M-Theory on G2-Manifolds:
Supersymmetry Breaking & Moduli-Axions Physics

with B. Acharya '11
& work in progress

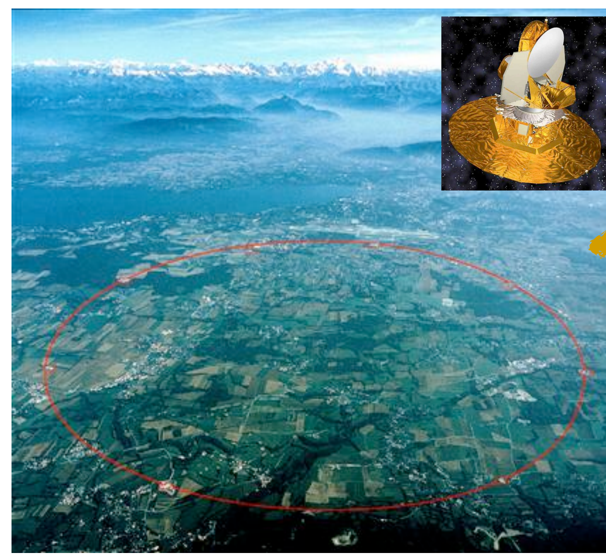
Mahdi Torabian
International Centre for Theoretical Physics, Trieste

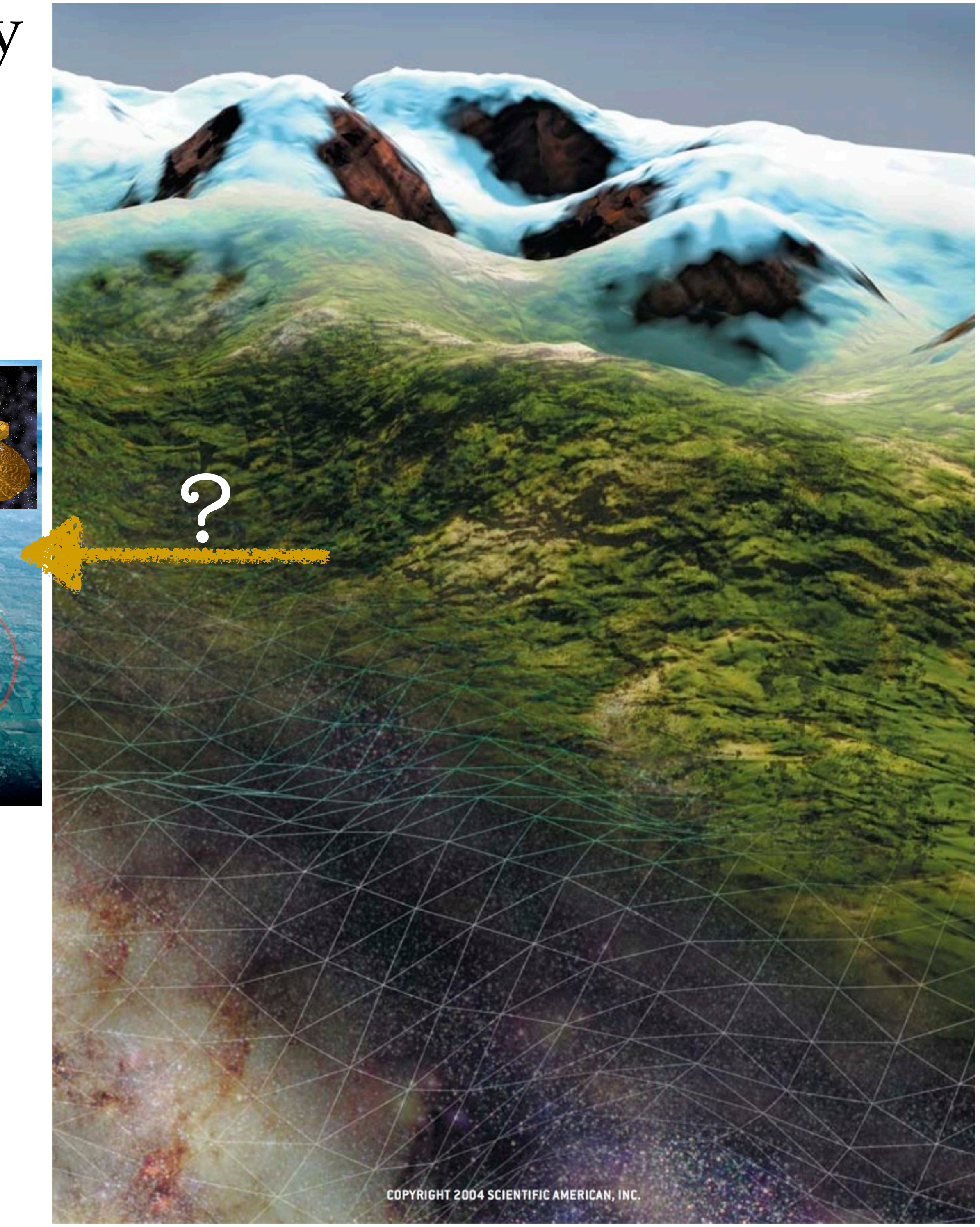
String Phenomenology TH Institute 2012
CERN

M-Theory on Singular G2 Holonomy Manifold

✔ Study 4 dimensional vacua of M-Theory compactified on a manifold of G2 holonomy and with particular kind of singularities.

? If there exists a vacuum which accommodates our empirical knowledge from colliders and satellites.





$$\mathcal{L}_{\rm SM}(24 \text{ dimensionless} + M_{\rm EW}, 60 \text{ dof's})$$
$$\Omega_\Lambda = 0.725 \pm 0.016 \quad w = 1.10 \pm 0.14 (68\%{\rm CL})$$
$$\Omega_b h_0^2 = 0.02255 \pm 0.00054$$
$$\Omega_{\rm DM} h_0^2 = 0.1126 \pm 0.0036$$
$$\Delta_{\mathcal{R}}^2 = (2.430 \pm 0.091) \times 10^{-9} \quad n_s = 0.968 \pm 0.012$$

✔ If it is so, it may provide a framework for studying physics beyond (and explaining prop's of) the SM's particle physics and hot cosmology.

M-Theory on Singular G2 Holonomy Manifold

✔ Riemannian 7-dimensional G2-Manifolds are natural choices for M-Theory compactifications.

