpotential W_{GVW}:

$$W_{GVW}(z) = \int_{X_4} G_4 \wedge \Omega_4(z)$$

Gukov,Vafa,Witten '99
Becker,Becker '96

- $\Omega_4(z)$ is $(4,0)$-form depending on complex structure z on X_4.
- G_4 is specified by flux quanta n^A along cycles C_A:



$$\longrightarrow \quad G_4 = n^A [C_A]$$

$$\longrightarrow \quad \Pi_A(z) = \int_{C_A} \Omega_4(z)$$

- W_{GVW} is sum of periods $\Pi_A(z)$ of X_4 measuring holomorphic volumes of cycles C_A:

$$W_{GVW}(z) = n^A \Pi_A(z)$$

- Exact calculation of W_{GVW} possible in almost all known fourfold X_4.

THE FLUXSUPERPOTENTIAL

- Many F-theory fourfolds explicitly constructable as toric hypersurface (CICY)

$X_4 = \{P = 0\}$ in a toric variety $\mathbb{P}^5_\Delta$

Klemm,Lian,Roan,Yau '97; Mayr '96

- $\Pi(z_4)$ can be obtained from Picard-Fuchs differential equations:

$W_{GVW}(z_4)$ is exactly calculable by algebraic methods

- Representation by residue integrals for CY-form Ω_4.
- Application of Griffiths-Dwork reduction method.

$\Longrightarrow$ Picard-Fuchs equations for $W_{GVW}(z_4)$

- Determination of exact perturbative superpotential for toric examples
  - few moduli z (Calabi-Yau threefold fibered Calabi-Yau fourfolds),
  - simplification of Picard-Fuchs: GKZ hyper-geometric system of Δ^*.

Grimm,Ha,Klemm,DK I '09

THE FLUXSUPERPOTENTIAL

- Calculation of exact Type IIB superpotentials in weak coupling limit:

  Grimm,Ha,Klemm,DK I '09

$$W_{GVW}(z) \mapsto W^{IIB}_{flux} + W^{IIB}_{7-brane}$$

- W_{GVW} for stabilization of bulk and brane moduli in F-theory and IIB: From toy models to F-theory GUTs? Moduli stabilization with $SU(5)$?
- W_{GVW} relevant for fourfold mirror symmetry: Generalizes N=2 prepotential.

  Greene,Morrison,Plesser;Klemm,Lian,Roan,Yau;Mayr

  Klemm,Pandharipande '07
- By heterotic/F-theory duality W_{GVW} calculates heterotic superpotentials: explicitly obtain heterotic flux, M5-brane and vector bundle superpotential.

  Grimm,Ha,Klemm,DK II'09

  Jockers,Mayr,Walcher'09

Chirality and flux back-reaction

VERTICAL FLUXES

VERTICAL FLUXES , CHIRALITY & ANOMALIES

- The vertical fluxes G_4 determines chirality $\chi(\boldsymbol{R})$ of 4D matter in rep R of gauge group G:

Fundamental flux integrals:

$$\Theta_{IJ} := \int_{\hat{X}_4} G_4 \wedge \omega_I \wedge \omega_J, \qquad \omega_I \in H^{1,1}(\hat{X}_4)$$

Chiralities:

$$\chi(\boldsymbol{R}) = A^{IJ}_{\boldsymbol{R}} \Theta_{IJ} \equiv \int_{C(\boldsymbol{R})} G_4, \qquad C(\boldsymbol{R}) = \text{ Matter surface}$$

'11:Braun,Collinucci,Valandro;Marsano,Schäfer-Nameki;Krause,Mayrhofer,Weigand;Grimm ,Hayashi.

- Careful construction of resolved fourfold $\hat{X}_4$ crucial for evaluating Θ_{IJ}.

Esole,Yau'11; Marsano,Schäfer-Nameki'11

- Spectrum of F-theory potentially anomalous: check anomaly cancellation.

