

# NNLO Parton Distributions and $\alpha_s(M_Z^2)$ for the LHC

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[[arXiv:1202.2700](https://arxiv.org/abs/1202.2700), Phys. Rev. D (2012) and in preparation]

Physics in Collision

Strbske Pleso, Slovakia

September 2012



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# References

- ABKM09: S. Alekhin, JB, S. Klein, S.-O. Moch [arXiv:0908.2766](#)
- ABM10: S. Alekhin, JB, S.-O. Moch [arXiv:1007.3657](#)
- ABM11: S. Alekhin, JB, S.-O. Moch [arXiv:1202.2281](#)
- NNLO benchmark cs at the Terascale: S. Alekhin, JB, P. Jimenez-Delgado, S.-O. Moch, E. Reya, [arXiv:1011.6259](#)
- Fixed target data and Higgs Prod. Rates: S. Alekhin, JB, S.-O. Moch [arXiv:1101.5261](#)

References below will mostly be used in terms of acronyms, like "MSTW08" etc.



# Introduction

- Precision determinations of **parton distribution functions** (PDFs) and  $\alpha_s(M_Z^2)$  are currently being performed at **NNLO** ( $\alpha_s^3$ ).
- NLO fits suffer from scale uncertainties being **too large**.
- The **heavy flavor corrections** are available in NLO + threshold corrections and the calculation of the NNLO corrections is making progress.
- Sensitive data, capable to constrain the known PDFs better, have to be selected, rather than performing global analyses using data with problematic systematics.
- Current data: DIS World data (including H1+ZEUS combined); Di-muon data ( $s$ ); Drell-Yan data ( $\bar{d} - \bar{u}$ );  $pp$ -jet data: LHC data will take over very soon; Likewise:  $W^\pm$ ,  $Z$  off-resonance Drell-Yan at LHC.
- $\alpha_s$  and **HT terms** have to be fitted along with the non-pert. parameters of the PDFs.



# Introduction

- Heavy flavor treatment: BMSN-interpolation from threshold to asymptotia; **observe all relations implied by the RGE Bierenbaum, JB, Klein, 2009.**
- Deuteron wave function corrections and Trajet Mass corrections are mandatory.
- Characterize the fit publishing the covariance matrices.
- **Soft Resummation:** heavy flavor threshold; NS Wilson coefficients beyond 3-loop; Effect on the HT-terms.
- We will compare most of the results with those obtained by other fitting groups.
- Predictions are made for main scattering cross sections at Tevatron and the LHC.



# The Data

	Experiment	NDP	$\chi^2(\text{NNLO})$	$\chi^2(\text{NLO})$
DIS inclusive	H1&ZEUS	486	537	531
	H1	130	137	132
	BCDMS	605	705	695
	NMC	490	665	661
	SLAC-E-49a	118	63	63
	SLAC-E-49b	299	357	357
	SLAC-E-87	218	210	219
	SLAC-E-89a	148	219	215
	SLAC-E-89b	162	133	132
	SLAC-E-139	17	11	11
SLAC-E-140	26	28	29	
Drell-Yan	FNAL-E-605	119	167	167
	FNAL-E-866	39	52	55
DIS di-muon	NuTeV	89	46	49
	CCFR	89	61	62
Total		3036	3391	3378

Tabelle 2: The value of  $\chi^2$  obtained in the NNLO and NLO fits for different data sets.



# Deuteron Corrections

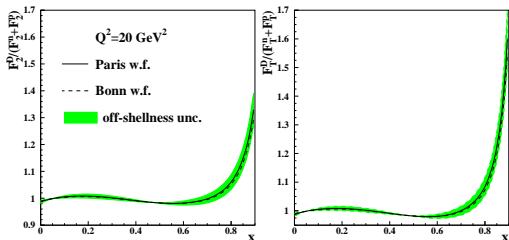


Figure 1: The ratio of the deuteron structure function  $F_2$  (left) and  $F_T$  (right) with account of the Fermi-motion and off-shellness effects of [Kulagin 2004] calculated for the Paris potential of (solid) and the Bonn potential (dashes) at the momentum transfer of  $20 \text{ GeV}^2$  to the sum of those for free proton and neutron versus  $x$ . The shaded area around the solid line gives the uncertainty due to a variation of the off-shell effects by 50%. The calculations are performed at NNLO QCD accuracy using the PDFs and the twist-4 terms obtained by ABKM.



# The Fit Parameters

$$\begin{aligned}
 Q_0^2 &= 9\text{GeV}^2 \\
 xq_v(x, Q_0^2) &= A_v x^{(a_v + P_v(x))} (1-x)^{b_v} \\
 xU_s(x, Q_0^2) &= A_{Us} x^{(a_{Us} + P_{Us}(x))} (1-x)^{b_{Us}} \\
 \Delta(x, Q_0^2) &= x\bar{d} - x\bar{u} = A_\Delta x^{a_\Delta} (1-x)^{b_\Delta} \\
 xS(x, Q_0^2) &= A_s x^{a_s} (1-x)^{b_s} \\
 xg(x, Q_0^2) &= A_g x^{(a_g + P_g(x))} (1-x)^{b_g}
 \end{aligned}$$

	$a$	$b$	$\gamma_1$	$\gamma_2$	$\gamma_3$	$A$
$u_v$	$0.712 \pm 0.081$	$3.637 \pm 0.138$	$0.593 \pm 0.774$	$-3.607 \pm 0.762$	$3.718 \pm 1.148$	
$d_v$	$0.741 \pm 0.157$	$5.123 \pm 0.394$	$1.122 \pm 1.232$	$-2.984 \pm 1.077$		
$u_s$	$-0.363 \pm 0.035$	$7.861 \pm 0.433$	$4.339 \pm 1.790$		$0.0280 \pm 0.0036$	$0.0808 \pm 0.0122$
$\Delta$	$0.70 \pm 0.28$	$11.75 \pm 1.97$	$-2.57 \pm 3.12$			$0.316 \pm 0.385$
$s$	$-0.240 \pm 0.055$	$7.98 \pm 0.65$				$0.085 \pm 0.017$
$g$	$-0.170 \pm 0.012$	$10.71 \pm 1.43$	$4.00 \pm 4.21$			

Tabelle 1: The parameters of the PDFs in and their  $1\sigma$  errors obtained in the scheme with  $n_f = 5$  flavors.





## Covariance Matrix

Table 12: The covariance matrix for the PDF parameters,  $\alpha_s$ ,  $m_c$  and  $m_b$ .

	$a_u$	$b_u$	$\gamma_{1,u}$	$\gamma_{2,u}$	$a_d$	$b_d$	$A_\Delta$	$b_\Delta$	$A_{us}$	$a_{us}$	$b_{us}$	$a_g$	$b_g$	$\gamma_{1,g}$
$a_u$	1.0000	0.9692	0.9787	-0.7929	0.7194	0.5279	-0.1460	-0.1007	0.7481	0.6835	-0.4236	-0.2963	0.3391	0.3761
$b_u$		1.0000	0.9396	-0.7244	0.6792	0.4939	-0.1146	-0.1099	0.7404	0.6840	-0.4146	-0.3138	0.3464	0.3738
$\gamma_{1,u}$			1.0000	-0.8940	0.6506	0.4646	-0.1865	-0.0539	0.6728	0.6093	-0.4799	-0.2755	0.3441	0.3717
$\gamma_{2,u}$				1.0000	-0.4102	-0.2267	0.2357	-0.0182	-0.4075	-0.3495	0.4543	0.1713	-0.3156	-0.3149
$a_d$					1.0000	0.8827	-0.2155	-0.1964	0.6875	0.6435	-0.3030	-0.3354	0.2635	0.3500
$b_d$						1.0000	-0.2462	-0.0979	0.5359	0.5099	-0.2957	-0.3443	0.3157	0.3763
$A_\Delta$							1.0000	-0.2068	-0.0689	-0.0698	0.2381	-0.0168	0.0384	0.0453
$b_\Delta$								1.0000	0.1015	0.1279	-0.4146	-0.0852	-0.1185	-0.0892
$A_{us}$									1.0000	0.9884	-0.4678	-0.4679	0.1961	0.2504
$a_{us}$										1.0000	-0.4520	-0.5195	0.1982	0.2596
$b_{us}$											1.0000	0.1436	0.0444	-0.0180
$a_g$												1.0000	-0.6289	-0.7662
$b_g$													1.0000	0.9392
$\gamma_{1,g}$														1.0000



## Covariance Matrix

Table 13: The covariance matrix for the PDF parameters,  $\alpha_s$ ,  $m_c$  and  $m_b$ .