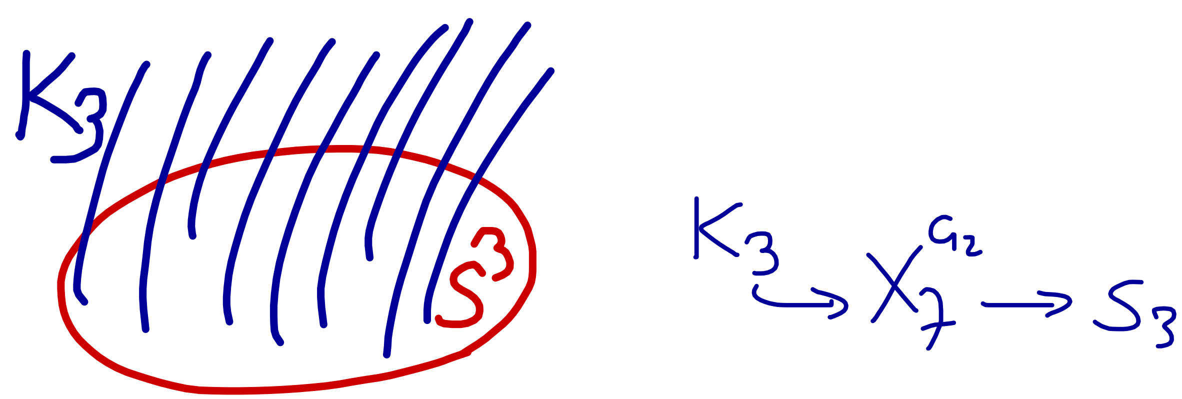
[Acharya-Gukov '04 report and ref's]

✔ No Yau-like existence theorem of G2-holonomy metric, as yet.

✔ Compact/smooth examples have been constructed.

G2-manifold as the total space of fibrationth: Fibers K3 surfaces vary over base S3.

[Kovalev, '02]



✔ There is a covariantly constant spinor and one covariantly constant 3-form but no covariantly constant vector.

✔ A G2-manifold is characterized by a G2-structure, aka (co)associative calibration.

✔ Low energy limit of KK reduction of 11-dimensional SUGRA: 4-dimensional SUGRA coupled to neutral chiral supermultiplets and Abelian vector supermultiplets.

M-Theory on Singular G2 Holonomy Manifold

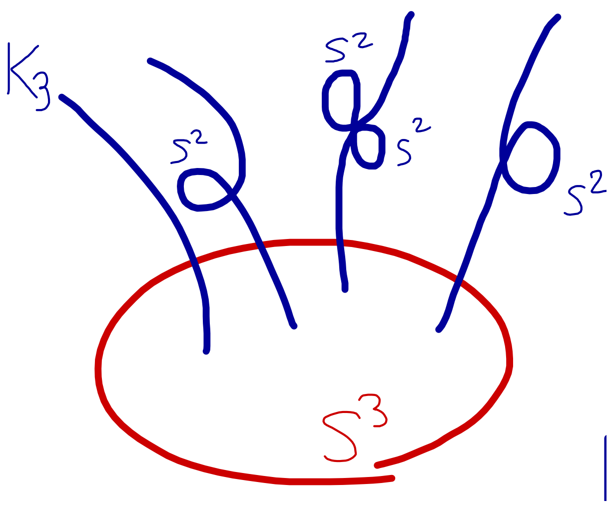
✔ To accommodate the SM degrees of freedom, singularities are needed. (Gauge symmetry enhancement at singularities.)

✔ Non-Abelian gauge symmetries are supported at co-dimension 4 orbifold singularities of ADE types. They are 3-dimensional submanifolds.

✔ Chiral matter is supported at co-dimension 7 conical singularities. At local enhancement of orbifold singularity by rank 1.

[Acharya; Atiyah; Witten, '98,'01, '02]

✔ Fields localized at singularities in the compact manifold.



$$S^3 \times \mathbb{C}^2/\Gamma \quad \Gamma \subset SU(2)$$

Γ	$\mathbb{Z}_{n+1}$	$\mathbb{D}_{k-2}$	$\mathbb{T}$	$\mathbb{O}$	$\mathbb{I}$
O	n	$4(k-2)$	24	48	120
G	A_n	D_k	E_6	E_7	E_8

✔ Yukawa-type superpotential interaction through Euclidean M2-branes (wrapping supersymmetric 3-cycles) connecting disparate conical singularities. Suppressed by the volume of instanton: Hierarchical.

KK Compactification and The Moduli Space

✔ The low energy effective theory from M-Theory compactification on a singular G2-manifold has essential elements to resemble our vacuum. But, it has more; e.g. many modulus/axion fields, SUSY, ubiquitous U(1)'s

✔ In G2-compactification there is ONE type of moduli, PAIRED with axion

$$z_i = t_i + is_i$$

The lowest components of the chiral superfields,
(parametrize the complexified moduli space of dimension b_3)

✔ Moduli: periods of the associative 3-from.

✔ Axions: periods of the 3-form field, transform under a *shift symmetry.*

$$g_{ij} = \det(s)^{-1/9} s_{ij}$$

$$s_{ij} = \frac{1}{144} \Phi_{ikl} \Phi_{jmn} \Phi_{pqr} \epsilon^{klmnpqr}$$

$$\Phi = \sum_i^{b_3} s_i \Omega_i$$

$$C^{(3)} = \sum_i^{b_3} t_i \Omega_i + \sum_I^{b_2} A_I \wedge \omega_I$$

Moduli Stabilization & SUSY Breaking

✔ The scalar potential of the low energy effective theory

$$V(s_i,t_i) = m_{\rm Pl}^4\, e^G\left(\sum G^i G_i - 3\right)$$

✔ We use experimental/observational knowledge from phenomenology and cosmology to look for our vacuum in the vast moduli space of vacua.

0. Moduli/axions must get vacuum expectation values (low scale phenomenological parameters are functions of them)

$$V'(s_i)_{\rm @min} = 0 \qquad V'(t_i)_{\rm @min} = 0$$

1. Moduli/axions must receive mass in range compatible with cosmology.

$$V''(s_i)^{1/2}_{\rm @min} \sim m_s \gtrsim 10'{\rm s\ TeV} \quad {\rm or} \quad m_s \lesssim 10^{-27}\,{\rm eV} \quad m_a \sim {\rm more\ options}$$

2. Our vacuum has a tiny cosmological constant. To explain the smallness, in the absence of any better mechanism, it must be tunable at tree level.

$$V_{@min}(s_i,t_i) \lesssim \mathcal{O}({\rm meV}^4)$$

3. In our vacuum we see hierarchical scales: $M_{\rm EW} \lll m_{\rm Pl}$

4. SUSY, if exists, must be broken at TeV scale in the visible sector, to stabilized the scale of the SM.

Moduli Stabilization & SUSY Breaking

✔ A mechanism to satisfy the above criteria...

✔ The classic idea: strong gauge dynamics in the hidden sector generates a potential for the moduli and breaks supersymmetry at low scale.

$$\Lambda_{\rm LS} \sim m_{\rm Pl} e^{-2\pi/b_0 \alpha}$$

[Witten '81]

✔ Fluxless G2 compactification implements this nice idea.

