The Lambda parameter of two flavor QCD

Björn Leder
Bergische Universität Wuppertal

Nucl. Phys. B 865 (2012) 397-429
Fritzsch, FK, BL, Marinkovic, Schaefer, Sommer, Virotta

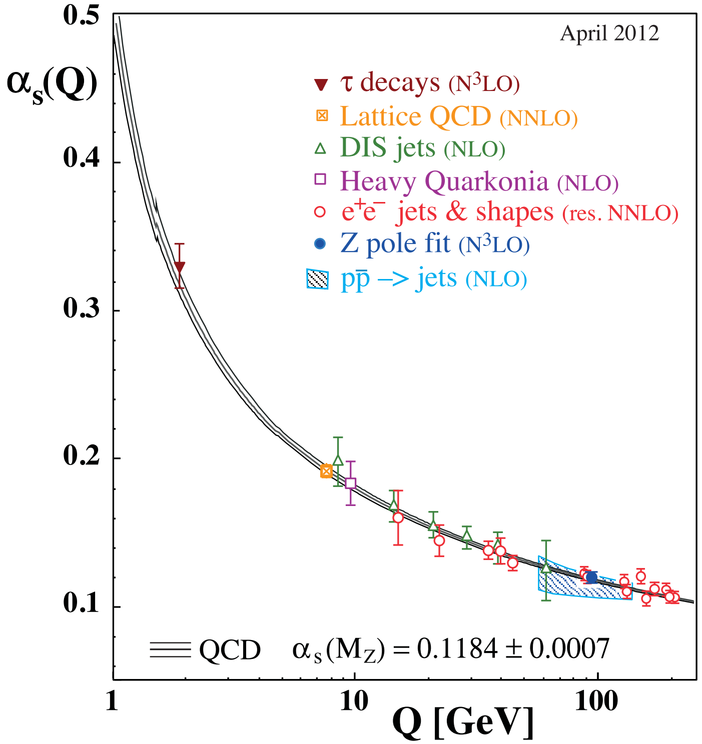
XXXII Physics in Collision 2012
September 12 - 15, 2012, Štrbské Pleso, Slovakia

Determining α_{S}

- $\alpha_{\overline{\mathsf{MS}}}(\mu)$ only defined perturbatively
- not directly accessible in experiment
  $\rightarrow$ match to perturbation theory (PT)
- uncertainties vanish only as $\mu \rightarrow \infty$

… or the Lambda parameter

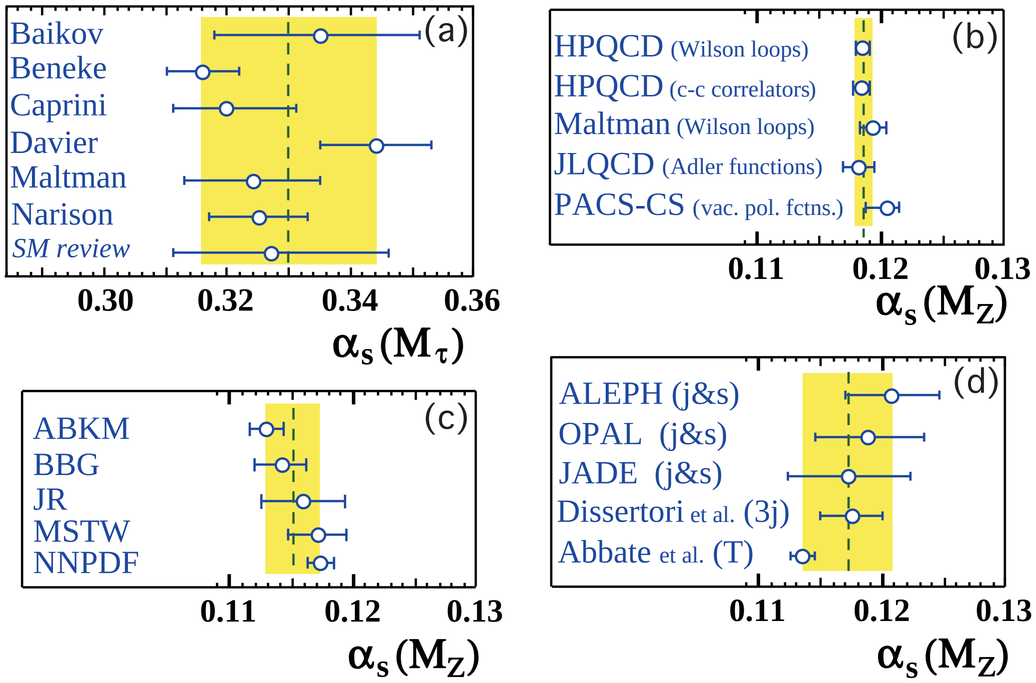
- scale independent integration constant (but scheme dependent)
- well defined from asymptotic behaviour of the coupling



