

# Precision Tests of QCD

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on behalf of the H1 and ZEUS collaborations

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# QCD and Precision

## QCD

$$\mathcal{L} = \sum_f^{n_f} \bar{q}_f (i\gamma^\mu \mathcal{D}_\mu - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

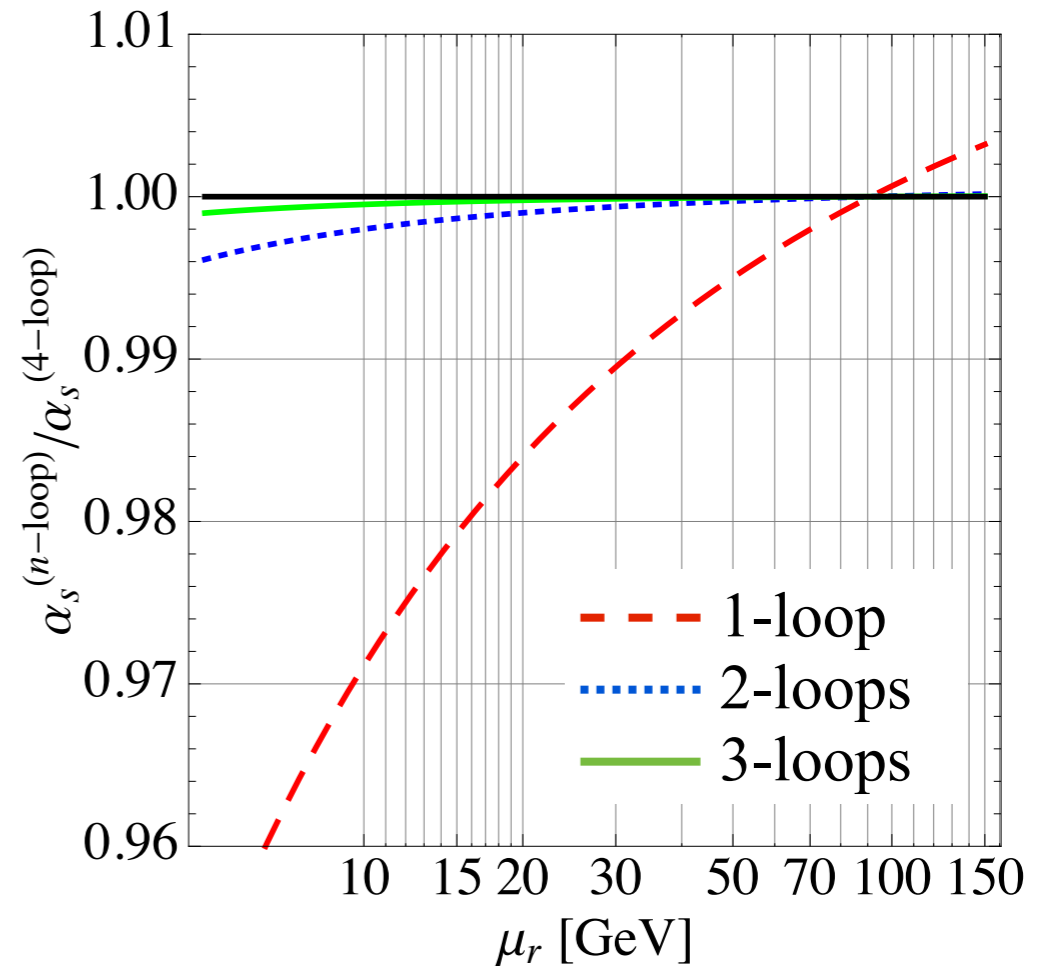
Beautiful theory, fully determined by the heavy quark masses  $m_f$  and  $\alpha_s$

## The Strong Coupling $\alpha_s$

Solution of the Renormalisation Group Equation (RGE) leads to the  $\beta$ -function

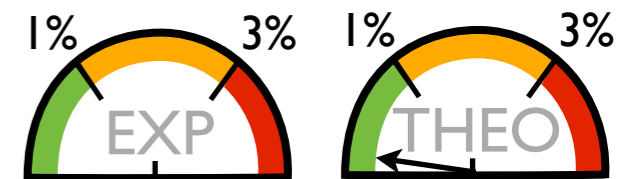
$$\beta(\alpha_s) = -\alpha_s \sum_{n=0}^{\infty} \beta_n \left( \frac{\alpha_s}{4\pi} \right)^{(n+1)}$$

with  $\beta_n$  known up to 4 loops (T. van Ritbergen et al., Phys. Lett. B400, 379 (1997))



Computed with RunDec by Chetyrkin, K.G. et al.,  
Comp. Phys. Comm. 133, 43(2000)

→ Test universality and energy behaviour of  $\alpha_s$



# $\alpha_s(M_Z)$ from $Z \rightarrow \text{hadrons}$

- ▶ Fit of electroweak precision observables
- ▶ Input mostly from LEP data from the Z-peak
- ▶ Determination of  $\alpha_s$ : most sensitivity through total hadronic cross section at the Z-pole and the partial leptonic width

$$\sigma_{\text{had}}^0 \equiv \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\text{had}}}{\Gamma_Z^2} \quad R_\ell^0 \equiv \Gamma_{\text{had}}/\Gamma_{\ell\ell}$$

obtained from the four LEP experiments, 17 million Z decays

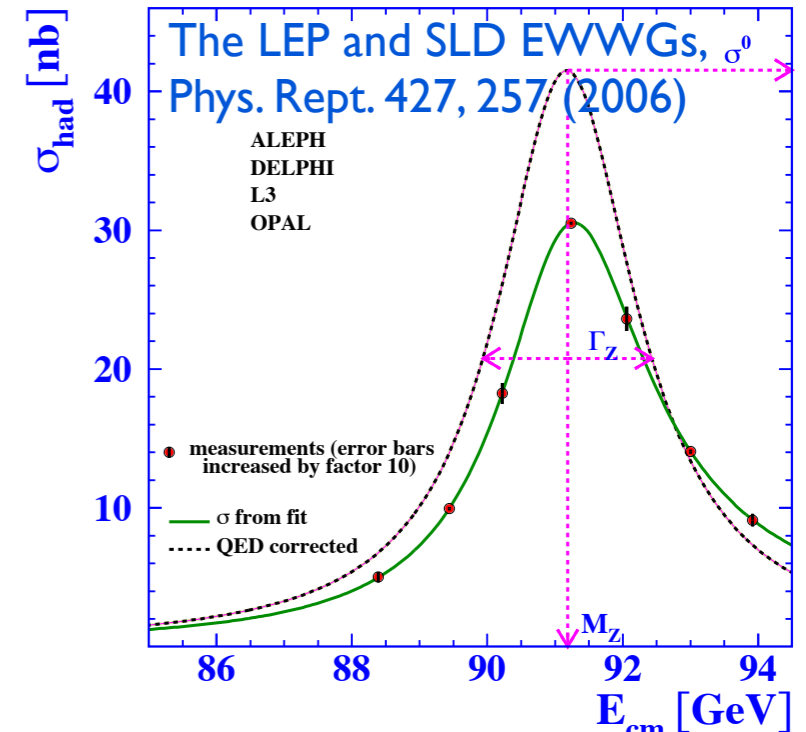
Complete  $O(\alpha_s^4)$  calculation available:

P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)

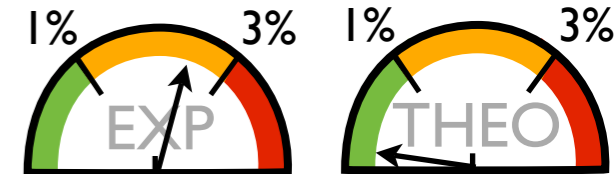
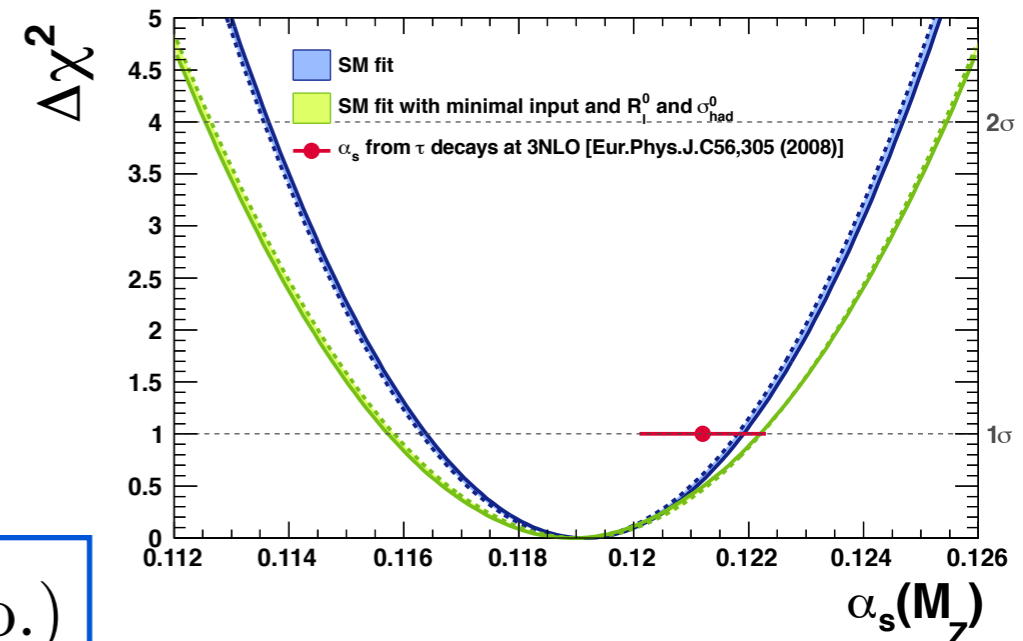
Gfitter:

$$\alpha_s(M_Z) = 0.1191 \pm 0.0028 \text{ (exp.)} \pm 0.0001 \text{ (theo.)}$$

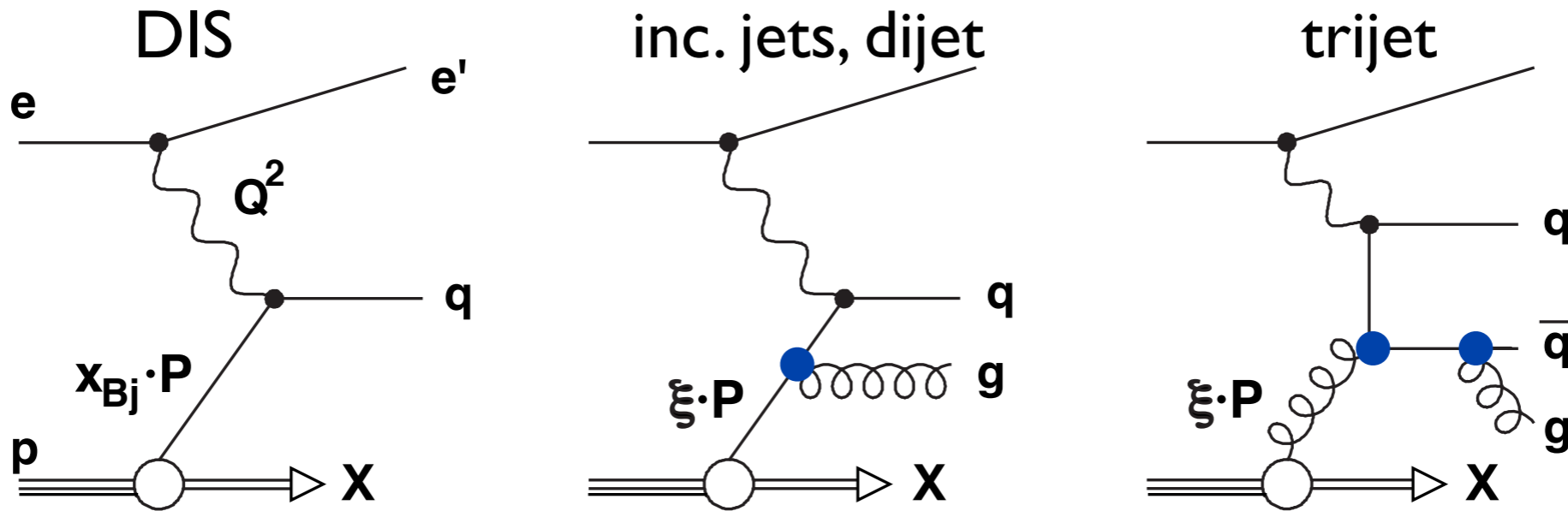
Improvement in precision only with ILC/GigaZ expected



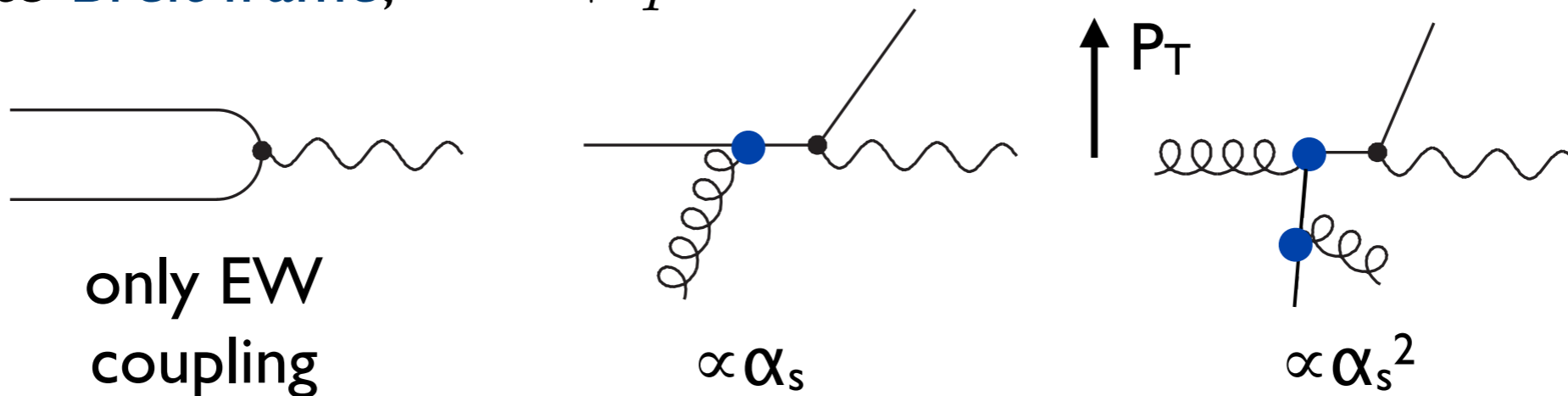
Gfitter Group, EPJ C72, 2003 (2012)



# Jet Production in DIS



Boost to Breit frame,  $2xP + q = 0$



Momentum fraction of struck parton (in LO):  $\xi = x \left( 1 + \frac{M_{12}^2}{Q^2} \right)$