[Acharya,et al '06]

✔ Phenomenology: fluxes are not used to give the moduli vev's
✔ All moduli being paired with axions enjoy PQ shift symmetry.
✔ The only contributions to W are of non-perturbative nature.

Moduli Stabilization & SUSY Breaking

✔ The non-perturbative superpotential

$$\begin{aligned}\mathcal{W} \supset\ & C N_c e^{2\pi i f/N_c} \\ & + C(N_c - N_f)\,\mathrm{det}(q\bar{q})^{-1/(N_c-N_f)} e^{2\pi i f/(N_c-N_f)} \\ & + \lambda e^{2\pi i N\cdot z} \\ & + \lambda T_1 T_2 T_3 e^{2\pi i N\cdot z}\end{aligned}$$

[Seiberg '94]

[Acharya,et al '06,'07]
[Acharya-MT '11]

✔ Single pure gauge dynamics, breaks SUSY and stabilizes all the moduli. BUT not in SUGRA limit!

✔ At least two non-perturbative effects are needed.

✔ Solo moduli dynamics does not generate de Sitter vacuum. Matter fields (not-from gravity sector) with high scale vev's are needed to uplift.

✔ Matter: quarks in the hidden sector, singlet exotic at singularities.

✔ Sufficiently large number of instantons are needed to stabilize all axion.

Moduli Stabilization & SUSY Breaking

✔ A working model [Acharya-MT '11]

✔ The superpotential

$$\mathcal{W}_{\rm HS} = \left(A\, e^{2\pi i f_{\rm HS}/P} + \lambda T_1 T_2 T_3 e^{2\pi i \tilde{N}_i z^i} \right) + \sum_{r=2}^{b_3+3} B_r\, e^{2\pi N_i^r z^i}$$

✔ The gauge kinetic function

$$f_{\rm HS} = \sum_{i=1}^{b^3} N^i_{\rm HS} z_i = \frac{\theta}{2\pi} + i\,\alpha^{-1}_{\rm HS} \qquad N_i = \int_{\gamma_3^{SU(P)}} \Phi \quad \tilde{N}_i = \int_{\gamma_3^{mem}} \Phi$$

✔ The Kahler potential [Beasley,Witten'02]

$$K^{G_2}(s_i) = -3\ln(4\pi^{1/3} V_X)$$

$$\sum_i^{b_3} s_i \partial_i V_X = 7/3 \qquad \sum_i^{b_3} s_i \partial_i K = -7 \qquad \sum_{i,j}^{b_3} s_i s_j \partial_i \partial_j K = 7$$

✔ Matter Kahler potential $K^{U_1} = \sum_{a=1}^{3} \frac{\overline{T}_a T_a}{V_X}$ [Acharya, Bobkov '07]

Moduli Stabilization & SUSY Breaking

✔ Having worked out the scalar potential, we found metastable de Sitter solutions for the scalar potential IF for some (at least one) i, we have to the leading order the following relation

$$\frac{\lambda}{A} T_{01} T_{02} T_{03} e^{\sum_i (2\pi/P - 2\pi/(N_i/\tilde{N}_i)) N_i s_i} = \frac{N_i/\tilde{N}_i}{P} < 1 \qquad \frac{N_i}{\tilde{N}_i} = n + \mathcal{O}(\alpha_{HS})$$

$$\langle s \rangle \sim m_{\rm Pl} \qquad \langle e^{K/2} F_s \rangle \sim \alpha_{\rm HS} m_{3/2} \langle s \rangle \sim m_{1/2} m_{\rm Pl}$$

$$\langle T \rangle \sim m_{\rm Pl} \qquad \langle e^{K/2} F_T \rangle \sim m_{3/2} \langle T \rangle \sim m_{3/2} m_{\rm Pl}$$

