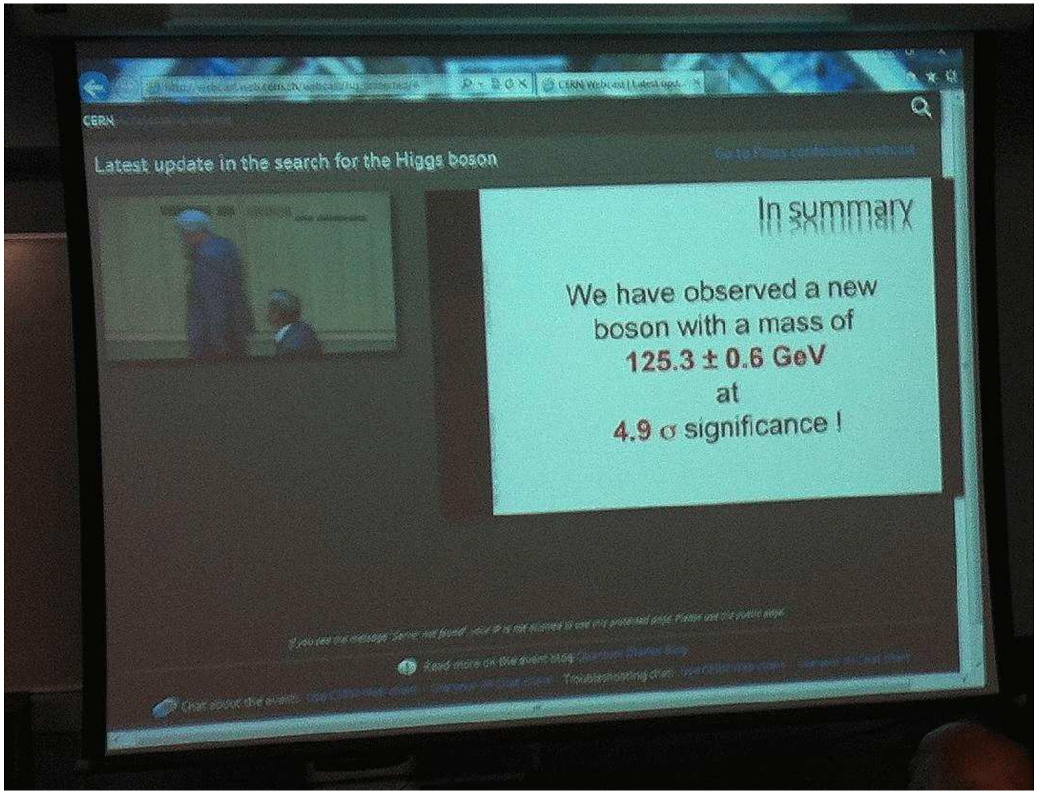
String Phenomenology: past, present and future

Progress Report: The early years 1989–2012

With: Dimitri Nanopoulos, Kajia Yuan, Edi Halyo, Ben Grinstein, Keith Dienes, Claudio Coriano, Sangyeon Chang, Marco Matone, Jogesh Pati, John Ellis, Gerald Cleaver, Per Berglund, Zongan Qiu, Maxim Pospelov, Joel Walker, Richard Garavuso, Jose Isidro, Alessandro Cafarella, Sander Nooij, John Rizos, Costas Kounnas, Cristina Timirgaziu, Elisa Manno, Marco Guzzi, Ben Assel, Kyriakos Christodoulides, Mirian Tsulaia

String Phenomenology Institute, CERN, 10 July 2011

... Before



4 July 2012

After ...

DATA $\longrightarrow$ STANDARD MODEL

$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$

$$\left[\begin{pmatrix} \nu \\ e \end{pmatrix} + D^c_L \right] + \left[U^c_L + \begin{pmatrix} u \\ d \end{pmatrix} + E^c_L \right] + N^c_L$$

$$\bar{5} \quad + \quad 10 \quad \quad 1 \quad \quad \overline{} \; 16$$

STANDARD MODEL -> UNIFICATION

ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

PRIMARY GUIDES:

3 generations

SO(10) embedding

Realistic free fermionic models

'Phenomenology of the Standard Model and string unification'

- Top quark mass $\sim$ 175–180GeV — PLB 274 (1992) 47
- Generation mass hierarchy — NPB 407 (1993) 57
- CKM mixing — NPB 416 (1994) 63 (with Halyo)
- Stringy seesaw mechanism — PLB 307 (1993) 311 (with Halyo)
- Gauge coupling unification — NPB 457 (1995) 409 (with Dienes)
- Proton stability — NPB 428 (1994) 111
- Squark degeneracy — NPB 526 (1998) 21 (with Pati)
- Minimal Superstring Standard Model — PLB 455 (1999) 135 (with Cleaver & Nanopoulos)
- Moduli fixing — NPB 728 (2005) 83
- Exophobia — PLB 683 (2010) 306 (with Assel, Christodoulides, Kounnas & Rizos)

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

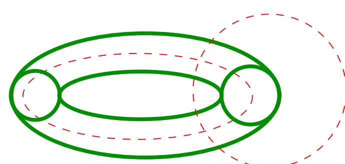
Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

Free Fermionic Construction

Left-Movers: $\psi^{\mu}_{1,2}, \quad \chi_i, \quad y_i, \; \omega_i \quad (i=1,\cdots,6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \; , \; \bar{\omega}_i & i=1,\cdots,6 \\ \\ \bar{\eta}_i & i=1,2,3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$



$V \longrightarrow V \qquad\qquad f \longrightarrow -e^{i\pi\alpha(f)} f$

$$Z = \sum_{\substack{all\ spin \\ structures}} c\binom{\vec{\alpha}}{\vec{\beta}} \; Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

The NAHE set:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$ $N = 4$ Vacua

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\},$$ $N = 4 \rightarrow N = 2$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, \omega^{56} | \bar{y}^{12}, \bar{\omega}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\},$$ $N = 2 \rightarrow N = 1$

$$b_3 = \{\chi^{12}, \chi^{34}, \omega^{12}, \omega^{34} | \bar{\omega}^{12}, \bar{\omega}^{34}, \bar{\eta}^3, \bar{\psi}^{1,\dots,5}\},$$ $N = 2 \rightarrow N = 1$

$Z_2 \times Z_2$ orbifold compactification

$\Longrightarrow$ Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set

Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y = \frac{1}{2}(B-L) + T_{3_R} \in SO(10) \,!$$

$$SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$$

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdots V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\text{Tr}Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j \; = 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

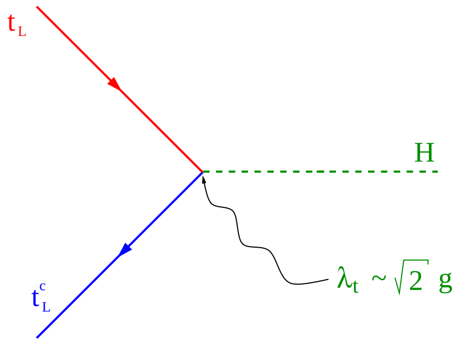
$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdots \cdot$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\longrightarrow$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdots V_N^b \;\rightarrow\; V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

Top Quark Mass Prediction



Only $\lambda_t = \langle Q t_L^c H \rangle = \sqrt{2} g$ at $N = 3$

mass of lighter quarks and leptons $\rightarrow$ nonrenormalizable terms

$$\longrightarrow \qquad \lambda_b = \lambda_\tau = 0.35 g^3 \sim \frac{1}{8}\lambda_t$$

Evolve λ_t , λ_b to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}} \sin\beta \qquad m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}} \cos\beta$$

where $v_0 = \dfrac{2 m_W}{g_2(M_Z)} = 246\mathrm{GeV}$ and $v_1^2 + v_2^2 = \dfrac{v_0^2}{2}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan\beta}{(1+\tan^2\beta)^{\frac{1}{2}}} \Longrightarrow$$

ıysics Letters B 274 (1992) 47–52
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PHYSICS LETTERS B

Hierarchical top–bottom mass relation in a superstring derived standard-like model

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Received 30 September 1991

I propose a mechanism in a class of superstring standard-like models which explains the mass hierarchy between the top and bottom quarks. At the trilinear level of the superpotential only the top quark gets a nonvanishing mass term while the bottom quarks and tau lepton mass terms are obtained from nonrenormalizable terms. I construct a model which realized this mechanism. In this model the bottom quark and tau lepton Yukawa couplings are obtained from quartic order terms. I show that $\lambda_b = \lambda_\tau \sim \frac{1}{8}\lambda_t$ at the unification scale. A naive estimate yields $m_t \sim 175$–180 GeV.

One of the unresolved puzzles of the standard model is the mass splitting between the top quark and the lighter ıuarks and leptons. Especially difficult to understand within the context of the standard model is the big splitıng in the heaviest generation. Experimental limits [1] indicate the top mass to be above 80 GeV, while the ›ottom and tau lepton masses are found at 5 GeV and 1.78 GeV respectively. Possible extensions to the standard nodel are grand unified theories. Although the main prediction of GUTs, proton decay, has not yet been oberved, calculations of $\sin^2\theta_W$ and of the mass ratio m_b/m_τ support their validity. Recent calculations seem to upport supersymmetric GUTs versus nonsupersymmetric ones [2]. In spite of the success of SUSY GUTs in onfronting LEP data [2], an understanding of the mass splitting between the top quark and the lighter quarks ınd leptons is still lacking. The next level in which such an understanding may be developed is in the context of uperstring theory [3].

