A Note on Kahler Potential of Charged Chiral Matter in F-theory

CERN Theory Institute '12

July 17 (Tue) at CERN

Taizan Watari (IPMU)

based on arXiv:1112.2987 [hep-th] (Phys.Lett.B)

in collaboration with **T. Kawano, Y. Tsuchiya**

IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Motivation

- Kahler potential of squarks and sleptons
  - SUSY breaking
    - Flavor problem, sequestering

  - Coupling with moduli, inflaton sector
- Papers in early days ’02—’04
    - Tend to use N=4 SYM, Het (2,2) model, D3-brane as “matter fields”

My thinking framework

- Coupling unification (not accidentally)
- SU(5) GUT breaking at KK scale < M_{str}
  - Small hierarchy between GUT and Planck
  - Field theory / intuitive analysis for Yukawa

- Up-type Yukawa in E_6 algebra Tatar TW '06
  - Het: how to compute $H^1(Z;V) \times H^1(Z;V) \times H^1(Z;\wedge^2 V^\times)$?
  - M/G_2: up-type Yukawa generally approx. rank-2
  - F-theory: tune cpx str. of g =1 10-repr. curve. Hayashi et.al. 0910.
- Up-type Yukawa from instanton effects.

by many experts

- Comments:
  - Jockers—Louis hep-th/0409098 [Nucl.Phys.B705]

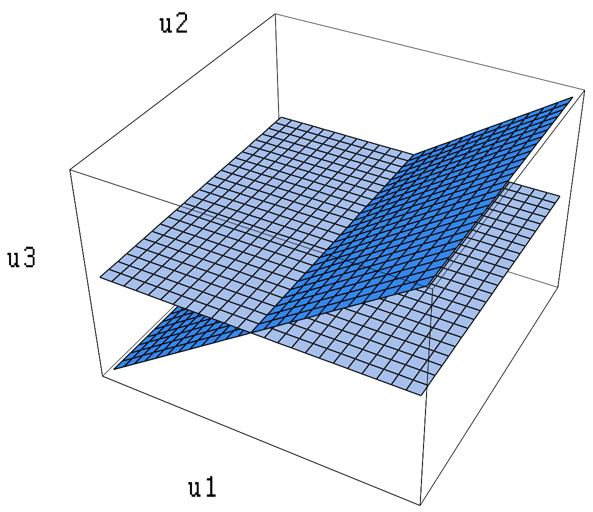
    reduction of Type IIB CY orientifold
    with D7-branes in SUGRA

    unfortunately studied kinetic term of matter
    of the form $H^1(S;\mathbb{C})$ and $H^0(S;K_S)$.

  - IIB or not IIB (F-theory) for Kahler of matter on D7-D7'

In IIB CY orientifolds
w/ matter from D7-D7 intersection

- intersecting D7-D7 (local picture) in CY_3



Kahler potential (kin. term) of such matter

combine disc amplitude for

$$77+77' \to (77')^* \to 77+77'.$$

$$77+77' \to 77'.$$

in CY_3 non-lin sigma model.