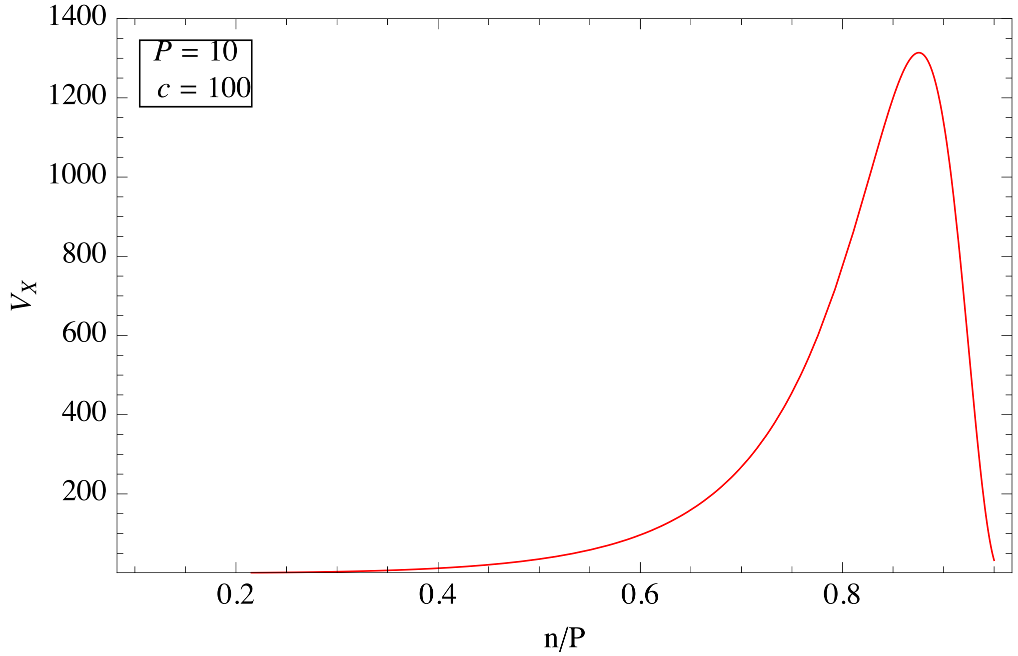
$$F_s \ll F_T$$

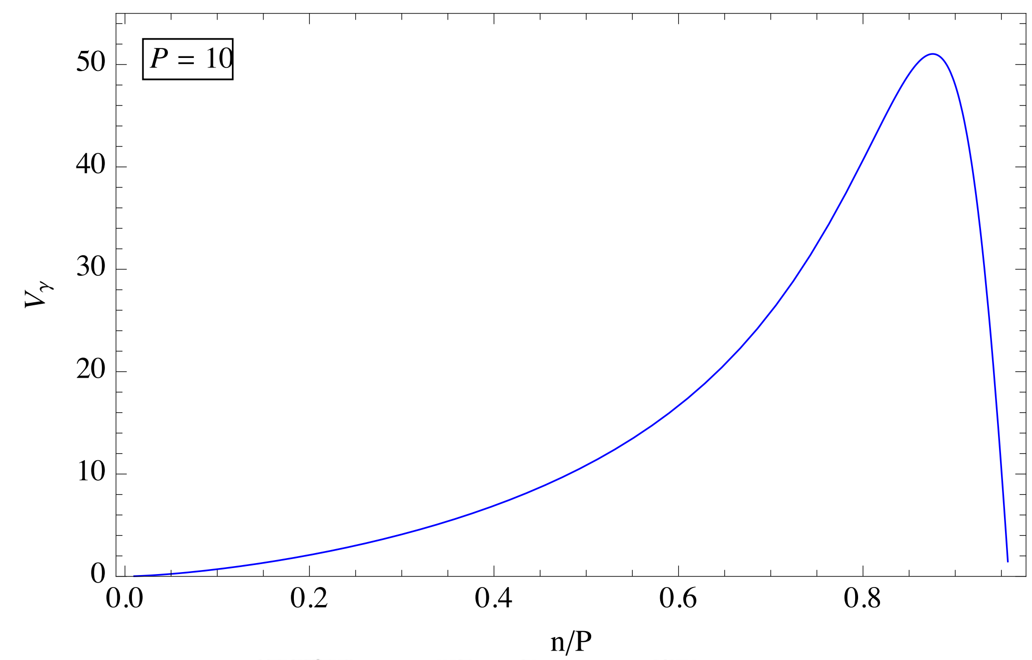
✔ Moduli get vev's in a regime where geometry makes sense, SUSY is broken via non-zero F-terms in a dS vacuum, moreover one can tune the cosmological constant to zero.

$$V_{@min} = \langle e^K (F_i^2 + F_T^2 - 3|W|^2) \rangle \approx 0$$

Moduli Stabilization & SUSY Breaking

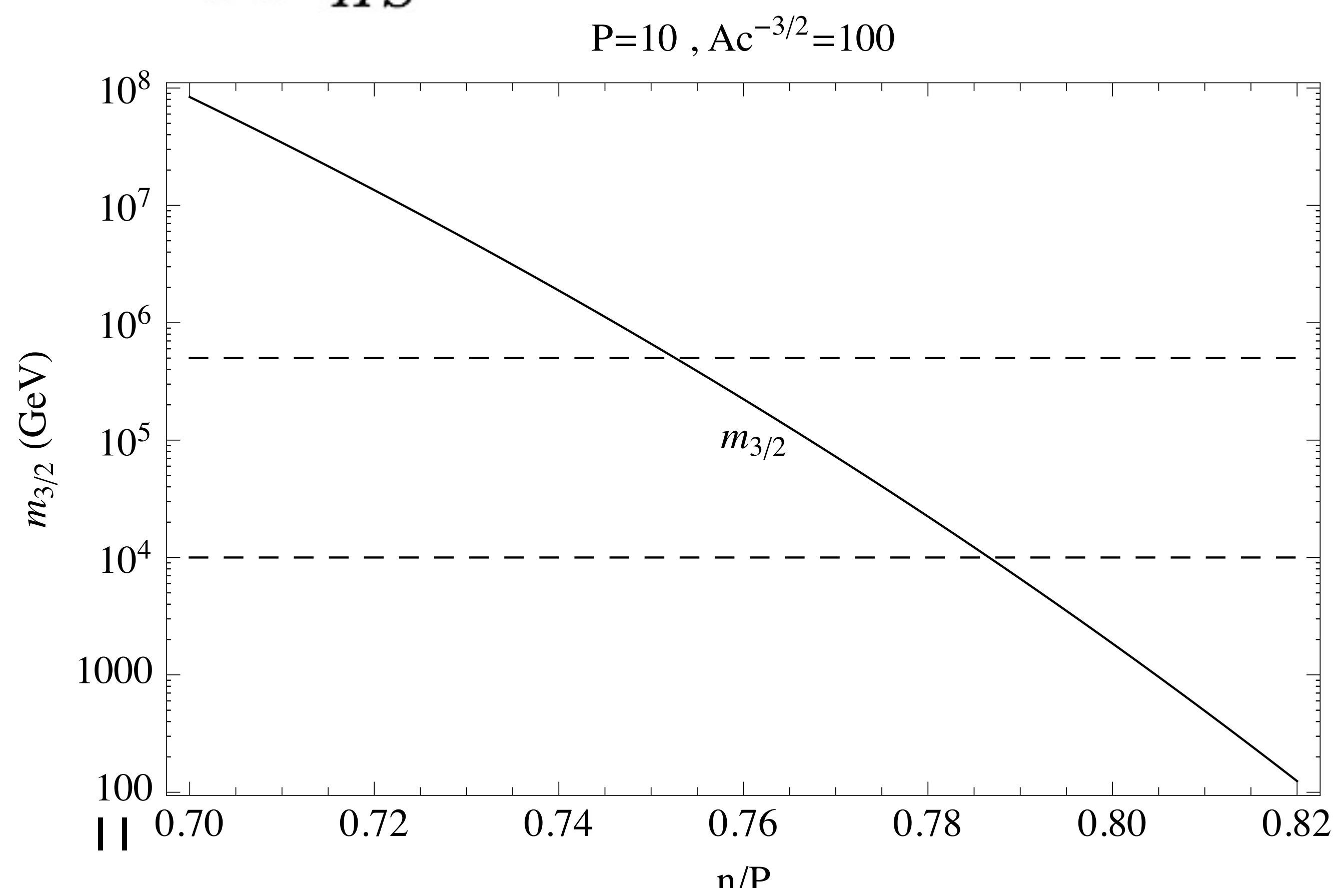
$$V_X(s_i) \sim \prod_{i=1}^{b_X^3} s_i^{a_i} \sim {\alpha_{HS}}^{-7/3} \quad , \quad V_{\gamma_3}(s_i) \sim \sum_{i=1}^{b_X^3} N_i s_i = {\alpha_{HS}}^{-1}$$





$$m_{3/2} = m_{Pl} \langle e^G \rangle \sim m_{Pl} V_X^{-3/2} e^{-2\pi V_Q/P} \sim m_{Pl} \alpha_{HS}^{-7/3} e^{-2\pi/\alpha_{HS} P}$$

✔ The gravitino mass:



✔ The same for moduli:

$$m_{ij} = \frac{\partial^2 V_{@min}(s_i)}{\partial s_i \partial s_j}$$

Phenomenology of the Vacua

✔ Add the visible (the GUT-MSSM) sector.

$$\lim_{m_{\rm Pl}\to\infty} \mathcal{L}_{\rm local}(\langle s\rangle, \langle F_s\rangle) = \mathcal{L}_{\rm global} + \mathcal{L}_{\rm soft}$$

[Nilles'84, Brignole et.al. '97, ...

$$\mathcal{L}_{\rm local} = \mathcal{L}_{\rm HS} + \mathcal{L}_{\rm GUT}$$

$$K = K_{\rm HS} + K_{\rm GUT} \qquad \mathcal{W} = \mathcal{W}_{\rm HS} + \mathcal{W}_{\rm GUT} \qquad f_{\rm GUT} = \sum_{i=1}^{b^3} N^i_{\rm GUT} z_i = \frac{\theta}{2\pi} + i\,\alpha^{-1}_{\rm GUT}$$

✔ The MSSM sits on another 3-manifold with appropriate singularities.

