Recent progress in Lattice QCD

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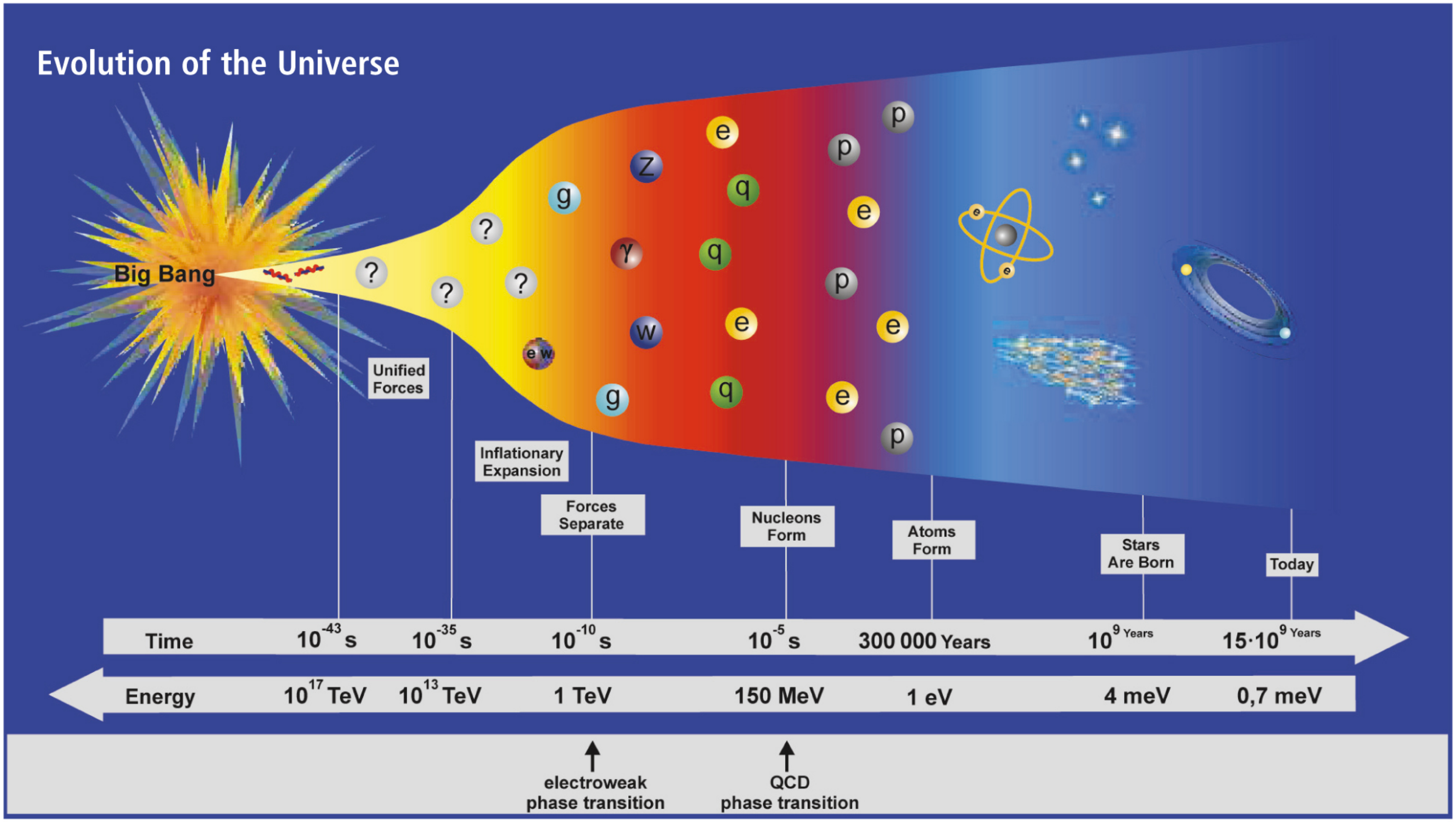
Origin of mass: EW versus QCD phase transition



- EW symmetry breaking (times Yukawa couplings) generates quark masses: $m_u = 2.4 \pm 0.7\,\mathrm{MeV}$, $m_d = 4.9 \pm 0.8\,\mathrm{MeV}$, $m_s = 105 \pm 25\,\mathrm{MeV}$ [PDG'10]
- QCD chiral/conformal symmetry breaking generates nucleon mass: $M_N \simeq 870\,\mathrm{MeV}$ at $m_{ud}\!=\!0$ (to be compared to $940\,\mathrm{MeV}$ at m_{ud}^{phys})