	$\alpha_s(\mu_0)$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	$A_s$	$b_s$	$a_s$	$\gamma_{3,u}$	$m_c(m_c)$	$\gamma_{3,us}$	$m_b(m_b)$	$a_\Delta$
$a_u$	-0.0435	0.0000	-0.8480	0.6008	0.1535	-0.0034	-0.0437	-0.0355	0.8111	0.0796	-0.4797	0.0044	-0.1718
$b_u$	-0.1251	0.0316	-0.8375	0.5537	0.1806	0.0008	-0.0345	-0.0276	0.7001	0.0625	-0.4889	-0.0005	-0.1452
$\gamma_{1,u}$	-0.0849	-0.0637	-0.8133	0.5422	0.1667	-0.0324	-0.0671	-0.0638	0.8948	0.0726	-0.4033	0.0075	-0.2028
$\gamma_{2,u}$	0.0920	0.1659	0.5760	-0.3308	-0.2276	0.0799	0.0966	0.1098	-0.9749	-0.0631	0.1728	-0.0142	0.2353
$a_d$	-0.0321	-0.0137	-0.7618	0.9630	-0.1842	0.0007	-0.0414	-0.0167	0.4878	0.0227	-0.4735	-0.0078	-0.2088
$b_d$	-0.1666	-0.1167	-0.6060	0.9351	-0.5969	-0.0064	-0.0249	-0.0203	0.3007	-0.0045	-0.3782	-0.0132	-0.2121
$A_\Delta$	0.0206	0.8718	0.1649	-0.2544	0.1916	-0.0232	-0.0212	-0.0294	-0.2398	0.0202	0.0667	0.0034	0.9721
$b_\Delta$	0.0086	-0.6291	-0.1067	-0.1834	-0.1103	0.0594	0.0577	0.0711	0.0052	-0.0063	-0.1768	-0.0083	-0.0662
$A_{us}$	0.0043	-0.0481	-0.8662	0.5862	0.0768	-0.0341	-0.0659	-0.0493	0.4485	0.1559	-0.8164	-0.0008	-0.0417
$a_{us}$	-0.0459	-0.0650	-0.8255	0.5493	0.0606	-0.0119	-0.0441	-0.0255	0.3870	0.0940	-0.8628	-0.0055	-0.0375
$b_{us}$	-0.0382	0.3783	0.7032	-0.3288	0.1278	-0.0734	-0.0445	-0.0807	-0.4262	-0.0100	0.3911	0.0040	0.1782
$a_g$	0.3785	0.0061	0.3050	-0.3280	0.1338	0.0936	0.0718	0.1165	-0.1744	-0.0137	0.4886	0.0323	-0.0360
$b_g$	-0.6085	0.1017	-0.0873	0.2827	-0.2104	-0.0543	-0.0114	-0.1223	0.2973	0.1560	-0.1337	0.0141	0.0066
$\gamma_{1,g}$	-0.4642	0.1021	-0.1778	0.3605	-0.1962	-0.0708	-0.0396	-0.1230	0.3132	0.0425	-0.1977	0.0071	0.0201



## Covariance Matrix

Table 14: The covariance matrix for the PDF parameters,  $\alpha_s$ ,  $m_c$  and  $m_b$ .

	$\alpha_s(\mu_0)$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	$A_s$	$b_s$	$a_s$	$\gamma_{3,u}$	$m_c(m_c)$	$\gamma_{3,us}$	$m_b(m_b)$	$a_\Delta$
$\alpha_s(\mu_0)$	1.0000	0.0176	-0.0394	-0.0798	0.2357	-0.0018	-0.0982	-0.0075	-0.0291	0.1904	0.0676	0.0562	0.0136
$\gamma_{1,\Delta}$		1.0000	0.1183	-0.0802	0.2640	-0.0427	-0.0489	-0.0550	-0.1595	0.0193	0.0985	0.0069	0.7657
$\gamma_{1,us}$			1.0000	-0.6753	-0.0493	-0.0525	0.0158	-0.0445	-0.6039	-0.0656	0.6590	0.0017	0.1487
$\gamma_{1,d}$				1.0000	-0.4041	-0.0213	-0.0513	-0.0366	0.4145	0.0148	-0.3931	-0.0086	-0.2284
$\gamma_{2,d}$					1.0000	0.0308	-0.0016	0.0326	0.1801	0.0276	-0.0510	0.0111	0.1212
$A_s$						1.0000	0.8570	0.9749	-0.0664	-0.0206	-0.4355	0.0017	-0.0139
$b_s$							1.0000	0.8730	-0.0894	-0.0706	-0.3708	0.0005	-0.0127
$a_s$								1.0000	-0.0967	-0.1234	-0.4403	-0.0050	-0.0172
$\gamma_{3,u}$									1.0000	0.0674	-0.2082	0.0153	-0.2378
$m_c(m_c)$										1.0000	-0.0010	0.0505	0.0141
$\gamma_{3,us}$											1.0000	0.0083	0.0276
$m_b(m_b)$												1.0000	0.0006
$a_\Delta$													1.0000



## HERA NC Data

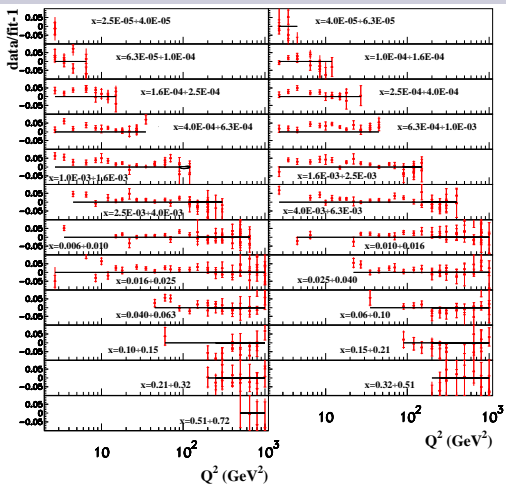


Figure 2: The pulls versus momentum transfer  $Q^2$  for the HERA neutral-current inclusive DIS cross section data of [H1ZEUS09] binned in  $x$  with respect to our NNLO fit. The data points with different inelasticity  $y$  still may overlap in the plot. The inner bars show statistical errors in data and the outer bars the statistical and systematic errors combined in quadrature.



## HERA CC Data

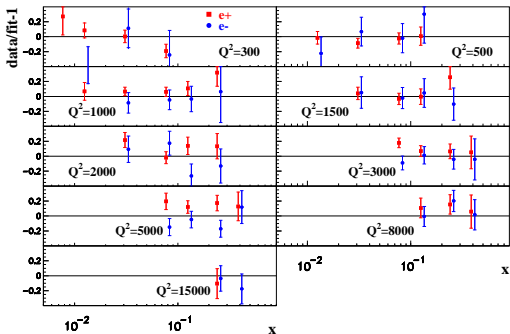


Figure 3: The same as Fig. 2 for the pulls of the HERA charged-current inclusive DIS cross section data of [H1ZEUS09] binned in the momentum transfer  $Q^2$  in units of  $\text{GeV}^2$  versus  $x$  (squares: positron beam; circles: electron beam).



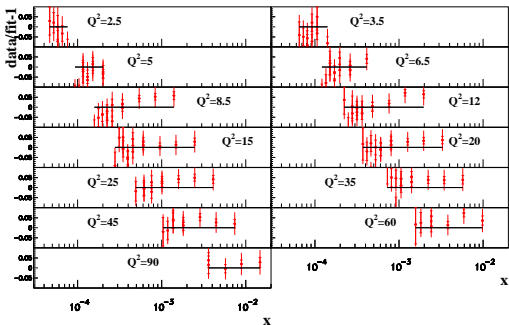
HERA Low  $Q^2$  Data

Figure 4: The same as Fig. 2 for the pulls of the H1 neutral-current inclusive DIS cross section data of [H12010] binned in the momentum transfer  $Q^2$  in units of  $\text{GeV}^2$  versus  $x$ .