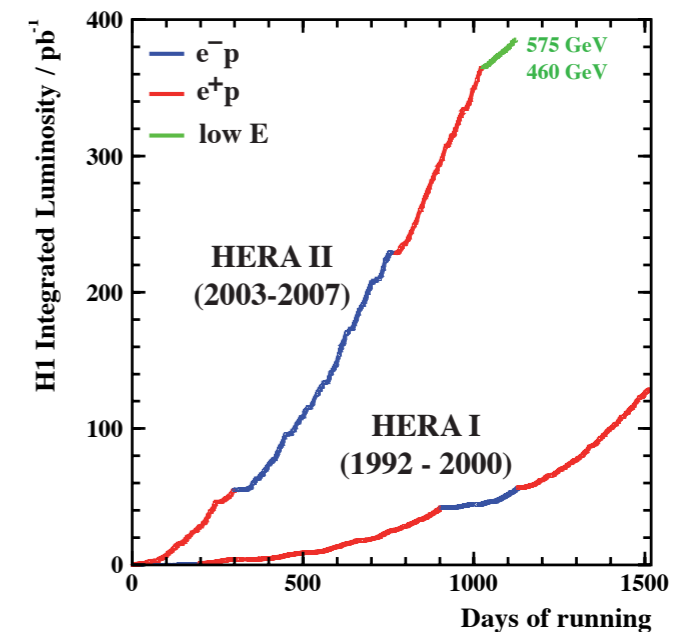
Direct sensitivity to  $\alpha_s$  and gluon PDF

# Precision Jet Measurements at HERA

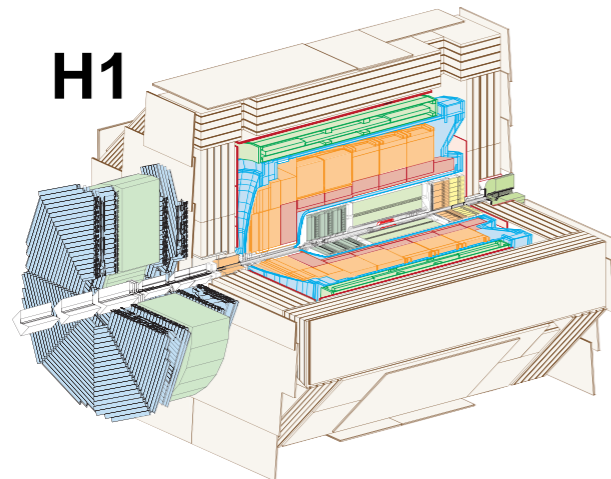
## HERA-2 jet measurements

### High statistics

$L = 300\text{-}500 \text{ pb}^{-1}$ : small statistical uncertainties, even at high  $Q^2$  and high  $P_T$



### Excellent control over systematic uncertainties



electron measurement: 0.5 – 1% scale uncertainty

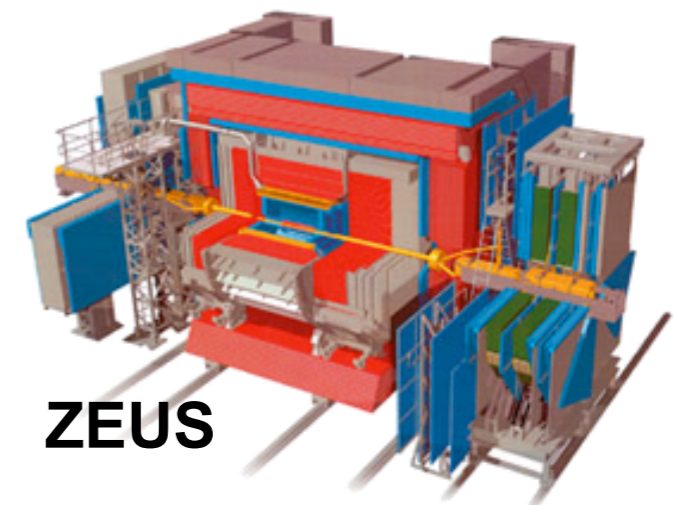
jet energy scale: 1% uncertainty!

effect on jet cross sections: 3 – 10%

acceptance correction:  
4 – 5% uncertainty

trigger: 1 – 2% normalisation uncertainty

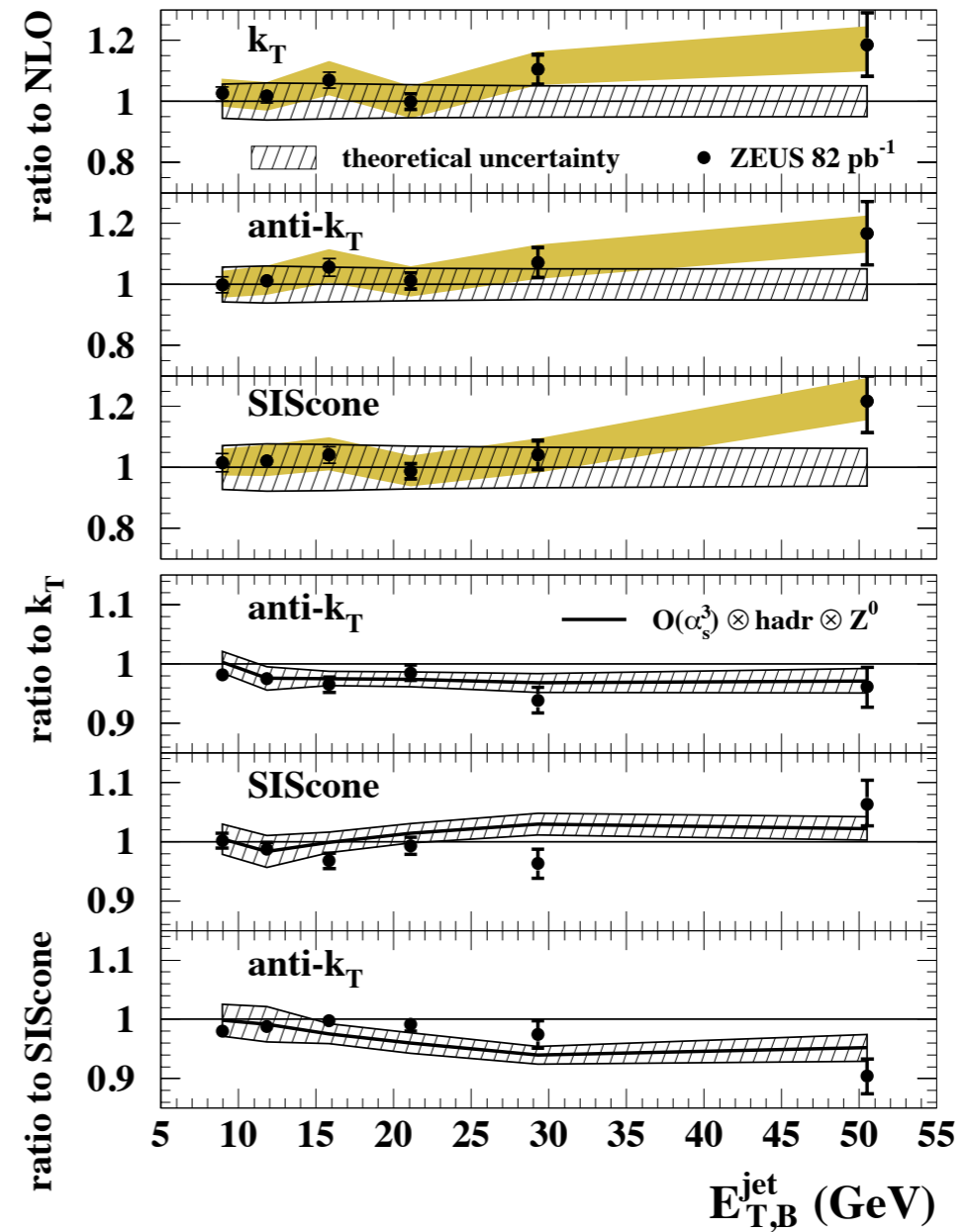
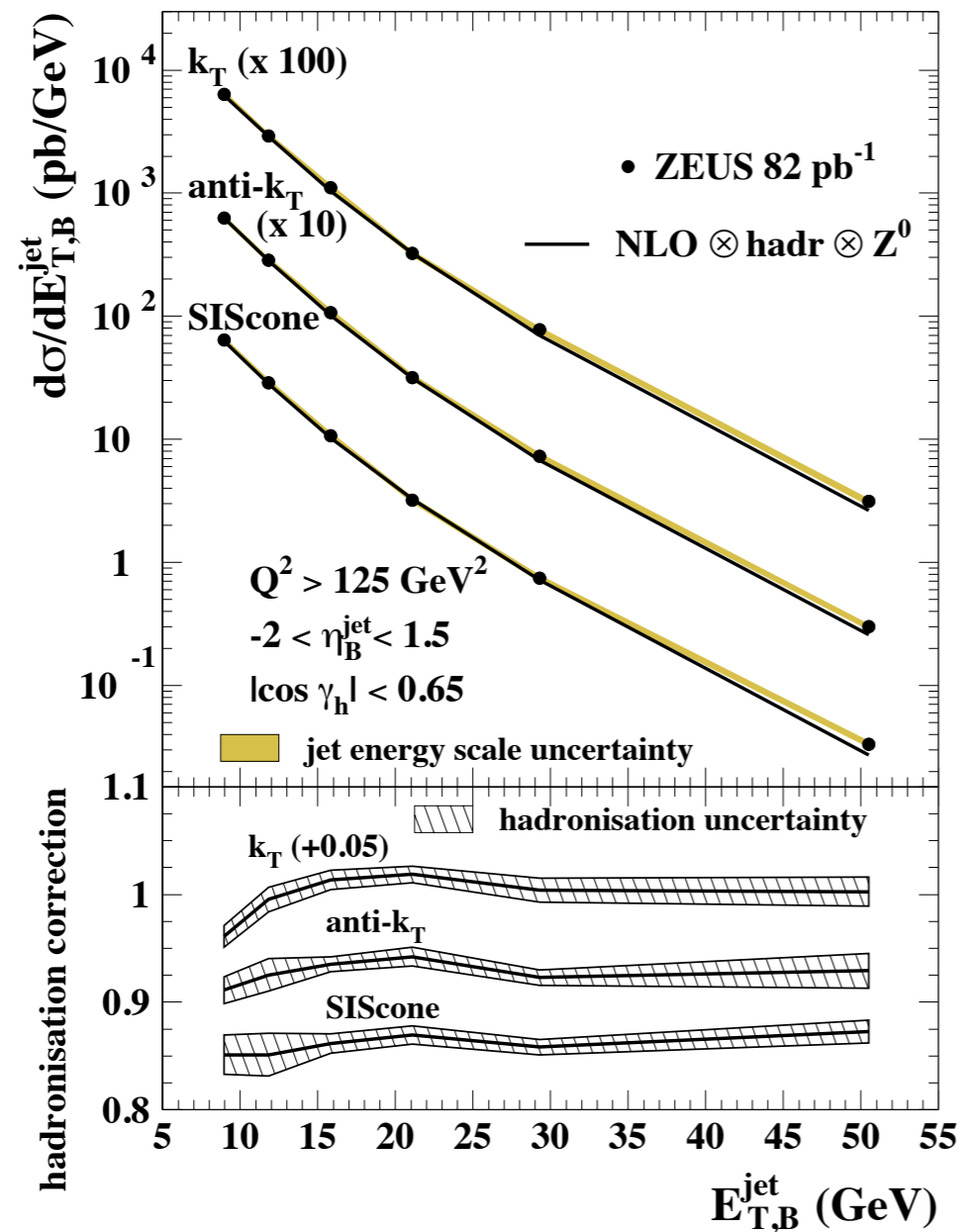
luminosity: 2 – 2.5% normalisation uncertainty



**ZEUS**

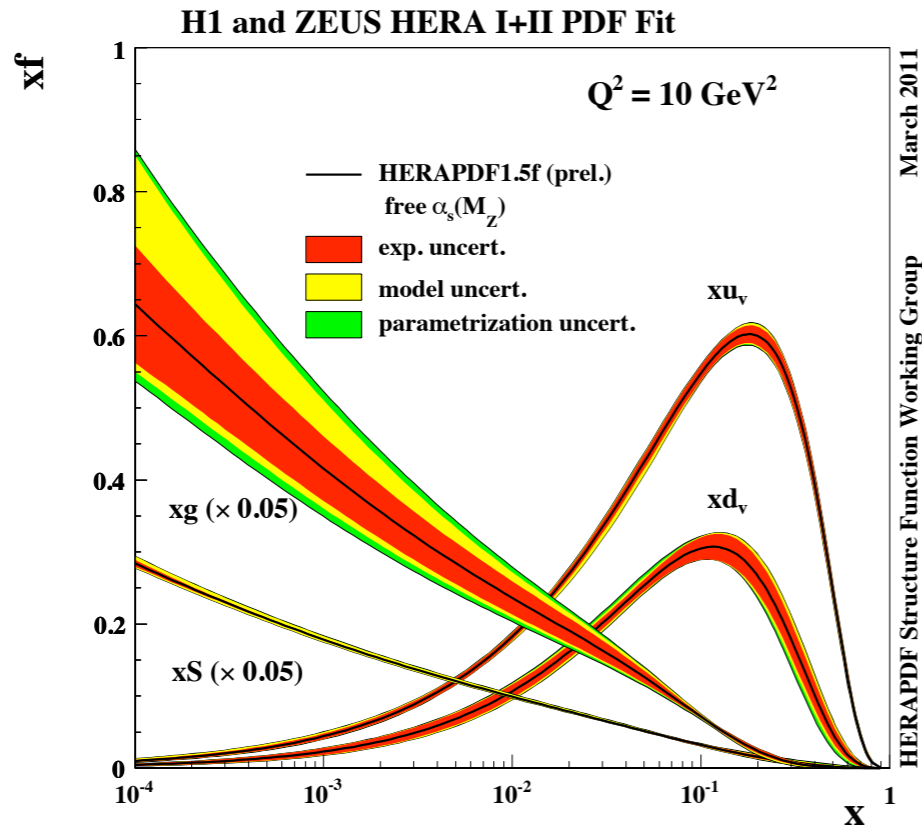
# Jet Algorithms in DIS

ZEUS, Phys. Lett. B691, 127 (2010)