Lattice QCD (1): combined UV/IR regulator

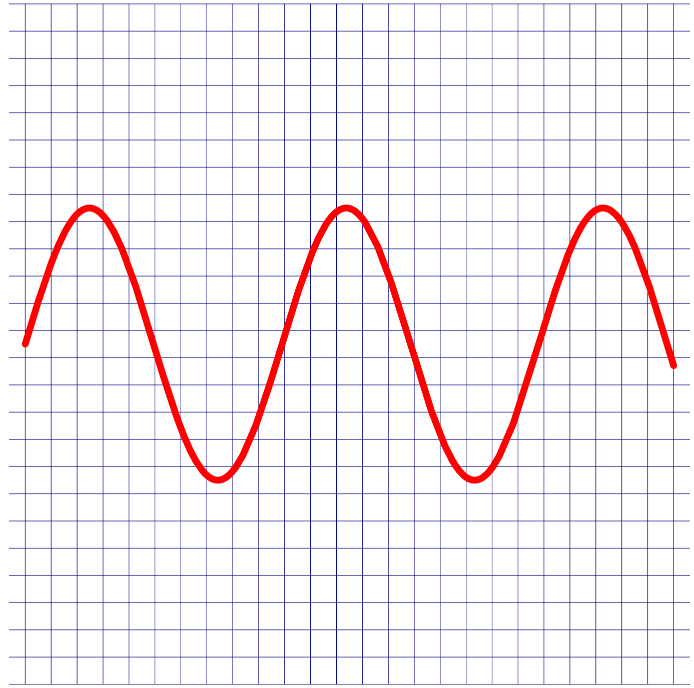
QCD Lagrangian contains quarks and gluons [Fritzsch, Gell-Mann and Leutwyler (1973)]

$$\mathcal{L}_{\rm QCD} = \frac{1}{4}{\rm Tr}(F_{\mu\nu}F_{\mu\nu}) + \sum_{i=1}^{N_f} \bar{q}^{(i)}(\not{D}+m^{(i)})q^{(i)} + i\theta\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}{\rm Tr}(F_{\mu\nu}F_{\rho\sigma})$$

- QCD must be regulated both in the UV and in the IR to make it well-defined.
- The lattice does the job through $a>0$ and $V=L^4<\infty$, but other options are possible. In fact, each gauge/fermion action is a different regulator.
- For $a\to 0$ correlation lengths diverge, but ratios ξ_π/ξ_Ω stay finite (renormalization). The extrapolations $a\to 0$ and $V\to\infty$ are performed in dimensionless observables.
- The result is independent of the action, thanks to universality [Wilson].

The lattice is not a model of QCD,
it is (one possible) *definition* of QCD !

Lattice QCD (2): scale hierarchies



typical spacing: $0.05\,\mathrm{fm} \leq a \leq 0.20\,\mathrm{fm}$

$1\,\mathrm{GeV} \leq a^{-1} \leq 4\,\mathrm{GeV}$

typical length: $2\,\mathrm{fm} \leq L \leq 6\,\mathrm{fm}$

require (UV): $a\,m_q \ll 1$

require (IR): $M_\pi L \geq 4$

u, d	c, s	(t), b
work near	interpolate to	extrapolate
$m_{ud} \gtrsim m_{ud}^{\mathrm{phys}}$	$m_s^{\mathrm{phys}}, m_c^{\mathrm{phys}}$	$m_b \to m_b^{\mathrm{phys}}$

In QCD with N_f flavors, N_f+1 observables used to set quark masses and scale.

Lattice QCD (3): quick consumer guide

Points to be considered when using/comparing LQCD results:

(1) Has the continuum limit ($a \to 0$) been taken ?

(2) Are the finite-volume effects (from $L < \infty$) under control ?

(3) Are the simulations performed anywhere close to $M_\pi = 135\,\mathrm{MeV}$?

(4) Advanced: are theoretical uncertainties properly assessed/propagated ?