## BCDMS Data

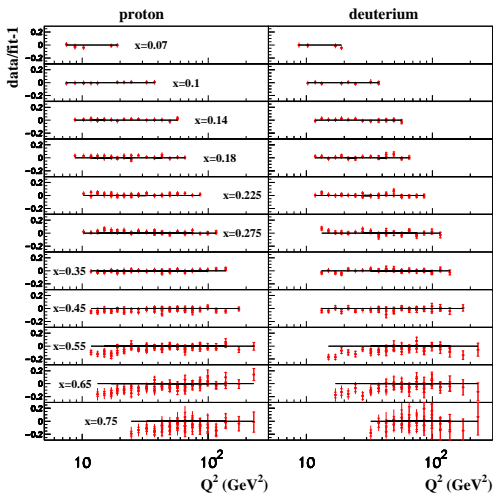


Figure 5: The same as Fig. 2 for the pulls of the BCDMS inclusive DIS cross section data for the proton target (left) and for the deuterium target (right).



## NMC Data

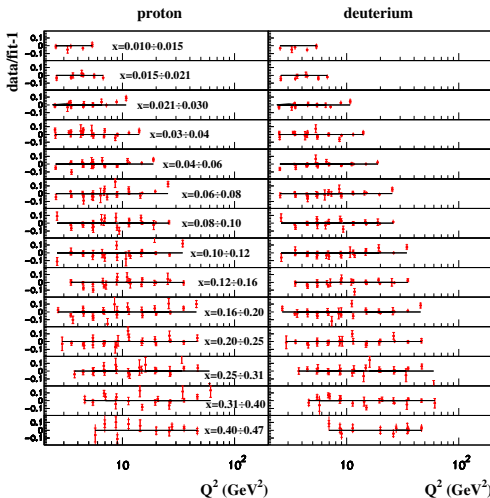


Figure 6: The same as Fig. 2 for the pulls of the NMC inclusive DIS cross section data for the proton target (left) and for the deuterium target (right).





## SLAC Data

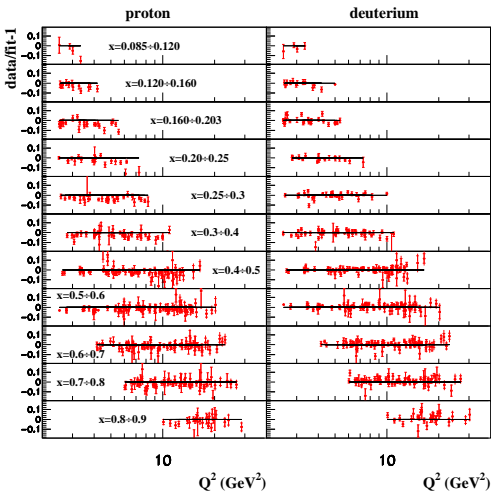


Figure 7: The same as Fig. 2 for the pulls of the SLAC inclusive DIS cross section data for the proton target (left) and for the deuterium target (right).



## Di-Muon Data

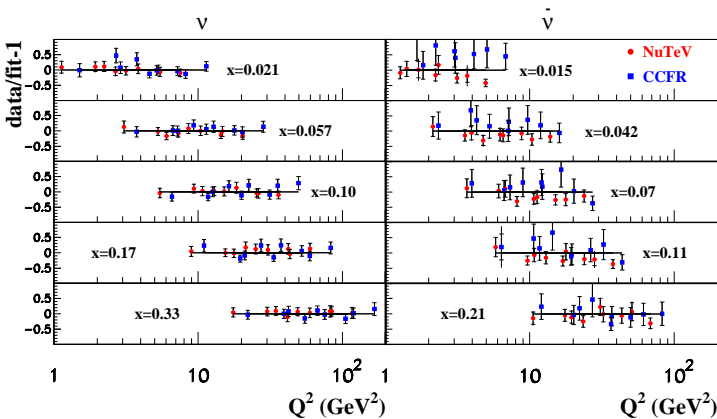


Figure 8: The same as Fig. 2 for the pulls of neutrino (left) and anti-neutrino (right) induced di-muon production cross section data (circles: NuTeV experiment, squares: CCFR experiment).



# The Drell-Yan Data

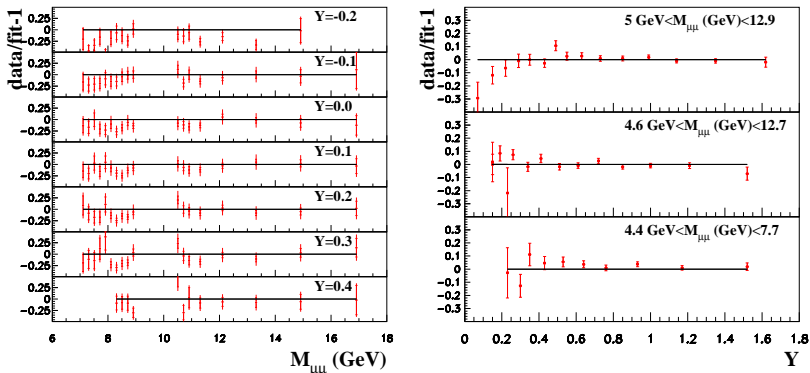


Figure 9: The same as Fig. 2 for the pulls of the DY process cross section data [Moreno90] binned in the muon pair rapidity  $Y$  versus the invariant mass  $M_{\mu\mu}$  of the muon pair (left) and the ones of [Towell01] binned in  $M_{\mu\mu}$  versus  $Y$  (right).



## ABM11: Fit Results

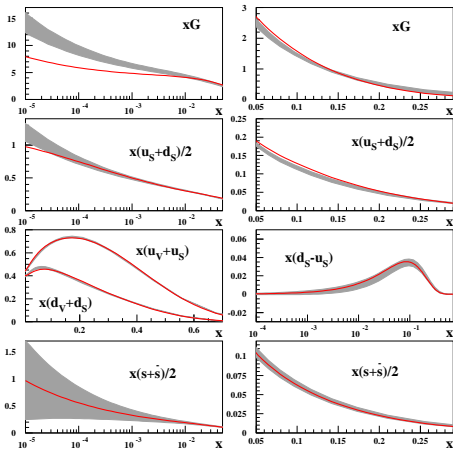
 $\mu=2 \text{ GeV}, n_f=4$ 

Figure 13: The  $1\sigma$  band for the 4-flavor NNLO ABKM09 PDFs at the scale of  $\mu = 2 \text{ GeV}$  versus  $x$  (shaded area) compared with the central values for ones of this analysis (solid lines).



## ABM11: Fit Results

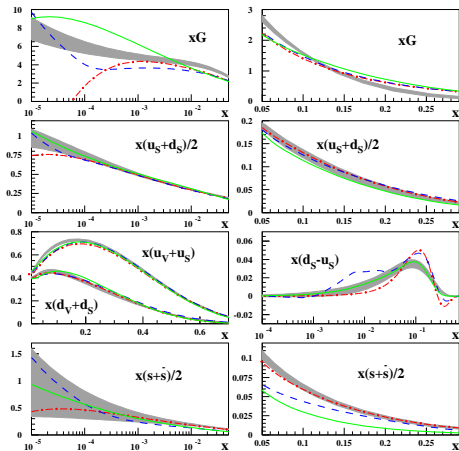
 $\mu=2 \text{ GeV}, n_f=4$ 

Figure 14: The  $1\sigma$  band for the 4-flavor NNLO ABM11 PDFs at the scale of  $\mu = 2 \text{ GeV}$  versus  $x$  obtained in this analysis (shaded area) compared with the ones obtained by other groups (solid lines: JR09, dashed dots: MSTW08, dashes: NN21).