CHIRALITY & ANOMALIES

1. Simple gauge group, e.g. $G = \mathrm{SU}(5)$ with reps $\boldsymbol{R} = \bar{\mathbf{5}}, \mathbf{10}.$
   - Purely non-Abelian gauge anomaly cancellation condition

$$\chi(\mathbf{10}) \equiv \chi(\bar{\mathbf{5}}): \quad A^{IJ}_{\mathbf{10}} \Theta_{\mathrm{IJ}} \equiv A^{IJ}_{\bar{\mathbf{5}}} \Theta_{\mathrm{IJ}}$$

   - This is obeyed for valid F-theory chirality inducing flux G_4.

2. Additional U(1)-factors, e.g. $G = \mathrm{SU}(5) \times \mathrm{U}(1); \boldsymbol{R} = \mathbf{1}_5, \bar{\mathbf{5}}_{\mathbf{2}}, \bar{\mathbf{5}}_{-\mathbf{3}}, \overline{\mathbf{10}}_{-\mathbf{1}}$
   - Purely non-Abelian gauge anomaly cancelled, Abelian/mixed not

$$I_6 = F^m_{U(1)} \wedge \left(X\, Tr(F \wedge F) + Y F^m_{U(1)} \wedge F^m_{U(1)} + Z\, tr(R \wedge R)\right)$$

   - Anomaly coefficients X, Y, Z determined by chiralities $\chi(\boldsymbol{R}) = A^{IJ}_{\boldsymbol{R}} \Theta_{\mathrm{IJ}}$ (+ group theory factors).
   - Generalized Green-Schwarz mechanism cancels factorizable anomalies.

GS-MECHANISM IN F-THEORY

- Chirality inducing G_4 induces gaugings of axions ρ_α by U(1)-vector $A^m_{U(1)}$:

$$D\rho_\alpha = d\rho_\alpha + \Theta_{\alpha m} A^m_{U(1)}, \quad \Theta_{\alpha m} = \int_{\hat{X}_4} G_4 \wedge \omega_\alpha \wedge \omega_m$$

Grimm'10;
Grimm,Kerstan,Palti,Weigand'11

- Green-Schwarz counter-terms take the form:

$$S_{GS} = (b_1 \cdot \rho) Tr(F \wedge F) + (b_2 \cdot \rho) F^m_{U(1)} \wedge F^m_{U(1)} + (a \cdot \rho) tr(R \wedge R)$$

- Coefficients b_1, b_2 and a determined by geometry of $\hat{X}_4$:
  location of resolved singularities (7-branes) + canonical bundle

- S_{GS} has the anomalous variation yielding:

$$I_6^{GS} = F^m_{U(1)} \Theta_{m\alpha} \wedge \left[b_1^\alpha Tr(F \wedge F) + b_2^\alpha F^m_{U(1)} \wedge F^m_{U(1)} + a^\alpha tr(R \wedge R) \right]$$

GS-MECHANISM IN F-THEORY

Anomaly cancellation

- Cancellation of Abelian/mixed anomaly requires relations:

$$X = \Theta_{m\alpha} b_1^\alpha, \qquad Y = \Theta_{m\alpha} b_2^\alpha, \qquad Z = \Theta_{m\alpha} a^\alpha$$

- This implies relations between $\chi(\boldsymbol{R}) = A_{\boldsymbol{R}}^{IJ} \Theta_{\mathrm{IJ}}$ and $a, b_1, b_2, \Theta_{\alpha m}$: non-trivial intertwining of geometry and flux G_4.

Grimm,DK: work in progress

CHIRALITY FROM 3D

- Derivation of chirality formula in 3D N=2 theory via duality:

F-theory on $X_4 \times S^1$ = M-theory on $\hat{X}_4$

- Compare F-theory effective action with M-theory effective action.