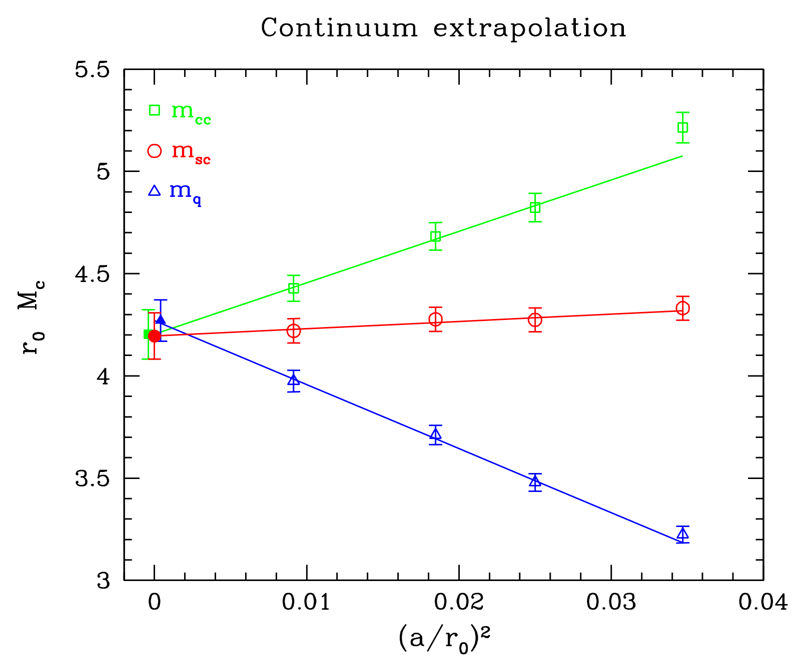
(5) Expert: algorithm details, treatment of isospin breakings, resonances, ...

Example regarding the first point:

- continuum limit is universal [Wilson]
- deviation at finite a may be substantial

Interesting limits tend to be expensive:

$\mathrm{CPU} \propto 1/a^{4-6}$, $\mathrm{CPU} \propto L^5$, $\mathrm{CPU} \propto 1/m_q^{1-2}$



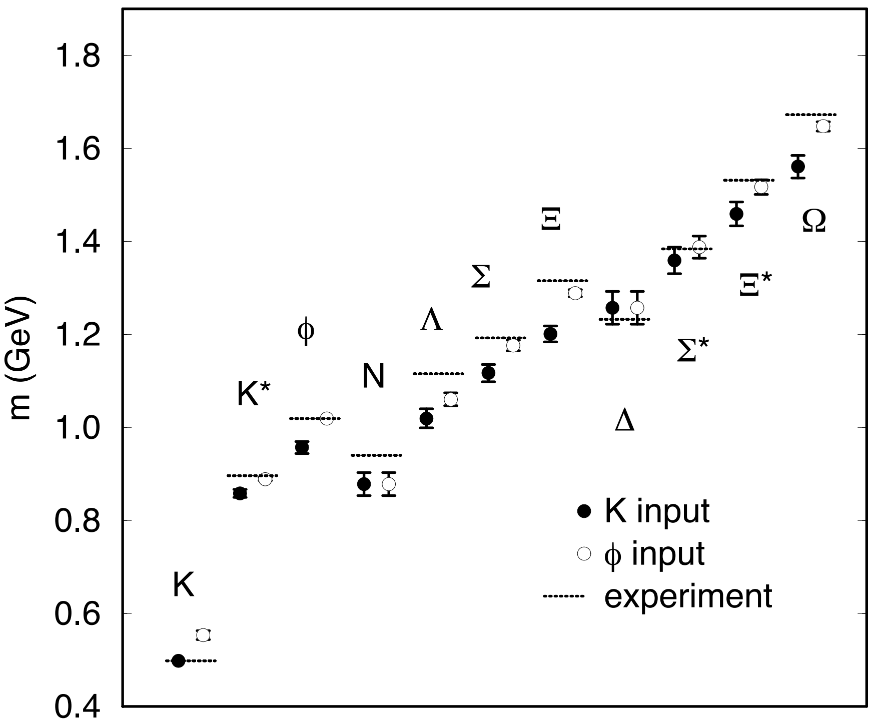
J.Rolf, S.Sint [ALPHA], JHEP 0212, 007 (2002)