# $R(x, Q^2)$

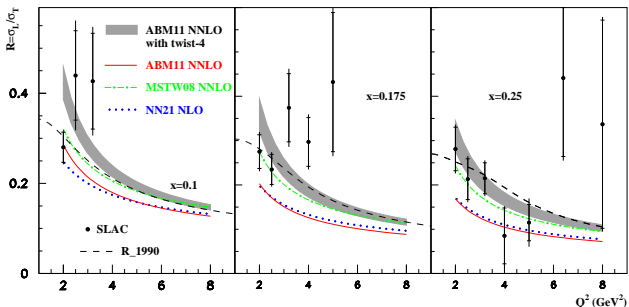


Figure 12: The shaded area gives  $1\sigma$  band of the NNLO predictions for the ratio  $R = \sigma_L/\sigma_T$  based on the ABM11 PDFs and the twist-4 terms obtained from our fit at different values of  $x$  versus the momentum transfer  $Q^2$ . The central values of the NNLO predictions for  $R$  based on the MSTW PDFs (dashed dots), the NNLO predictions based on the ABM11 PDFs (solid line), and the NLO predictions based on the NN21 PDFs (dots), all taken in the 3-flavor scheme and without twist-4 terms, are given for comparison. The data points show values of  $R$  extracted from the SLAC proton and deuterium data with the empirical parameterization of those data  $R_{1990}$  superimposed (dashes).



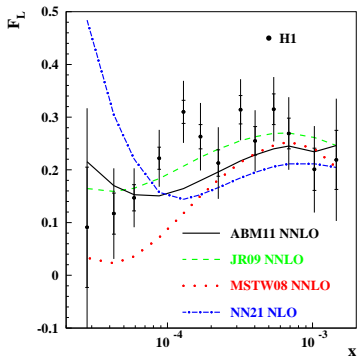
$F_L(x, Q^2)$ 

Figure 15: The data on  $F_L$  versus  $x$  obtained by the H1 collaboration [H12010] confronted with the 3-flavor scheme NNLO predictions based on the different PDFs (solid line: this analysis, dashes: JR09, dots: MSTW08). The NLO predictions based on the 3-flavor NN21 PDFs are given for comparison (dashed dots). The value of  $Q^2$  for the data points and the curves in the plot rises with  $x$  in the range of  $1.5 \div 45 \text{ GeV}^2$ .



## 2nd Moment for Valence PDFs

$$xV(x, Q^2) = x[u(x, Q^2) + \bar{u}(x, Q^2) - u(d, Q^2) + \bar{d}(x, Q^2)]$$

	$\langle xu_v(x) \rangle$	$\langle xd_v(x) \rangle$	$\langle x[u_v - d_v](x) \rangle$	$\langle xV(x) \rangle$
ABM11	$0.2971 \pm 0.0039$	$0.1174 \pm 0.0050$	$0.1797 \pm 0.0042$	$0.1655 \pm 0.0039$
ABKM09	$0.2981 \pm 0.0025$	$0.1191 \pm 0.0023$	$0.1790 \pm 0.0023$	$0.1647 \pm 0.0022$
HERAPDF1.5	$0.2938 \begin{smallmatrix} + 0.0031 \\ - 0.0052 \end{smallmatrix}$	$0.1264 \begin{smallmatrix} + 0.0054 \\ - 0.0059 \end{smallmatrix}$	$0.1674 \begin{smallmatrix} + 0.0043 \\ - 0.0052 \end{smallmatrix}$	$0.1706 \begin{smallmatrix} + 0.0071 \\ - 0.0103 \end{smallmatrix}$
JR09	$0.2897 \pm 0.0035$	$0.1253 \pm 0.0052$	$0.1645 \pm 0.0063$	$0.1513 \pm 0.0118$
MSTW08	$0.2816 \begin{smallmatrix} + 0.0051 \\ - 0.0042 \end{smallmatrix}$	$0.1171 \begin{smallmatrix} + 0.0027 \\ - 0.0028 \end{smallmatrix}$	$0.1645 \begin{smallmatrix} + 0.0046 \\ - 0.0034 \end{smallmatrix}$	$0.1533 \begin{smallmatrix} + 0.0041 \\ - 0.0033 \end{smallmatrix}$
NN21	$0.2913 \pm 0.0038$	$0.1218 \pm 0.0042$	$0.1695 \pm 0.0040$	$0.1539 \pm 0.0030$
BBG	$0.2986 \pm 0.0029$	$0.1239 \pm 0.0026$	$0.1747 \pm 0.0039$	
BBG [N <sup>3</sup> LO]	$0.3006 \pm 0.0031$	$0.1252 \pm 0.0027$	$0.1754 \pm 0.0041$	

Table 11: Comparison of the second moment of the valence quark distributions at NNLO and N<sup>3</sup>LO obtained in different analyses at  $Q^2 = 4 \text{ GeV}^2$ .





# Comparison with Lattice Results

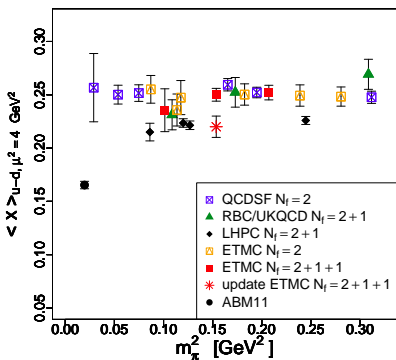


Figure 24: Comparison of lattice computations for the second moment of the non-singlet distribution as a function of the pion mass  $m_\pi$  with the result of ABM11 along with the uncertainties of the respective measurement.



# Higher Twist Contributions

ABM11:

$$F_i(x, Q^2) = F_i^{TMC, \tau=2}(x, Q^2) + \frac{H_i^4(x)}{Q^2} + \frac{H_i^6(x)}{Q^4} + \dots$$

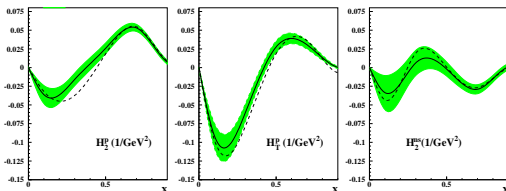


Figure 10: The central values (solid line) and the  $1\sigma$  bands (shaded area) for the coefficients of the twist-4 terms of the inclusive DIS structure functions obtained from our NNLO fit (left panel:  $F_2$  of the proton, central panel:  $F_T$  of the proton, right panel: non-singlet  $F_2$ ). The central values of the twist-4 coefficients obtained from our NLO fit are shown for comparison (dashes).



# Higher Twist Contributions

ABM11 :

To completely remove the HT-terms the cut  $W^2 > 12.5 \text{ GeV}^2$ ,  $Q^2 > 10 \text{ GeV}^2$  is necessary!

	$H_2^p(x)/\text{GeV}^2$	$H_2^{\text{ns}}(x)/\text{GeV}^2$	$H_T^p(x)/\text{GeV}^2$
$x = 0.1$	$-0.036 \pm 0.012$	$-0.034 \pm 0.023$	$-0.091 \pm 0.017$
$x = 0.3$	$-0.016 \pm 0.008$	$0.006 \pm 0.017$	$-0.061 \pm 0.012$
$x = 0.5$	$0.026 \pm 0.007$	$-0.0020 \pm 0.0094$	$0.0276 \pm 0.0081$
$x = 0.7$	$0.053 \pm 0.005$	$-0.029 \pm 0.006$	$0.031 \pm 0.006$
$x = 0.9$	$0.0071 \pm 0.0026$	$0.0009 \pm 0.0041$	$0.0002 \pm 0.0015$

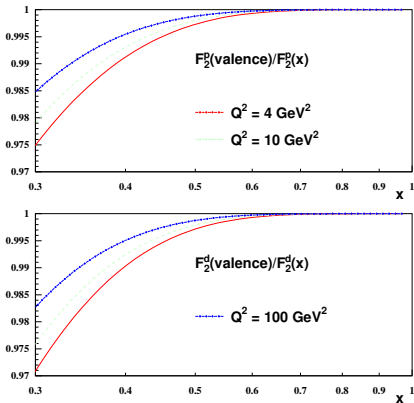
Table 3: The parameters of the twist-4 contribution to the DIS structure functions in for the fit to NNLO accuracy in QCD.