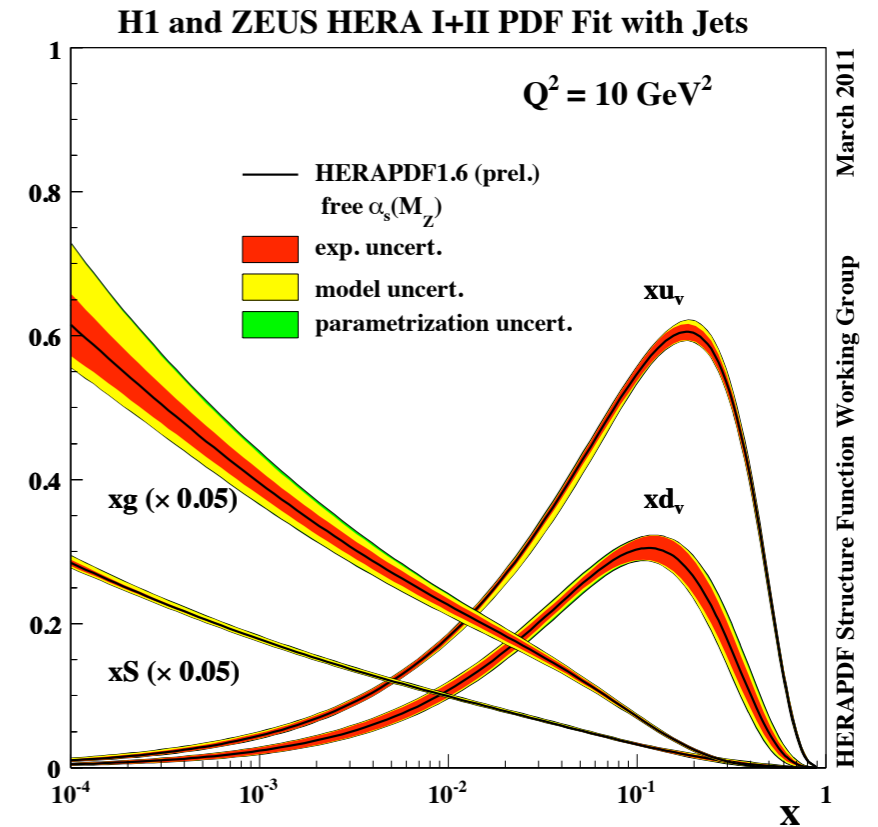


- ▶ No pile-up subtraction or corrections for the underlying event
- ▶ Differences very small, within uncertainties
- ▶ HERA provides a very clean environment to study QCD

# HERA Jet Data in PDF Fits



including  
jets

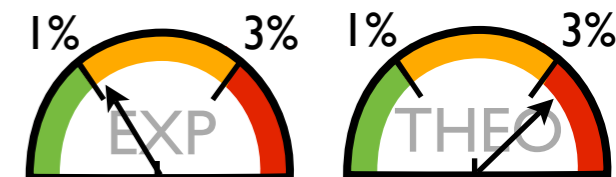
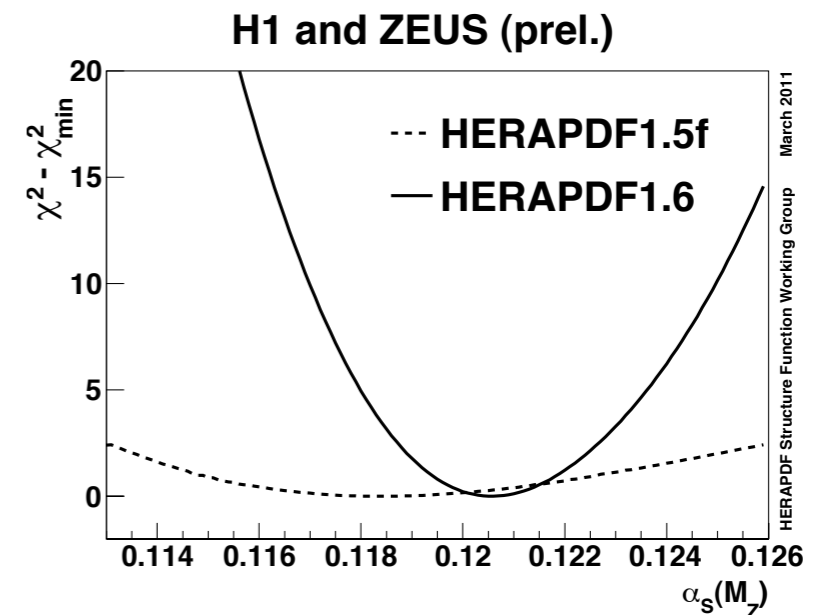


H1prelim-II-034, ZEUS-Prel-II-001

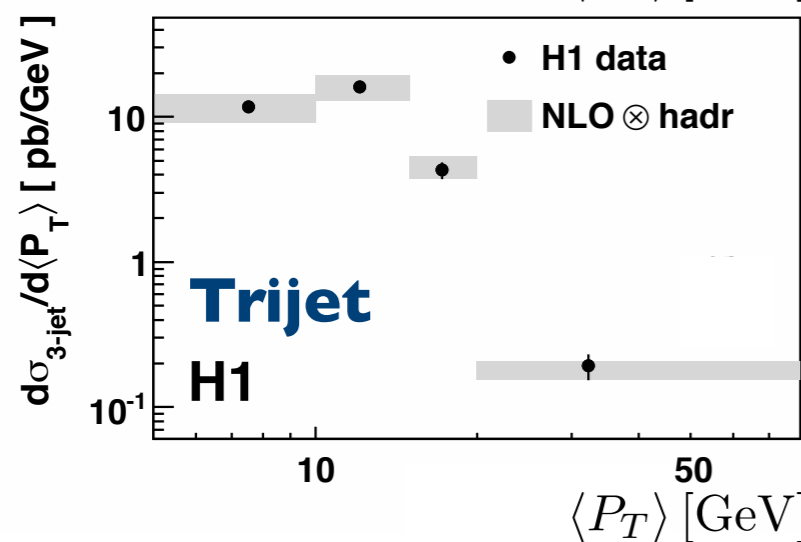
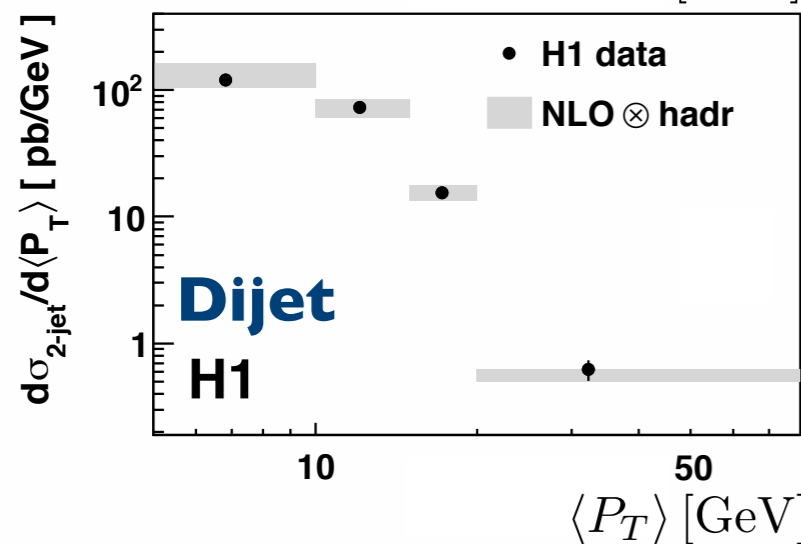
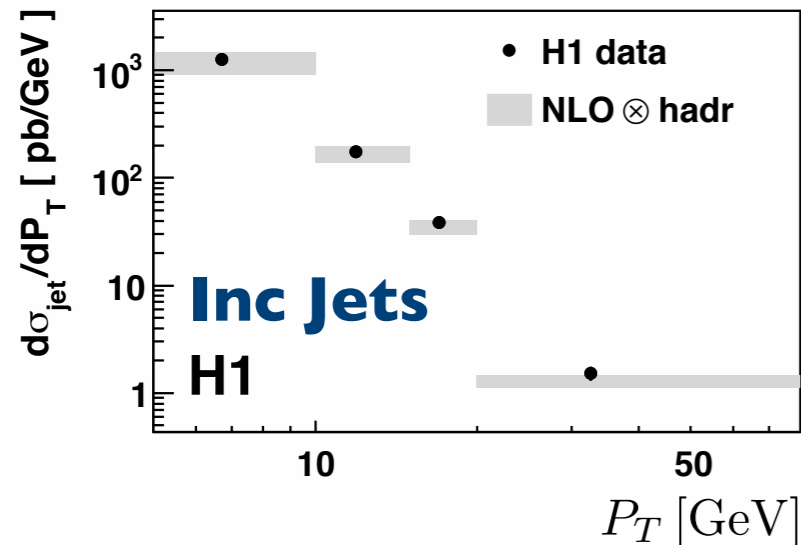
Only inclusive jet cross sections included in HERAPDF fits so far

Large potential shown: correlation between gluon PDF and  $\alpha_s(M_Z)$  disentangled

$$\alpha_s(M_Z) = 0.1202 \pm 0.0013(\text{exp.}) \pm 0.0012(\text{had}) \pm 0.0045(\text{scale})$$



# Multijet Cross Sections At Low $Q^2$



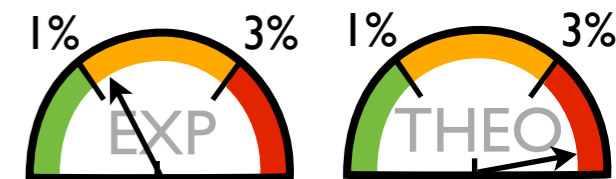
- Double-differential inclusive jet, dijet and trijet measurement, small experimental uncertainties of 6 – 10 %
- Data well described by NLO,  $\mu_r = \sqrt{(Q^2 + P_T^2)}/2$  theoretical uncertainties dominated by missing higher orders: 30% at low  $Q^2, P_T$  and 10% at high  $Q^2, P_T$
- choice of  $\mu_r = \langle P_T \rangle$  disfavoured by data
- Simultaneous  $\alpha_s(M_Z)$  fit to 62 data points:

$$\alpha_s(M_Z) = 0.1160 \pm 0.0014(\text{exp.})$$

$$+0.0093 \text{ (th.)} \pm 0.0016(\text{pdf})$$

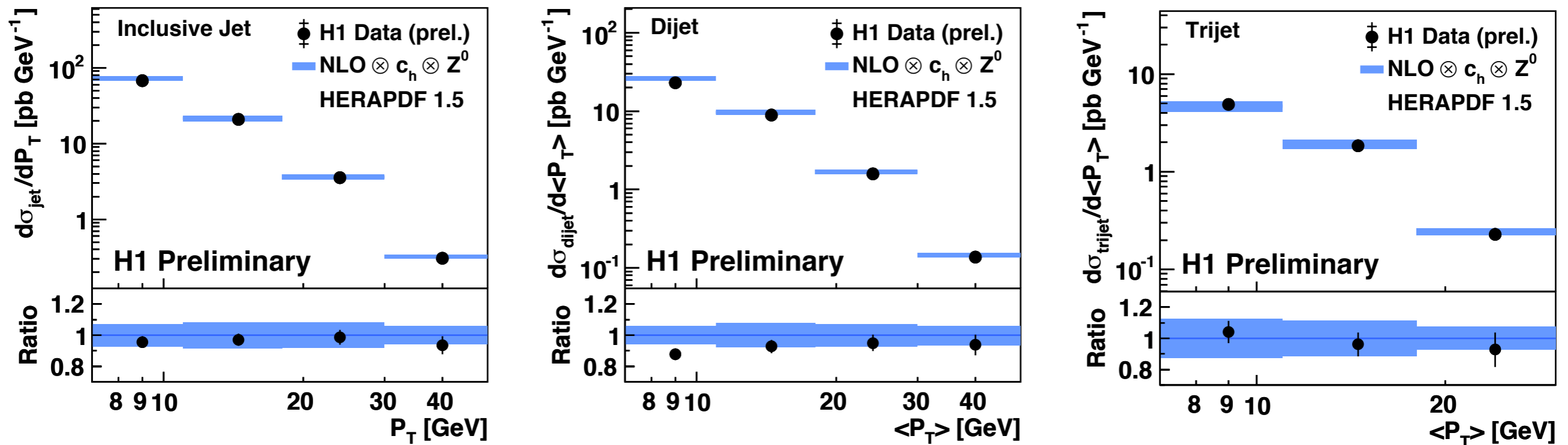
$$-0.0077$$

H1, EPJ C67, 1 (2010)



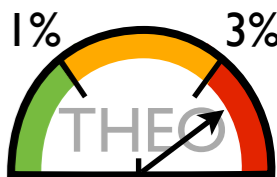


# Multijet Cross Sections At High $Q^2$



- Double-differential inclusive jet, dijet and trijet measurement at  $150 < Q^2 < 15000 \text{ GeV}^2$
- Reduced scale dependence compared to low  $Q^2$  measurement
- Data are well described by NLO calculations,  $\mu_r = \sqrt{(Q^2 + P_T^2)}/2$   
independent test of HERAPDF 1.5
- Determination of  $\alpha_s(M_Z)$  from individual observables  
with  $\sim 2\%$  experimental and  $\sim 3.5\%$  theoretical uncertainty