Cabibbo mixing PLB 307 (1993) 305

Find anomaly free solution

$$M_d \sim \begin{pmatrix} \epsilon & \frac{V_2\bar{V}_3\Phi_{\alpha\beta}}{M^3} & 0 \\ \frac{V_2\bar{V}_3\Phi_{\alpha\beta}\xi_1}{M^4} & \frac{\bar{\Phi}_2^-\xi_1}{M^2} & 0 \\ 0 & 0 & \frac{\Phi_1^+\xi_2}{M^2} \end{pmatrix} v_2,$$

$$\epsilon < 10^{-8} \qquad \frac{V_2\bar{V}_3\Phi_{\alpha\beta}}{M^3} = \frac{\sqrt{5}\, g^6}{64\,\pi^3} \approx 2 - 3 \times 10^{-4}.$$

$$\Rightarrow \quad |V| \sim \begin{pmatrix} 0.98 & 0.2 & 0 \\ 0.2 & 0.98 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Three generation mixing $\longrightarrow$ NPB 416 (1994) 63 $\quad |J| \sim 10^{-6}$

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in FMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi–realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

vanishing one–loop partition function, perturbatively broken SUSY

Fixed geometrical, twisted and SUSY moduli

Cleaver $etal$, $SO(10)$ and FSU5 analysis $->$ stringent flat directions

Classification of fermionic $Z_2 \times Z_2$ orbifolds

Basis vectors:

$$1 = \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$$

$$S = \{\psi^\mu, \chi^{1,\dots,6}\},$$

$$z_1 = \{\bar{\phi}^{1,\dots,4}\},$$

$$z_2 = \{\bar{\phi}^{5,\dots,8}\},$$

$$e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1,\dots,6, \qquad N = 4 \text{ Vacua}$$

$$b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \qquad N = 4 \rightarrow N = 2$$

$$b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, \qquad N = 2 \rightarrow N = 1$$

$$\alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$$

& Gauge group $SO(6) \times SO(4) \times U(1)^3 \times \text{hidden}$

Independent phases $c\binom{v_i}{v_j} = \exp[i\pi(v_i|v_j)]$: upper block

	1	S	e_1	e_2	e_3	e_4	e_5	e_6	z_1	z_2	b_1	b_2	α
1	-1	-1	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
S			-1	-1	-1	-1	-1	-1	-1	-1	1	1	-1
e_1				$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_2					$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_3						$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_4							$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_5								$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
e_6									$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
z_1										$\pm$	$\pm$	$\pm$	$\pm$
z_2											$\pm$	$\pm$	$\pm$
b_1												$\pm$	$\pm$
b_2												-1	$\pm$
α													

A priori 66 independent coefficients $\rightarrow$ 2^{66} distinct vacua

The twisted matter spectrum:

$$B^1_{\ell^1_3\ell^1_4\ell^1_5\ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$

$$B^2_{\ell^2_1\ell^2_2\ell^2_5\ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$

$$B^3_{\ell^3_1\ell^3_2\ell^3_3\ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4 \qquad l^j_i = 0, 1$$

sectors B^i_{pqrs} $\quad\longrightarrow\quad$ 16 or $\overline{16}$ of $SO(10)$ with multiplicity $(1, 0, -1)$

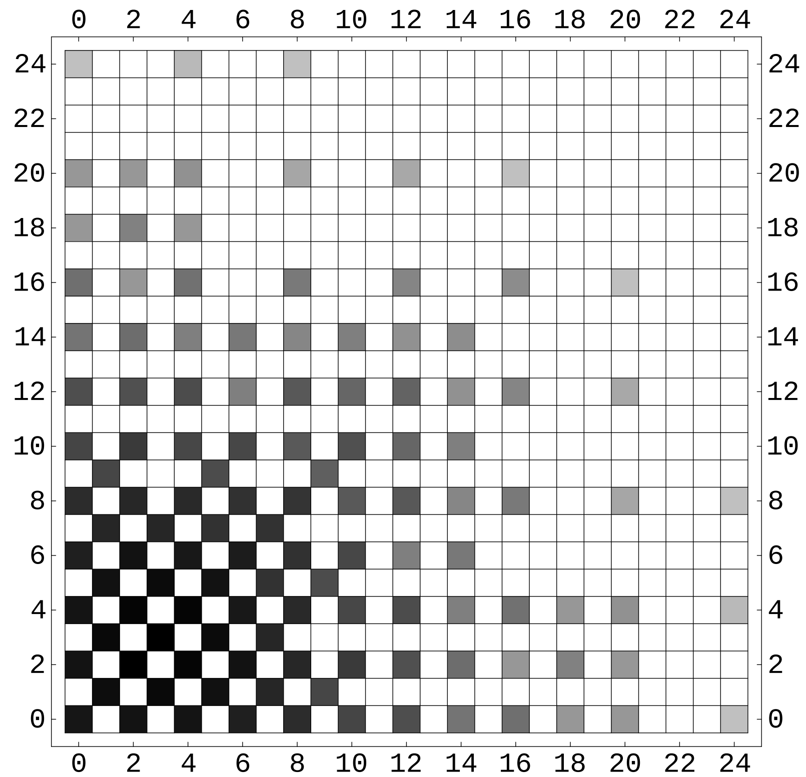
$B^i_{pqrs} + x \quad\longrightarrow\quad 10$ of $SO(10)$ with multiplicity $(1, 0)$

$x = \{\bar\psi^{1,\cdots,5}, \bar\eta^1, \bar\eta^2, \bar\eta^3\}$ $\qquad$ $x -$ map $\leftrightarrow$ spinor–vector map

Algebraic formulas for $\quad S = \sum_{i=1}^3 S_+^{(i)} - S_-^{(i)}$ $\quad$ and $\quad V = \sum_{i=1}^3 V^{(i)}$

Spinor–vector duality:

Invariance under exchange of $\#(16+\overline{16}) \;<-> \;\; \#(10)$



Symmetric under exchange of rows and columns

$$E_6 \;\; : \qquad 27 = 16 + 10 + 1 \qquad \overline{27} = \overline{16} + 10 + 1$$

Self-dual: $\#(16+\overline{16}) \;\; = \;\; \#(10)$ without E_6 symmetry

Spinor–Vector duality in Orbifolds:

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \frac{1}{2}\left(\frac{\theta_3^n}{\eta^n} + \frac{\theta_4^n}{\eta^n}\right), \qquad V_{2n} = \frac{1}{2}\left(\frac{\theta_3^n}{\eta^n} - \frac{\theta_4^n}{\eta^n}\right),$$

$$S_{2n} = \frac{1}{2}\left(\frac{\theta_2^n}{\eta^n} + i^{-n}\frac{\theta_1^n}{\eta^n}\right), \qquad C_{2n} = \frac{1}{2}\left(\frac{\theta_2^n}{\eta^n} - i^{-n}\frac{\theta_1^n}{\eta^n}\right).$$

where

$$\theta_3 \equiv Z_f\binom{0}{0} \quad \theta_4 \equiv Z_f\binom{0}{1}$$

$$\theta_2 \equiv Z_f\binom{1}{0} \quad \theta_1 \equiv Z_f\binom{1}{1}$$

Starting from:

$$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} (\bar{O}_{16} + \bar{S}_{16}) \, (\bar{O}_{16} + \bar{S}_{16}) \ ,$$

where as usual, for each circle,

$$p^i_{\rm L,R} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \, ,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p^2_{\rm L}} \, \bar{q}^{\frac{\alpha'}{4} p^2_{\rm R}}}{|\eta|^2} \, .$$

apply $Z_2 \times Z_2' \; : \; g \times g'$

$$g \; : \; (-1)^{(F_1+F_2)}\delta$$

$$F_{1,2} \; : \; (\, \overline{O}_{16}^{1,2},\, \overline{V}_{16}^{1,2},\, \overline{S}_{16}^{1,2},\, \overline{C}_{16}^{1,2} \,) \longrightarrow (\, \overline{O}_{16}^{1,2},\, \overline{V}_{16}^{1,2},\, -\overline{S}_{16}^{1,2},\, -\overline{C}_{16}^{1,2} \,)$$

with $\delta X_9 = X_9 + \pi R_9$,

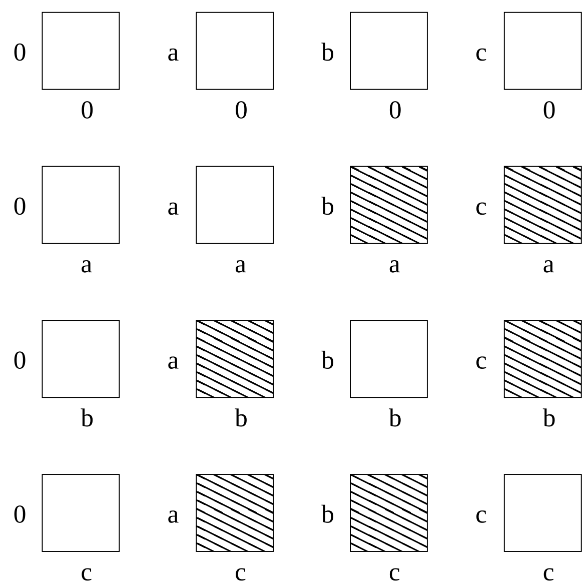
$$\delta \; : \; \Lambda^9_{m,n} \longrightarrow (-1)^m \Lambda^9_{m,n}$$

$$g' \; : \; (x_4, x_5, x_6, x_7, x_8, x_9) \longrightarrow (-x_4, -x_5, -x_6, -x_7, +x_8, +x_9)$$

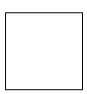
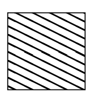
Note: A single space twisting Z_2' $\Rightarrow$ $N = 4 \rightarrow N = 2$

$E_7 \rightarrow SO(12) \times SU(2)$

$$\Rightarrow \quad \textsf{Analyze} \qquad Z = \left(\frac{Z_+}{Z_g \times Z_{g'}} \right) = \left[\frac{(1+g)}{2} \frac{(1+g')}{2} \right] Z_+$$



a = g ; b = g' ; c = gg'

P.F. = ( + ε ) = $\Lambda_{m,n}$ • ( ) + $\Lambda_{m,n+1/2}$ • ( )

$\varepsilon = \pm 1$

massless massive

• sector b

$$\Lambda_{p,q}\left\{\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4+\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{V}_{12}\overline{C}_4\overline{O}_{16}+P_\epsilon^- Q_s\overline{S}_{12}\overline{O}_4\overline{O}_{16}\right]+\right.$$

$$\left.\frac{1}{2}\left(\left|\frac{2\eta}{\theta_4}\right|^4-\left|\frac{2\eta}{\theta_3}\right|^4\right)\left[P_\epsilon^+ Q_s\overline{O}_{12}\overline{S}_4\overline{O}_{16}\right]\right\} \quad + \quad \text{massive}$$

where

$$P_\epsilon^+ = \left(\frac{1+\epsilon(-1)^m}{2}\right)\Lambda_{m,n} \qquad P_\epsilon^- = \left(\frac{1-\epsilon(-1)^m}{2}\right)\Lambda_m$$

$$\epsilon = +1 \quad \Rightarrow \quad P_\epsilon^+ = \Lambda_{2m,n} \qquad P_\epsilon^- = \Lambda_{2m+1,n}$$

$$\epsilon = -1 \quad \Rightarrow \quad P_\epsilon^+ = \Lambda_{2m+1,n} \qquad P_\epsilon^- = \Lambda_{2m,n}$$

and $\quad 12\cdot 2 \; + \; 4\cdot 2 \quad = \quad 32$

Further :

- The spinor–vector duality in this model is realised in terms of a continuous interpolation between two discrete Wilson lines.