# Higher Twist Terms in the Valence Region

J.B. and H. Böttcher, 2012 (BB)

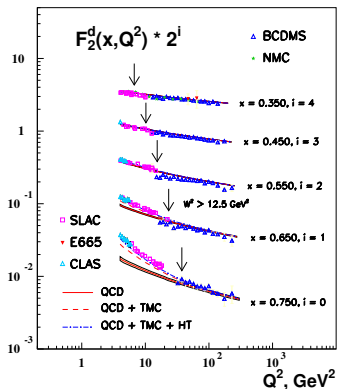
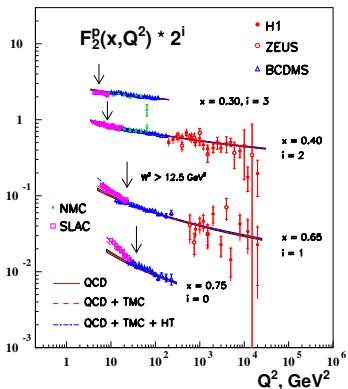
NS-tails at NNLO :



Using ABKM09 we corrected for non NS-tails in  $F_2(x, Q^2)$ .



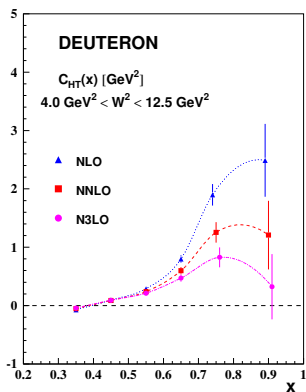
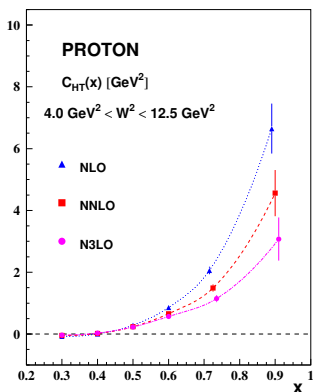
# $F_2(x, Q^2)$ in the region of larger $x$



Valence region: HT-terms are removed by the cut  
 $W^2 > 12.5 \text{ GeV}^2$ ,  $Q^2 > 4 \text{ GeV}^2$ .



# Extracting the Higher Twist Contributions to $F_2(x, Q^2)$



$$F_2(x, Q^2) = F_2^{\tau=2}(x, Q^2) \left[ 1 + \frac{C(\langle Q^2 \rangle, x)}{Q^2} \right]$$



$$\alpha_s(M_Z^2)$$

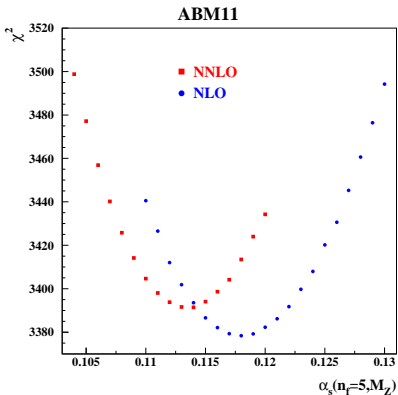


Figure 17: The  $\chi^2$ -profile as a function of  $\alpha_s(M_Z)$  in the present analysis. At NLO (circles) and NNLO (squares).



$$\alpha_s(M_Z^2)$$

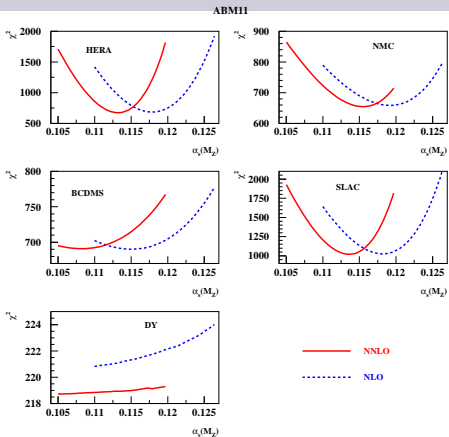


Figure 19: The  $\chi^2$ -profile versus the value of  $\alpha_s(M_Z)$  for the data sets used, all calculated with the PDF and HT parameters fixed at the values obtained from the fits with  $\alpha_s(M_Z)$  released (solid lines: NNLO fit, dashes: NLO one).





$$\alpha_s(M_Z^2)$$

ABM11

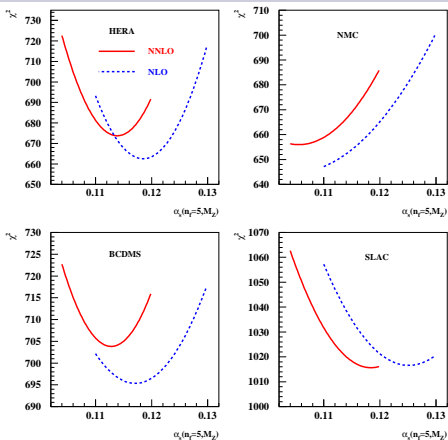


Figure 18: The  $\chi^2$ -profile versus the value of  $\alpha_s(M_Z)$  for the data sets used all obtained in variants of the present analysis with the value of  $\alpha_s$  fixed and all other parameters fitted (solid lines: NNLO fit, dashes: NLO fit).



$\alpha_s(M_Z^2)$ : ABM11

Experiment	$\alpha_s(M_Z)$		
	NLO <sub>exp</sub>	NLO	NNLO
BCDMS	$0.1111 \pm 0.0018$	$0.1150 \pm 0.0012$	$0.1084 \pm 0.0013$
NMC	$0.117 \begin{smallmatrix} +0.011 \\ -0.016 \end{smallmatrix}$	$0.1182 \pm 0.0007$	$0.1152 \pm 0.0007$
SLAC		$0.1173 \pm 0.0003$	$0.1128 \pm 0.0003$
HERA comb.		$0.1174 \pm 0.0003$	$0.1126 \pm 0.0002$
DY		$0.108 \pm 0.010$	$0.101 \pm 0.025$
ABM11		$0.1180 \pm 0.0012$	$0.1134 \pm 0.0011$

Tabelle 4: Comparison of the values of  $\alpha_s(M_Z)$  obtained by BCDMS and NMC at NLO with the individual results of the fit in the present analysis at NLO and NNLO for the HERA data the NMC data the BCDMS data the SLAC data and the DY data.



$\alpha_s(M_Z^2)$ : ABM11 + Tevatron jets

Experiment	$\alpha_s(M_Z)$		
	NLO <sub>exp</sub>	NLO	NNLO*
D0 1 jet	0.1161 <sup>+0.0041</sup> <sub>-0.0048</sub>	0.1190 ± 0.0011	0.1149 ± 0.0012
D0 2 jet		0.1174 ± 0.0009	0.1145 ± 0.0009
CDF 1 jet (cone)		0.1181 ± 0.0009	0.1134 ± 0.0009
CDF 1 jet ( $k_\perp$ )		0.1181 ± 0.0010	0.1143 ± 0.0009
ABM11		0.1180 ± 0.0012	0.1134 ± 0.0011

Tabelle 5: Comparison of the values of  $\alpha_s(M_Z)$  obtained by D0 with the ones based on including individual data sets of Tevatron jet data into the analysis at NLO. The NNLO\* fit refers to the NNLO analysis of the DIS and DY data together with the NLO and soft gluon resummation corrections (next-to-leading logarithmic accuracy) for the 1 jet inclusive data.



$\alpha_s(M_Z^2)$  : BBG/BB

Experiment	$\alpha_s(M_Z)$			
	NLO <sub>exp</sub>	NLO	NNLO	N <sup>3</sup> LO*
BCDMS	$0.1111 \pm 0.0018$	$0.1138 \pm 0.0007$	$0.1126 \pm 0.0007$	$0.1128 \pm 0.0006$
NMC	$0.117 \begin{smallmatrix} +0.011 \\ -0.016 \end{smallmatrix}$	$0.1166 \pm 0.0039$	$0.1153 \pm 0.0039$	$0.1153 \pm 0.0035$
SLAC		$0.1147 \pm 0.0029$	$0.1158 \pm 0.0033$	$0.1152 \pm 0.0027$
BBG		$0.1148 \pm 0.0019$	$0.1134 \pm 0.0020$	$0.1141 \pm 0.0021$
BB		$0.1147 \pm 0.0021$	$0.1132 \pm 0.0022$	$0.1137 \pm 0.0022$

Tabelle 6: Comparison of the values of  $\alpha_s(M_Z)$  obtained by BCDMS and NMC at NLO with the results of the flavor non-singlet fits BBG and BB of the DIS flavor non-singlet world data, at NLO, NNLO, and N<sup>3</sup>LO\* with the response of the individual data sets, combined for the experiments BCDMS NMC SLAC.