0-mode of X^μ in the disc amplitudes

only in $\mu = 0,1,2,3$ & (matter curve). → localized expression

In F-theory

$(X_4, C_{(3)})$

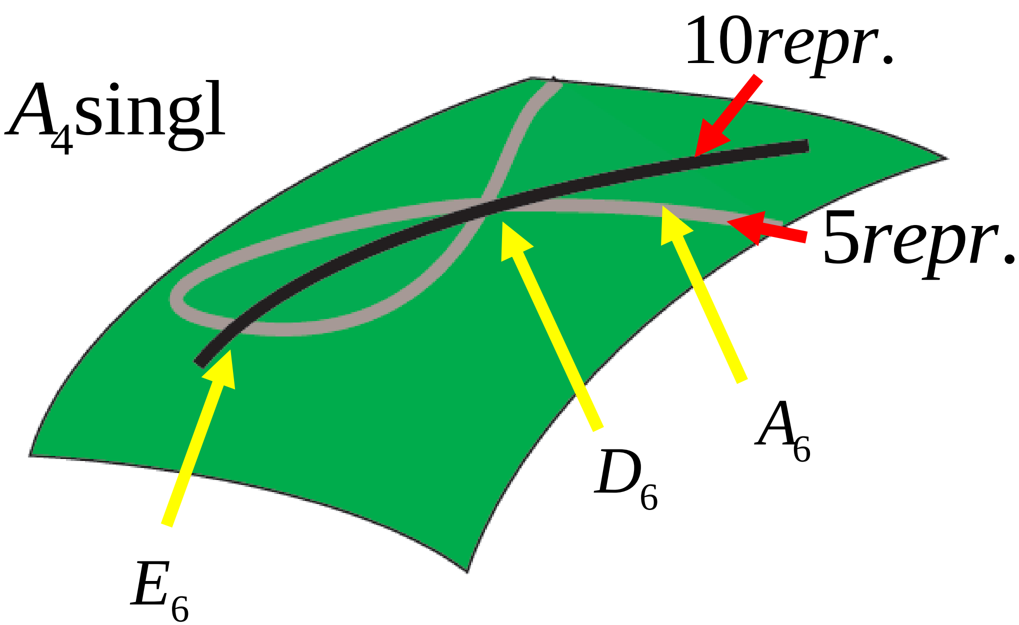
approx. for local geometry

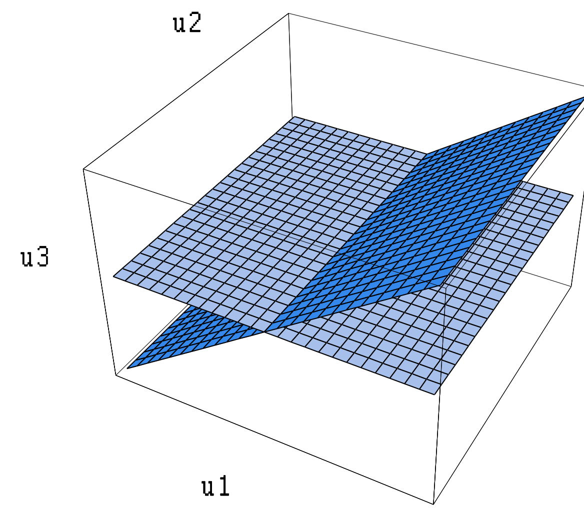
Vafa, Morrison-Vafa, Bershadsky et.al. '96

gauge theory on 7+1-dim.

Katz-Vafa '96

Hitchin map recovers partial information





7-brane intersection

scalar (Higgs) field config.

matter kinetic term

- Limitation for now
  - dim. reduction
  - as fcn. of moduli vev

$$\omega \wedge \bar{D}'\bar{\psi} + \frac{|\alpha|^2}{2}\bar{\tilde{\varphi}}\,\bar{\chi} = 0,$$
$$\bar{\partial}\bar{\tilde{\psi}} = 0,$$
$$\bar{\partial}\bar{\chi} - i\bar{\tilde{\varphi}}\,\bar{\psi} = 0.$$

bg. cpx str., $\tilde{\varphi}$
Kahler ω
flux $C_{(3)}$ $\tilde{D}'$

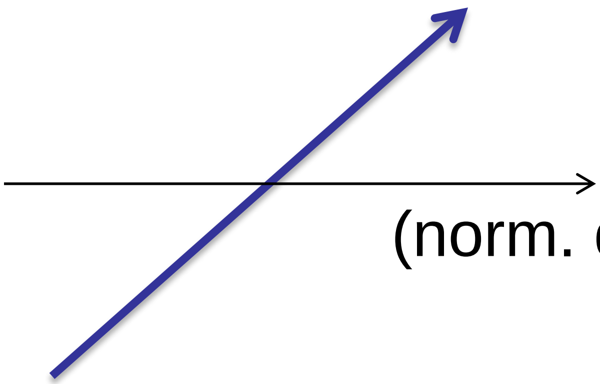
0-mode eq. $(\psi, \chi)_\rho$ in hol. frame

$$K_{\text{eff.}} = K^{(R_I)}_{i\bar{j}}\ \Phi^\dagger_{(R_I);\bar{j}} e^{\rho_{R_I}(V)} \Phi_{(R_I);i} + \cdots,$$
$$K^{(R_I)}_{i\bar{j}} \simeq \frac{c_{(U_I,R_I)} M_*^4}{4\pi} \int_U i\omega \wedge [\bar{\psi}_{\bar{j}} \wedge \psi_i] + \frac{|\alpha|^2}{2}[\overline{\chi_{\bar{j}}} \wedge \chi_i].$$

$U \subset S_{GUT}$

An easy case (Beasley Heckman Vafa 0802.)