✔ A generic bundle has $\mathbb{Z}_p$ and $\mathbb{T}$ type orbifold singularities.

✔ Generically 3-dim manifolds inside 7 dimensions do not intersect. SUSY breaking effects are gravity mediated.

✔ In these vacua a combination of non-perturbative effects and non-renormalizable effects generate hierarchical scales.

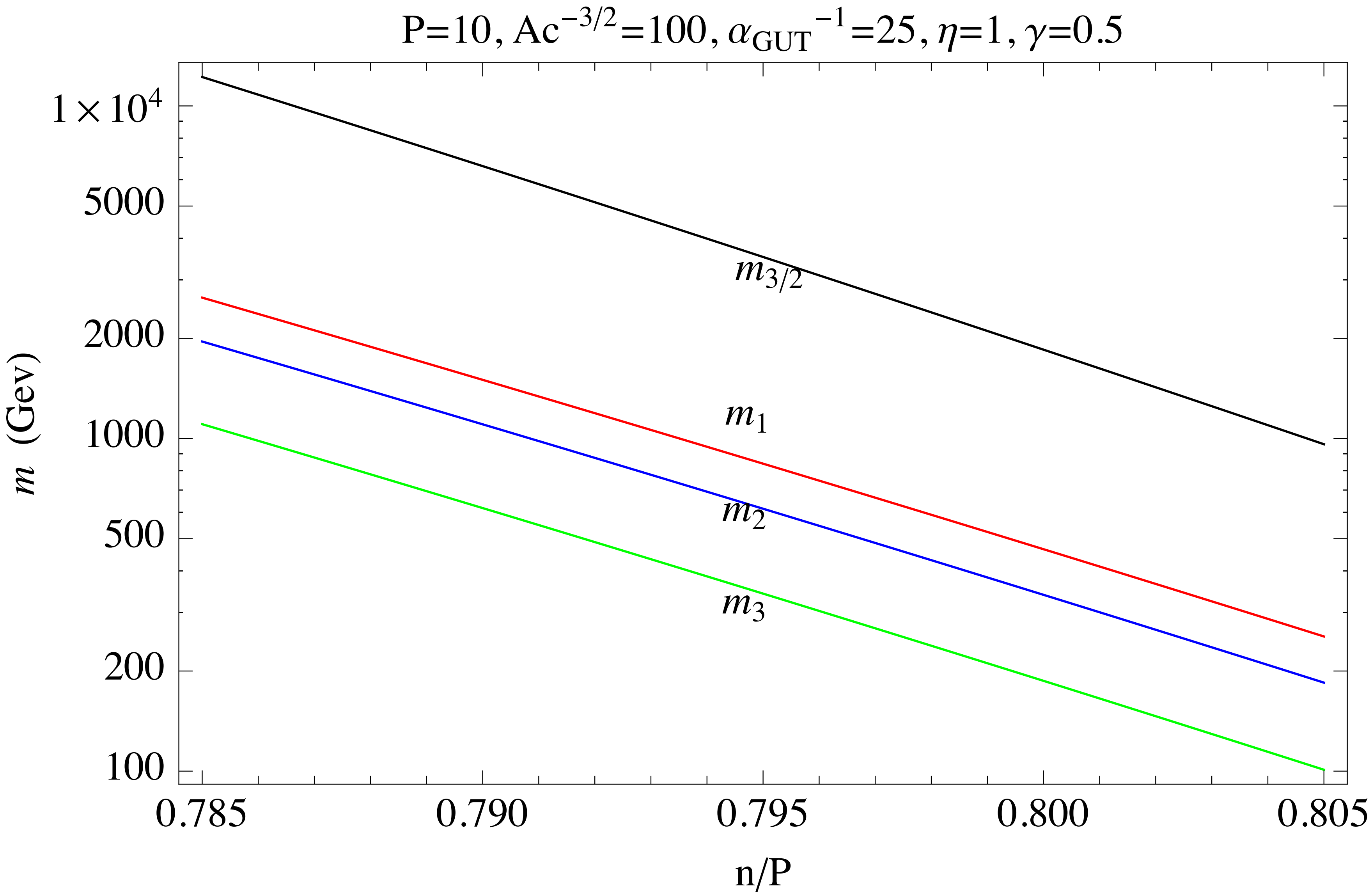
Phenomenology of the Vacua

✔ The MSSM spectrum at the GUT scale: gaugino mass

$$m_{1/2}^{\text{tree}} \sim m_{\text{Pl}} \langle e^{K/2} F^i \partial_i f^{\text{SM}} / 2\text{Im} f^{\text{SM}} \rangle \sim \alpha_{\text{HS}} m_{3/2}$$