4D F-theory on X_4 ⟶	F-theory on S^1	3D M-theory on $\hat{X}_4$
$N = 1$ gauge theory →	$N = 2$ gauge theory on Coulomb branch	
Chiral matter in rep R →	**massive matter** **No CS-term**	**no chiral matter** (massive M2) **CS-term $A^I \wedge F^J$**

- Match of **CS-terms** at 1-loop: Integrating out massive chiral matter

Grimm,Hayashi '11

$$\chi(\boldsymbol{R}) = \int_{C(\boldsymbol{R})} G_4$$

- Algorithmic computation of chiral indices from **CS-terms** in toric examples.

Grimm,DK:work in progress

Charged instantons in M-theory

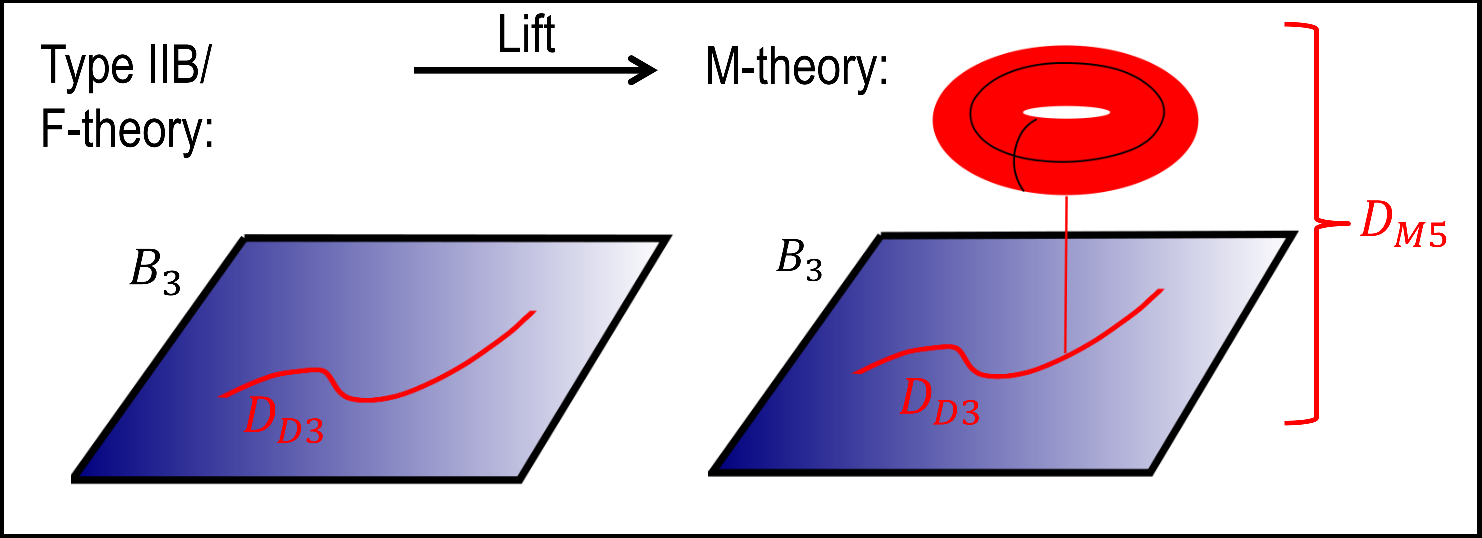
INSTANTONS IN F-THEORY FROM 3D

INSTANTONS IN F-& M-THEORY

We can analyze more questions in 3D: non-perturbative effects in F-theory

- M-/F-theory instantons: M5-instantons on vertical divisor D_{M5} in M-theory are D3-instantons on divisor D_{D3} in Type IIB/F-theory