- single component matter 0-mode $(\psi, \chi)_\rho$



$$i\psi_{\bar{n}} = -\frac{1}{\sqrt{h_{t\bar{t}}}} e^{-i\mathrm{Arg}(F)} \exp\left[-|F|\frac{|z_n|^2}{\sqrt{h_{t\bar{t}}}}\right] f_{(R_I);i;a}(z_t),$$

$$(2\alpha\chi_{tn}) = \exp\left[-|F|\frac{|z_n|^2}{\sqrt{h_{t\bar{t}}}}\right] f_{(R_I);i;a}(z_t),$$

$$\Delta K_{i\bar{j}}^{(R_I)} \propto \omega_{\bar{c}_{(R_I)}} \left[(2\alpha(\bar{f}_j)_{uv})^* \frac{\sqrt{h^{\bar{u}u}} M_*^4}{2|F|} (2\alpha \bar{f}_i)_{uv}\right]$$

BHV-I 0802. Hayashi et.al. 0910.

- 1-comp. approx. (BHV-1 0802.)

valid everywhere only if [matter curve] $\sim K_S$ (effective)

Hayashi et.al. 0805., 0901.

Reminder: how to describe 0-modes

$$H^0\left(\bar{\tilde{c}}_{(R_I)}; \mathcal{L}_{G^{(4)}} \otimes K^{1/2}_{\bar{\tilde{c}}_{(R_I)}}\right) = Ext^1_{\mathbb{K}_S}\left(i_*\mathcal{O}_S, i_*\mathcal{N}_{U_I}\right)$$

$$\mathrm{Coker}\left[(\varphi\times) : H^0(\widetilde{C}_{U_I}; \widetilde{\mathcal{N}}_{U_I}) \to H^0(\widetilde{C}_{U_I}; \widetilde{\mathcal{N}}_{U_I} \otimes \tilde{\pi}^*_{C_{U_I}}(K_U))\right].$$

$$\mathrm{Coker}\left(H^0(U_a; \rho_{U_I}(V)) \to H^0(U_a; \rho_{U_I}(V) \otimes K_S)\right).$$

single comp. fcn
on "matter curves"

Curio '98, Diaconescu Ionesei '98
Donagi et.al. '04, Blumenhagen et.al. '06
Hayashi et.al. 0805.

single comp. fcn on "spec. surface"

or multi-comp. fcn on GUT 7-brane

Hayashi et.al. 0901., 0910.
Ceccotti et.al. 0910.

Reminder: how to describe 0-modes

$$H^0\left(\bar{\bar{\mathcal{C}}}_{(R_I)}; \mathcal{L}_{G^{(4)}} \otimes K^{1/2}_{\bar{\bar{\mathcal{C}}}_{(R_I)}}\right) = Ext^1_{\mathbb{K}_S}(i_*\mathcal{O}_S, i_*\mathcal{N}_{U_I})$$

$$\mathrm{Coker}\left[(\varphi\times): H^0(\widetilde{C}_{U_I};\widetilde{\mathcal{N}}_{U_I}) \to H^0(\widetilde{C}_{U_I};\widetilde{\mathcal{N}}_{U_I}\otimes \tilde{\pi}^*_{C_{U_I}}(K_U))\right].$$

$$\mathrm{Coker}\left(H^0(U_a;\rho_{U_I}(V)) \to H^0(U_a;\rho_{U_I}(V)\otimes K_S)\right).$$

$$\tilde{\psi} = \bar{\partial}\ {}^{\exists}\tilde{\Lambda},$$

$$\tilde{\chi} = i\tilde{\varphi}\tilde{\Lambda} + \tilde{f}.$$

$$[\tilde{f}]\ \mathrm{mod}\ + i\tilde{\varphi}\tilde{k}$$

$$\bar{\partial}\tilde{\psi} = 0,$$

$$\bar{\partial}\tilde{\chi} - i\tilde{\varphi}\tilde{\psi} = 0.$$

0-mode eq. (multi comp.)

fcn $k(\xi)$ on $(\xi^2 - v = 0)$

$$k(\xi) = k_+(v) + \xi k_-(v).$$

$$k(v) = \begin{pmatrix} k_+(v) \\ k_-(v) \end{pmatrix}.$$

Hayashi et.al. 0910.
appendix C

$$[\xi\times] = [\varphi\times]$$

$$[\tilde{\varphi}\times] = \begin{pmatrix} & v \\ 1 & \end{pmatrix}\times$$

regular spectral cover (def.)
Donagi et.al. math/0005132

kinetic term localized on matter curve

Kawano Tsuchiya TW '11

$$K_{i\bar{j}}^{(R_I)} \simeq \frac{M_*^4}{4\pi}\int_U i\omega\wedge\left[(\partial\tilde{\Lambda}_{\bar{j}}^\dagger)\wedge H(\bar{\partial}\tilde{\Lambda}_i)\right]+\frac{|\alpha|^2}{2}\left[\left\{\tilde{f}_{\bar{j}}^\dagger - i\tilde{\Lambda}_{\bar{j}}^\dagger(\tilde{\varphi})^\dagger\right\}\wedge H\left\{\tilde{f}_i+i\tilde{\varphi}\tilde{\Lambda}_i\right\}\right]$$