H1-Prelim-11-032



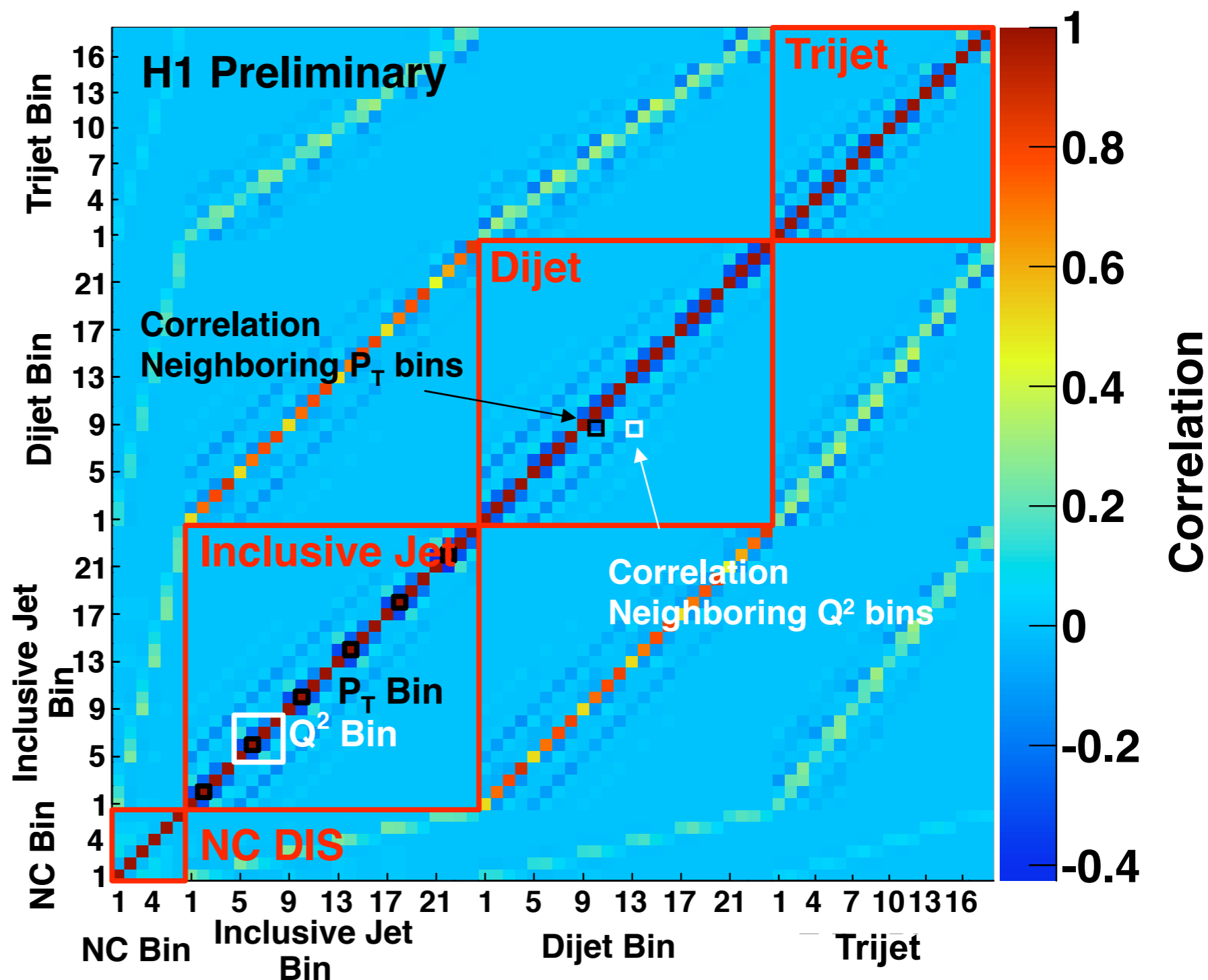
# Unfolding of Multijet Cross Sections

Take correlations between observables into account

Full, partly anti-correlated, covariance matrix available after unfolding

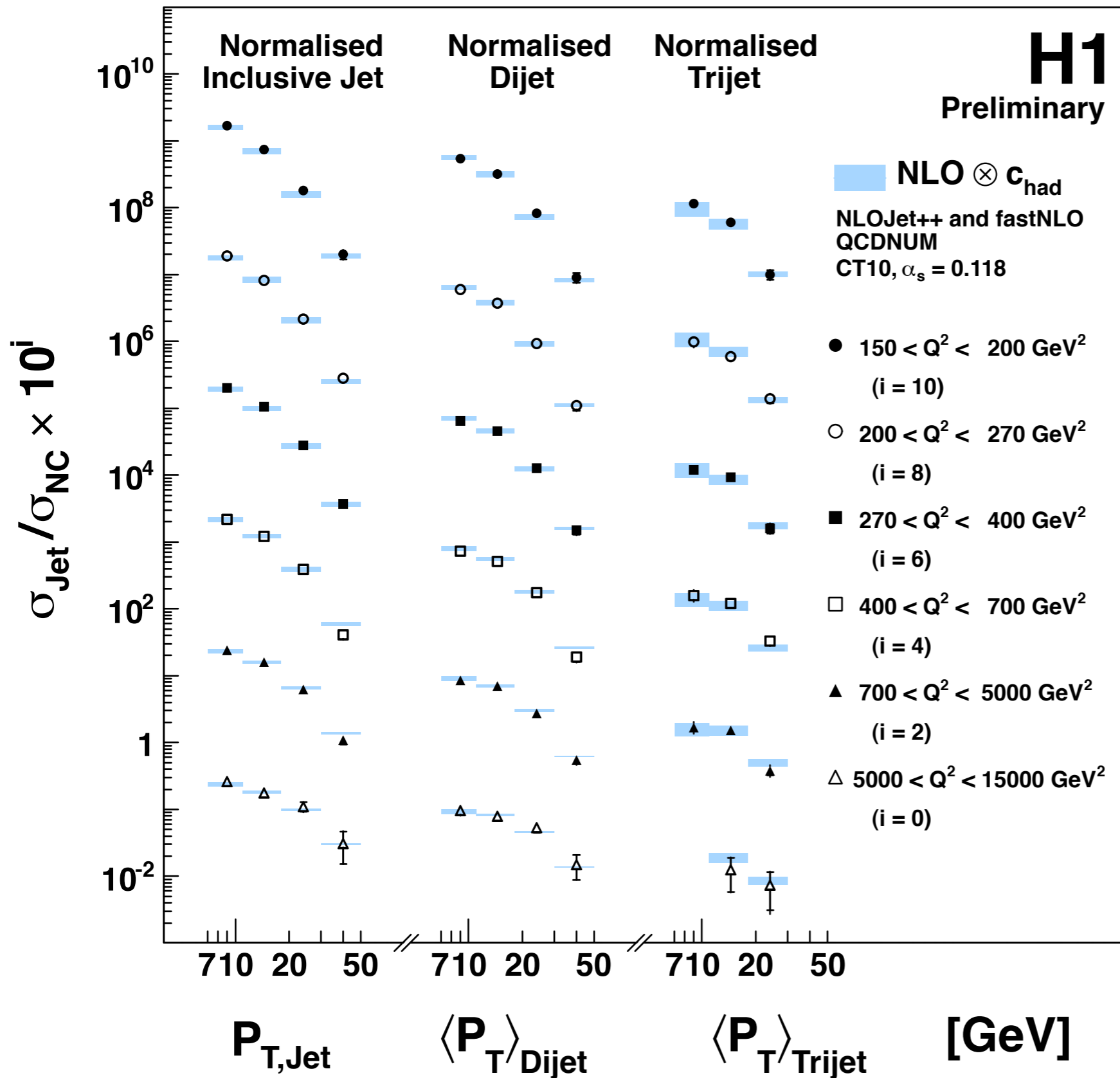
Normalisation of individual measurements possible using full error propagation

Valuable information for QCD fits



H1-Prelim-12-03 I

# Normalised Multijet Cross Sections



## NLO Calculation

NLOJet++ and  
QCDNUM

corrected for  
hadronisation effects

## Scale choice:

$$\mu_f^2 = Q^2$$

$$\mu_r^2 = \frac{1}{2} (Q^2 + P_T^2)$$

H1-Prelim-12-031

# Determination of $\alpha_s(M_Z)$

## Normalized Inclusive Jet

$$\alpha_s = 0.1197 \pm 0.0008 \text{ (exp)} \pm 0.0014 \text{ (PDF)} \pm 0.0011 \text{ (had)} \pm 0.0053 \text{ (theo)}$$
$$\chi^2 / \text{ndf} = 28.7/23 = 1.24$$

## Normalized Dijet

$$\alpha_s = 0.1142 \pm 0.0010 \text{ (exp)} \pm 0.0016 \text{ (PDF)} \pm 0.0009 \text{ (had)} \pm 0.0048 \text{ (theo)}$$
$$\chi^2 / \text{ndf} = 27.0/23 = 1.17$$

## Normalized Trijet

$$\alpha_s = 0.1185 \pm 0.0018 \text{ (exp)} \pm 0.0013 \text{ (PDF)} \pm 0.0016 \text{ (had)} \pm 0.0042 \text{ (theo)}$$
$$\chi^2 / \text{ndf} = 12.0/16 = 0.75$$

Good  $\chi^2/\text{ndf}$  for each individual observable

Tension between  $\alpha_s$  from dijets and inclusive/trijets observed, but  $\alpha_s$  values well within theoretical uncertainties

## Combined Fit

$$\alpha_s = 0.1177 \pm 0.0008 \text{ (exp)}$$
$$\chi^2 / \text{ndf} = 104.608 / 64 = 1.634$$

**Fit to all data points**

Relatively large  $\chi^2/\text{ndf}$

H1-Prelim-12-03 I

# Combined Fit

## Largest benefit is from a combined fit

simultaneous fit to normalised inclusive jet, dijet and trijet cross sections

## Sensitivity to higher orders

theoretical uncertainty estimated by variation of scale, use k-factor as indicator for higher order contributions

$$k = \sigma_{\text{NLO}} / \sigma_{\text{LO}} \quad \text{range of k-factor: } 1.05 < k < 1.45$$

## Restrict analysis to $k < 1.3$

faster convergence of perturbative series

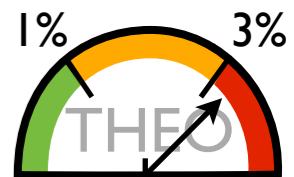
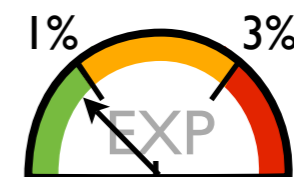
trade-off between number of data points and smaller theoretical uncertainty

## Normalised Multijets with $k < 1.3$

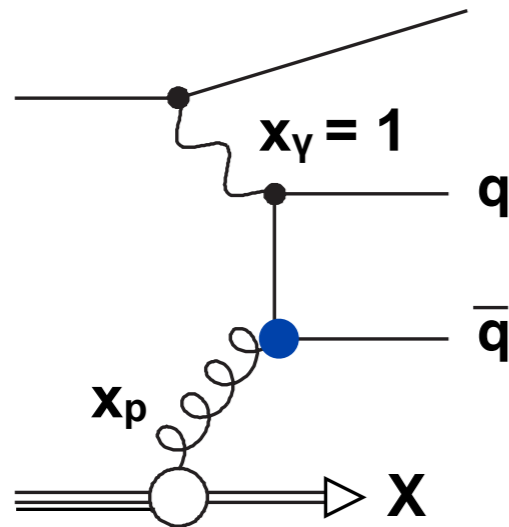
$$\alpha_s = 0.1163 \pm 0.0011 \text{ (exp)} \pm 0.0014 \text{ (PDF)} \pm 0.0008 \text{ (had)} \pm 0.0039 \text{ (theo)}$$

much better  $\chi^2/\text{ndf}$ :  $53.2 / 41 = 1.30$

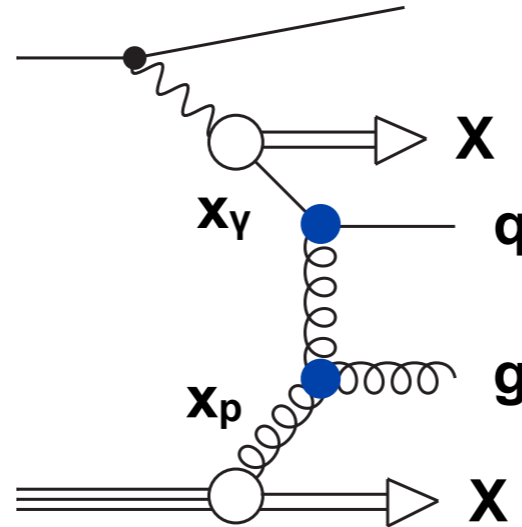
H1-Prelim-12-031



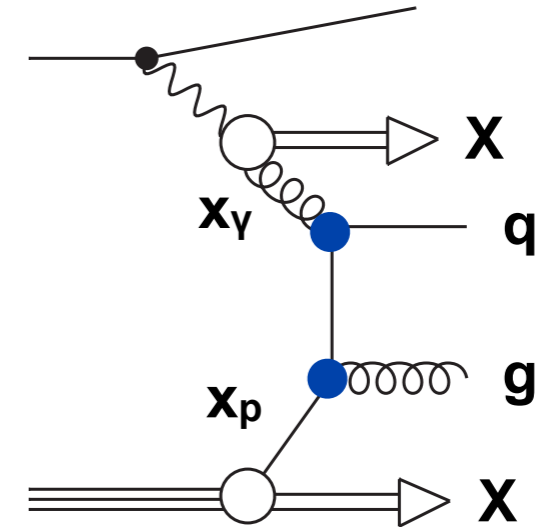
# Jet Production in Photoproduction



direct photoproduction



resolved photoproduction



$$\sigma_{n\text{-jet}}^{\gamma p} = \sum_m \alpha_s^m(\mu_r) \sum_{i,j=q,\bar{q},g} \int dx_p \int dx_\gamma f_{i/p}(x_p, \mu_f) f_{j/\gamma}(x_\gamma, \mu_f) \cdot \hat{\sigma}_{ij}^m(x_p, x_\gamma, \mu_r, \mu_f)$$