- The spinor-vector duality is realised in terms of a spectral flow operator that operates in the bosonic side of the heterotic string. In the case of enhanced E_6 symmetry, the spectral flow operator acts as an internal E_6 generator. When E_6 is broken the spectral flow operator induces the spinor–vector duality map.

Pati–Salam models statistics with respect to phenomenological constraints

constraint	# of models	probability	# of models
None	100000000000	1	2.25×10^{15}
+ No gauge group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
+ Complete families	22497003372	2.25×10^{-1}	5.07×10^{14}
+ 3 generations	298140621	2.98×10^{-3}	6.71×10^{12}
+ PS breaking Higgs	23694017	2.37×10^{-4}	5.34×10^{11}
+ SM breaking Higgs	19191088	1.92×10^{-4}	4.32×10^{11}
+ No massless exotics	121669	1.22×10^{-6}	2.74×10^{9}

Constraints in second column act additionally.

Exemplary Model (Christodoulides, AEF, Rizos, PLB 702 (2011) 81)

- A specific choice of one–loop GSO phases
- Analysis of cubic level superpotential and flat directions
- Only one Yukawa coupling at cubic level -> heavy family
- All extra colour triplets -> massive
- One light Higgs bi–doublet
- Solely MSSM below PS breaking scale

Away from the free fermionic point:

$$Z = \int \frac{d^2\tau}{\tau_2^2} \frac{\tau_2^{-1}}{\eta^{12}\bar{\eta}^{24}} \frac{1}{2^3} \left(\sum (-)^{a+b+ab} \vartheta \begin{bmatrix} a \\ b \end{bmatrix} \vartheta \begin{bmatrix} a+h_1 \\ b+g_1 \end{bmatrix} \vartheta \begin{bmatrix} a+h_2 \\ b+g_2 \end{bmatrix} \vartheta \begin{bmatrix} a+h_3 \\ b+g_3 \end{bmatrix} \right)_{\psi^\mu,}$$

$$\times \left(\frac{1}{2} \sum_{\epsilon,\xi} \bar{\vartheta} \begin{bmatrix} \epsilon \\ \xi \end{bmatrix}^5 \bar{\vartheta} \begin{bmatrix} \epsilon+h_1 \\ \xi+g_1 \end{bmatrix} \bar{\vartheta} \begin{bmatrix} \epsilon+h_2 \\ \xi+g_2 \end{bmatrix} \bar{\vartheta} \begin{bmatrix} \epsilon+h_3 \\ \xi+g_3 \end{bmatrix} \right)_{\bar{\psi}^{1\dots5},\bar{\eta}^{1,2,3}}$$

$$\times \left(\frac{1}{2} \sum_{H_1,G_1} \frac{1}{2} \sum_{H_2,G_2} (-)^{H_1G_1+H_2G_2} \bar{\vartheta} \begin{bmatrix} \epsilon+H_1 \\ \xi+G_1 \end{bmatrix}^4 \bar{\vartheta} \begin{bmatrix} \epsilon+H_2 \\ \xi+G_2 \end{bmatrix}^4 \right)_{\bar{\phi}^{1\dots8}}$$

$$\times \left(\sum_{s_i,t_i} \Gamma_{6,6} \begin{bmatrix} h_i|s_i \\ g_i|t_i \end{bmatrix} \right)_{(y\omega\bar{y}\bar{\omega})^{1\dots6}} \times e^{i\pi\Phi(\gamma,\delta,s_i,t_i,\epsilon,\xi,h_i,g_i,H_1,G_1,H_2,G_2)}$$

$$\Gamma_{1,1}\left[{}^h_g\right] = \frac{R}{\sqrt{\tau_2}} \sum_{\tilde{m},n} \exp\left[-\frac{\pi R^2}{\tau_2} \left|(2\tilde{m}+g)+(2n+h)\,\tau\right|^2 \right]$$

Towards String Predictions

1. Low energy supersymmetry

   Specific SUSY breaking patterns $\longrightarrow$ Collider implications

2. Additional (non–GUT) gauge bosons

   Proton Stability and low–scale Z' $\longrightarrow$ Collider signatures

3. Exotic matter

   In realistic string models

   Unifying gauge group $\Rightarrow$ broken by "Wilson lines".

   $\Rightarrow$ non–GUT physical states.

   $\Rightarrow$ Meta-stable heavy string relics

   $\rightarrow$ Dark Mater ; UHECR candidates

Conclusions

- DATA $\longrightarrow$ UNIFICATION

- STRINGS $\longrightarrow$ GAUGE & GRAVITY UNIFICATION

- EXPERIMENTAL PREDICTIONS ?

- FUNDAMENTAL PRINCIPLES ?

  $e.g.$ spinor–vector duality $\rightarrow$ Physics & Geometry

  phase–space duality & the equivalence postulate of QM

Proton stability and superstring Z' (with Coriano and Guzzi)

Standard Model: $(SU321) \oplus (Q\ L\ U\ D\ E\ N) \oplus (h)$:

Effective renormalizable QFT below a cutoff

baryon and lepton numbers protected by accidental global symmetries

Non-renormalizable operators suppressed by cutoff M

B & L numbers violating operators

Dimension six: $QQQL\frac{1}{M^2}$ $\Rightarrow$ $M \sim 10^{16} GeV$

supersymmetry:

Dimension four: $\eta_1 QLD\ \ \&\ \ \eta_2 UDD$ $\Rightarrow$ $(\eta_1 \cdot \eta_2 \leq 10^{-24})$

Dimension five: $\lambda QQQL\frac{1}{M}$ $\Rightarrow$ $(\frac{\lambda}{M}) \leq 10^{-26}$

Appealing Proposition: Low scale gauged $U(1)$ symmetry

Additional Facts

Standard Model: $\longrightarrow$ Unification $\longleftrightarrow$ $SO(10)$

Dimension four: $16^4 \Rightarrow \eta'_1 UDD\frac{\langle N\rangle}{M} + \eta'_2 QLD\frac{\langle N\rangle}{M}$

$+$ ... dimension five ; dimension six

left–handed neutrino masses: $M_{\nu_L} \approx M_{\text{Up}} \left(\frac{M_{\text{weak}}}{M_{\langle N\rangle}}\right)^2$

Fermion masses: $\lambda_{ij}^{\text{Up}} Q^i U^j \bar{h}$; $\lambda_{ij}^{\text{Down}} Q^i D^j h$; $\lambda_{ij}^{\text{CLepton}} L^i E^j h$;

Flavour universality

Freedom from anomalies

What can we learn from string constructions?

Extra $U(1)$'s beyond the Standard Model

$e.g.$ PLB 278 (1992) 131 $\longrightarrow$ seven extra $U(1)$'s

$$U(1)_{Z'} = \frac{B-L}{2} - \frac{2}{3}T_{3_R}$$

The $U(1)$ combinations

$$U_A = \frac{1}{\sqrt{15}}(2(U_1+U_2+U_3)-(U_4+U_5+U_6))$$
$$U_\chi = \frac{1}{\sqrt{15}}(U_1+U_2+U_3+2U_4+2U_5+2U_6)$$
$$U_{12} = \frac{1}{\sqrt{2}}(U_1-U_2) \quad , \quad U_\psi = \frac{1}{\sqrt{6}}(U_1+U_2-2U_3),$$
$$U_{45} = \frac{1}{\sqrt{2}}(U_4-U_5) \quad , \quad U_\zeta = \frac{1}{\sqrt{6}}(U_4+U_5-2U_6)$$

Pati, PLB388 (1996) 532; $U(1)_\psi$ in conjuction with $U(1)_{B-L}$ or $U(1)_\chi$ provides adequate protection as well as allowing light neutrino masses

PLB499 (2001) 147; the extra $U(1)$s are broken at high scale

Patterns of $SO(10)$ symmetry breaking

The $SO(10) \rightarrow \text{subgroup} \quad b(\bar{\psi}^{1\cdots5}_{\frac{1}{2}})$:

1. $b\{\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\ \bar{\eta}^1\ \bar{\eta}^2\ \bar{\eta}^3\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \Rightarrow\ SU(5)\times U(1)\ \ U(1)\ U(1)\ U(1)$
2. $b\{\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\ \bar{\eta}^1\ \bar{\eta}^2\ \bar{\eta}^3\} = \{\ 11100\ 000\ \} \Rightarrow SO(6)\times SO(4)\ U(1)\ U(1)\ U(1)$

$$(1. + 2.) \ \Rightarrow\ SO(10) \ \ \rightarrow SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$$

$$SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

2. $b\{\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\bar{\eta}^1\ \bar{\eta}^2\ \bar{\eta}^3\} = \{\ 11100000\ \} \Rightarrow SO(6)\times SO(4)\ U(1)\ U(1)\ U(1)$
3. $b\{\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\ \bar{\eta}^1\ \bar{\eta}^2\ \bar{\eta}^3\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}00\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \Rightarrow$

$$SU(\ 3\ )_C \times U(1)_C \times SU(2)_L \times SU(2)_R\ U(1)\ U(1)\ U(1)$$

$U(1)$ matter charges

in cases 1. 2.