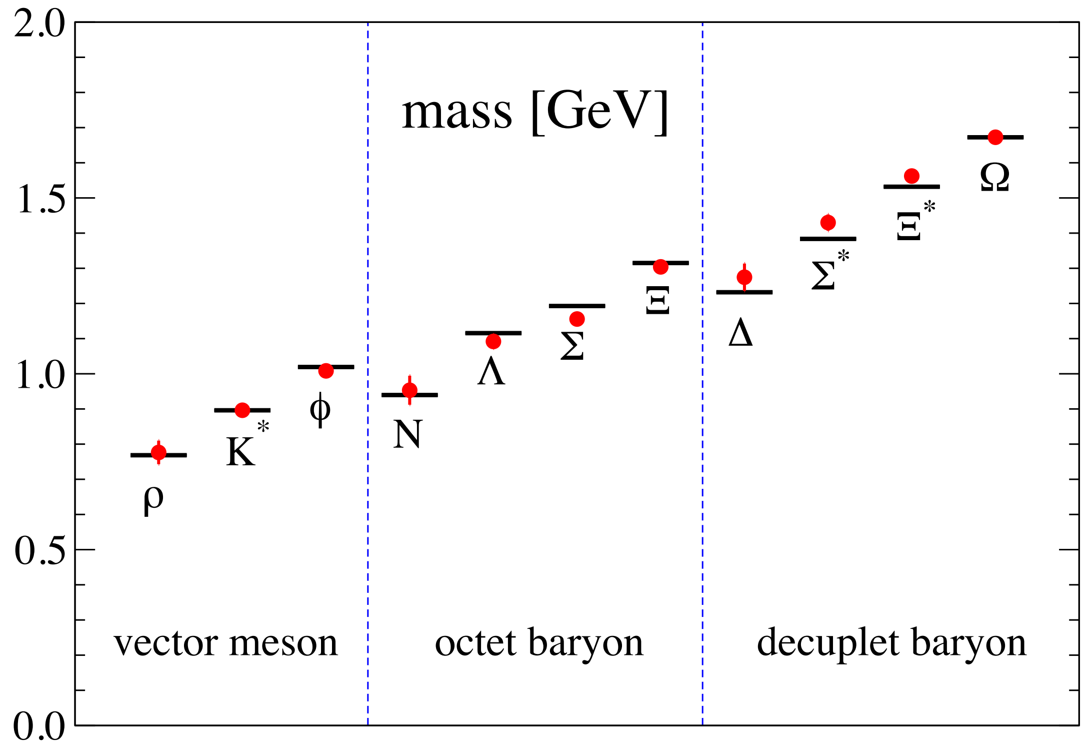
Talk outline

- Lattice QCD
- Hadron spectroscopy
  - Spectra of stable versus unstable/mixing hadrons
  - Strangeness in the nucleon and dark matter
  - Scattering of $\pi\pi$, πK, KK, πN, NN and nuclear physics
- Flavor physics and FLAG effort
  - Quark masses: m_u, m_d, m_s, m_c
  - Decay constants, form factors and CKM-unitarity
  - Kaon mixing: B_K, B_{BSM}, $K \to 2\pi$ amplitude
- Interlude: algorithms/machines
- Other topics
  - QCD thermodynamics at $\mu = 0$ and $\mu > 0$
  - Large N_c, large N_f, different fermion representations
  - $N_f = 1+1+1+1$ simulations with electromagnetism
- Epilogue: (clusters of) topics not covered