[PDG 2012]

$$\frac{\Lambda}{\mu} = \left(b_0 \bar{g}(\mu)^2\right)^{-b_1/(2b_0^2)} \mathrm{e}^{-1/(2b_0\bar{g}(\mu)^2)} \exp\left\{-\int_0^{\bar{g}(\mu)} \mathrm{d}x \left[\frac{1}{\beta_{\mathrm{S}}(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right]\right\}$$

Determining α_S



[PDG 2012]

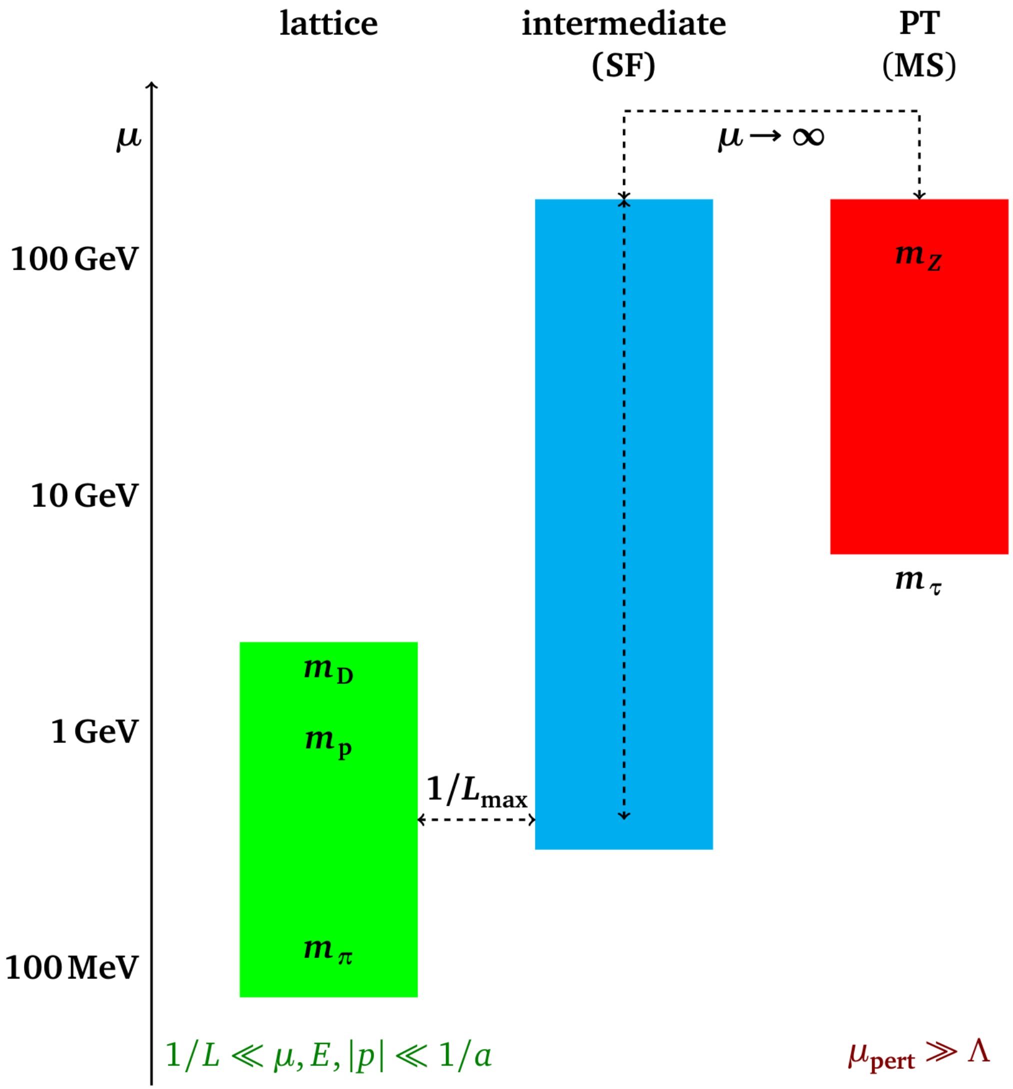
… in lattice QCD

- error of world average dominated by one lattice result ($N_f = 2 + 1$, PT matching)
- use lattice QCD to its full potential
  $\rightarrow$ fully non-perturbative (NP) determination