$$m_{1/2}^{\text{AMSB}} \sim (\alpha_{\text{GUT}}/4\pi) m_{3/2}$$

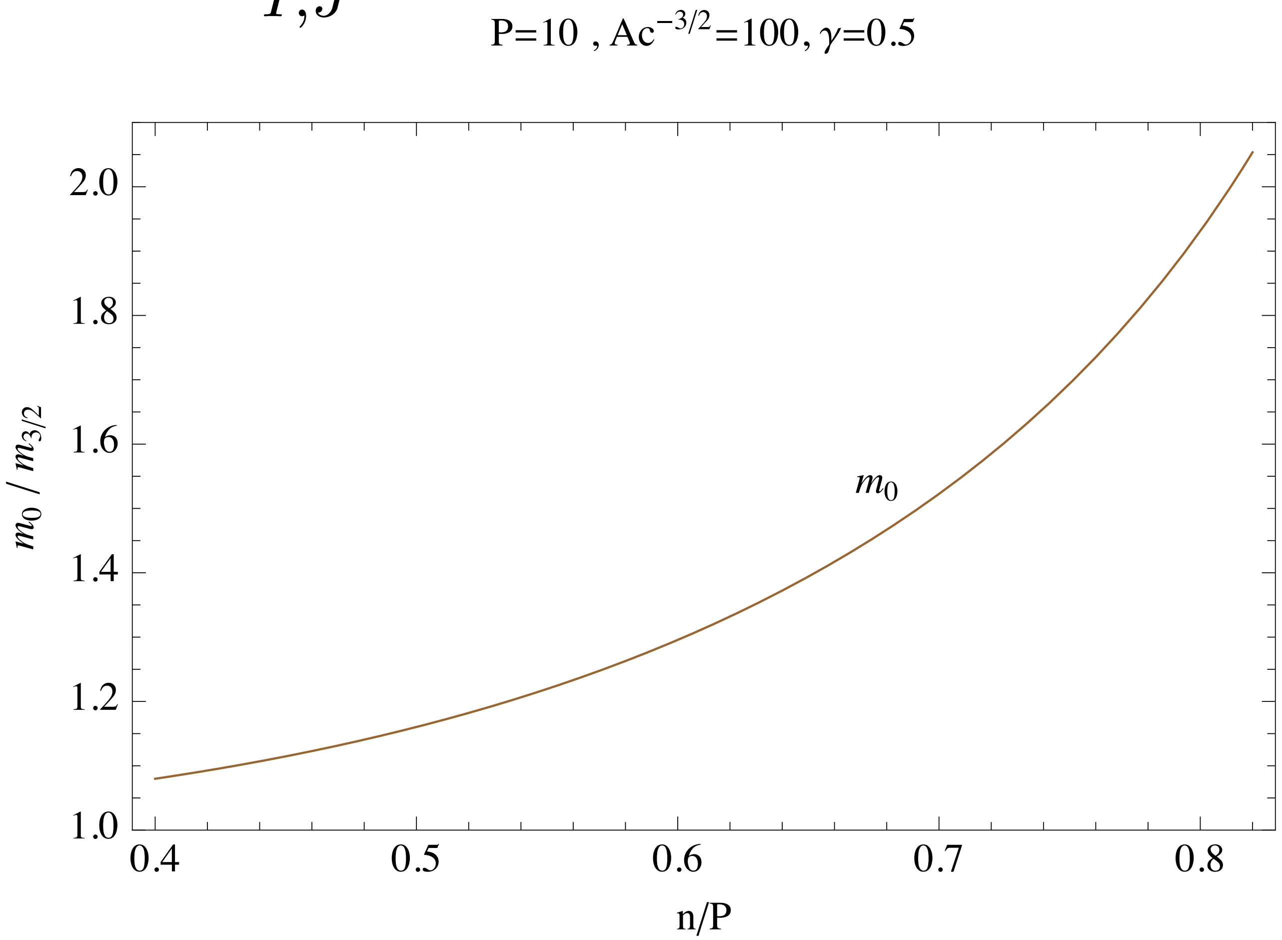
[Gaillard et al'99, Bagger et al'99]



Phenomenology of the Vacua

✔ The MSSM spectrum at the GUT scale: scalar mass

$$m^2_{\alpha\bar{\beta}} = \left(m^2_{3/2} - \sum_{I,\bar{J}} e^{K^{X+U_1}} F^I F^{\bar{J}} \partial_I \partial_{\bar{J}} \ln \tilde{K}^{\rm VS}\right)\delta_{\alpha\bar{\beta}}$$

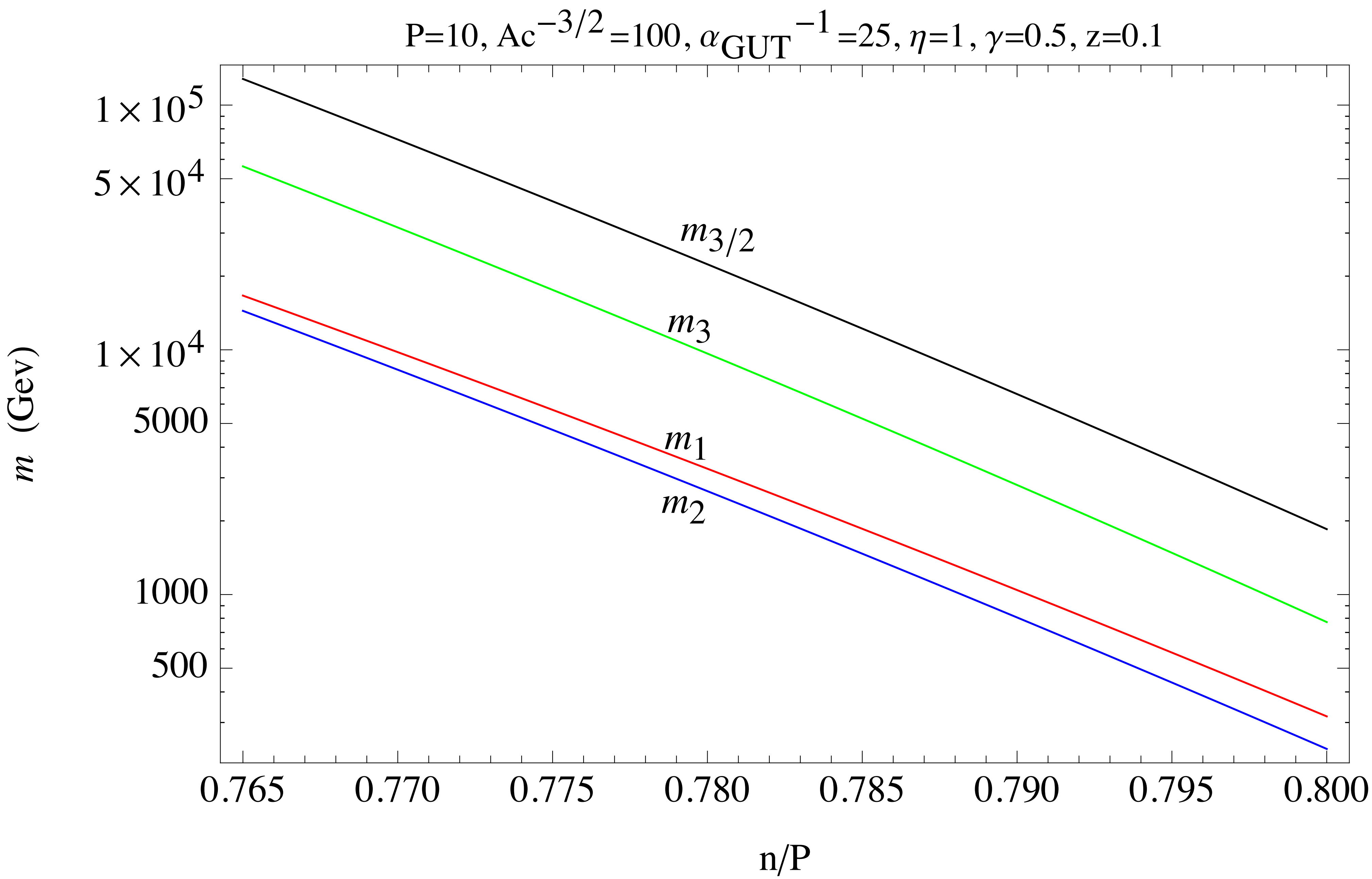


✔ The MSSM spectrum at the GUT scale: the rest of soft parameters

$$A \sim B_\mu \sim \mu \sim m_{3/2}$$

Phenomenology of the Vacua

✔ The MSSM spectrum at the EW scale: gaugino mass



[See Kane et al ’11,’12 for more collider phenomenology]

Phenomenology of the Vacua

✔ The Axion mass

There are $b_3 + 3$ axions. One linear combination of axions is stabilized along with moduli and gets mass. The rest are stabilized by exponentially smaller instanton corrections.