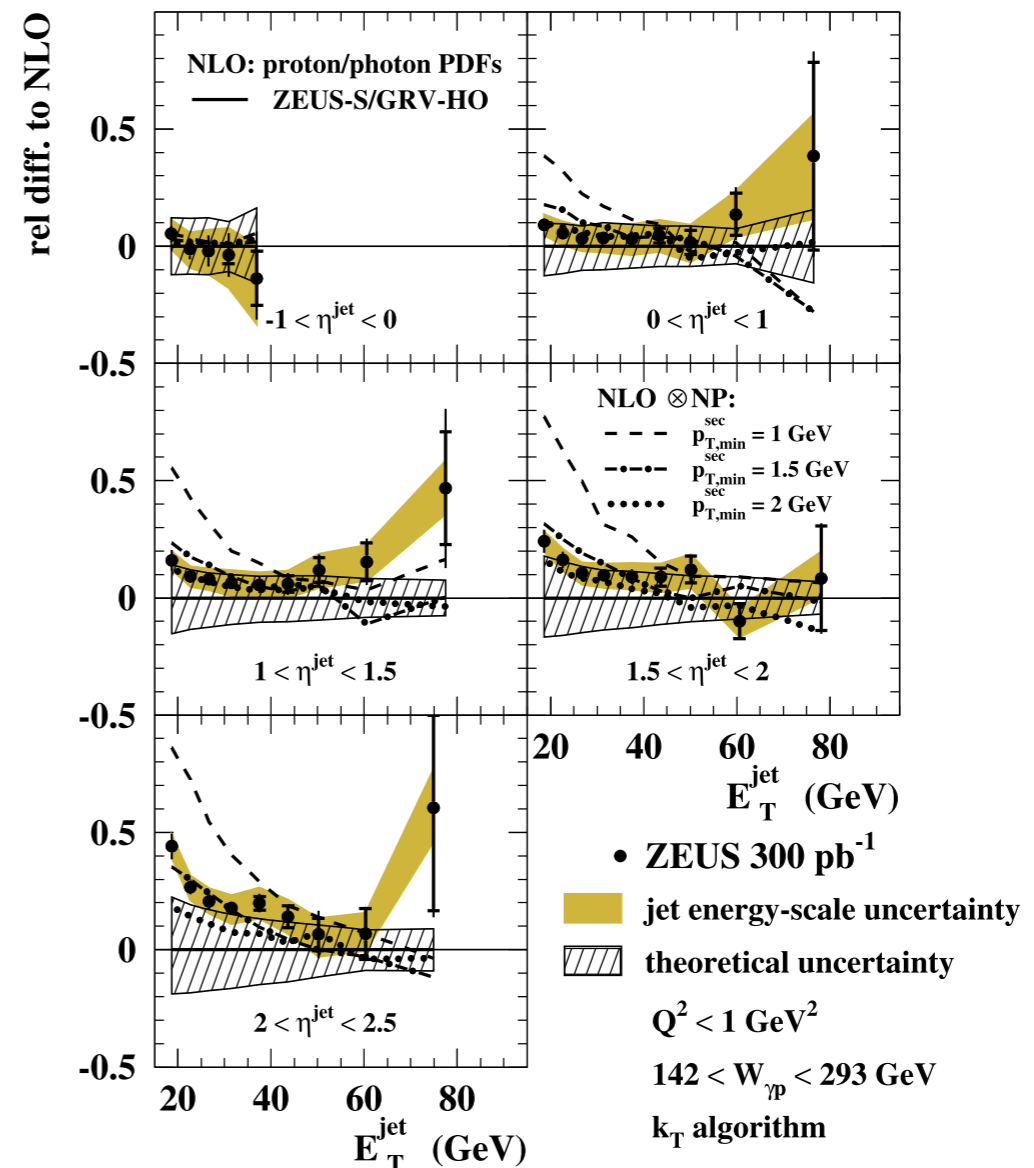
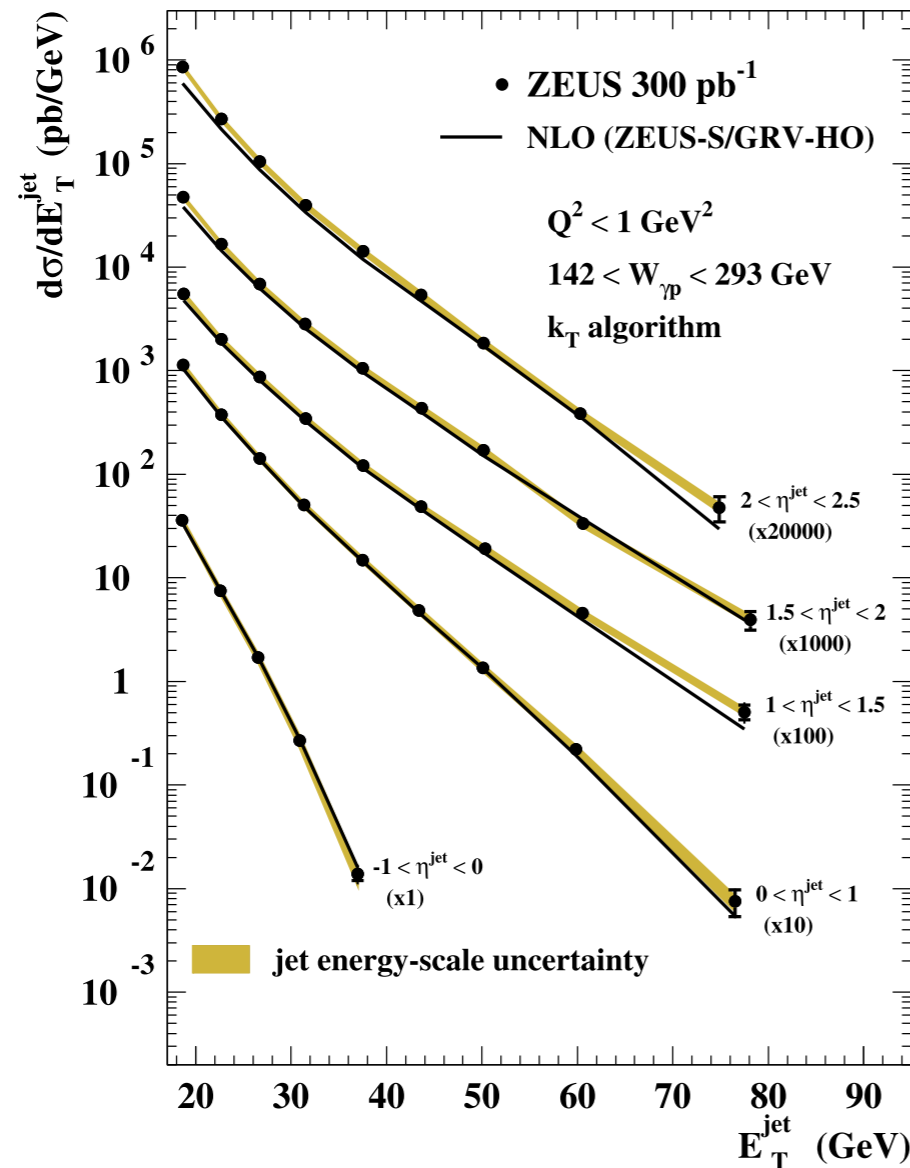
Photon flux: Weizsäcker-Williams approximation

Direct case:  $f_{j/\gamma}(x_\gamma, \mu_f) = \delta(x_\gamma - 1)$

Direct sensitivity to  $\alpha_s$ , gluon and photon PDFs

# Jet Production in Photoproduction

ZEUS, Nucl. Phys. B, 864, 1 (2012)



## Double differential measurement in $P_T$ and $\eta$

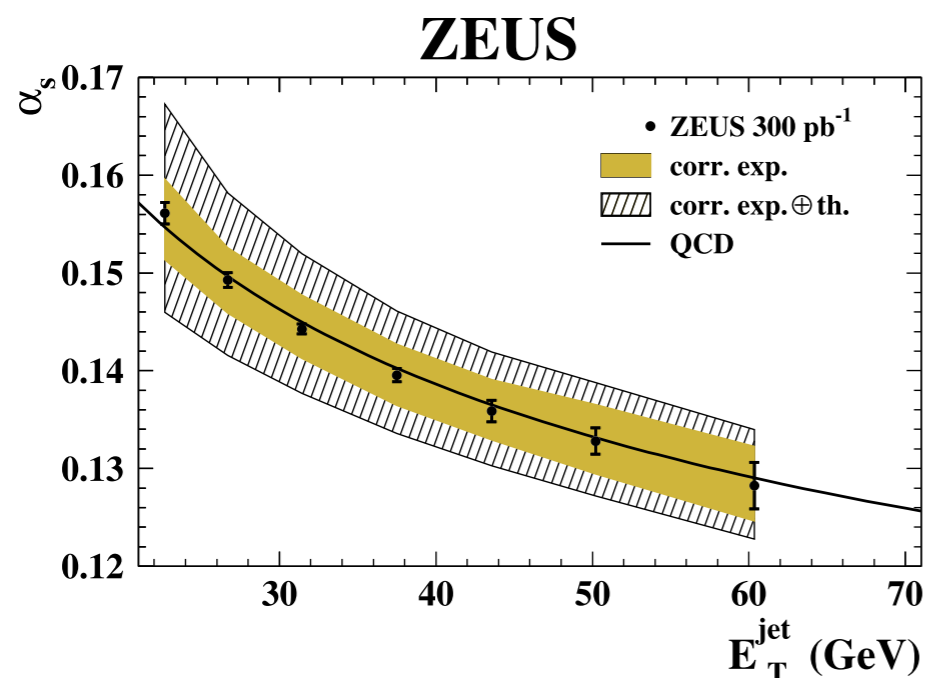
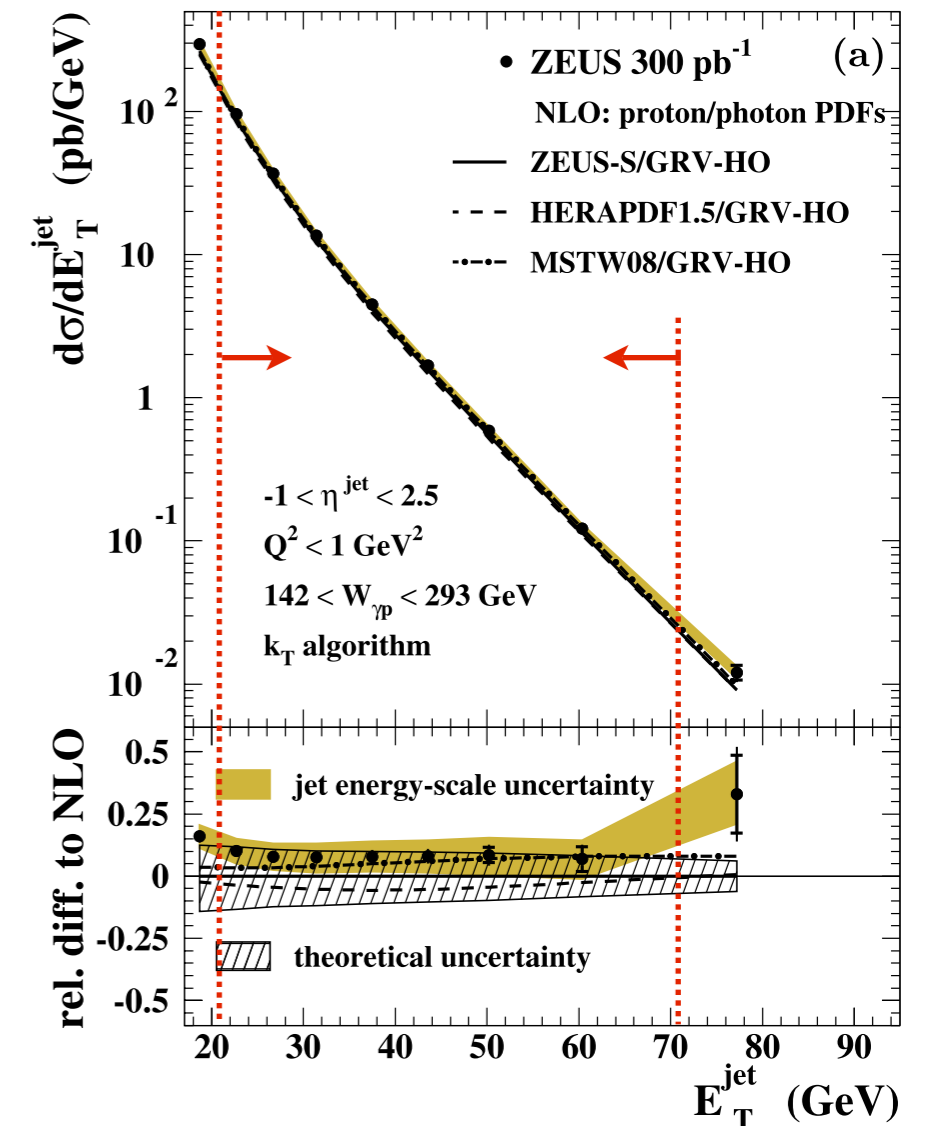
- ▶ Higher  $P_T$  reach than in the DIS case
- ▶ Differences between data and NLO at low  $P_T$ /large  $\eta$  could be from photon PDFs or non-perturbative effects

# Jet Production in Photoproduction

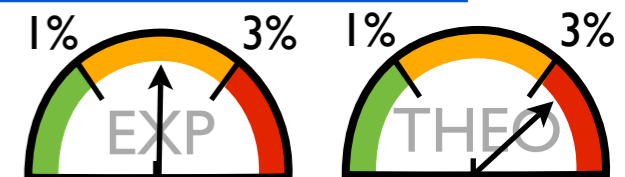
ZEUS, Nucl. Phys. B, 864, 1 (2012)

## Determination of $\alpha_s$

- ▶ restricted  $P_T$  region of  $21 < P_T < 71$  GeV to reduce non-perturbative effects and dependence on proton PDF
- ▶ similar values obtained of  $k_T$ , anti- $k_T$  and SISCone algorithms
- ▶ reduced uncertainty due to missing higher orders, but uncertainty due to photon PDFs non-negligible



$$\alpha_s(M_Z) = 0.1206^{+0.0023}_{-0.0022} \text{ (exp.) } ^{+0.0042}_{-0.0035} \text{ (th.)}$$

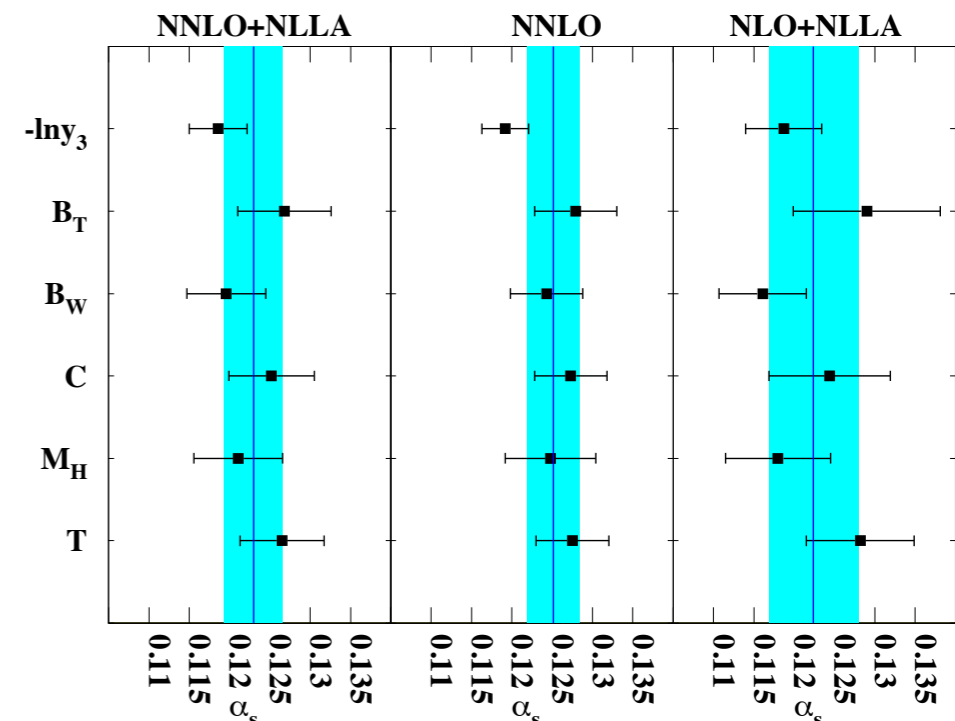




# Extending the Reach: $e^+e^-$

## Use event shapes at $\mu \neq M_Z$

- ▶  $-\ln(y_3)$ , the two-to-three jet transition
- ▶  $B_T$  and  $B_W$ , the total and wide jet broadening
- ▶  $C$ , the C-parameter derived from the linearised momentum tensor
- ▶  $M_H$ , the heavy jet mass
- ▶  $T$ , thrust