$\alpha_s(M_Z^2)$ : NNPDF

Experiment	$\alpha_s(M_Z)$		
	NLO <sub>exp</sub>	NLO	NNLO
BCDMS	$0.1111 \pm 0.0018$	$0.1204 \pm 0.0015$	$0.1158 \pm 0.0015$
NMC <sub>p</sub>	$0.117 \begin{smallmatrix} + 0.011 \\ - 0.016 \end{smallmatrix}$	$0.1192 \pm 0.0018$	$0.1150 \pm 0.0020$
NMC <sub>pd</sub>			$0.1146 \pm 0.0107$
SLAC		$> 0.124$	$> 0.124$
HERA I		$0.1223 \pm 0.0018$	$0.1199 \pm 0.0019$
ZEUS H2		$0.1170 \pm 0.0027$	$0.1231 \pm 0.0030$
ZEUS F2C		$0.1144 \pm 0.0060$	
NuTeV		$0.1252 \pm 0.0068$	$0.1177 \pm 0.0039$
E605		$0.1168 \pm 0.0100$	
E866		$0.1135 \pm 0.0029$	
CDF Wasy		$0.1181 \pm 0.0060$	
CDF Zrap		$0.1150 \pm 0.0034$	$0.1205 \pm 0.0081$
D0 Zrap		$0.1227 \pm 0.0067$	
CDF R2KT	$0.1161 \begin{smallmatrix} + 0.0041 \\ - 0.0048 \end{smallmatrix}$	$0.1228 \pm 0.0021$	$0.1225 \pm 0.0021$
D0 R2CON		$0.1141 \pm 0.0031$	$0.1111 \pm 0.0029$
NN21		$0.1191 \pm 0.0006$	$0.1173 \pm 0.0007$

Tabelle 7: Comparison of the values of  $\alpha_s(M_Z)$  obtained by BCDMS, NMC, and D0 at NLO with the results of NN21 for the fits to DIS and other hard scattering data at NLO and NNLO and the corresponding response of the different data sets analysed.



$\alpha_s(M_Z^2)$  : MSTW08/09

Experiment	$\alpha_s(M_Z)$		
	NLO <sub>exp</sub>	NLO	NNLO
BCDMS $\mu p, F_2$	$0.1111 \pm 0.0018$	–	$0.1085 \pm 0.0095$
BCDMS $\mu d, F_2$		$0.1135 \pm 0.0155$	$0.1117 \pm 0.0093$
NMC $\mu p, F_2$	$0.117 \pm_{0.016}^{0.011}$	$0.1275 \pm 0.0105$	$0.1217 \pm 0.0077$
NMC $\mu d, F_2$		$0.1265 \pm 0.0115$	$0.1215 \pm 0.0070$
NMC $\mu n/\mu p$		0.1280	0.1160
E665 $\mu p, F_2$		0.1203	–
E665 $\mu d, F_2$		–	–
SLAC $ep, F_2$		$0.1180 \pm 0.0060$	$0.1140 \pm 0.0060$
SLAC $ed, F_2$		$0.1270 \pm 0.0090$	$0.1220 \pm 0.0060$
NMC,BCDMS,SLAC, $F_L$		$0.1285 \pm 0.0115$	$0.1200 \pm 0.0060$
E886/NuSea $pp, DY$ %citeWebb:2003bj		–	$0.1132 \pm 0.0088$
E886/NuSea $pd/pp, DY$		$0.1173 \pm 0.107$	$0.1140 \pm 0.0110$
NuTeV $\nu N, F_2$		$0.1207 \pm 0.0067$	$0.1170 \pm 0.0060$
CHORUS $\nu N, F_2$		$0.1230 \pm 0.0110$	$0.1150 \pm 0.0090$
NuTeV $\nu N, xF_3$		$0.1270 \pm 0.0090$	$0.1225 \pm 0.0075$
CHORUS $\nu N, xF_3$		$0.1215 \pm 0.0105$	$0.1185 \pm 0.0075$
CCFR		0.1190	–
NuTeV $\nu N \rightarrow \mu\mu X$		$0.1150 \pm 0.0170$	–
H1 $ep$ 97-00, $\sigma_r^{NC}$		$0.1250 \pm 0.0070$	$0.1205 \pm 0.0055$
ZEUS $ep$ 95-00, $\sigma_r^{NC}$		$0.1235 \pm 0.0065$	$0.1210 \pm 0.0060$
H1 $ep$ 99-00, $\sigma_r^{CC}$		$0.1285 \pm 0.0225$	$0.1270 \pm 0.0200$
ZEUS $ep$ 99-00, $\sigma_r^{CC}$		$0.1125 \pm 0.0195$	$0.1165 \pm 0.0095$
H1/ZEUS $ep, F_3^{\text{charm}}$		–	$0.1165 \pm 0.0095$
H1 $ep$ 99-00 incl. jets	$0.1168 \pm_{0.0034}^{0.0037}$	$0.1127 \pm 0.0093$	
ZEUS $ep$ 96-00 incl. jets	$0.1208 \pm_{0.0040}^{0.0048}$	$0.1175 \pm 0.0055$	
D0 II $p\bar{p}$ incl. jets	$0.1161 \pm_{0.0048}^{0.0041}$	$0.1185 \pm 0.0055$	$0.1133 \pm 0.0063$
CDF II $p\bar{p}$ incl. jets		$0.1205 \pm 0.0045$	$0.1165 \pm 0.0025$
D0 II $W \rightarrow l\nu$ asym.		–	–
CDF II $W \rightarrow l\nu$ asym.		–	–
D0 II $Z$ rap.		$0.1125 \pm 0.0100$	$0.1136 \pm 0.0084$
CDF II $Z$ rap.		$0.1160 \pm 0.0070$	$0.1157 \pm 0.0067$
MSTW		$0.1202 \pm_{0.0015}^{0.0012}$	$0.1171 \pm 0.0014$

Table 8: Comparison of the values of  $\alpha_s(M_Z)$  obtained by BCDMS, NMC, HERA-jet and D0 at NLO with the results of the MSTW fits to DIS and other hard scattering data at NLO and NNLO and the corresponding response of the different data sets analysed, cf. Figs. 7a and 7b in MSTW08. Entries not given correspond to  $\alpha_s(M_Z)$  central values below 0.110 or above 0.130; in case no errors are assigned these are larger than the bounds provided in form of the plots in MSTW08.



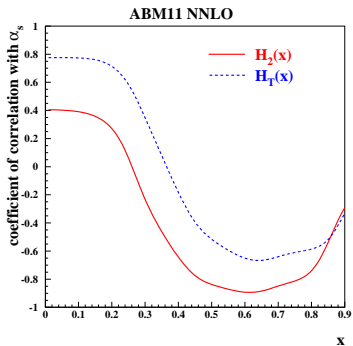
$\alpha_s(M_Z^2)$ : Correlation with HT

Figure 20: The correlation coefficient of  $\alpha_s(M_Z)$  with the nucleon twist-4 coefficients  $H_2$  (solid line) and  $H_T$  (dashes) versus  $x$  as obtained in our NNLO fit.



$\alpha_s(M_Z^2)$ : Global DIS Comparison

Data Set	ABM11	BBG	NN21	MSTW
BCDMS	$0.1048 \pm 0.0013$	$0.1126 \pm 0.0007$	$0.1158 \pm 0.0015$	$0.1101 \pm 0.0094$
NMC	$0.1152 \pm 0.0007$	$0.1153 \pm 0.0039$	$0.1150 \pm 0.0020$	$0.1216 \pm 0.0074$
SLAC	$0.1128 \pm 0.0003$	$0.1158 \pm 0.0034$	$> 0.124$	$\left\{ \begin{array}{l} 0.1140 \pm 0.0060 \text{ ep} \\ 0.1220 \pm 0.0060 \text{ ed} \end{array} \right.$
HERA	$0.1126 \pm 0.0002$		$\left\{ \begin{array}{l} 0.1199 \pm 0.0019 \\ 0.1231 \pm 0.0030 \end{array} \right.$	$0.1208 \pm 0.0058$
DY	$0.101 \pm 0.025$	–	–	$0.1136 \pm 0.0100$
	$0.1134 \pm 0.0011$	$0.1134 \pm 0.0020$	$0.1173 \pm 0.0007$	$0.1171 \pm 0.0014$

Table 9: Comparison of the pulls in  $\alpha_s(M_Z)$  per data set between the ABM11, BBG, NN21, MSTW analyses at NNLO.