Hadron spectroscopy

Spectra of stable hadrons (1): $N_f = 0$ versus $N_f = 2+1$

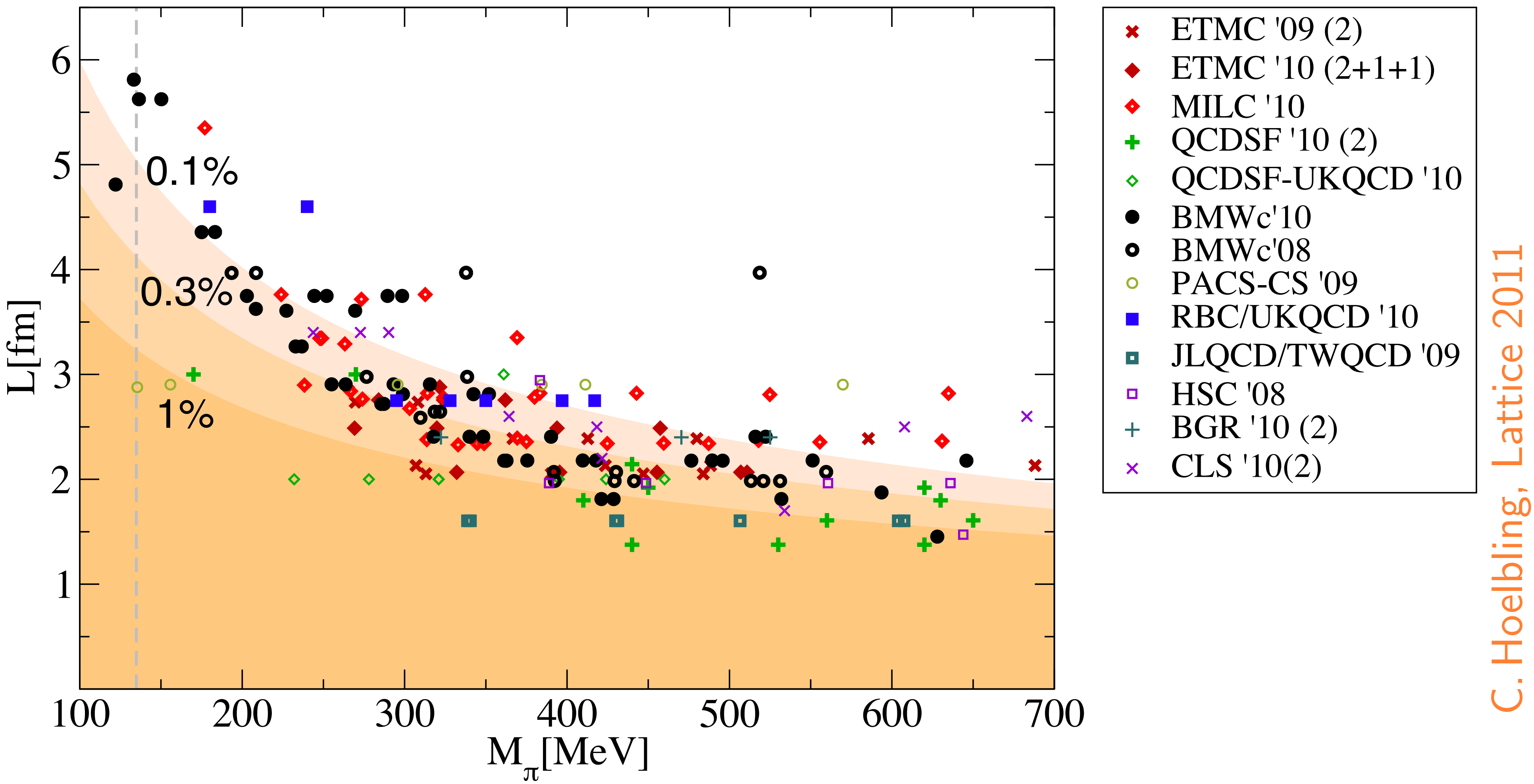




CP-PACS (2000, left, $N_f=0$) versus PACS-CS (2009, right, $N_f=2+1$)

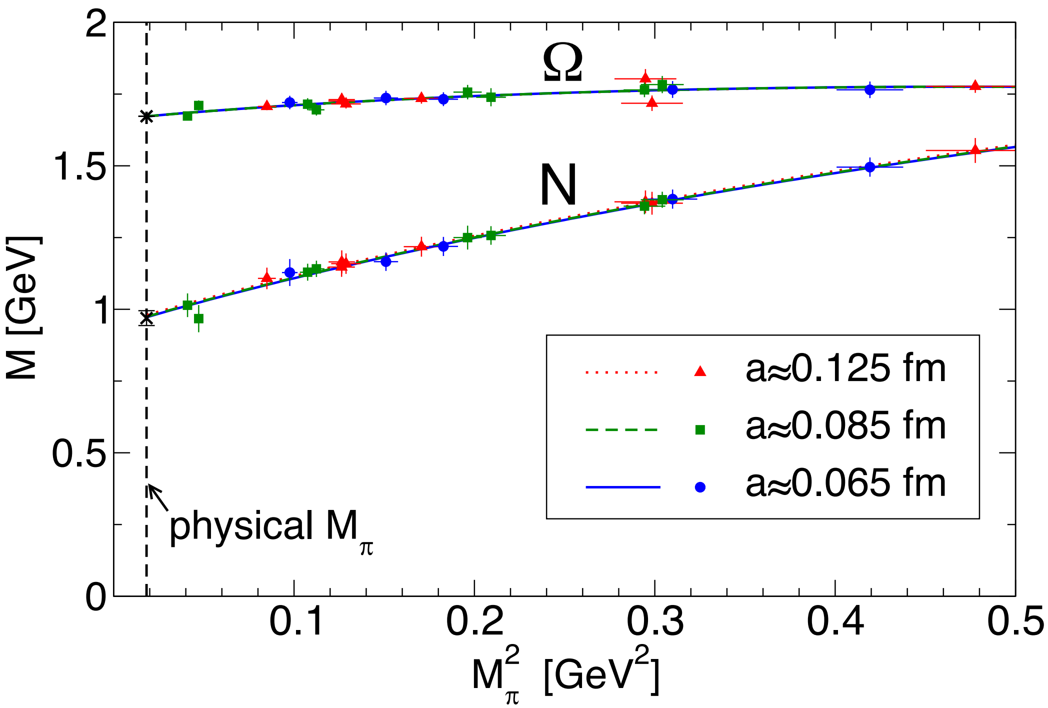
$\longrightarrow$ Quenched approximation is qualitatively good, but differs from real world (2000)