$$\Longrightarrow \qquad Q_{U(1)_j}(16 = \{Q, L, U, D, E, N\}) \;=\; +\tfrac{1}{2}$$

$\Longrightarrow$ the $U(1)_{1,2,3}$ are anomalous

In the LRS model of case 3.

$$\Longrightarrow \qquad Q_{U(1)_j}(Q_L, L_L) \;=\; -\tfrac{1}{2}$$

$$Q_{U(1)_j}(Q_R = \{U, D\}, L_R = \{E, N\}) \;=\; +\tfrac{1}{2}$$

$\Longrightarrow$ the $U(1)_{1,2,3}$ are anomaly free

The $U(1)$ combination $U(1) = U(1)_1 + U(1)_2 + U(1)_3$ is :

a. family universal
b. anomaly free

The Baryon number violating terms :

$$Q_L Q_L Q_L L_L \rightarrow QQQL$$
$$Q_R Q_R Q_R L_R \rightarrow \{UDDN, UUDE\}$$

are forbidden

The Lepton number violating terms :

$$Q_L Q_R L_L L_R \rightarrow QDLN$$
$$L_L L_L L_R L_R \rightarrow LLEN$$

are allowed

The fermion mass terms:

$$Q_L Q_R h \quad \text{and} \quad L_L L_R h \quad \text{and} \quad N N \bar{N}_H \bar{N}_H \ .$$

are allowed

STRING DERIVED LEFT-RIGHT SYMMETRIC MODEL

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,...,6}$	$\bar{y}^{3,...,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,...,4}$	$\bar{\omega}^{1,...,4}$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,...,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,...}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1 1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1 0 0
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

Gerald Cleaver, AEF and Christopher Savage, PRD 63:066001,2001.

3 generations;
3 untwisted Higgs bi–doublets;
Fermion mass terms arise from $N = 3$ and $N = 5$ superpotential terms

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.

Moduli?

Untwisted moduli $->$ shape & size of the internal dimensions

Twisted moduli $->$ arise from the twisted sectors

models

$$\frac{T^6}{Z_2 \times Z_2}$$

$$T^6 \;:\quad G_{IJ} \quad ; \quad B_{IJ} \qquad I, J \;=\; 1, \cdots, 6 \;.$$

untwisted moduli: coefficients of exactly marginal operators

moduli fields: massless chiral superfields with flat scalar potential

Scalar couplings of $N = 4$ SUGRA

$$\underbrace{\frac{SO(6,6)}{SO(6) \times SO(6)}}_{\text{internal manifold}} \times \underbrace{\frac{SU(1,1)}{U(1)}}_{\text{dilaton}}$$

Up to orbifold projections

$$\frac{T^6}{Z_2 \times Z_2} \Longrightarrow \frac{SO(6,6)}{SO(6) \times SO(6)} \longrightarrow \left(\frac{SO(2,2)}{SO(2) \times SO(2)}\right)^3$$

$\Rightarrow$ 3 complex structures $+$ 3 Kähler moduli

In all symmetric $Z_2 \times Z_2$ orbifolds

We would like to identify these moduli in the fermionic formalism

In symmetric orbifolds:

$$\text{EMO} \rightarrow \partial X^I \bar{\partial} X^J$$

$$X^I \quad I = 1, \cdots, 6 \quad \rightarrow \quad T^6$$

$$S = \frac{1}{8\pi} \int d^2\sigma \;\; (G_{IJ}\, \partial X^I \bar{\partial} X^J \; + \; B_{IJ}\, \partial X^I \bar{\partial} X^J)$$

In FFF $\partial X_L^I \;\rightarrow\; y^i\omega^I$

$i\partial X_L^I \;\rightarrow\; U(1)$ current in the Cartan subalgebra

In the fermionic language

$$i\partial X_L^I \;\rightarrow\; J_L^I = y^I\omega^I$$

$$\Rightarrow\quad \partial X^I\bar{\partial}X^J \rightarrow J_L^I(z)\bar{J}_R(\bar{z})$$

$\rightarrow$ WS Thirring interactions $\quad (R-\frac{1}{R})J_L(z)\bar{J}(\bar{z})$

Thirring interactions vanish at free fermionic point with $R=\frac{1}{R}$

To identify the untwisted moduli in the free fermionic models

$\rightarrow$ find the operators of the form

$$J_L^I(z)\bar{J}_R^J(\bar{z})$$

that are allowed by the orbifold (fermionic) symmetry group

Bosonization

$$\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = e^{iX_i}, \eta_i = \sqrt{\frac{i}{2}}(y_i - i\omega_i) = ie^{-iX_i}$$

simlarly for the right-movers

Boundary conditions translate to twists and shifts

$$X^I(z,\bar{z}) = X^I_L(z) + X^I_R(\bar{z})$$

Complex internal coordinates

$$Z_k^{\pm} = (X^{2k-1} \pm iX^{2k}), \quad \psi_k^{\pm} = (\chi^{2k-1} \pm i\chi^{2k}) \quad (k = 1, 2, 3)$$

Untwisted moduli

$$S = \int d^2 z h_{ij}(X) J_L^i(z) \bar{J}_R^j(\bar{z})$$

$J_L^i \sim y^i \omega^i$ $\qquad i = 1, \cdots, 6$ are chiral currents of $U(1)_L^6$

$J_R^j \sim \bar{\phi}^j \bar{\phi}^{*j}$ $\qquad j = 1, \cdots, 22$ are chiral currents of $U(1)_R^{22}$

$h_{ij} \rightarrow$ scalar components of untwisted moduli

some of these operators are projected out in concrete models

$\Rightarrow$ some of the EMO may not be invariant

Models $\{\mathbf{1}, S\}$ $N = 4$ $SO(44)$

$$\frac{SO(6,22)}{SO(6)\times SO(22)}$$ Moduli space

$$\chi^i \otimes \bar{\phi}^a \bar{\phi}^{*^a} |0\rangle$$ moduli fields

$$6 \ \times \ 22$$ scalar fields

$Z_2 \times Z_2$ $\{ \ 1 \ , \ S \ , \ \xi_1 \ , \ \xi_2 \ \} \ + \ \{ \ b_1 \ , \ b_2 \ \}$

$$SO(12)\times E_8 \times E_8 \qquad Z_2 \times Z_2$$

$$\rightarrow \quad SO(4)^3 \times E_6 \times U(1)^2 \times E_8$$

The Thirring interactions that remain invariant are

$$J_L^{1,2} \bar{J}_R^{1,2} \quad ; \quad J_L^{3,4} \bar{J}_R^{3,4} \quad ; \quad J_L^{5,6} \bar{J}_R^{5,6}$$

$$y^{1,2}\omega^{1,2}\bar{y}^{1,2}\bar{\omega}^{1,2} \quad ; \quad y^{3,4}\omega^{3,4}\bar{y}^{3,4}\bar{\omega}^{3,4} \quad ; \quad y^{5,6}\omega^{5,6}\bar{y}^{5,6}\bar{\omega}^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds in realistic models

$$\{\,1\,,\;S\,,\;\xi_1\,,\;\xi_2\,\}\;\oplus\;\{\,b_1\,,\;b_2\,\}\;\oplus\;\{\,\alpha\,,\;\beta\,,\;\gamma\}$$

$$N=4 \qquad\qquad N=1$$

$$E_8\times E_8 \qquad\qquad Z_2\times Z_2$$

new feature   Asymmetric orbifold

the key focus: boundary conditions of the internal fermions

$$\{\;y\;,\;\omega\;\mid\;\bar{y}\;,\;\omega\;\}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\longrightarrow$ Ising model $\longrightarrow$ symmetric real fermions

pairing of LL & RR fermions $\longrightarrow$ complex fermions $\longrightarrow$ asymmetric

STRING DERIVED STANDARD-LIKE MODEL (PLB278)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$y^{3,\dots,6}$	$\bar{y}^{3,\dots,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,\dots,4}$	$\bar{\omega}^{1,\dots,4}$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
1	1	1	1	1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1	1,...,1	0	0	1	0,...,0

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1

Asymmetric $BC \Rightarrow$ all untwisted moduli are projected out!

all $y_i\omega_i\bar{y}_i\bar{\omega}_i$ are disallowed

can be translated to asymmetric bosonic identifications

$$X_L + X_R \rightarrow X_L - X_R$$

moduli fixed at enhanced symmetry point

Twisted moduli

(2,2) $b_j \oplus b_j + \xi_2 \quad \longrightarrow \quad (16 \oplus 10 + 1) + 1 = 27 + 1 \Rightarrow$ twisted moduli

(2,0) $b_j + b_j + 2\gamma \quad \longrightarrow 16_{SO(10)} + 16_{SO(16)}$

all twisted moduli are projected out

A STRINGY DOUBLET–TRIPLET SPLITTING MECHANISM

NAHE $\rightarrow$ $\chi_j \bar{\psi}^{1,\cdots,5} \bar{\eta}_j + c.c. \rightarrow (5+\bar{5})_j = 10_j$ of $SO(10)$

$$\alpha \quad \rightarrow \quad SO(10) \quad \rightarrow \quad SO(6)\times SO(4)$$