Early works: Witten'96;Donagi,Grassi,Witten'96; Katz,Vafa'96, Diaconescu,Gukov'98,..
New works: Blumenhagen,Collinucci,Jurke'09;Cvetic,Garcia-Etxebarria,Halverson'10/11;Donagi,Wijnholt'10; Aparicio,Font,Ibanez,Marchesano'11;Grimm,Kerstan,Palti,Weigand'11;Marsano,Saulina,SchäferNameki'11; Bianchi,Collinucci,Martucci'12; Kerstan,Weigand'12…



- Induced non-perturbative superpotential: Kähler modulus T_{M5} of D_{M5}

$$W_{3D}^{M} = Pfaff \cdot \mathrm{e}^{-\mathrm{T}_{\mathrm{M5}}}$$

INSTANTONS IN F-& M-THEORY

Central question for instanton physics is field dependence of Pfaffian $Pfaff$.

Analysis in 3D: match of F-theory on $X_4 \times S^1$ with M-theory on smooth $\hat{X}_4$

- Focus on instanton generated U(1)-charged matter couplings in W_{4D}

Blumenhagen,Cvetic,Weigand'06;Ibanez,Uranga'06

$$W_{4D} \sim \Phi^{\mathrm{k}} \mathrm{e}^{-\mathrm{T}_{\mathrm{D3}}}$$

Gauging: $DT_{D3} = dT_{D3} + i\Theta_{D3m}\mathrm{A}^{\mathrm{m}}_{\mathrm{U(1)}}$

Matter: $\nabla\Phi = \mathrm{d}\Phi + iq_m\Phi\mathrm{A}^{\mathrm{m}}_{\mathrm{U(1)}}$

Alternative approach: Heterotic/F-theory duality & spectral cover techniques

- Calculation of complex structure dependence of $Pfaff$ in F-theory via heterotic/F-theory duality + study of vanishings at enhanced symmetry points in moduli space.

  Heterotic:Buchbinder,Donagi,Ovrut'02; Heterotic/F-theory: Cvetic,Garcia-Etxebarria,Halverson'11
- Extension to direct F-theory calculations with G-fluxes without heterotic dual.

  Cvetic,Donagi,Halverson,Marsano: to appear

INSTANTONS IN F-& M-THEORY

Strategy: Follow non-perturbative effects in 4D IIB/F-theory to 3D M-theory

1) 4D F-Theory with U(1) gauge symmetries:

- D3-instanton on divisor D_{D3} with gauged T_{D3} requires charged matter Φ as prefactor

$$W_{4D} = \Phi^{k} e^{-T_{D3}}$$

Gauging: $DT_{D3} = dT_{D3} + i\Theta_{D3m} A^{m}_{U(1)}$

Matter: $\nabla\Phi = d\Phi + i q_m \Phi A^{m}_{U(1)}$

- Integer k determined by U(1) charge q_m of Φ and Θ_{D3m} by gauge invariance: microscopically k is chirality of charged zero-modes.

Question:

- What gauged 4D instantons are there?
- What are the criterions for an existing instanton correction.

Cvetic,Grimm,Halverson,DK : work in progress

GAUGINGS & INSTANTONS IN 3D

2) Answer on 3D Coulomb branch:

Superpotential in F-theory on S^1: lift $T_{D3} \to T_{DM5}$, new exceptional divisor T_m

$$W_{3D}^F = \Phi^{\mathrm{k}} \mathrm{e}^{-\mathrm{T_{M5}}} + \Phi^{\mathrm{l}} \mathrm{e}^{-\mathrm{T_m}}$$

Gauging: $DT_{M5} = dT_{M5} + i\Theta_{M5m} A_{U(1)}^m$

$DT_m = dT_m + i\Theta_{mm} A_{U(1)}^m$

Matter: $\nabla\Phi = \mathrm{d}\Phi + i\mathrm{q_m}\Phi\mathrm{A_{U(1)}^m}$

- Integrate out massive Φ on Coulomb branch in 3D effective action with W_{3D}^F:
- Determination of new effective superpotential $\widetilde{W}_{3D}$

$$\widetilde{W}_{3D} \sim \mathrm{e}^{-\Theta_{mm}\mathrm{T_{M5}} + \Theta_{M5m} T_m} \equiv e^{-\tilde{T}_{M5}}$$

No U(1) Gauging: $D\tilde{T}_{M5} = dT_{M5}$

No Matter.