Minimal assumptions:

asymptotic freedom, existence of continuum limit of lattice theory, NP corrections vanish for $\mu \to \infty$

Connecting low and high energies non-perturbatively



Connecting low and high energies non-perturbatively

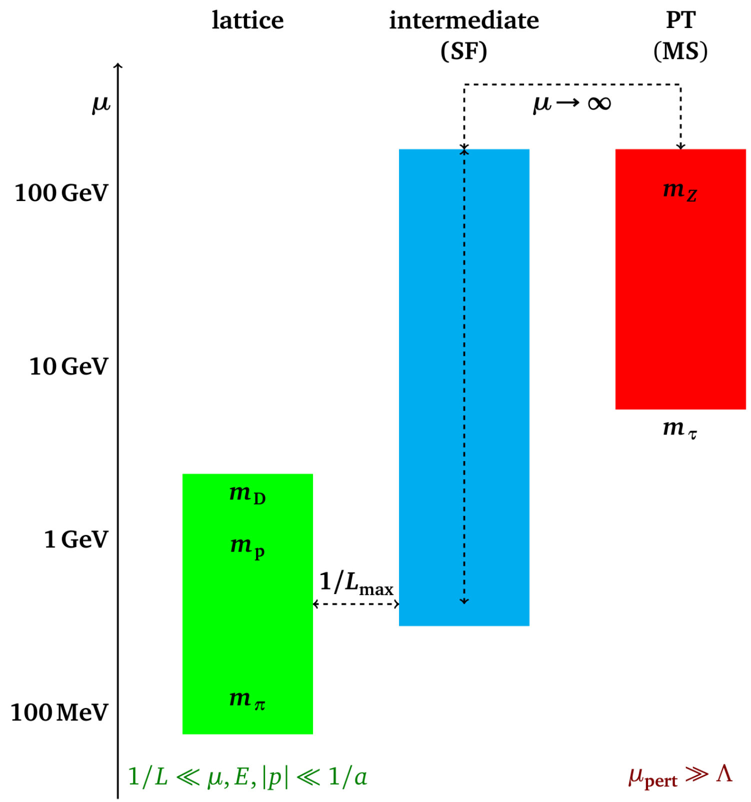
not possible on single lattice since $L/a \lesssim 100$

- $\rightarrow$ use intermediate coupling that scales with box size: $\mu = 1/L$ (Schrödinger functional (SF))
- $\rightarrow$ bridge low/high energies by several steps of scaling by factor two: $L \rightarrow L/2$
- $\rightarrow$ continuum scheme, yields integrated version of β-function

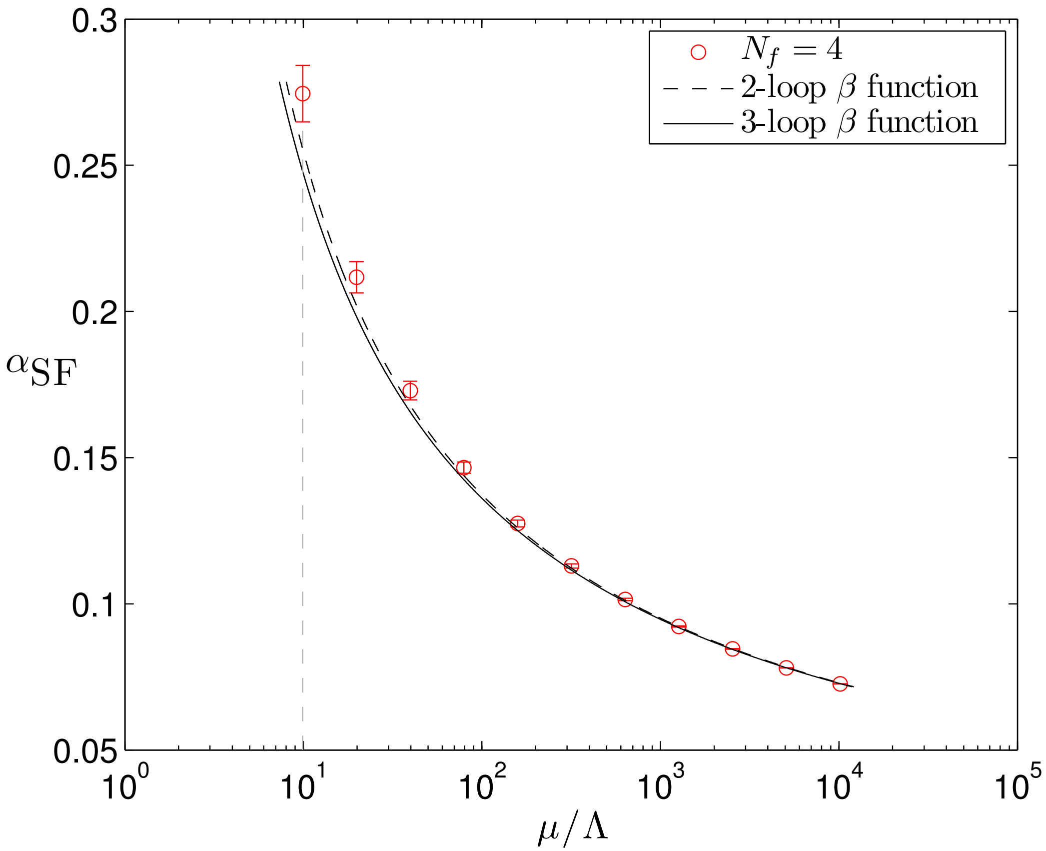
Master formula:

$$\frac{\Lambda^{(5)}_{\overline{\mathrm{MS}}}}{f_{\mathrm{K}}} = \frac{1}{f_{\mathrm{K}} L_{\mathrm{max}}} \times \frac{L_{\mathrm{max}}}{L_k} \times \Lambda^{(4)}_{\mathrm{SF}} L_k \times \frac{\Lambda^{(4)}_{\overline{\mathrm{MS}}}}{\Lambda^{(4)}_{\mathrm{SF}}} \times \frac{\Lambda^{(5)}_{\overline{\mathrm{MS}}}}{\Lambda^{(4)}_{\overline{\mathrm{MS}}}}$$

$\Lambda^{(4)}_{\overline{\mathrm{MS}}}/\Lambda^{(4)}_{\mathrm{SF}} = 2.9065$,
$L_{\mathrm{max}} \approx 0.5$ fm, $L_k = 2^{-k} L_{\mathrm{max}}$,
Kaon decay constant $f_{\mathrm{K}} = 155$ MeV



Connecting low and high energies non-perturbatively



[ALPHA 2010]

- NP running yields $\alpha_{\mathsf{SF}}(1/L_{\mathsf{max}}) \mapsto \alpha_{\mathsf{SF}}(1/L_k)$
- get $\Lambda_{\mathsf{SF}}^{(4)} L_k$ at $\alpha_{\mathsf{SF}}(1/L_k) \approx 0.1$ from PT

Flavour dependence and other lattice determinations

Perturbation theory states: $\Lambda_{\overline{\rm MS}}^{(N_f)} < \Lambda_{\overline{\rm MS}}^{(N_f-1)}$ (precise for $N_f = 5$, i.e., decoupling of bottom)

N_f	$\Lambda_{\overline{\rm MS}}$	experiment	theory / Ref.
0	$238(19)$ MeV	$m_{\rm K}$, $K \to \mu\nu_\mu$, $K \to \pi\mu\nu_\mu$	[ALPHA 1999]
2	$310(20)$ MeV	$m_{\rm K}$, $K \to \mu\nu_\mu$, $K \to \pi\mu\nu_\mu$	[ALPHA 2012]
5	sub-percent level uncertainty for $\alpha_{\overline{\rm MS}}(M_Z)$ envisaged		
5	$212(12)$ MeV	world average	perturb. theory [Bethke 2011]

$\Rightarrow \Lambda_{\overline{\rm MS}}^{(2)} > \Lambda_{\overline{\rm MS}}^{(0)}$ is a non-perturbative effect

Comparison in units of r_0 (avoids ambiguity in scale setting), selection:

$\Lambda_{\overline{\rm MS}}^{(2)} r_0$	method	Ref.
$0.790(51)$	non-perturbative running	[ALPHA 2012]
$0.59(2)\left(^{+4}_{\ 0}\right)$	Adler functions, perturbative matching	[JLQCD 2009]
$0.658(55)$	static potential, perturbative matching	[ETMC 2011]