$y_3\bar{y}_3$	$y_4\bar{y}_4$	$y_5\bar{y}_5$	$y_6\bar{y}_6$		$y_3 y_6$	$y_4\bar{y}_4$	$y_5\bar{y}_5$	$\bar{y}_3\bar{y}_6$
1	0	0	1		1	0	0	0

$$\Delta_j = |\alpha_L(T_2^j) - \alpha_R(T_2^j)|$$

$\Delta_j = 0 \rightarrow D_j, \bar{D}_j$ $\qquad\qquad$ $\Delta_j = 1 \rightarrow h_j, \bar{h}_j$

$$\Delta_{1,2,3} = 1 \Rightarrow h_j \quad \bar{h}_j \qquad j = 1,2,3$$

A superstring solution to the GUT hierarchy problem

MINIMAL DOUBLET HIGGS CONTENT

(w Manno & Timirgaziu (SLM); Christodoulides (FSU5) in progress)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$
α	0	0	0	0	1	0	0	1	0	0	1	1
β	0	0	0	0	0	0	1	1	1	0	0	1
γ	0	0	0	0	0	1	0	0	0	1	0	0

	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0
β	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1
γ	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

SYMMETRIC $\leftrightarrow$ ASYMMETRIC

with respect to b_1 & b_2

$h_1,\ \bar{h}_1,\ D_1,\ \bar{D}_1$, $h_2,\ \bar{h}_2,\ D_2,\ \bar{D}_2$ are projected out

$h_3,\ \bar{h}_3$ remain in the spectrum

$$\lambda_t Q_3 t_3^c \bar{h}_3 \ \text{ with } \ \lambda_t\ O(1)$$

No Phenomenologically viable flat directions

Cleaver, Faraggi, Manno, Timirgaziu, PRD 78 (2008) 046009

Classification of F and D flat directions in EMT reduced Higgs model

No D flat direction which is F-flat up to order eight in the superpotential

no stringent flat directions to all orders

Suggesting no supersymmetric flat directions in this model (class of models)

implying no supersymmetric moduli

only remaining perturbative moduli is the dilaton

quasi–realistic model: SLM; 3 gen; $SO(10)$ embed; Higgs & $\lambda_t \sim 1$; ...

vanishing one–loop partition function

Fixed geometrical, twisted and SUSY moduli

NAHE-based partition functions:

w Carlo Angelantonj, Mirian Tsulaia

Question:

$$\frac{T^6}{Z_2 \times Z_2} \;\rightarrow\; 48 \;\text{ fixed points} \qquad\qquad \frac{SO(12)}{Z_2 \times Z_2} \;\rightarrow\; 24 \;\text{ fixed points}$$

$$Z_2 \;\text{ shift } \;: 48 \;\longleftrightarrow\; 24$$

Is this the same model? In general, no.

shift that reproduces the $SO(12)$ lattice at the free fermionic point?

Possible shifts:

$$A_1 \ : \ \ X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\pi R \,,$$

$$A_2 \ : \ \ X_{\rm L,R} \to X_{\rm L,R} + \tfrac{1}{2}\left(\pi R \pm \frac{\pi\alpha'}{R}\right) \,,$$

$$A_3 \ : \ \ X_{\rm L,R} \to X_{\rm L,R} \pm \tfrac{1}{2}\frac{\pi\alpha'}{R} \,.$$

Using the level-one $\mathrm{SO}(2n)$ characters

$$O_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} + \frac{\vartheta_4^n}{\eta^n}\right) \,, \qquad V_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_3^n}{\eta^n} - \frac{\vartheta_4^n}{\eta^n}\right) \,,$$

$$S_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} + i^{-n}\frac{\vartheta_1^n}{\eta^n}\right) \,, \qquad C_{2n} = \tfrac{1}{2}\left(\frac{\vartheta_2^n}{\eta^n} - i^{-n}\frac{\vartheta_1^n}{\eta^n}\right) \,.$$

Starting from:

$$Z_+ = (V_8 - S_8) \left(\sum_{m,n} \Lambda_{m,n} \right)^{\otimes 6} \left(\bar{O}_{16} + \bar{S}_{16}\right) \left(\bar{O}_{16} + \bar{S}_{16}\right) \, ,$$

where as usual, for each circle,

$$p^i_{\mathrm{L,R}} = \frac{m_i}{R_i} \pm \frac{n_i R_i}{\alpha'} \, ,$$

and

$$\Lambda_{m,n} = \frac{q^{\frac{\alpha'}{4} p_{\mathrm{L}}^2} \, \bar{q}^{\frac{\alpha'}{4} p_{\mathrm{R}}^2}}{|\eta|^2} \, .$$

Add shifts : (A_1, A_1, A_1) , (A_3, A_3, A_3)

($48 \rightarrow 24$ yes)
($SO(12)$? no)

Uniquely:

$$g \;:\; (A_2, A_2, 0)\,,$$

$$h \;:\; (0, A_2, A_2)\,,$$

where each A_2 acts on a complex coordinate

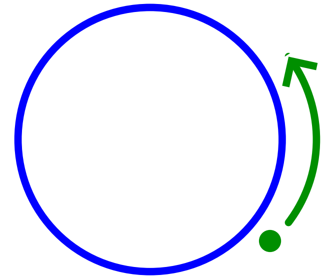
($48 \longrightarrow 24$ yes)

($SO(12)$? yes)

$$R = \sqrt{\alpha'}$$

T1 − COMPACTIFICATION

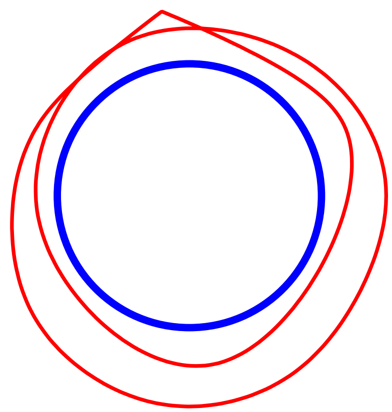
$$X \longrightarrow X \sim X + 2\pi R m$$



Point particle

$$\Psi \sim \mathrm{Exp}(i\, P\, X) \quad \Rightarrow P = \frac{m}{R}$$

String



$$P_{L,R} = \frac{m}{R} \pm \frac{n\, R}{\alpha'}$$

T − DUALITY

$$\text{mass}^2 = \left(\frac{n}{R}\right)^2 + \left(\frac{m\,R}{\alpha'}\right)^2$$

Invariant under

$$\frac{1}{R} \longleftrightarrow \frac{R}{\alpha'} \qquad \text{with} \qquad m \longleftrightarrow n$$

An exact symmetry in string perturbation theory!

Self−dual point $R = \frac{\alpha'}{R}$ = free fermionic point

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdot \cdot \cdot V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\text{Tr}Q_A \neq 0 \Rightarrow \quad D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$

$$D_j \; = 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdot \cdot \cdot \cdot$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms $\longrightarrow$ effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdot \cdot \cdot V_N^b \; \rightarrow \; V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdot \cdot \cdot V_N^b \rangle}{M^{N-3}}$$

The massless spectrum

Three twisted generations $b_1, \quad b_2, \quad b_3$

Untwisted Higgs doublets

$$h_{1_{1,0,0}} \qquad \bar{h}_{1_{-1,0,0}}$$
$$h_{2_{0,1,0}} \qquad \bar{h}_{2_{0,-1,0}}$$
$$h_{3_{0,0,1}} \qquad \bar{h}_{3_{0,0,-1}}$$

Sector $b_1 + b_2 + \alpha + \beta$ $\quad h_{\alpha\beta_{-\frac{1}{2},-\frac{1}{2},0,0,0,0}} \qquad \bar{h}_{\alpha\beta_{\frac{1}{2},\frac{1}{2},0,0,0,0}}$

$\oplus$ $SO(10)$ singlets

Sectors $b_j + 2\gamma \quad j = 1, 2, 3 \quad \longrightarrow$ hidden matter multiplets

"standard" $SO(10)$ representations

NAHE + $\{ \alpha , \beta , \gamma \}$ $\rightarrow$ exotic vector–like matter $\rightarrow$ superheavy

$\oplus$ Quasi-realistic phenomenology

Fermion mass hierarchy

Fermion mass terms

$$c g f_i f_j h \left(\frac{\langle\phi\rangle}{M}\right)^{N-3}$$

c - calculable coefficients g - gauge coupling

$$f_i,\, f_j \;\in\; b_j \quad j \;=\; 1, 2, 3$$

$h \rightarrow$ light Higgs multiplets

$$M \;\sim\; 10^{18}\, GeV$$

$\langle\phi\rangle$ generalized VEVs, several sources

Up/Down–type Yukawa Selection Mechanism

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	0	0	0	1	1	0	0	1	1	1 1 1 0 0	0	0	0	1 1 1
β	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	1	1 1 1 0 0	0	0	0	1 1 1
γ	0	0	0	0	0	1	0	1	0	1	0	1	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

At the cubic level $\longrightarrow$ γ selects trilevel Yukawa coupling:

$$\Delta_j = |\gamma(U(1)_{L_{j+3}}) - \gamma(U(1)_{R_{j+3}})| = 0, 1 \qquad j = 1, 2, 3$$

$$\Delta_j = 1 \Rightarrow u_j Q_j \bar{h}_j \qquad\qquad \Delta_j = 0 \Rightarrow d_j Q_j h_j \; ; \; e_j L_j h_j$$

b_1 :	$y_3 y_6$	$y_4 \bar{y}_4$	$y_5 \bar{y}_5$	$\bar{y}_3 \bar{y}_6$		$y_3 y_6$	$y_4 \bar{y}_4$	$y_5 \bar{y}_5$	$\bar{y}_3 \bar{y}_6$
γ :	1	0	0	0		1	0	0	1

$\Delta_1 = 1$ $\qquad\qquad$ $\Delta_1 = 0$

$\underline{\lambda_j t_j^c Q_j \bar{h}_j}$

Top quark mass prediction

only $\lambda_t = \langle t^c Q_t \bar{h}_1 \rangle = \sqrt{2} g \neq 0$ at $N = 3$

$$W_4 \longrightarrow b^c Q_b h_{\alpha\beta} \Phi_1 + \tau^c L_\tau h_{\alpha\beta} \Phi_1$$

$$\Longrightarrow \qquad \lambda_b = \left(c_b \frac{\langle \phi \rangle}{M} \right) \qquad \lambda_\tau = \left(c_\tau \frac{\langle \phi \rangle}{M} \right)$$

$$\longrightarrow \qquad \lambda_b = \lambda_\tau = 0.35 g^3 \sim \frac{1}{8} \lambda_t$$