- Comparison to classical geometric calculation in M-theory.

GAUGINGS & INSTANTONS IN 3D

Superpotential in M-theory on $\hat{X}_4$: Prediction of $\widetilde{W}_{3D}$

- Certain sectors of 3D non-perturbative superpotential determined by classical geometric calculation in M-theory.
- M-theory conditions for M5-instanton corrections from divisors D

1) D vertical with arithmetic genus $\chi_0(D) = 1$. (fermionic zero modes: $h^{0,i}(D) = (1,0,0,0)$)
2) No U(1)-charge of T_D.

$$W_{3D}^M \sim \sum_D e^{T_D}$$

Witten'96

- Match of M- and F-theory superpotentials:

Effective action: $\widetilde{W}_{3D} \equiv W_{3D}^M$:Geometry

- In general condition 2) only obeyed for non-generic flux G_4 necessary.
- Explicit matches performed in concrete geometries: X_4 with U(1), X_4 with U(1)xSU(5).

Cvetic,Grimm,Halverson,DK : work in progress

Calculating 7-brane gauge couplings in F-theory

WARPING IN F-THEORY

BACKREACTED FLUXES IN F-THEORY

- In M-theory (=F-theory on S^1) G-flux G_4 back-reacts on geometry
  - Warping: $$\Delta_{X_4} e^{3A/2} = *_{X_4} (G_4 \wedge G_4)$$ Becker,Becker '96; Haack,Louis '01
  - Change of KK-ansatz by warping: non-closed 3-from β $$C_3 = \text{Harmonic forms} + \beta$$ Dasgupta,Rajesh,Sethi '99;

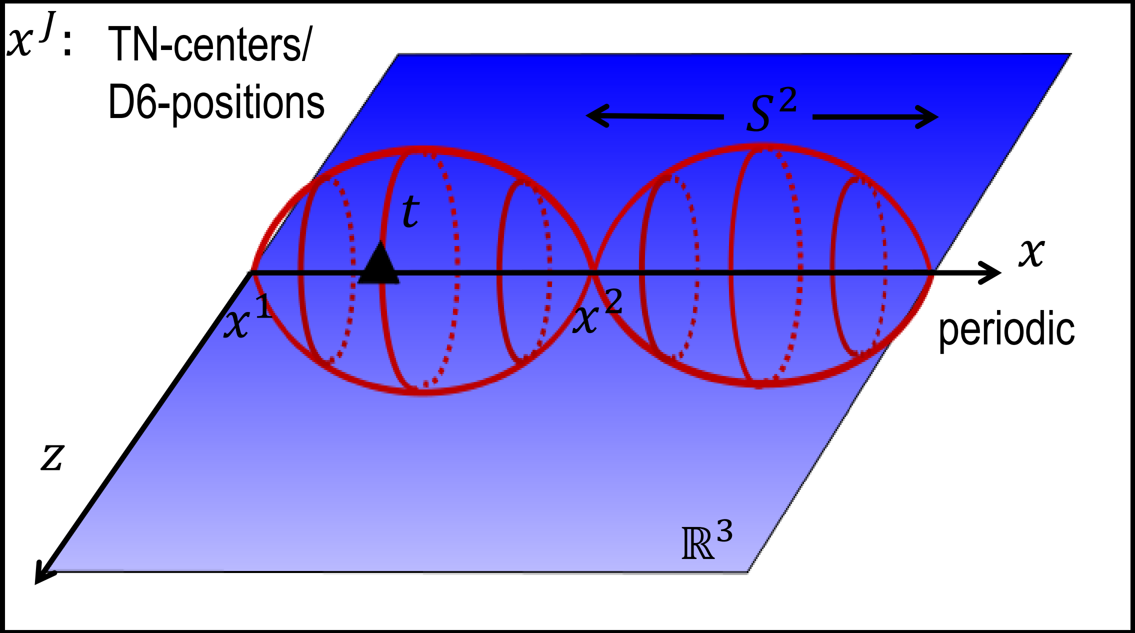
Goal:

- Solve warp-factor equation and construct 3-form β in local model for X_4.
- Understand corrections on 4d effective physics: 7-brane gauge coupling.