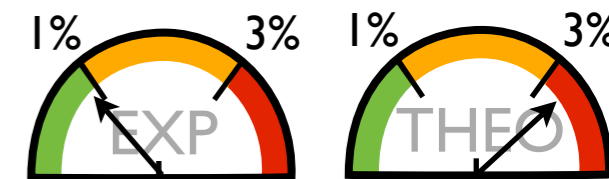
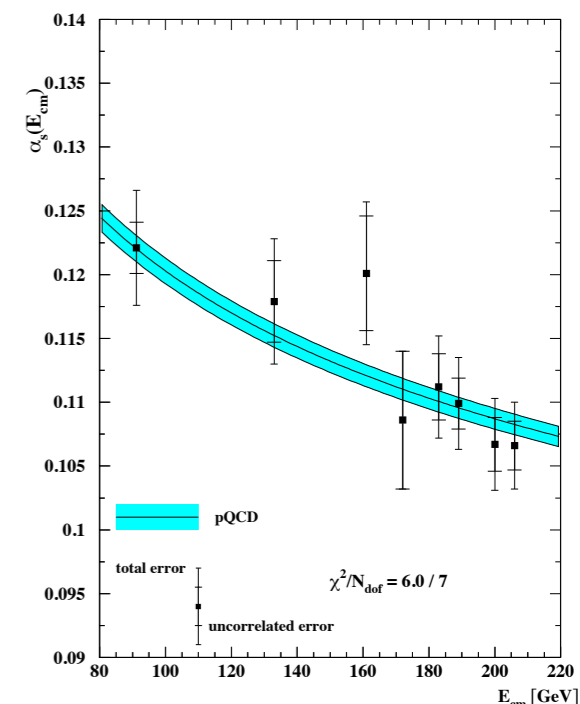


## NNLO Calculations have been performed

- ▶ Resummation of leading logs due to soft gluon radiation essential
- ▶ Partial re-introduction of scale dependence
- ▶ Running of  $\alpha_s$  probed up to  $\mu = 204$  GeV

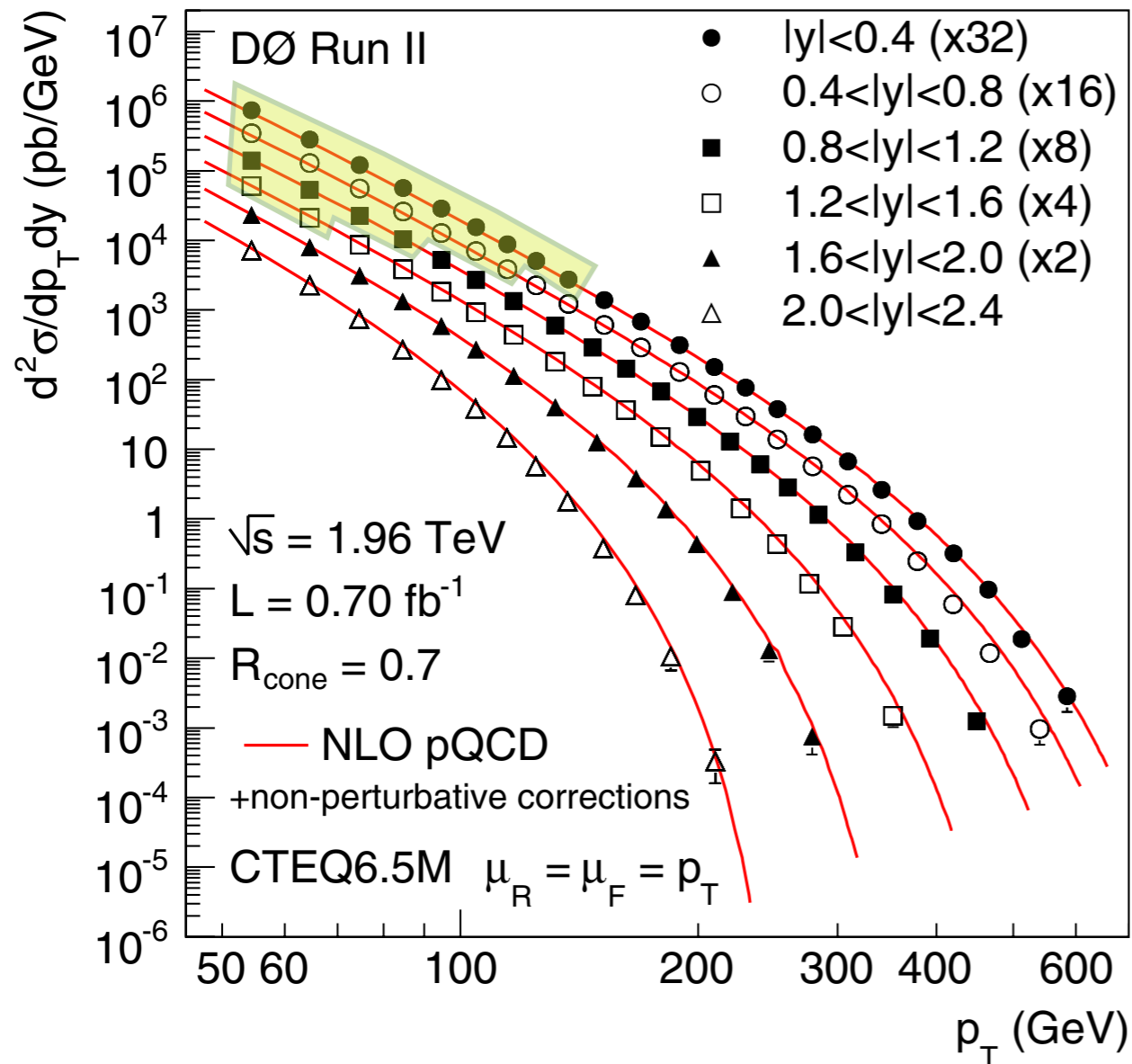
$$\alpha_s(M_Z) = 0.1224 \pm 0.0009 \text{ (stat)} \pm 0.0009 \text{ (exp)} \\ \pm 0.0012 \text{ (had)} \pm 0.0035 \text{ (theo)}$$

G. Dissertori et al., JHEP 08, 036 (2009)



# Extending the Reach: $p\bar{p}$

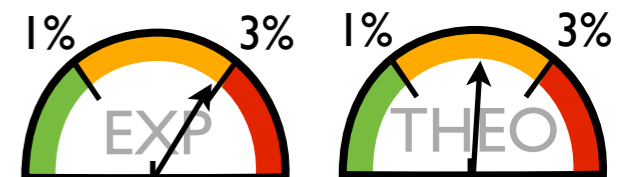
## D0 measurement of inclusive jet production



- ▶ Jet  $P_T$  measured up to 600 GeV
- ▶ Good description over full  $\eta$  and  $P_T$  range by NLO calculations
- ▶ Potential bias: data are input for PDF fits and influence gluon density at  $x > 0.2$
- ▶ Extraction of  $\alpha_s$  restricted to  $x_1, x_2 < 0.25$ , only 22 bins left at relatively small  $P_T$
- ▶ threshold corrections available: reduction of theoretical uncertainty

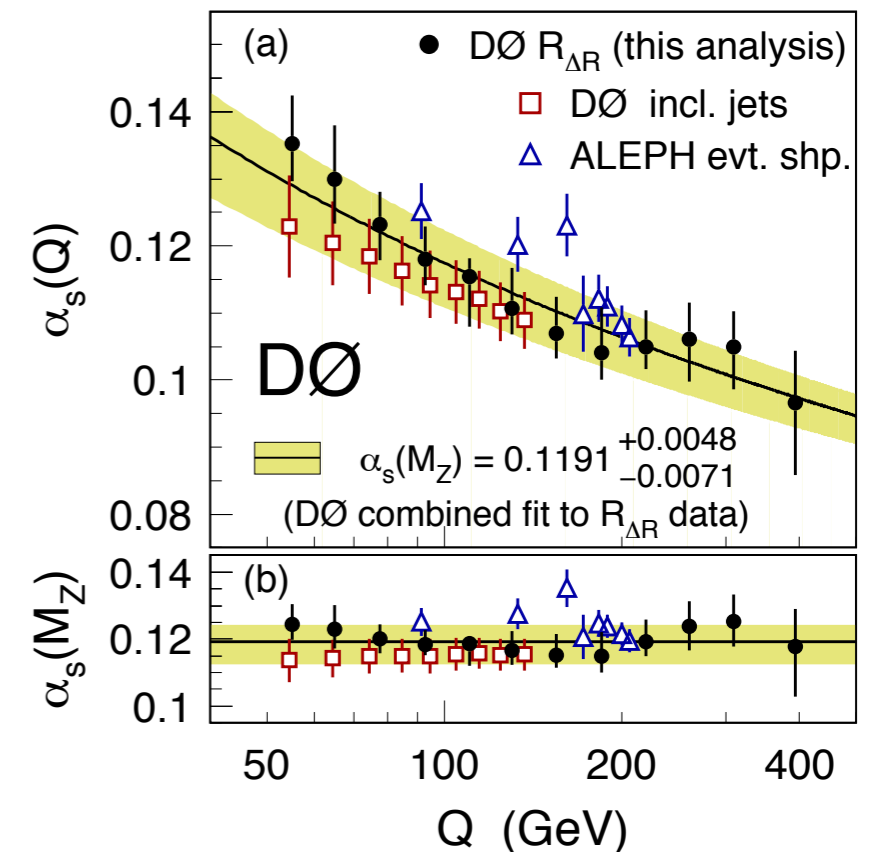
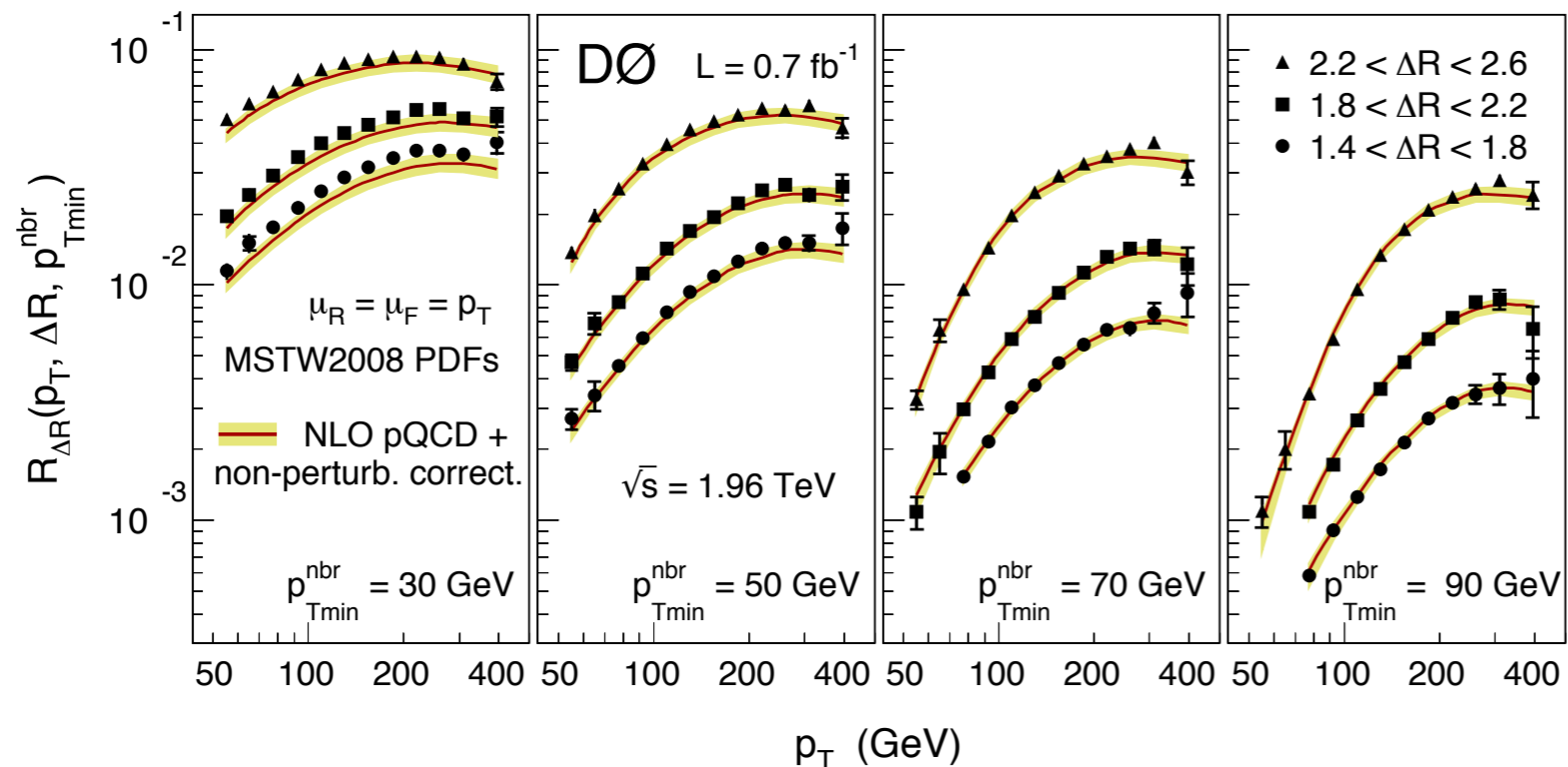
$$\alpha_s(M_Z) = 0.1161^{+0.0034}_{-0.0033} \text{ (exp.) } ^{+0.0029}_{-0.0035} \text{ (theo.)}$$

D0, Phys. Rev. D80, 111107 (2009)



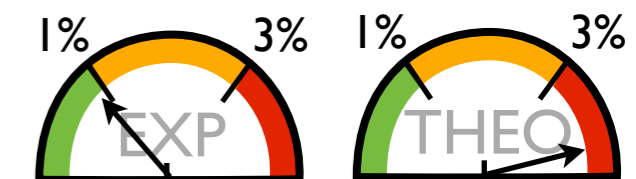
# Extending the Reach: $p\bar{p}$

## Normalised jet cross sections at the Tevatron



Measurement of  $R_{\Delta R} = \frac{\sum_{i=1}^{N_{\text{jet}}(p_T)} N_{\text{nbr}}^{(i)}(\Delta R, p_{T\text{min}}^{\text{nbr}})}{N_{\text{jet}}(p_T)}$

- ▶ Measure of hardness of neighbouring jets within  $\Delta R$
- ▶ Small sensitivity to proton PDFs
- ▶ Small experimental uncertainty due to partial cancellations
- ▶ Probing of scales up to 400 GeV, but large theoretical uncertainty (only NLO calculations available)