Evolve λ_t , λ_b to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}} \sin\beta \qquad\qquad m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}} \cos\beta$$

where $v_0 = \frac{2 m_W}{g_2(M_Z)} = 246 \mathrm{GeV}$ and $(v_1^2 + v_2^2) = \frac{v_0^2}{2}$

$$m_t = \lambda_t(m_t) \frac{v_0}{\sqrt{2}} \frac{\tan\beta}{(1 + \tan^2\beta)^{\frac{1}{2}}} \Longrightarrow \qquad m_t \sim 175 \mathrm{GeV} \quad \mathrm{PLB274(1992)47}$$

ıysics Letters B 274 (1992) 47–52
orth-Holland

PHYSICS LETTERS B

Hierarchical top–bottom mass relation in a superstring derived standard-like model

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Received 30 September 1991

I propose a mechanism in a class of superstring standard-like models which explains the mass hierarchy between the top and bottom quarks. At the trilinear level of the superpotential only the top quark gets a nonvanishing mass term while the bottom quarks and tau lepton mass terms are obtained from nonrenormalizable terms. I construct a model which realized this mechanism. In this model the bottom quark and tau lepton Yukawa couplings are obtained from quartic order terms. I show that $\lambda_b = \lambda_\tau \sim \frac{1}{8}\lambda_t$ at the unification scale. A naive estimate yields $m_t \sim 175$–180 GeV.

One of the unresolved puzzles of the standard model is the mass splitting between the top quark and the lighter ıuarks and leptons. Especially difficult to understand within the context of the standard model is the big split- ing in the heaviest generation. Experimental limits [1] indicate the top mass to be above 80 GeV, while the ·ottom and tau lepton masses are found at 5 GeV and 1.78 GeV respectively. Possible extensions to the standard nodel are grand unified theories. Although the main prediction of GUTs, proton decay, has not yet been ob- erved, calculations of $\sin^2\theta_W$ and of the mass ratio m_b/m_τ support their validity. Recent calculations seem to upport supersymmetric GUTs versus nonsupersymmetric ones [2]. In spite of the success of SUSY GUTs in onfronting LEP data [2], an understanding of the mass splitting between the top quark and the lighter quarks ınd leptons is still lacking. The next level in which such an understanding may be developed is in the context of uperstring theory [3].

Introducing the notation $c\binom{a_i}{a_j} = e^{i\pi(a_i|a_j)}$, $(a_i\,|a_j) = 0,1$

the projectors can be written as system of equations

$$\Delta^{(I)}\, U_{16}^{(I)} = Y_{16}^{(I)} \quad , \quad \Delta^{(I)}\, U_{10}^{(I)} = Y_{10}^{(I)} \;,\; I = 1,2,3$$

where the unknowns are the fixed point labels

$$U_{16}^{(I)} = \begin{bmatrix} p_{16}^I \\ q_{16}^I \\ r_{16}^I \\ s_{16}^I \end{bmatrix} \;, \qquad U_{10}^{(I)} = \begin{bmatrix} p_{10}^I \\ q_{10}^I \\ r_{10}^I \\ s_{10}^I \end{bmatrix}$$

and

$$\Delta^{(1)} = \begin{bmatrix} (e_1\,|e_3) & (e_1\,|e_4) & (e_1\,|e_5) & (e_1\,|e_6) \\ (e_2\,|e_3) & (e_2\,|e_4) & (e_2\,|e_5) & (e_2\,|e_6) \\ (z_1\,|e_3) & (z_1\,|e_4) & (z_1\,|e_5) & (z_1\,|e_6) \\ (z_2\,|e_3) & (z_2\,|e_4) & (z_2\,|e_5) & (z_2\,|e_6) \end{bmatrix} \quad ; \quad \Delta^{(2)} \ldots$$

$$Y_{16}^{(1)} = \begin{bmatrix} (e_1\,|b_1) \\ (e_2\,|b_1) \\ (z_1\,|b_1) \\ (z_2\,|b_2) \end{bmatrix} \quad , \quad Y_{16}^{(2)} = \ldots$$

$$Y_{10}^{(1)} = \begin{bmatrix} (e_1\,|b_1 + x) \\ (e_2\,|b_1 + x) \\ (z_1\,|b_1 + x) \\ (z_2\,|b_2 + x) \end{bmatrix} \quad , \quad Y_{10}^{(2)} = \ldots$$

The number of solutions per plane

$$S^{(I)} = \begin{cases} 2^{4-\text{rank}\left(\Delta^{(I)}\right)} & \text{rank}\left(\Delta^{(I)}\right) = \text{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \\ 0 & \text{rank}\left(\Delta^{(I)}\right) < \text{rank}\left[\Delta^{(I)}, Y_{16}^{(I)}\right] \end{cases}$$

$$V^{(I)} = \begin{cases} 2^{4-\text{rank}\left(\Delta^{(I)}\right)} & \text{rank}\left(\Delta^{(I)}\right) = \text{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right] \\ 0 & \text{rank}\left(\Delta^{(I)}\right) < \text{rank}\left[\Delta^{(I)}, Y_{10}^{(I)}\right] \end{cases}$$

rank $\left(\Delta^{(I)}\right)$	rank $\left[\Delta^{(I)}, Y_{16}^{(I)}\right]$	rank $\left[\Delta^{(I)}, Y_{10}^{(I)}\right]$	# of Spinorials	# of vectorials
4	4	4	1	1
3	4	4	0	0
	3	4	2	0
	4	3	0	2
	3	3	2	2
2	3	3	0	0
	2	3	4	0
	3	2	0	4
	3	3	4	4
1	2	2	0	0
	1	2	8	0
	2	1	0	8
	1	1	8	8
0	1	1	0	0
	0	1	16	0
	1	0	0	16
	0	0	16	16

Table 1: *Total number of $SO(10)$ spinorial and vectorial representations in a given orbifold plane $I = 1, 2, 3$ for all possible ranks of the projection matrices $\left(\Delta^{(I)}\right)$, $\left[\Delta^{(I)}, Y_{16}^{(I)}\right]$, and $\left[\Delta^{(I)}, Y_{10}^{(I)}\right]$.*

RESULTS:

7×10^9 models $\sim$ 15% with 3 gen FKRI

Exotics : (in FNY SLM–model, NPB 335 (1990) 347)

$$W_2 = \frac{1}{\sqrt{2}}\{ \ H_1 H_2 \phi_4 + H_3 H_4 \bar{\phi}_4 + H_5 H_6 \bar{\phi}_4 + (H_7 H_8 + H_9 H_{10})\phi'_4 + (H_{11} + H_{12})(H_{13} + H_{14})\bar{\phi}'_4 + V_{41} V_{42} \bar{\phi}_4 + V_{43} V_{44} \bar{\phi}_4 + V_{45} V_{46} \phi_4 + (V_{47} V_{48} + V_{49} V_{50})\bar{\phi}'_4 + V_{51} V_{52} \phi'_4 \}$$

$\langle \ \bar{\phi}_4, \bar{\phi}'_4, \phi_4, \phi'_4 \ \rangle \rightarrow$ massive exotic states at N=3 (PRD46 (1993) 3204)

CFN $\rightarrow$ Classification of flat directions (PLB 455 (1999) 135)

Example: $\{\phi_{12}, \phi_{23}, \bar{\phi}_{56}, \phi_4, \phi'_4, \bar{\phi}_4, \bar{\phi}'_4, H_{15}, H_{30}, H_{31}, H_{38}\}$

All Standard Model charged states beyond MSSM $\rightarrow \ \approx \ M_{\text{string}}$

MINIMAL STANDARD HETEROTIC STRING MODEL

Beyond $SO(10)$ Classification: Pati–Salam Heterotic–String :

$$SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R \to SU(3) \times SU(2) \times U(1)$$

single additional vector $v_{13} = \alpha = \{\bar{\psi}^{45}\bar{\phi}^{1,2}\}$

12 new GSO phases $c[\alpha, v_j], j = 1, \ldots, 12.$

$$\mathbf{16} = F_L(\mathbf{4},\mathbf{2},\mathbf{1}) + F_R(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) \to (Q_L + L_L) + (Q_R + L_R)$$

$$\mathbf{10} = D(\mathbf{6},\mathbf{1},\mathbf{1}) \ + \ \ h(\mathbf{1},\mathbf{2},\mathbf{2}) \ \to (D_3 + \bar{D}_3) + (h^d + h^u)$$

PS Symmetry broken by $\langle \nu^c_H \rangle$, $\langle \nu_H \rangle$

$$\bar{H}(\bar{\mathbf{4}},\mathbf{1},\mathbf{2}) \to u^c_H(\bar{\mathbf{3}},\mathbf{1},+\frac{2}{3}) + d^c_H(\bar{\mathbf{3}},\mathbf{1},-\frac{1}{3}) + \langle \nu^c_H(\mathbf{1},\mathbf{1},0) \rangle + e^c_H(\mathbf{1},\mathbf{1},-1)$$

$$H(\mathbf{4},\mathbf{1},\mathbf{2}) \to u_H(\mathbf{3},\mathbf{1},-\frac{2}{3}) + d_H(\mathbf{3},\mathbf{1},+\frac{1}{3}) + \langle \nu_H(\mathbf{1},\mathbf{1},0) \rangle + e_H(\mathbf{1},\mathbf{1},+1)$$

Models to date contain additional fractionally charged matter

ALGEBRAIC ANALYSIS OF ALL SECTORS

Search strategy

A model is characterized by 9 integers $\left(n_g, k_L, k_R, n_6, n_h, n_4, n_{\bar{4}}, n_{2L}, n_{2R}\right)$

$n_{4L} - n_{\bar{4}R} = n_{\bar{4}L} - n_{4R} = n_g =$ # of generations

$n_{\bar{4}L} = k_L =$ # of non chiral left pairs

$n_{4R} = k_R =$ # of non chiral right pairs

$n_6 =$ # of $(\mathbf{6}, \mathbf{1}, \mathbf{1})$

$n_h =$ # of $(\mathbf{1}, \mathbf{2}, \mathbf{2})$

$n_4 =$ # of $(\mathbf{4}, \mathbf{1}, \mathbf{1})$ (exotic)

$n_{\bar{4}} =$ # of $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{1})$ (exotic)

$n_{2L} =$ # of $(\mathbf{1}, \mathbf{2}, \mathbf{1})$ (exotic)

$n_{2R} =$ # of $(\mathbf{1}, \mathbf{1}, \mathbf{2})$ (exotic)

51 independent GSO phases, $\Longrightarrow \quad 2^{51} \sim 2 \times 10^{15}$ models.