BACKREACTED FLUXES IN F-THEORY

- Construct a local model of X_4 for a stack of k 7-branes on S as follows

k 7-brane stack on divisor S	$\xrightarrow[\text{T-duality}]{S^1}$	k 6-brane stack on divisor S	$\xrightarrow[\text{theory}]{\text{M-}}$	Periodic multi-center Taub-NUT over S



Grimm,DK,Poretschkin '12

- Taub-NUT with k-centers is resolved A_k-singularity: k resolving S^2 = SU(k) gauge group of k 7-branes.

PERIODIC TAUB-NUT FOR F-THEORY

- Explicit construction of metric on periodic Taub-NUT:

Gibbons-Hawking: $$ds^2 = \frac{1}{V}(dt + U)^2 + V d\vec{r}^{\,2}, \qquad \vec{r} \in \mathbb{R}^3$$

$$V = 1 + \sum_{I=1}^{k} V_I\,, \qquad U = \sum_{I=1}^{k} U_I\,, \qquad dV_I = *_3\, dU_I$$

- V_I explicitly constructed as infinite series

$$V_I = \log(|z|) - \sum_{n>0} K_0(2\pi|z|n)\cos(2\pi n(x - x_I))$$

Ooguri,Vafa'96

- Identification along periodic direction x yields elliptic fibration of F-theory X_4

$$ds^2 = \frac{v_0}{Im\,\tau}\left((dt + Re\,\tau dx)^2 + Im\,\tau dx^2\right) + ds_{\mathbb{R}^2\times S}$$

Leading F-theory axio-dilaton: $\tau(z) = \tau_0 + \frac{k}{2\pi i}\log z + \cdots$

- Recover familiar form of $\tau(z)$ for k D7-branes + new corrections.

THE WARP-FACTOR ON TAUB-NUT

- Explicit solution of warp-factor eq. $$\Delta_{X_4} e^{3A/2} = *_{X_4} (G_4 \wedge G_4)$$
  - Specializial G-flux: 7-brane flux $\mathfrak{F}^I$ on S

$$G_4 = \Omega_I \wedge \mathfrak{F}^I,$$

  - Use explicitly constructed basis of k self-dual 2-forms on Taub-NUT

$$\Omega_I = d\left(\frac{V_I}{V}(dt + U) - U_I\right)$$

Ruback '86

  - Focus on flux with only non-trivial instanton number on S

$$\int_S \mathfrak{F}^I \wedge \mathfrak{F}^J = n^I \delta^{IJ}$$

Warp-factor on periodic Taub-NUT analytically determined

$$e^{3A/2} = 1 - \frac{n^I}{vol(S)}\left(\frac{V_I^2}{V} - V_I\right)$$

Grimm,DK,Poretschkin '12

CORRECTIONS TO 4D GAUGE COUPLINGS

Calculation of corrections to effective action:

- KK-reduction with fluxes and warping: 3d fluctuations F^I of G_4^{11d}

$$C_3^{11d} = A^I \wedge \Omega_I + \beta,$$

- Altered KK-ansatz due to warping: non-harmonic 3-form β
  - "Chern-Simons" form to flux G_4 determines β

$$d_{X_4}\beta = G_4 = \mathfrak{F}^I \wedge \Omega_I$$

  - β depends on moduli $M^a = (C_0, x^J)$ of Taub-NUT

$$G_4^{11d} = dC_3^{11d} = F^I \wedge \Omega_I + dM^a \wedge \partial_{M^a}\beta + G_4$$