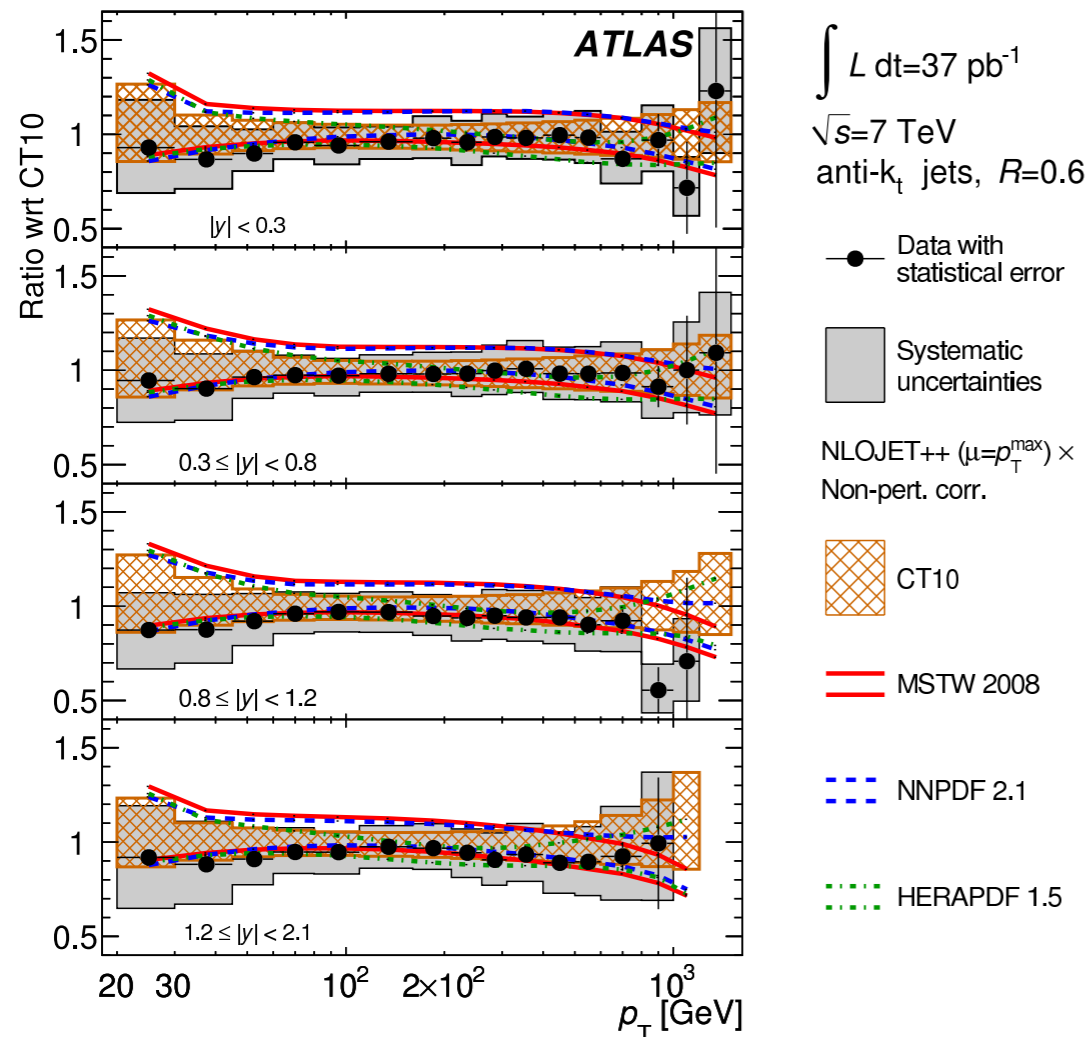


D0, arXiv:1207.4957

# LHC

## Jet measurements at the LHC are gaining in precision

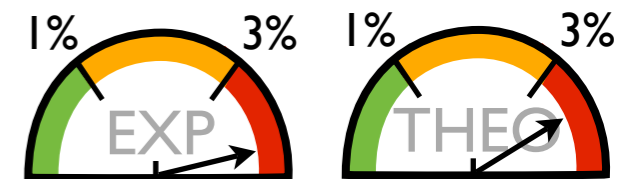
ATLAS, Phys. Rev. D86, 014022 (2012)



- ▶ Jet data available with scales up to 1-2 TeV
- ▶ Full unfolding of experimental effects of ATLAS inclusive jet data
- ▶ Experimental uncertainties of 10-20%
- ▶ Non-negligible non-perturbative corrections
- ▶ First determination of  $\alpha_s$  at scales up to 600 GeV
- ▶ Large uncertainty due to disagreement between  $R=0.4$  and  $R=0.6$

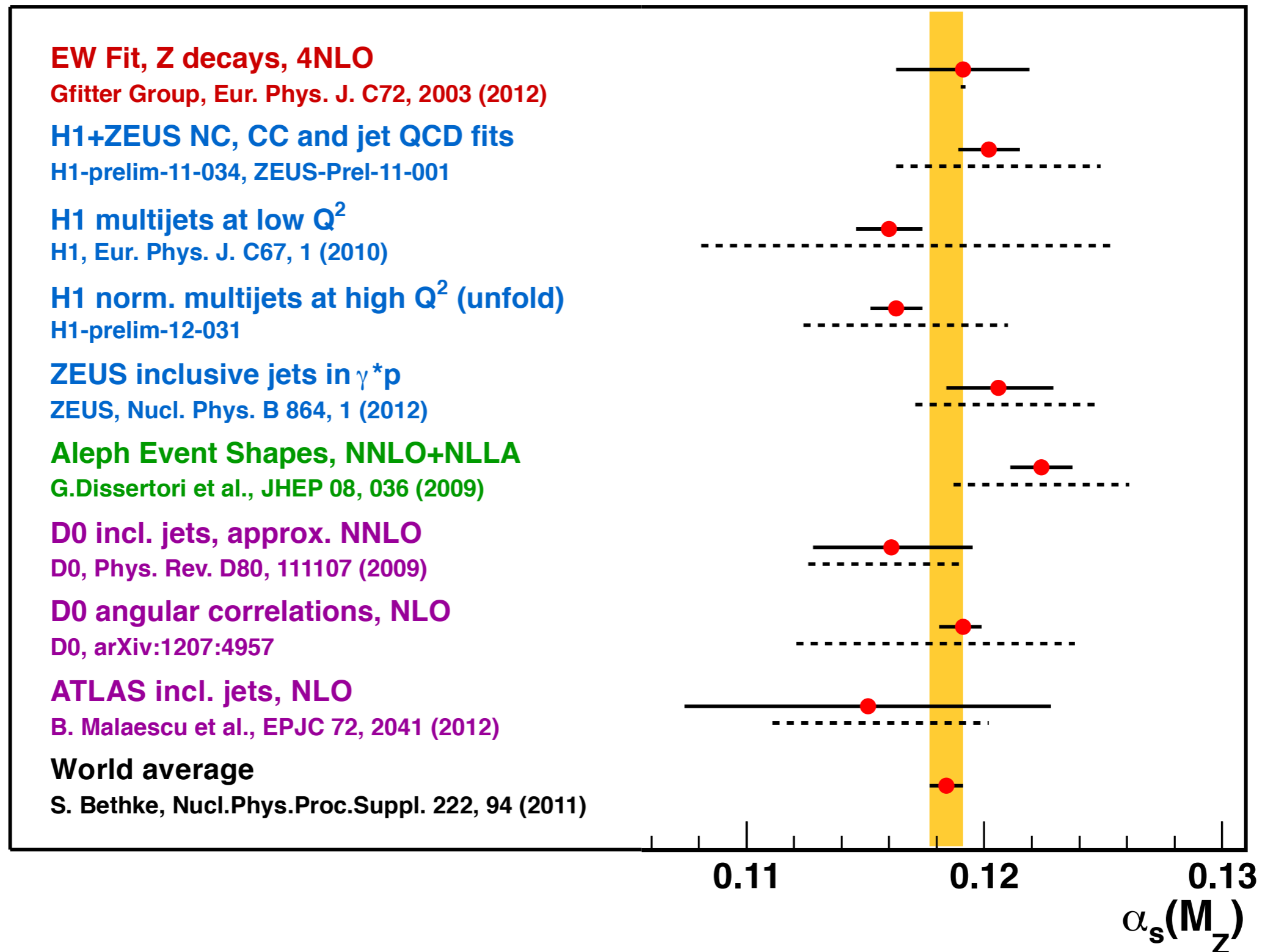
$$\alpha_s(M_Z) = 0.1151 \pm 0.0077 \text{ (exp.) } \begin{matrix} +0.0051 \\ -0.0040 \end{matrix} \text{ (theo.)}$$

B. Malaescu, P Starovoitov, EPJC 72, 2041 (2012)



# Comparison of $\alpha_s(M_Z)$ Values

Uncertainties: exp. ——— theo. - - - - -



# Summary

## The Strong Coupling $\alpha_s$

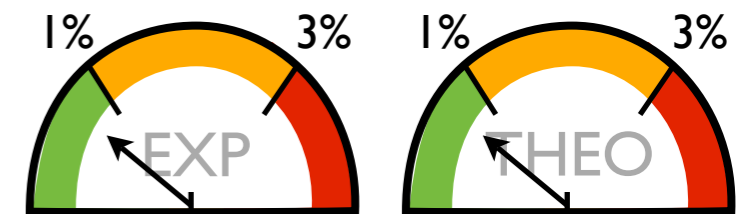
- ▶ Allows for stringent tests of QCD
- ▶ Universality impressively demonstrated by determination from very different processes at very different scales
- ▶ Deviations from the RGE could hint at new physics - precision needed!

## Experimental Data

- ▶ HERA jet data among the most precise data for precision tests of QCD
- ▶ New normalised Tevatron jet measurement with exp. uncertainty  $\sim 1\%$
- ▶ Probe highest scales with LHC jet data

## Theory

- ▶ Missing higher orders often the dominating source of uncertainty



**We are not there yet!**

# Additional Material

# Jet Observables

## Inclusive Jets

each jet above a given  $P_T$  requirement contributes to the cross section: large statistics, calculation needs contributions from higher-order configurations

## Dijets

events with at least two jets above a certain  $P_T$  contribute: reduced statistics but NLO calculations have smaller scale dependence

## Trijets

events with at least three jets above a certain  $P_T$  contribute: smaller statistics and slightly larger experimental uncertainties but high sensitivity to  $\alpha_s$  ( $O(\alpha_s^2)$  at LO)

## Normalised Jet Cross Sections

benefit from partial cancellations of experimental and theoretical uncertainties by measurement of  $\sigma_{\text{jet}}/\sigma_{\text{NC}}$





# Regularised Unfolding

Migration matrix  $\mathbf{A}$  describes the detector response

$$\mathbf{m} = \mathbf{A} \cdot \mathbf{x}$$

$\mathbf{m}$ : measured distribution (detector level)  
 $\mathbf{x}$ : true distribution (particle level)

Perform unfolding by analytic minimisation of

$$\chi^2 = \frac{1}{2} (\mathbf{m} - \mathbf{A}\mathbf{x})^T \mathbf{V}^{-1} (\mathbf{m} - \mathbf{A}\mathbf{x}) + \tau^2 \cdot \mathbf{L}$$

TUnfold (S. Schmitt), arXiv:1205.6201

Regularisation parameter  $\tau$  suppresses large fluctuations

Correlation of datasets contained in covariance matrix  $\mathbf{V}$

Possibility to unfold four measurements at once:

NC DIS, inclusive jet, dijet and trijet cross sections

# Unfolding of Jet Multiplicities

Particle level			<b>Trijet</b> $Q^2, \langle p_T \rangle_3, y,$ Trijet-cuts	$\epsilon_{J3}$
			<b>Dijet</b> $Q^2, \langle p_T \rangle_2, y,$ Dijet-cuts	$\epsilon_{J2}$
		<b>Incl. Jet</b> $p_T, Q^2, y, (\eta)$		$\epsilon_J$
	<b>DIS-Events</b> $(Q^2, y)$	Reconstructed jets without match to generator level	Reconstructed Dijet events which are not generated as Dijet event	Reconstructed Trijet events which are not generated as Trijet event
	Detector level			

## Migration Matrix

Multidimensional unfolding in  $Q^2, P_T$  and  $y$

Full treatment of migrations between jet observables

Normalisation preserved with inclusive NC DIS events

Detector response obtained from simulation

Dimension:  
about 600 x 2200 bins

# MC Test

## Performance test:

Test unfolded result w.r.t.  
MC truth

## Pull distribution:

$$P_i = \frac{x_i^{\text{unfold}} - x_i^{\text{true}}}{\delta_i^{\text{unfold}}}$$