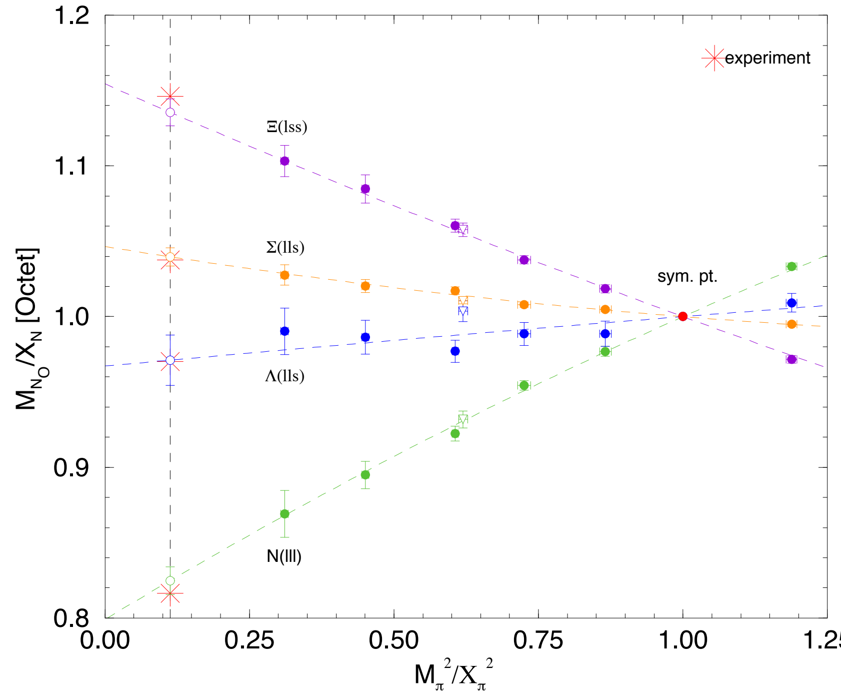
Spectra of stable hadrons (2): simulated M_π, L, a



Challenge: tune (m_{ud}, m_s) to the (a-priori unknown) physical value, keeping L large enough and a small enough in every simulation point

Spectra of stable hadrons (3): approaching $(m_{ud}^{\mathrm{phys}}, m_s^{\mathrm{phys}})$