RESULTS: of random search of over 10^{11} of 5×10^{15} vacua

RESULTS: of random search of over 10^{11} of 5×10^{15} vacua

Number of 3-generation models versus total number of exotic multiplets

Construction of the physical states

$$b_j \quad j = 1, \cdots, N \quad \rightarrow \quad \Xi = \sum_j n_j b_j$$

For $\vec{\alpha} = (\vec{\alpha}_L; \vec{\alpha}_R) \in \Xi \Rightarrow \mathrm{H}_{\vec{\alpha}}$

$$\alpha(f) = 1 \Rightarrow |\pm\rangle \quad ; \quad \alpha(f) \neq 1 \Rightarrow f, f^* \quad , \quad \nu_{f,f^*} = \frac{1 \mp \alpha(f)}{2}$$

$$M_L^2 = -\frac{1}{2} + \frac{\vec{\alpha}_L \cdot \vec{\alpha}_L}{8} + N_L = -1 + \frac{\vec{\alpha}_R \cdot \vec{\alpha}_R}{8} + N_R = M_R^2 \quad (\equiv 0)$$

GSO projections $\quad e^{i\pi(\vec{b}_i \cdot \vec{F}_\alpha)} |s\rangle_{\vec{\alpha}} = \delta_\alpha c^* \binom{\vec{\alpha}}{\vec{b}_i} |s\rangle_{\vec{\alpha}}$

$$F_\alpha(f) \rightarrow \text{fermion \# operator} = \begin{cases} -1, & |-\rangle \\ 0, & |+\rangle \end{cases} = \begin{cases} +1, \\ -1, \end{cases}$$

$Q(f) = \frac{1}{2}\alpha(f) + F(f) \rightarrow$ U(1) charges

Correspondence with $Z_2 \times Z_2$ orbifold

PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1) \quad \rightarrow \quad \{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & i < \\ 0 & i = \\ -g_{ij} & i > \end{cases}$$

$R_i \quad \rightarrow$ the free fermionic point $\rightarrow$ G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow$ Exact correspondence

In the realistic free fermionic models

replace $\xi_2 \equiv x = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$

with $2\gamma = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$SO(12) \times SO(16) \times SO(16)$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$ N=1 SUSY and

$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$

b_1, b_2, b_3 $\Rightarrow (3 \times 8) \cdot 16$ of $SO(10)_O$

$b_1 + 2\gamma$, $b_2 + 2\gamma$, $b_3 + 2\gamma$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

Alternatively, $c\binom{\xi_1}{\xi_2} = +1 \rightarrow -1$

$b_1 + \xi_1$, $b_2 + \xi_1$, $b_3 + \xi_1$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

The twisted matter spectrum:

$$B^1_{\ell^1_3\ell^1_4\ell^1_5\ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$

$$B^2_{\ell^2_1\ell^2_2\ell^2_5\ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$

$$B^3_{\ell^3_1\ell^3_2\ell^3_3\ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4 \qquad l^j_i = 0, 1$$

sectors $B^i_{pqrs} \rightarrow$ 16 or $\overline{16}$ of $SO(10)$ with multiplicity $(1, 0, -1)$

$B^i_{pqrs} + x \rightarrow$ 10 of $SO(10)$ with multiplicity $(1, 0)$

Counting: for each B^i_{pqrs}:

Projectors:

$$P^{(1)}_{p^1q^1r^1s^1} = \frac{1}{16} \prod \left(1 - c\binom{e_i}{B^{(1)}_{p^1q^1r^1s^1}} \right) \prod \left(1 - c\binom{z_i}{B^{(1)}_{p^1q^1r^1s^1}} \right)$$

$$S^{(i)}_{\pm} = \sum_{pqrs} \frac{1 \pm X^{(i)}_{p^iq^ir^is^i}}{2} P^{(i)}_{p^iq^ir^is^i} \, , \; i = 1, 2, 3$$

similarly for vectorials

Algebraic formulas for $S = \sum_{i=1}^{3} S^{(i)}_{+} - S^{(i)}_{-}$ and $V = \sum_{i=1}^{3} V^{(i)}$

Other approaches

Geometrical

Greene, Kirklin, Miron, Ross (1987)
Donagi, Ovrut, Pantev, Waldram (1999)
Blumenhagen, Moster, Reinbacher, Weigand (2006)
Heckman, Vafa (2008)
....

Orbifolds

Ibanez, Nilles, Quevedo (1987)
Bailin, Love, Thomas (1987)
Kobayashi, Raby, Zhang (2004)
Lebedev, Nilles, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter (2007)
Blaszczyk, Groot–Nibbelink, Ruehle, Trapletti, Vaudrevange (2010)
....

Other CFTs

Gepner (1987)
Schellekens, Yankielowicz (1989)
Gato–Rivera, Schellekens (2009)
....

Orientifolds

Cvetic, Shiu, Uranga (2001)
Ibanez, Marchesano, Rabadan (2001)
Kiristis, Schellekens, Tsulaia (2008)
....

Pati–Salam models statistics with respect to phenomenological constraints

constraint	# of models	probability	# of models
None	100000000000	1	2.25×10^{15}
+ No gauge group enhancements.	78977078333	7.90×10^{-1}	1.78×10^{15}
+ Complete families	22497003372	2.25×10^{-1}	5.07×10^{14}
+ 3 generations	298140621	2.98×10^{-3}	6.71×10^{12}
+ PS breaking Higgs	23694017	2.37×10^{-4}	5.34×10^{11}
+ SM breaking Higgs	19191088	1.92×10^{-4}	4.32×10^{11}
+ No massless exotics	121669	1.22×10^{-6}	2.74×10^{9}

Constraints in second column act additionally.

Exotics -> fractionally charged states

Leptophobic Z' (PLB388 (1996) 524; arXiv:1106.5422 with Viraf Mehta)

CDF -> enhancement in di–jet data at $\sim 4\sigma$; D0 -> no enhancement

possible interpretation -> a leptophobic Z' -> in heterotic string?

The NAHE set : $\{ \mathbf{1}, \ S \ , \quad b_1 \ , \quad b_2 \ , \quad b_3 \ \}$

$\Longrightarrow$ Gauge group $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$ $\rightarrow$ 3 generations

$SO(10) \longrightarrow \text{subgroup}$

$e.g. \ \ SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$

Patterns of $SO(10)$ symmetry breaking

The $SO(10) \rightarrow$ subgroup $\quad b(\bar{\psi}^{1\cdots5}_{\frac{1}{2}})$:

$$1.\ b\{\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\ \bar{\eta}^1\ \bar{\eta}^2\ \bar{\eta}^3\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \Rightarrow\ SU(5)\times U(1)\ \ U(1)\ U(1)\ U(1)$$

$$2.\ b\{\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\ \bar{\eta}^1\ \bar{\eta}^2\ \bar{\eta}^3\} = \{\ 11100\ 000\ \} \Rightarrow SO(6)\times SO(4)\ U(1)\ U(1)\ U(1)$$

$$(1.+2.)\ \Rightarrow\ SO(10)\ \ \rightarrow SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$$

$$SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$2.\ b\{\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\bar{\eta}^1\ \bar{\eta}^2\ \bar{\eta}^3\} = \{\ 11100000\ \} \Rightarrow SO(6)\times SO(4)\ U(1)\ U(1)\ U(1)$$

$$3.\ b\{\bar{\psi}^{1\cdots5}_{\frac{1}{2}}\ \bar{\eta}^1\ \bar{\eta}^2\ \bar{\eta}^3\} = \{\frac{1}{2}\frac{1}{2}\frac{1}{2}00\frac{1}{2}\frac{1}{2}\frac{1}{2}\} \Rightarrow$$

$$SU(\ 3\ )_C \times U(1)_C \times SU(2)_L \times SU(2)_R\ U(1)\ U(1)\ U(1)$$

$U(1)$ matter charges

in cases 1. 2.

$$\Longrightarrow \qquad Q_{U(1)_j}(16 = \{Q, L, U, D, E, N\}) \;=\; +\frac{1}{2}$$

$\Longrightarrow$ the $U(1)_{1,2,3}$ are anomalous

In the LRS model of case 3.

$$\Longrightarrow \qquad Q_{U(1)_j}(Q_L, L_L) \;=\; -\frac{1}{2}$$

$$Q_{U(1)_j}(Q_R = \{U, D\}, L_R = \{E, N\}) \;=\; +\frac{1}{2}$$

$\Longrightarrow$ the $U(1)_{1,2,3}$ are anomaly free

In the LRS models

$$U(1)_{B'} = \frac{1}{3}U_C - U_1 - U_2 - U_3$$

$$Q_{B'}(L_L) \quad = \quad -\frac{1}{2} + \frac{1}{2} = \quad 0 \qquad Q_{B'}(L_R) \quad = \quad +\frac{1}{2} - \frac{1}{2} = \quad 0$$

$$Q_{B'}(Q_L) \quad = \quad +\frac{1}{2} + \frac{1}{2} = +1 \qquad Q_{B'}(Q_R) \quad = \quad -\frac{1}{2} - \frac{1}{2} = -1$$

$$\longrightarrow$$

A Family Universal Anomaly Free Leptophobic $U(1)$

Correspondence with $Z_2 \times Z_2$ orbifold   PLB 326 (1994) 62

NAHE $\oplus$ $(\xi_2 = \{\bar{\psi}_{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1)$ $\rightarrow$ $\{1, S, \xi_1, \xi_2, b_1, b_2\}$