Use:  $W^2 > 12.5 \text{ GeV}^2$ ,  $Q^2 > 2.5 \text{ GeV}^2$  and no HT:  $\alpha_s(M_Z^2) = 0.1191 \pm 0.0016$

Use:  $W^2 > 12.5 \text{ GeV}^2$ ,  $Q^2 > 10 \text{ GeV}^2$  and no HT:  $\alpha_s(M_Z^2) = 0.1134 \pm 0.0008$

MSTW08 and NNPDF: no HT corrections.

Taking into account error correlations is of importance.





$\alpha_s(M_Z^2)$ : DIS & other NNLO Determinations

	$\alpha_s(M_Z)$	
BBG	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO
BB	$0.1132 \pm 0.0022$	valence analysis, NNLO
GRS	0.112	valence analysis, NNLO
ABKM	$0.1135 \pm 0.0014$	HQ: FFNS $n_f = 3$
ABKM	$0.1129 \pm 0.0014$	HQ: BSMN-approach
JR	$0.1124 \pm 0.0020$	dynamical approach
JR	$0.1158 \pm 0.0035$	standard fit
ABM11	$0.1134 \pm 0.0011$	
MSTW	$0.1171 \pm 0.0014$	
NN21	$0.1173 \pm 0.0007$	
CT10	$0.118 \pm 0.005$	
Gehrmann et al.	$0.1153 \pm 0.0017 \pm 0.0023$	$e^+e^-$ thrust
Abbate et al.	$0.1135 \pm 0.0011 \pm 0.0006$	$e^+e^-$ thrust
3 jet rate	$0.1175 \pm 0.0025$	Dissertori et al. 2009
Z-decay	$0.1189 \pm 0.0026$	BCK 2008/12 (N <sup>3</sup> LO)
$\tau$ decay	$0.1212 \pm 0.0019$	BCK 2008
$\tau$ decay	$0.1204 \pm 0.0016$	Pich 2011
$\tau$ decay	$0.1169 \pm 0.0025$	Boito et al. 2011
lattice	$0.1205 \pm 0.0010$	PACS-CS 2009 (2+1 fl.)
lattice	$0.1184 \pm 0.0006$	HPQCD 2010
lattice	$0.1200 \pm 0.0014$	ETM 2012 (2+1+1 fl.)
lattice	$0.1156 \pm 0.0022$	Brambilla et al. 2012 (2+1 fl.)
BBG	$0.1141^{+0.0020}_{-0.0022}$	valence analysis, N <sup>3</sup> LO(*)
BB	$0.1137 \pm 0.0022$	valence analysis, N <sup>3</sup> LO(*)
world average	$0.1184 \pm 0.0007$	(2009)
	$0.1183 \pm 0.0010$	(2011)

Table 10: Summary of recent NNLO QCD analyses of the DIS world data, supplemented by related measurements using other processes.



## Remark on a recent study on BCDMS

- proposal to reject low  $y$  data.  $\rightarrow \Delta\alpha_s(M_Z^2) = +0.0090$  w.r.t. BCDMS
- our analysis:  $\alpha_s(M_Z^2) = 0.1139 \pm 0.0012$  NNLO

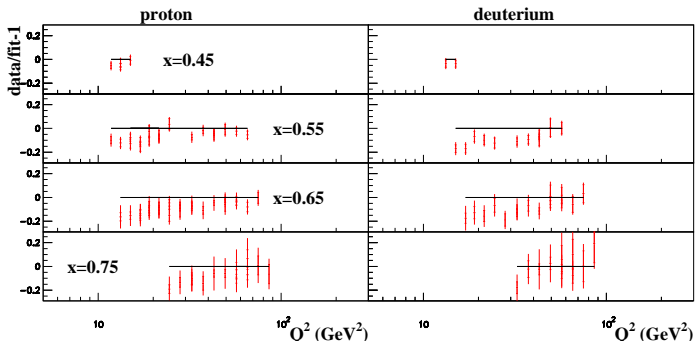


Figure 21: The same as in Fig. 5 for the data points rejected in the analysis of [Shaikhatdenov09].



# Heavy Flavor Structure Functions

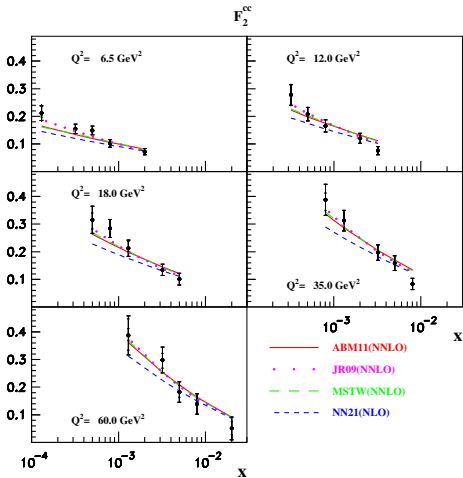


Figure 22: Comparison of the data from [H1-11] for the semi-inclusive structure function  $F_2^{cc}$  at different values of the momentum transfer  $Q^2$  versus  $x$  with predictions of various PDF sets at NLO and NNLO in QCD, all taken in the FFNS with  $n_f = 3$  and with a running-mass of  $m_c = 1.27$  GeV. The NNLO predictions for  $F_2^{cc}$  use the ABM11 PDFs (solid curves), the JR09 PDFs (dots), and the MSTW08 PDFs (long dashes). The NLO calculations are based on the NN21 PDFs (short dashes).



# Heavy Flavor Structure Functions

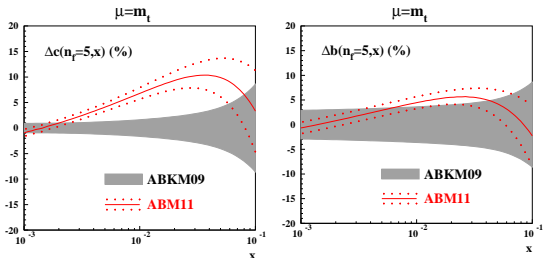


Figure 23: The charm- (left) and the bottom-quark (right) PDFs obtained in the global fit: The dotted (red) lines denote the  $\pm 1\sigma$  band of relative uncertainties (in percent) and the solid (red) line indicates the central prediction resulting from the fit with the running masses. For comparison the shaded (grey) area represents the results of ABKM09.



## D0 Jet Data

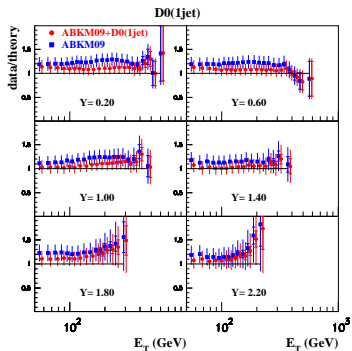


Figure 26: Cross section data for 1-jet inclusive production from the D0 collaboration [Abazov:08] as a function of the jet's transverse energy  $E_T$  for the renormalization and factorization scales equal to  $E_T$  compared to the result of ABKM09 (circles) and a re-fit including this data (squares) including the NNLO threshold resummation corrections to the jet production [Kidonakis:00].



# CDF Jet Data

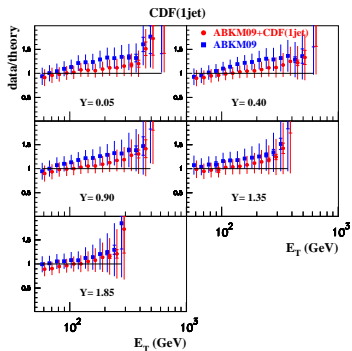


Figure 27: Same as Fig. 26 for the cross section data for 1-jet inclusive production from the CDF collaboration using a  $k_T$  jet algorithm [Abulencia:07].