[Acharya et al '10]

$$\mathcal{W} \supset \mathcal{W}_{\rm SUSY} + \sum_{r=2}^{b_3+2} B_r \, e^{2\pi N_i^r z^i}$$

$$V(s_i, t_i) = V(s_i, t_1) + V(t_i)$$

$$V(t_i) \sim m_{\rm Pl}^2 m_{3/2}^2 e^{-2\pi(\hat{V}_i - V/P)}$$

✔ Axions mass:

$$m_t \sim m_{3/2}(m_{\rm Pl}/f_a) e^{-\pi(\hat{V} - V/P)} \sim 10^{-32} m_{3/2} (\hat{V} = 25)$$

✔ Axions decay constant:

[Svrcek,Witten '06]

$$f_a \sim m_{\rm Pl}\sqrt{2K_{ij}} \sim m_{\rm GUT} \qquad K_{ij} \sim (\alpha_{\rm HS}/4\pi)^2$$

Cosmology of the Vacua

✔ Include gravity and consider a Friedman Universe.

✔ An epoch of non-thermal moduli dominated Universe. Moduli oscillate and eventually decay.

$$H_{\rm osc} \sim m_\varphi \sim m_{3/2}$$

$$H_{\rm decay} \sim \Gamma_\varphi \sim m_\varphi \left(\frac{m_\varphi}{m_{\rm Pl}}\right)^2$$

✔ Decay products scatter off each other and thermalize the Universe.

$$g_{*\rm rh}^{1/4} T_{\rm rh} \sim m_{\rm Pl} \left(\frac{m_\varphi}{m_{\rm Pl}}\right)^{3/2}$$

✔ Successful prediction of the (last) observable nucleosynthesis demands

$$T_{\rm rh} \gtrsim \mathcal{O}(1)\mathrm{MeV}$$

$$m_\varphi \gtrsim \mathcal{O}(1) \times 10 \ \mathrm{TeV}$$

Cosmology of the Vacua

✔ The Wino LSP abundance:

$$n_\chi \sim n_{\varphi,d}\mathrm{Br}_{\varphi\to\chi}\frac{m_\varphi}{m_\chi} \sim m_{\mathrm{Pl}}^3\mathrm{Br}_{\varphi\to\chi}\frac{m_\varphi}{m_\chi}\left(\frac{m_\varphi}{m_{\mathrm{Pl}}}\right)^5$$

$$\Gamma_{\mathrm{ann}} \sim n_\chi^c\langle\sigma_{\mathrm{ann}}v\rangle \lesssim H \qquad\qquad n_\chi^c \sim \frac{\Gamma_\varphi}{\langle\sigma_{\mathrm{ann}}v\rangle} \sim m_{\mathrm{Pl}}^3\frac{(m_\chi/m_\varphi)^2}{\langle\sigma_{\mathrm{ann}}v\rangle m_\chi^2}\left(\frac{m_\varphi}{m_{\mathrm{Pl}}}\right)^5$$

$$Y_f^\chi = \frac{n_c^\chi}{s_{\mathrm{rh}}} \sim \frac{(m_\chi/m_\varphi)^2}{\langle\sigma_{\mathrm{ann}}v\rangle m_\chi^2}\left(\frac{m_\varphi}{m_{\mathrm{Pl}}}\right)^{1/2}$$

$$\Omega_\chi h^2 = \frac{m_\chi Y_0^\chi}{\rho_c/s_0h^2} \sim 10^{-3}\left(\frac{m_\varphi}{m_{\mathrm{Pl}}}\right)^{-1/2}\frac{(m_\chi/m_\varphi)^2}{\langle\sigma_{\mathrm{ann}}v\rangle m_\chi^2} \lesssim 0.1$$

[Acharya et al '09]

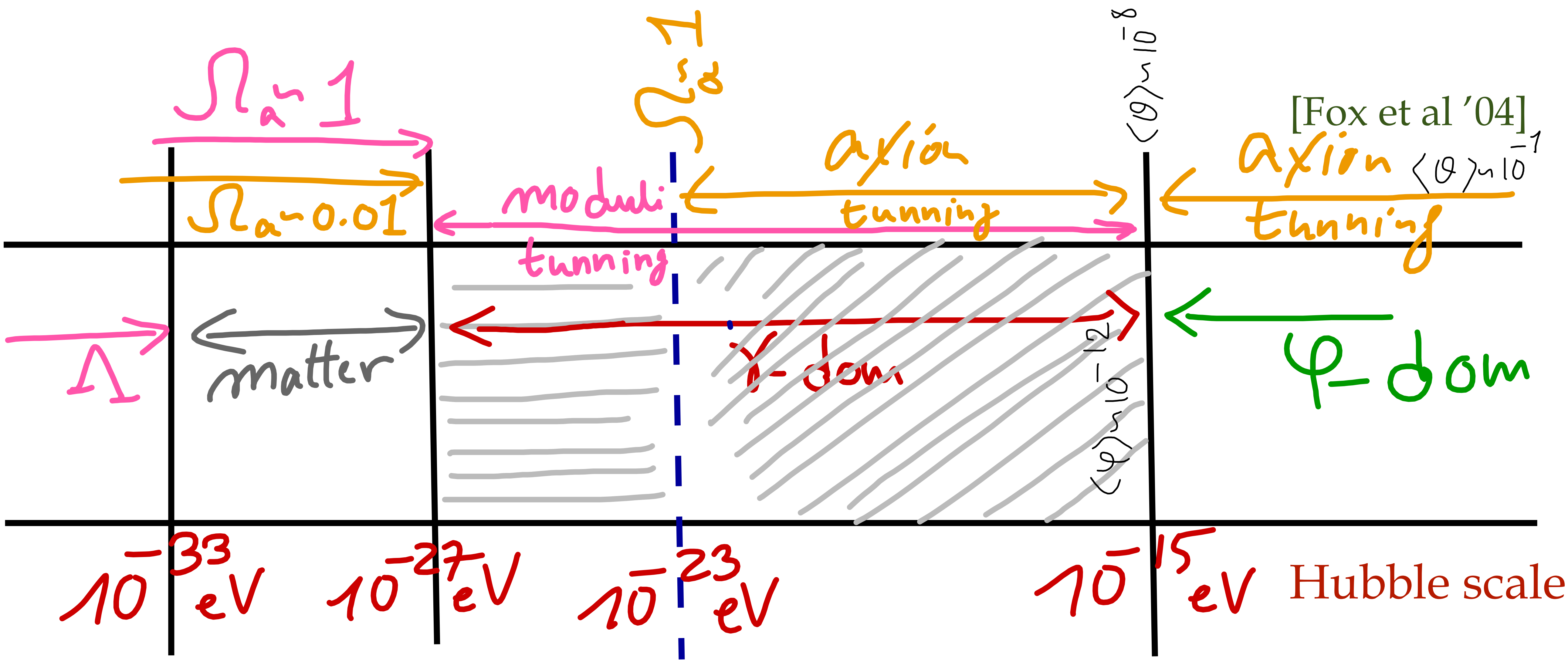
$$m_\varphi \sim \mathcal{O}(100\mathrm{TeV}) \qquad m_\chi \sim \mathcal{O}(100\mathrm{GeV}) \qquad \langle\sigma_{\mathrm{ann}}v\rangle \sim 3\times10^{-24}\mathrm{cm}^3\mathrm{s}^{-1}$$