Gauge group: $SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ and 24 generations.

toroidal compactification $(6_L + 6_R)$ g_{ij}, b_{ij}

$$g_{ij} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 \end{pmatrix} \qquad b_{ij} = \begin{cases} g_{ij} & i < j \\ 0 & i = j \\ -g_{ij} & i > j \end{cases}$$

R_i $\rightarrow$ the free fermionic point $\rightarrow$G.G. $SO(12) \times E_8 \times E_8$

mod out by a $Z_2 \times Z_2$ with standard embedding

$\Rightarrow SO(4)^3 \times E_6 \times U(1)^2 \times E_8$ with 24 generations

Exact correspondence

In the realistic free fermionic models

replace $X = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3\} = 1$

with $2\gamma = \{\bar{\psi}^{1,\cdots,5}, \bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{\phi}^{1,\cdots,4}\} = 1$

Then $\{\vec{1}, \vec{S}, \vec{\xi}_1 = \vec{1} + \vec{b}_1 + \vec{b}_2 + \vec{b}_3, 2\gamma\} \rightarrow$ N=4 SUSY and

$$SO(12) \times SO(16) \times SO(16)$$

apply $b_1 \times b_2 \rightarrow Z_2 \times Z_2 \rightarrow$ N=1 SUSY and

$$SO(4)^3 \times SO(10) \times U(1)^3 \times SO(16)$$

b_1, b_2, b_3 $\Rightarrow (3 \times 8) \cdot 16$ of $SO(10)_O$

$b_1 + 2\gamma$, $b_2 + 2\gamma$, $b_3 + 2\gamma$ $\Rightarrow (3 \times 8) \cdot 16$ of $SO(16)_H$

This will be important for the twisted moduli.

Moduli?

Untwisted moduli $->$ shape & size of the internal dimensions

Twisted moduli $->$ arise from the twisted sectors

models

$$\frac{T^6}{Z_2 \times Z_2}$$

$$T^6 \ : \qquad G_{IJ} \qquad ; \qquad B_{IJ} \qquad I, J \ = \ 1, \cdots, 6 \ .$$

untwisted moduli: coefficients of exactly marginal operators

moduli fields: massless chiral superfields with flat scalar potential

Scalar couplings of $N = 4$ SUGRA

$$\frac{SO(6,6)}{SO(6) \times SO(6)} \qquad \times \qquad \frac{SU(1,1)}{U(1)}$$

internal manifold — dilaton

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Scalar couplings of $N = 4$ SUGRA

$$\underset{\text{internal manifold}}{\frac{SO(6,6)}{SO(6) \times SO(6)}} \qquad \times \qquad \underset{\text{dilaton}}{\frac{SU(1,1)}{U(1)}}$$

Up to orbifold projections

$$\frac{T^6}{Z_2 \times Z_2} \Longrightarrow \frac{SO(6,6)}{SO(6)\times SO(6)} \longrightarrow \left(\frac{SO(2,2)}{SO(2)\times SO(2)}\right)^3$$

$\Rightarrow$ 3 complex structures $+$ 3 Kähler moduli

In all symmetric $Z_2 \times Z_2$ orbifolds

We would like to identify these moduli in the fermionic formalism

In symmetric orbifolds:

$$\text{EMO} \rightarrow \partial X^I \bar{\partial} X^J$$

$$X^I \quad I = 1, \cdots, 6 \quad \rightarrow \quad T^6$$

$$S = \frac{1}{8\pi}\int d^2\sigma \;\; (G_{IJ}\, \partial X^I \bar{\partial} X^J \; + \; B_{IJ}\, \partial X^I \bar{\partial} X^J)$$

In FFF $\partial X_L^I \;\rightarrow\; y^i\omega^I$

$i\partial X_L^I \;\rightarrow\; U(1)$ current in the Cartan subalgebra

In the fermionic language

$$i\partial X_L^I \;\rightarrow\; J_L^I = y^I\omega^I$$

$$\Rightarrow \;\; \partial X^I\bar{\partial}X^J \rightarrow J_L^I(z)\bar{J}_R(\bar{z})$$

$\rightarrow$ WS Thirring interactions $\quad (R-\frac{1}{R})J_L(z)\bar{J}(\bar{z})$

Thirring interactions vanish at free fermionic point with $R=\frac{1}{R}$

To identify the untwisted moduli in the free fermionic models

$\rightarrow$ find the operators of the form

$$J_L^I(z)\bar{J}_R^J(\bar{z})$$

that are allowed by the orbifold (fermionic) symmetry group

Bosonization

$$\xi_i = \sqrt{\frac{1}{2}}(y_i + i\omega_i) = e^{iX_i}, \eta_i = \sqrt{\frac{i}{2}}(y_i - i\omega_i) = ie^{-iX_i}$$

simlarly for the right-movers

Boundary conditions translate to twists and shifts

$$X^I(z,\bar{z}) = X_L^I(z) + X_R^I(\bar{z})$$

Complex internal coordinates

$$Z_k^{\pm} = (X^{2k-1} \pm iX^{2k}), \quad \psi_k^{\pm} = (\chi^{2k-1} \pm i\chi^{2k}) \quad (k = 1,2,3)$$

Untwisted moduli

$$S = \int d^2z h_{ij}(X) J_L^i(z) \bar{J}_R^j(\bar{z})$$

$J_L^i \sim y^i \omega^i$ $\quad i = 1, \cdots, 6$ are chiral currents of $U(1)_L^6$

$J_R^j \sim \bar{\phi}^j \bar{\phi}^{*j}$ $\quad j = 1, \cdots, 22$ are chiral currents of $U(1)_R^{22}$

$h_{ij} \rightarrow$ scalar components of untwisted moduli

some of these operators are projected out in concrete models

$\Rightarrow$ some of the EMO may not be invariant

Models $\{\mathbf{1}, S\}$ $N=4$ $SO(44)$

$$\frac{SO(6,22)}{SO(6)\times SO(22)}$$ Moduli space

$$\chi^i \otimes \bar{\phi}^a \bar{\phi}^{*^a} |0\rangle$$ moduli fields

$$6 \ \times \ 22$$ scalar fields

$Z_2 \times Z_2$ $\{ \ 1 \ , \ S \ , \ \xi_1 \ , \ \xi_2 \ \} \ + \ \{ \ b_1 \ , \ b_2 \ \}$

$$SO(12)\times E_8 \times E_8 \qquad Z_2 \times Z_2$$

$$\rightarrow \quad SO(4)^3 \times E_6 \times U(1)^2 \times E_8$$

The Thirring interactions that remain invariant are

$$J_L^{1,2}\bar{J}_R^{1,2} \quad ; \quad J_L^{3,4}\bar{J}_R^{3,4} \quad ; \quad J_L^{5,6}\bar{J}_R^{5,6}$$

$$y^{1,2}\omega^{1,2}\bar{y}^{1,2}\bar{\omega}^{1,2} \quad ; \quad y^{3,4}\omega^{3,4}\bar{y}^{3,4}\bar{\omega}^{3,4} \quad ; \quad y^{5,6}\omega^{5,6}\bar{y}^{5,6}\bar{\omega}^{5,6}$$

These moduli are always present in symmetric $Z_2 \times Z_2$ orbifolds in realistic models

$$\{\,1\,,\;S\,,\;\xi_1\,,\;\xi_2\,\}\;\oplus\;\{\,b_1\,,\;b_2\,\}\;\oplus\;\{\,\alpha\,,\;\beta\,,\;\gamma\}$$

$$N=4 \qquad\qquad N=1$$

$$E_8\times E_8 \qquad\qquad Z_2\times Z_2$$

new feature Asymmetric orbifold

the key focus: boundary conditions of the internal fermions

$$\{\,y\,,\;\omega\;\mid\;\bar{y}\,,\;\bar{\omega}\,\}$$

WS fermions that have same B.C. in all basis vectors are paired

pairing of LR fermions $\longrightarrow$ Ising model $\longrightarrow$ symmetric real fermions

pairing of LL & RR fermions $\longrightarrow$ complex fermions $\longrightarrow$ asymmetric

Twisted moduli

(2,2) $b_j \oplus b_j + \xi_2 \quad \longrightarrow \quad (16 \oplus 10 + 1) + 1 = 27 + 1 \Rightarrow$ twisted moduli

(2,0) $b_j + b_j + 2\gamma \quad \longrightarrow 16_{SO(10)} + 16_{SO(16)}$

all twisted moduli are projected out

MINIMAL DOUBLET HIGGS CONTENT (EJPC50)

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^5$	$y^2\bar{y}^2$	$\omega^6\bar{\omega}^6$	$\bar{y}^1\bar{\omega}^5$	$\omega^2\omega^4$	$\omega^1\bar{\omega}^1$	$\omega^3\bar{\omega}^3$	$\bar{\omega}^2\bar{\omega}^4$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}$
α	0	0	0	0	1	0	0	1	0	0	1	1	0	0	1	1	1 1 1 0 0	1	0	0	1 1 0
β	0	0	0	0	0	0	1	1	1	0	0	1	0	1	0	1	1 1 1 0 0	0	1	0	0 0 1
γ	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 0 0

(With Elisa Manno and Cristina Timirgaziu)

SYMMETRIC $\leftrightarrow$ ASYMMETRIC

with respect to b_1 & b_2

$h_1,\ \bar{h}_1,\ D_1,\ \bar{D}_1$, $h_2,\ \bar{h}_2,\ D_2,\ \bar{D}_2$ are projected out

$h_3,\ \bar{h}_3$ remain in the spectrum

$$\lambda_t Q_3 t_3^c \bar{h}_3 \ \text{ with } \ \lambda_t\ O(1)$$

No Phenomenologically viable flat directions