$$= \frac{M_*^4}{4\pi}\int_U \partial\left(i\omega\wedge\left[\tilde{\Lambda}_{\bar{j}}^\dagger\wedge H(\bar{\partial}\tilde{\Lambda}_i)\right]\right)+\frac{|\alpha|^2}{2}\left[\tilde{f}_{\bar{j}}^\dagger\wedge H\left\{\tilde{f}_i+i\tilde{\varphi}\tilde{\Lambda}_i\right\}\right]$$

$$+ \frac{M_*^4}{4\pi}\int_U -i\omega\wedge\left[\tilde{\Lambda}_{\bar{j}}^\dagger\wedge\partial\left(H(\bar{\partial}\tilde{\Lambda}_i)\right)\right]-i\frac{|\alpha|^2}{2}\left[\tilde{\Lambda}_{\bar{j}}^\dagger(\tilde{\varphi})^\dagger\wedge H\left\{\tilde{f}_i+i\tilde{\varphi}\tilde{\Lambda}_i\right\}\right].$$ ⟶ e.o.m.

$$K_{i\bar{j}}^{(R_I)} \simeq \frac{M_*^4}{4\pi}\int_U \frac{|\alpha|^2}{2}\left[\tilde{f}_{\bar{j}}^\dagger\wedge H\left\{\tilde{f}_i+i\tilde{\varphi}\tilde{\Lambda}_i\right\}\right],$$

$$= \frac{M_*^4}{4\pi}\int_U \frac{|\alpha|^2}{2}\left[((\tilde{f}_{\bar{j}})_{uv})^\dagger(\tilde{\varphi}_{uv})^{\dagger-1}(\tilde{\varphi})^\dagger\wedge H\left\{\tilde{f}_i+i\tilde{\varphi}\tilde{\Lambda}_i\right\}\right],$$

$$= -\frac{M_*^4}{4\pi}\int_U \left[((\tilde{f}_{\bar{j}})_{uv})^\dagger(\tilde{\varphi}_{uv})^{\dagger-1}\omega\wedge\partial\left(H\tilde{\psi}_i\right)\right],$$

$$= \frac{M_*^4}{4\pi}\int_U \left[\underline{\partial\left\{((\tilde{f}_{\bar{j}})_{uv})^\dagger(\tilde{\varphi}_{uv})^{\dagger-1}\right\}}\,\omega\wedge H\tilde{\psi}_i\right];$$

localized on $\det(\tilde{\varphi})=0$ locus.

Residue: $\tilde{f}\,|_{\bar{c}_{(R)}}$

exploiting holomorphicity
like in F-term (Yukawa): Cecotti et.al. 0910.

Kawano Tsuchiya TW '11

effort to get more explicit expression

using single comp.[$\tilde{f}$]

- a slice of matter curve with $\xi - Fv \sim 0.$
- a slice of matter curve with $\xi^2 - c^2 v = 0.$

$$\Delta K^{(R_I)}_{i\bar{j}} \simeq \omega_{\bar{c}_{(R_I)}} \left[(2\alpha(\bar{f}_j)_{uv})^* \frac{\sqrt{h^{\bar{u}u}}|e_{uv}|M_*^2}{2|c|^2} (2\alpha(\bar{f}_i)_{uv}) \right] \sqrt{H_{0*}} \left(-\frac{A_1}{A_0} \right)_* ;$$

Hayashi et.al. 0910. appendix

$$\left(-\frac{A_1}{A_0} \right)_* = \frac{1}{\sqrt{H_{0*}}}.$$

Cecotti Cordova Heckman Vafa 1010.
(nature of Painleve III eq.)

$$\Delta K^{(R_I)}_{i\bar{j}} = \omega_{\bar{c}_{(R_I)}} \left[(2\alpha(\bar{f}_j)_{uv})^* \frac{\sqrt{h^{\bar{u}u}}|e_{uv}|M_*^2}{2|c|^2} (2\alpha(\bar{f}_i)_{uv}) \right] .$$

cf.

$$\Delta K^{(R_I)}_{i\bar{j}} \propto \omega_{\bar{c}_{(R_I)}} \left[(2\alpha(\bar{f}_j)_{uv})^* \frac{\sqrt{h^{\bar{u}u}}M_*^4}{2|F|} (2\alpha\bar{f}_i)_{uv} \right]$$

Presumably somehow in the form of norm of sections of $\mathcal{L}_G \otimes K^{1/2}_{\bar{c}_{(R)}}$