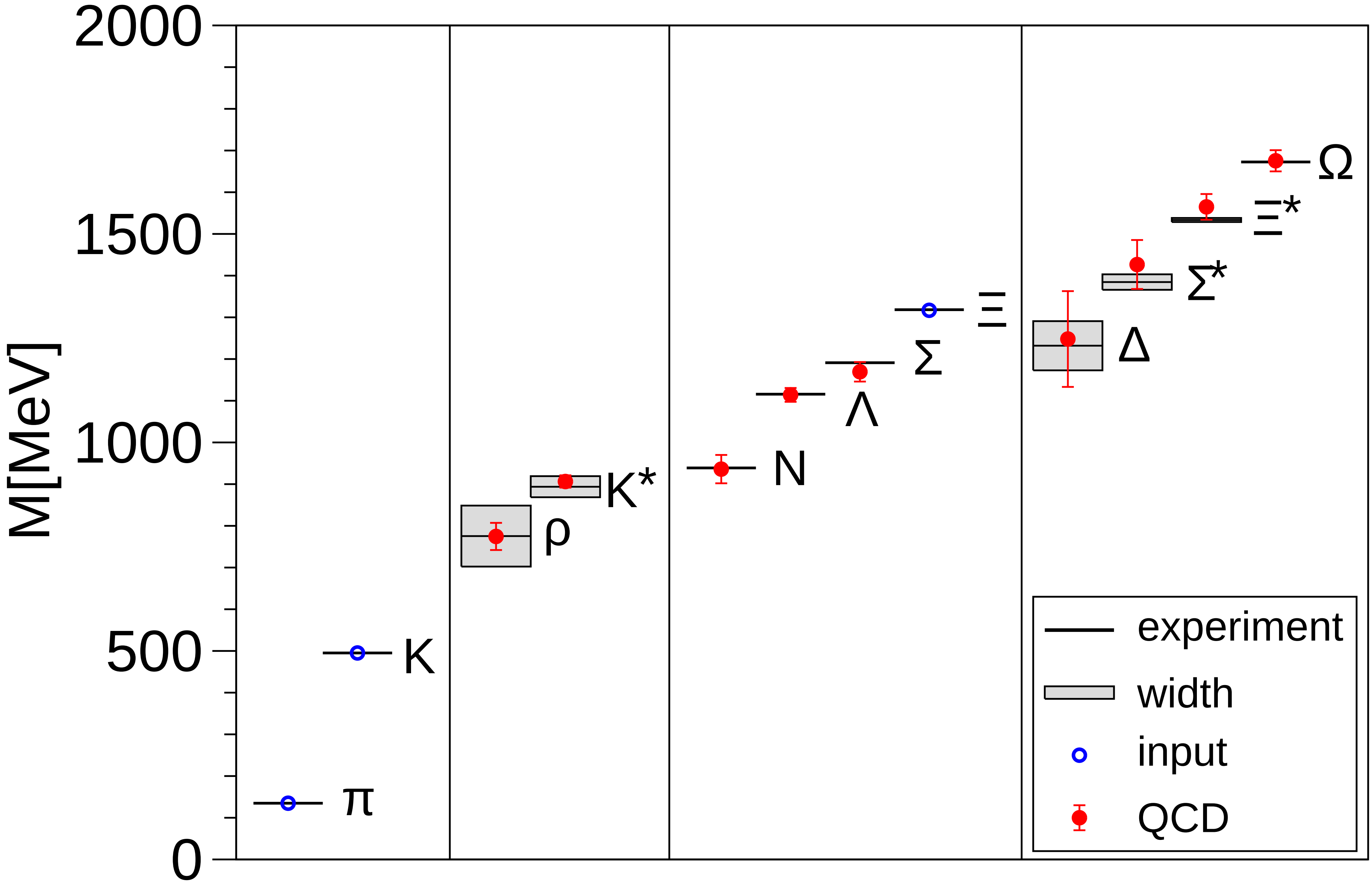




Strategy 1: PACS-CS/BMW-c/... lower m_{ud} while keeping m_s (roughly) constant

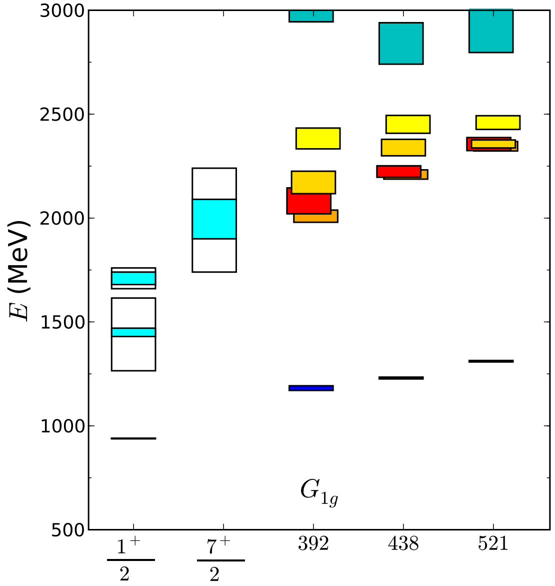
Strategy 2: QCDSF lower m_{ud} while keeping $2m_{ud}+m_s$ constant

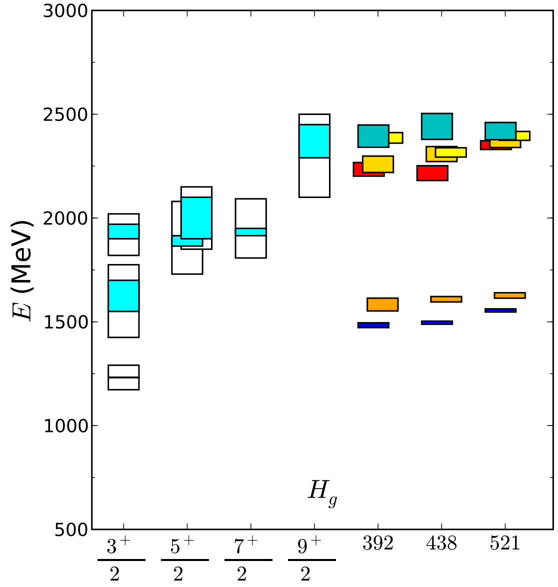
Spectra of stable hadrons (4): final result

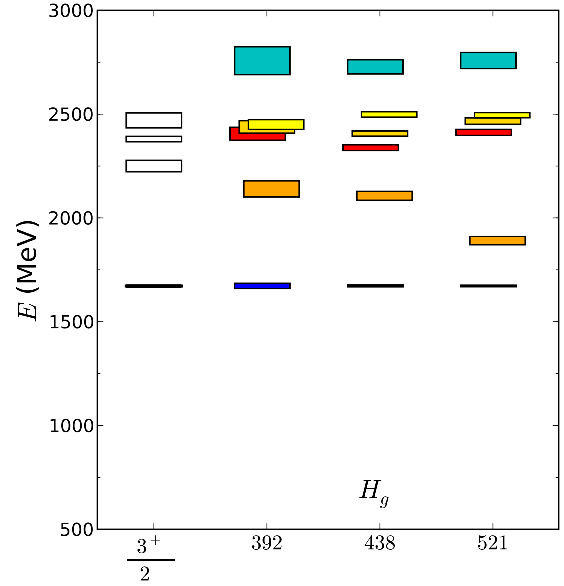


After $a \to 0$, $L \to \infty$, $M_\pi = 135\,\mathrm{MeV}$ agreement with experiment [S. Dürr *et al.*, Science 322, 1224 (2008)]