Cosmology of the Vacua

✔ The axion abundance (in axiverse):

✔ $H \sim m_a$ In matter epoch, scalar condensate of a single axion coherent oscillations contributes about 0.01% to total dark matter.($f_a/m_{\rm Pl} \sim 10^{-2}$)

✔ Single axion can contribute more (up to 100%), if it starts oscillations in the radiation dominated universe.



Cosmology of the Vacua

✔ Holography and Cosmology

✔ Covariant holographic bound:

The entropy of matter on a light-sheet orthogonal to a spatial surface, cannot exceed the surface area.
It holds for any co-dimension 2 space-like surface in any spacetime (satisfying Einstein eq's).

[Fischler-Susskind '98]
[Bousso '99]
[Kaloper-Linde '99]
[Banks-Fischler'03]
[Linde-Vanchurin'09]
[MT, in preparation]

✔ The entropy in the reheating volume $(T_{\rm RH}L_{\rm RH})^3 \leq (m_{\rm Pl}L_{\rm RH})^2$

✔ Adiabatic expansion $(T_{\rm RH}L_{\rm RH})^3 = (T_0 L_0)^3$

$$L_{\rm RH} = H_0^{-1}\frac{T_0}{T_{\rm RH}}{\rm N}_0^{1/3} \qquad {\rm N}_0 \gtrsim 10^3$$

[Silk et.al '11]

✔ Upper bound on the reheat temperature

$$T_{\rm RH} \leq m_{\rm Pl}\left(\frac{H_0}{T_0}\right)^{1/2}{\rm N}_0^{-1/6} \sim 5\ {\rm TeV}\times {\rm N}_0^{-1/6}$$

$$m_\varphi \lesssim 4\times 10^8 {\rm GeV}\times {\rm N}_0^{-1/6}$$

✔ Saturating the bound $T_{\rm RH} \sim 5\ {\rm TeV}\times {\rm N}_0^{-1/6}$

Cosmology of the Vacua

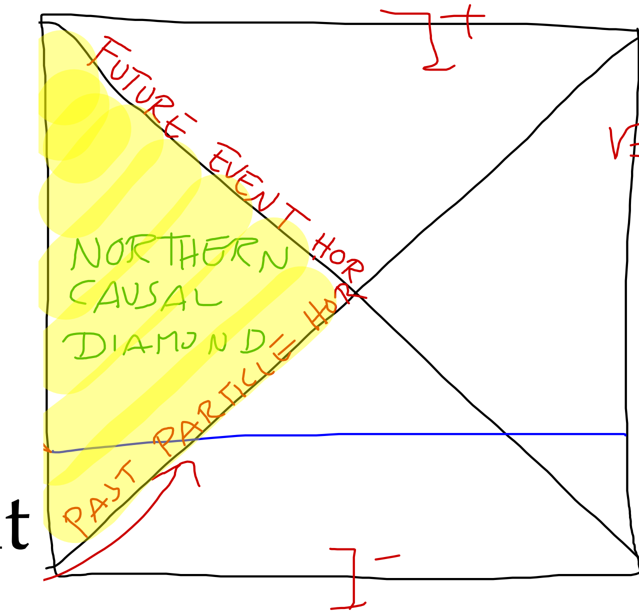
✔ Observations: asymptotically de Sitter Universe in the far future.

✔ An observer is surrounded by a cosmological event horizon.

✔ The de Sitter horizon has finite non-zero entropy.

[Gibbons-Hawking'77]

$$S_{\rm Universe} = S_{\rm dS} = S_{\rm bulk} + S_{\rm horizon} = \left(\frac{m_{\rm Pl}}{H_0}\right)^2$$



✔ A bound on the entropy of any matter system can be put in a causal domain of an asymptotically de Sitter universe.

$$(T_{\rm RH} L_{\rm RH})^3 \leq \left(\frac{m_{\rm Pl}}{H_0}\right)^2$$

✔ Adiabatic expansion $\quad (T_{\rm RH} L_{\rm RH})^3 = (T_0 L_0)^3$

$$\mathrm{N}_0 \leq \left(\frac{m_{\rm Pl}}{H_0}\right)^2 \left(\frac{H_0}{T_0}\right)^3$$ bound saturates for $\mathrm{N}_0 \sim 10^{33}$

$$T_{\rm RH} \sim 5 \text{ TeV} \times \mathrm{N}_0^{-1/6} \sim 15 \text{ MeV} \qquad m_\varphi \sim 80 \text{ TeV}$$

Conclusions

Non-perturbative effects plus matter have been studied:

✔ All the moduli and axions are stabilized in SUGRA approximation.

✔ SUSY is spontaneously broken in a de Sitter vacuum.

✔ U(1) symmetries are Higgsed.

✔ Hierarchical scales produced.

✔ Interesting phenomenology and cosmology.

? We need to learn more about the G2-manifolds to do more precise analysis: challenge for G_2eometer friends!

? Inflation in the G2 vacua. Slow-roll, N-flation do not work! Higher curvature correction: inflation happens in another vacuum.

? Same stabilizing mechanism in the heterotic string theory. Implemented successfully in IIB string theory on CY orientifold.

[Stuart Raby and company '10]

? Implications of holographic principle in compactifications.

Thanks for Attention