# ATLAS Jet Data

NNLO: ABM11:  $\alpha_s(M_Z^2) = 0.1134(11)$

ABM11+ATLAS jets:  $\alpha_s(M_Z^2) = 0.1141(8)$

NLO: ATLAS:  $\alpha_s(M_Z^2) = 0.1151 \pm 0.0047(exp) \pm 0.0023(pdf)$

Malaescu/Starovoitov '12

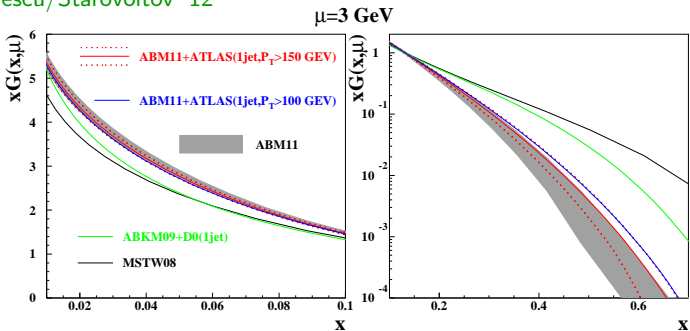
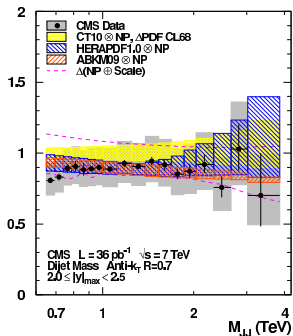
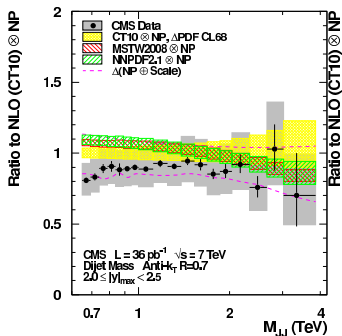


Figure 28: Gluon distribution obtained by including the ATLAS jet data into the ABM11 analysis.



## CMS Jets



K. Rabbertz et al.: CMS-Note 2011-004.





# Charged-lepton Asymmetry

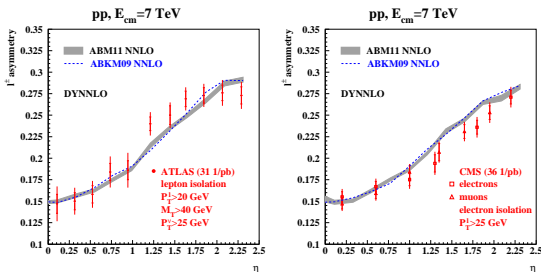
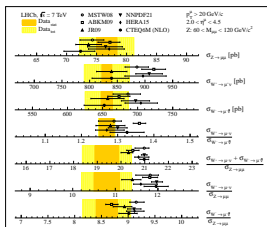
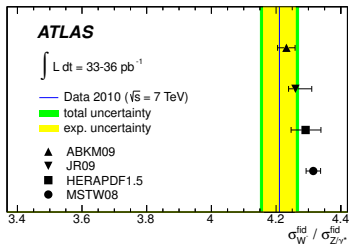
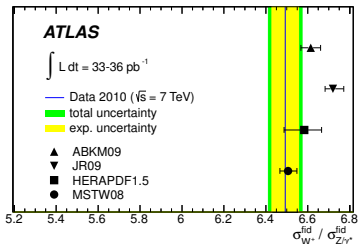


Figure 25: The data on charged-lepton asymmetry versus the lepton pseudo-rapidity  $\eta$  obtained by the ATLAS [Aad:11] (left panel) and CMS [Chatrchyan:11] (right panel) experiments compared to the NNLO predictions based on the DYNL0 code [Catani:09/10] and the ABM11 NNLO PDFs with the shaded area showing the integration uncertainties. The ABM09 NNLO predictions are given for comparison by dashes, without the integration uncertainties shown.



# ATLAS & LHCb $W^\pm, Z$



ATLAS: [arXiv:1109.5141](https://arxiv.org/abs/1109.5141); LHCb: [arXiv:1204.1620](https://arxiv.org/abs/1204.1620)



$t\bar{t}$ 

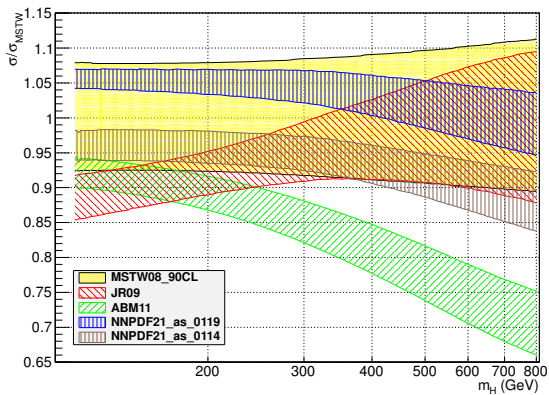
nb	ABM11	JR09	MSTW	NNPDF21
$\sigma_{t\bar{t}}$ fit $m_t$	145.5	170.4	175.6	172.7
$\sigma_{t\bar{t}}$ fit $m_t = 160$ GeV	154.4	188.4	192.0	196.9

Calculated with **HATHOR**: [Aliev et al. arXiv:1007.1327](#)



# Higgs

$$\sigma(pp \rightarrow H^0) \propto \alpha_s^2 x G^2(x, Q^2).$$



Anastasiou et al., arXiv:1202.3638



# Conclusions

- New ABM11: theory improvements, inclusion of jet data; predicts hadron collider data well
- NNLO:  $\alpha_s(M_Z^2) = 0.1134 \pm 0.0011$  vs ABKM09  
 $\alpha_s(M_Z^2) = 0.1135 \pm 0.0014$
- The different  $\alpha_s$ -values given by MSTW and NNPDF are understood.
- The correct inclusion of **systematic error correlations** and **higher twist effects** are important.
- ATLAS jet data analyzed  $\rightarrow \alpha_s(M_Z^2) = 0.1141 \pm 0.0008$ ; the gluon obtained is softer than that in case of Tevatron; Higgs production cross section almost unchanged.
- Inclusive top-cross section: still differences between the groups:  $\rightarrow \alpha_s, xg(x, Q^2)$
- LHC data start to constrain sea-quarks  **$W^\pm, Z$ , off-resonance Drell-Yan**



# Conclusions

- LHC data start to constrain the gluon **LHC 2,3-jets**; these data will improve  $\alpha_s(M_Z)$ .
- Need of continuous benchmarking
- **The “pdf4lhc recommendation” ignored 2 of 3 NNLO analyses since 2009.**  
The combined HERA data were not considered in the fits chosen there (but were available).  
All NNLO pdfs are available in LHAPDF and are run very fast.
- **Recommendation:**  
Use all NNLO pdfs in your final analyses, together with the right value of  $\alpha_s(M_Z)$  being correlated, to find out yourself about the remaining systematics.



# Comparison to MSTW and NN21: SLAC Data

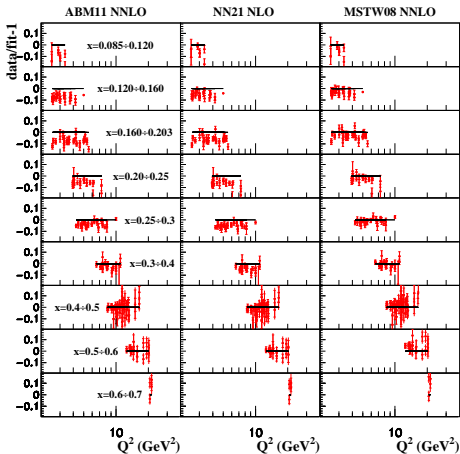


Figure 11: The same as Fig. 7 without the HT terms taken into account and for various 3-flavor PDFs (left panel: present analysis, right panels: MSTW. The NLO calculations based on the 3-flavor NLO NN21 PDFs are given for comparison (central panels)). Only the data surviving after the HT-cut are shown; the proton and deuterium data points are superimposed.