## Two theo. models:

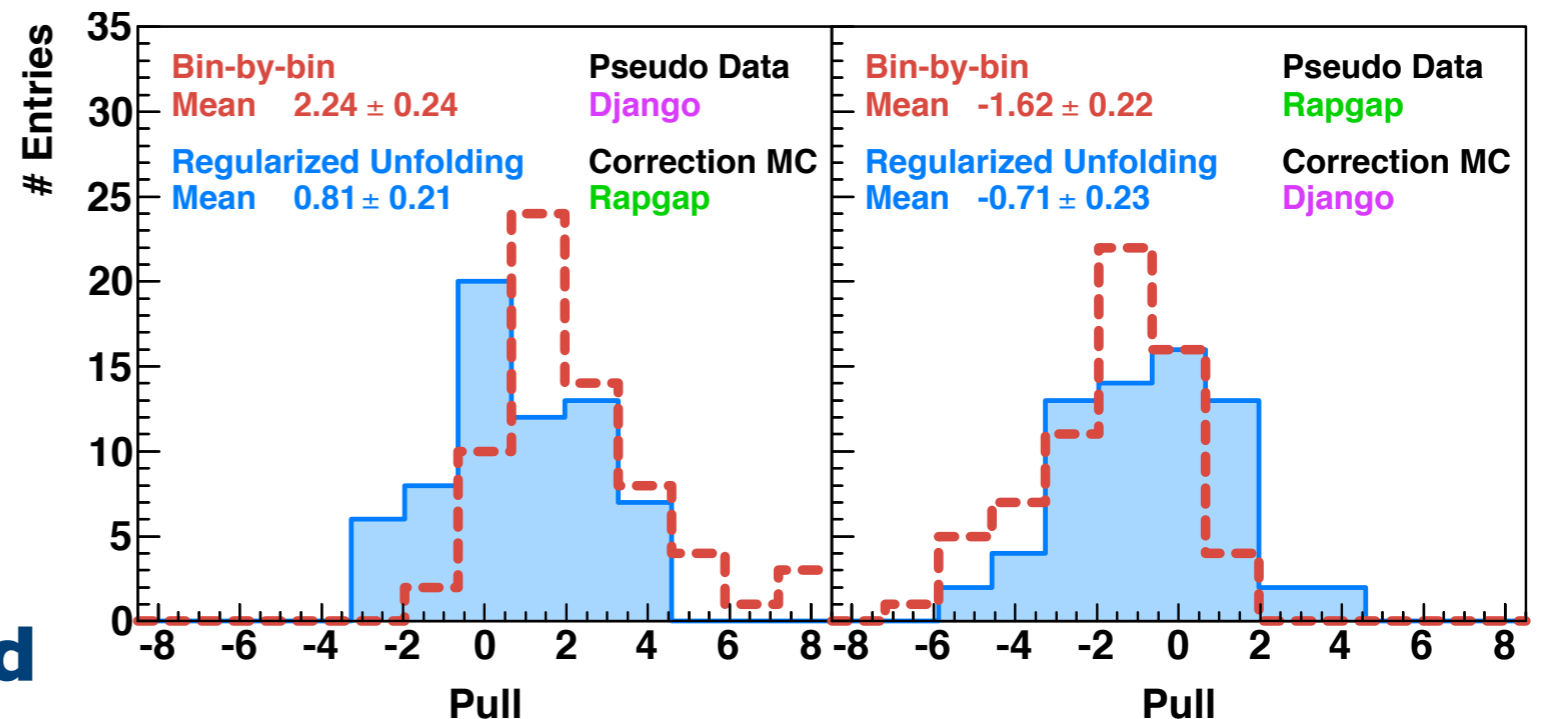
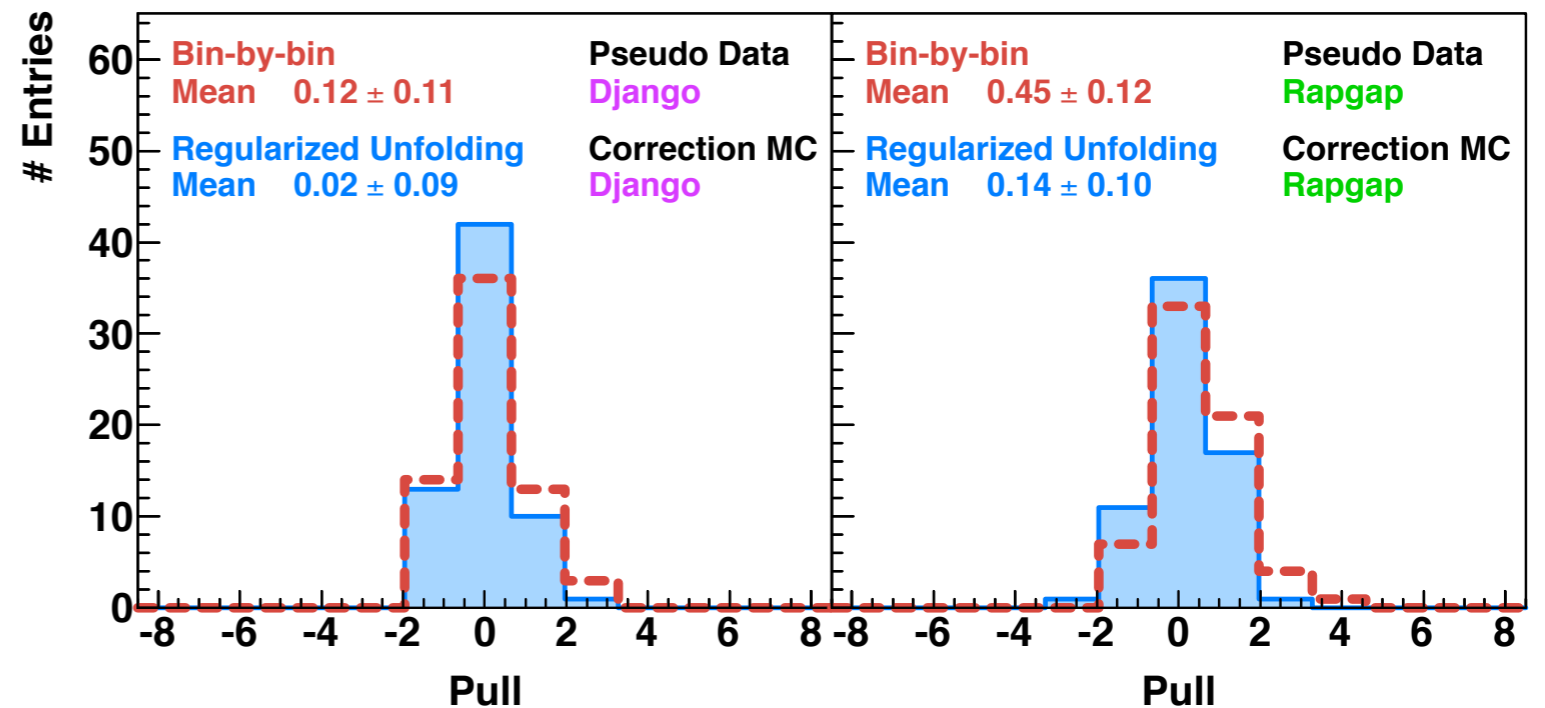
Djangoh (CDM)

Rapgap (MEPS)

## Comparison:

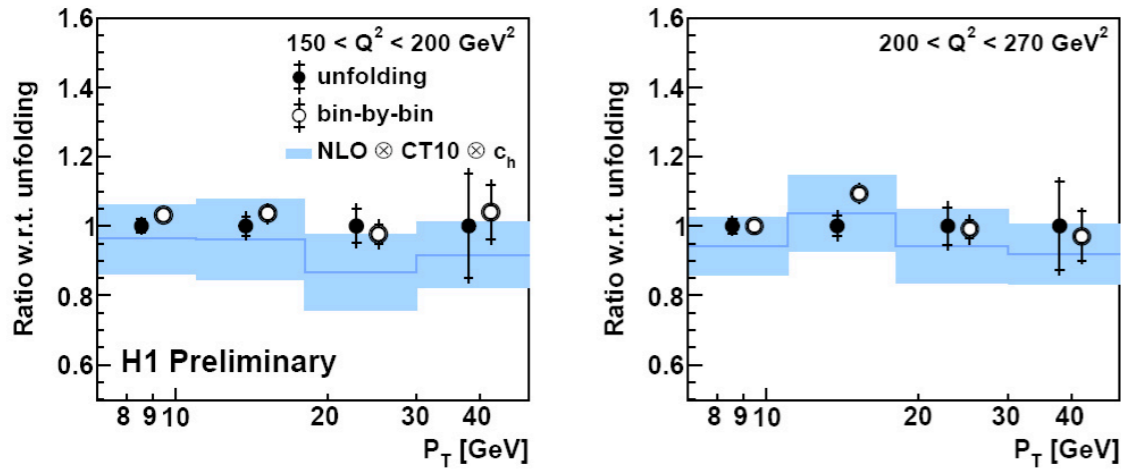
Unfolded results with  
results obtained **bin-wise**  
**derived correction** factors

⇒ **Unfolding less biased**

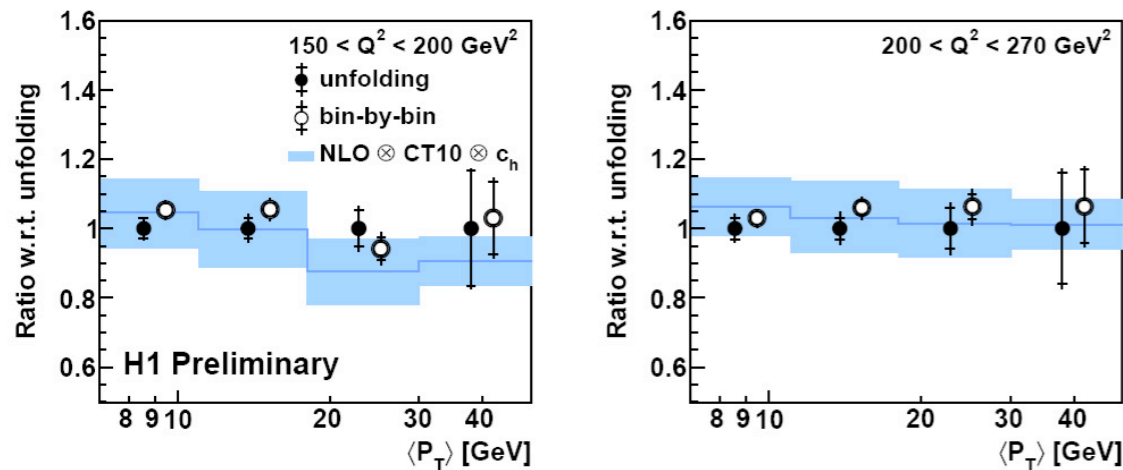


# Comparison to “bin-by-bin”

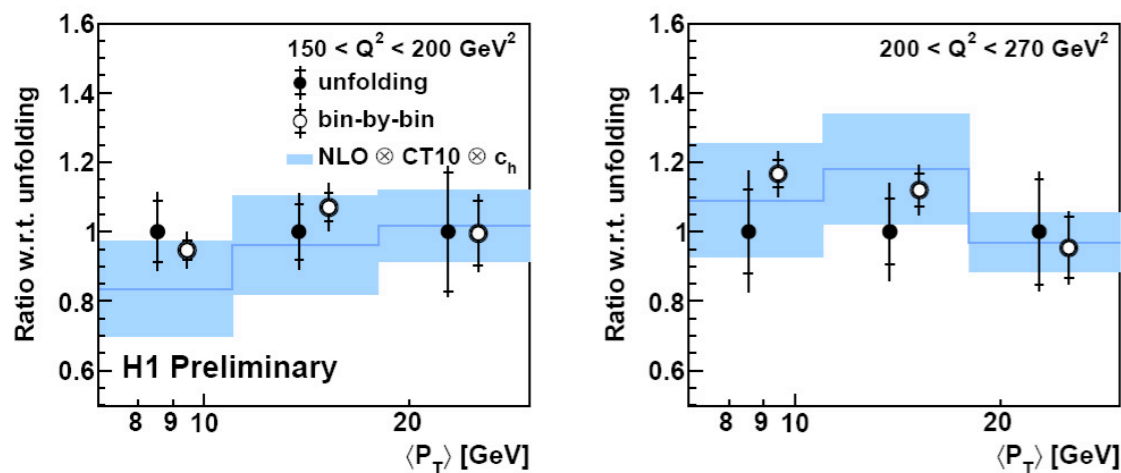
Normalised Inclusive Jet Cross Section



Normalised Dijet Cross Section



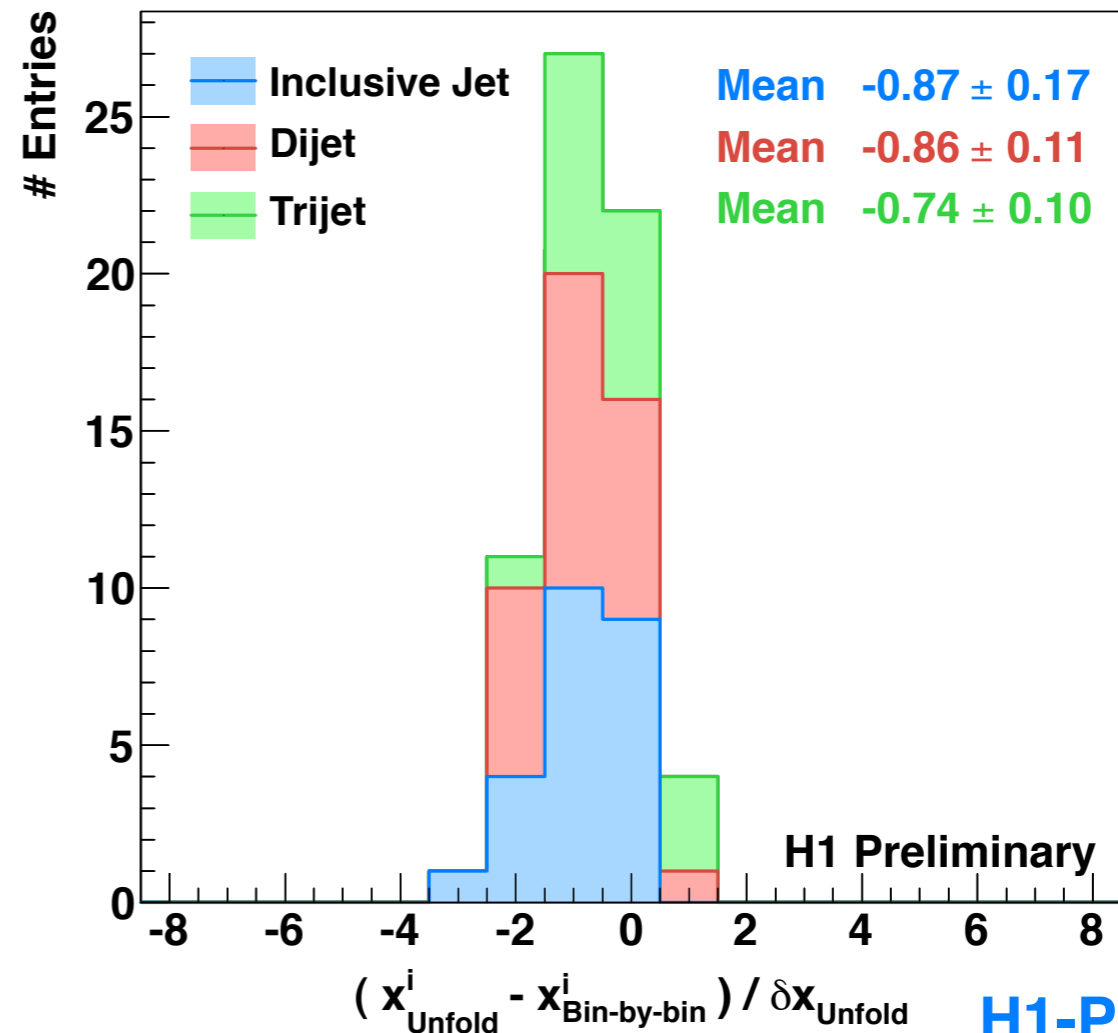
Normalised Trijet Cross Section



## Performance on data

- ▶ bin-by-bin result gives slightly higher cross section ( $\sim 0.8\sigma$ )
- ▶ larger stat. error - but full covariance matrix available

Pull between two Correction Methods



H1-Prel-12-031

# Determination of $\alpha_s(M_Z)$

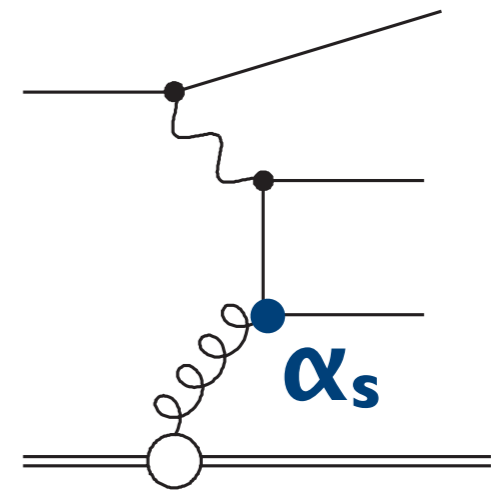
NLO calculation depends on PDF and  $\alpha_s(M_Z)$

⇒ Keep PDF fixed and fit  $\alpha_s(M_Z)$

Hessian method: Minimise  $\chi^2(\alpha_s)$

$$\chi^2(\alpha_s) = \mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} + \sum_k \epsilon_k^2$$

$$u_i = \sigma_i^{\text{exp}} - \sigma_i^{\text{theo}}(\alpha_s, \text{pdf}) \left( 1 - \sum_k \Delta_{ik} \epsilon_k \right)$$



- Experimental uncertainty obtained by  $\chi^2 = \chi^2_{\text{min}} + 1$
- Theoretical uncertainty obtained by offset method:
  - ▶ Repeat fit for  $\mu_r$  and  $\mu_f$  varied by a factor of 1/2 and 2
- PDF uncertainty calculated with PDF eigenvalues
- Consistency with PDF sets with varied  $\alpha_s(M_Z)$  checked