- Comparison of 3d effective action determined by dimensional reduction of M-theory on warped X_4 and F-theory on S^1:

$$S^{11d} \supset \int_{\mathbb{R}^{1,10}} G_4 \wedge * G_4 + \frac{1}{3} C_3 \wedge G_4 \wedge G_4$$

CORRECTIONS TO 4D GAUGE COUPLINGS

Warped M-theory: $S^{3d}_{G_4} \supset \int_{\mathbb{R}^3} G_{IJ} F^I \wedge * F^J + d_{abI}\left(M^a dM^b \wedge F^I\right)$

F-theory on S^1: $S^{3d}_{F} \supset \int_{\mathbb{R}^3} Re\, f_{IJ} F^I \wedge * F^J + Im\, f^{flux}_{IJ} d\zeta^I \wedge F^J$

Warping corrected couplings: $G_{IJ} = \int_{X_4} e^{3A/2}\Omega_I \wedge \Omega_J\,, \qquad d_{abI} = \int_{X_4} \Omega_I \wedge \partial_{M^a}\beta \wedge \partial_{M^b}\beta$

- f_{IJ} = D7-brane gauge coupling, 3d vector multiplet (x^I, A^I)
- Small string coupling $g_s \to 0$: full D7-brane gauge coupling + corrections

Jockers,Louis '04

$$Re\, f_{IJ} = \int_{X_4} e^{3A/2}\Omega_I \wedge \Omega_J = \delta_{IJ}\left(vol(S) + g_s^{-1} n^I\right) + O(g_s)$$

$$Im\, f^{flux}_{IJ} = \int_{X_4} \Omega_I \wedge \partial_{C_0}\beta \wedge \partial_{x^J}\beta = \delta_{IJ}\left(C_0 g_s^{-1} n^I\right) + O(g_s)$$

$$\Rightarrow \quad f_{IJ} = \delta_{IJ}\left(\int_S (J \wedge J + iC_4) - i\tau n^I\right) + O(g_s)$$

Grimm,DK,Poretschkin '12

CORRECTIONS TO 4D GAUGE COUPLINGS

- Warp-factor dependence on auxiliary torus direction x is essential.
- Only possible origin for flux corrections to f_{IJ}:

Flux-correction is quartic in F: $S^{4d} \supset \int_{S\times\mathbb{R}^{1,3}} \mathfrak{F}^I \wedge \mathfrak{F}^I \wedge F^I \wedge F^I$

M-theory at most cubic: $S^{11d} \supset \int_{\mathbb{R}^{1,10}} G_4 \wedge * G_4 + \frac{1}{3} C_3 \wedge G_4 \wedge G_4$

- Understanding of flux correction essential e.g. for fate of unification in MSSM.
  Blumenhagen'08; Ibanez,Marchesano,Regalado,Valenzuela'12
- Physical understanding and more quantitative study of g_s corrections (Comparison to Dixon,Kapulnowski,Louis…).
- Inclusion of other corrections (M2-branes, curvature…) to warp-factor eq. and determination of corrected 3d/4d effective action.

Summary

- G-flux required by consistency and for phenomenology.
- Horizontal flux:
  - 4d superpotential W_{GVW} allows moduli stabilization,
  - W_{GVW} exactly calculable by algebraic methods.
- Vertical flux:
  - generation of 4D chiral, anomaly-free spectrum,
  - relation at one-loop between chiral index and CS-terms in 3D.
- Gauged instantons with charged matter prefactors analyzed in 3D.
- Back-reaction of fluxes induces warping & changes KK-ansatz
  - analytic solution of warp-factor eq. on local 7-brane geometry,
  - Calculation of full 4d of gauge coupling function in F-theory.