Spectra of unstable/mixing hadrons (1): excited baryons





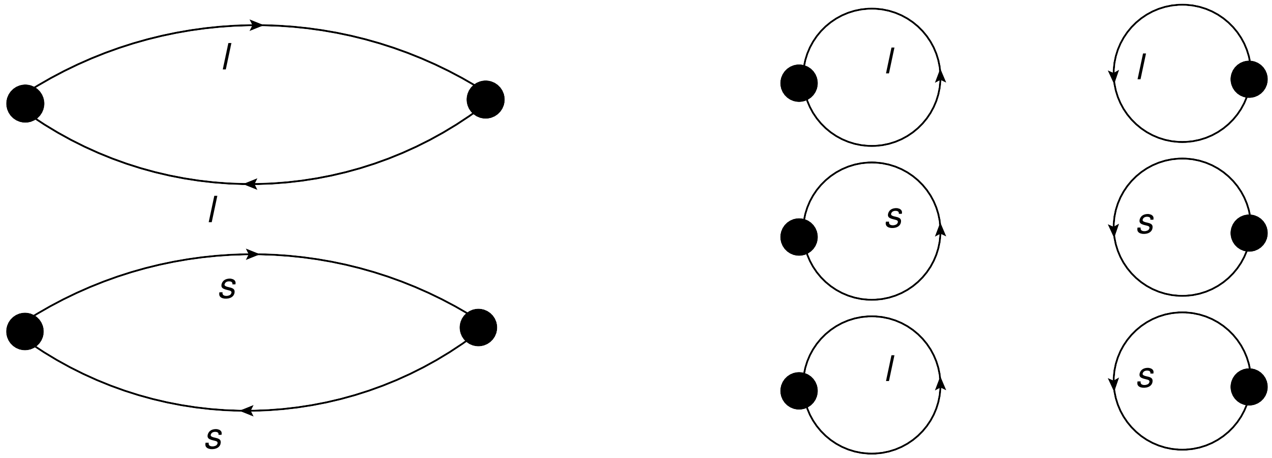


Excited state spectrum of the N (left), Δ (middle), Ω (right)

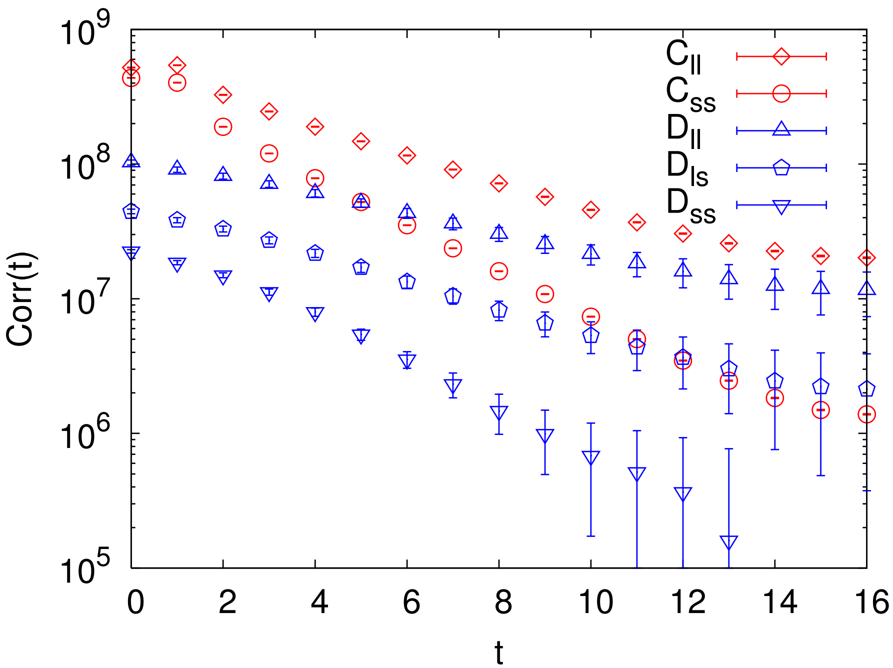
Bulava et al. [Hadron Spectrum Collaboration], Phys. Rev. D 82, 014507 (2010)

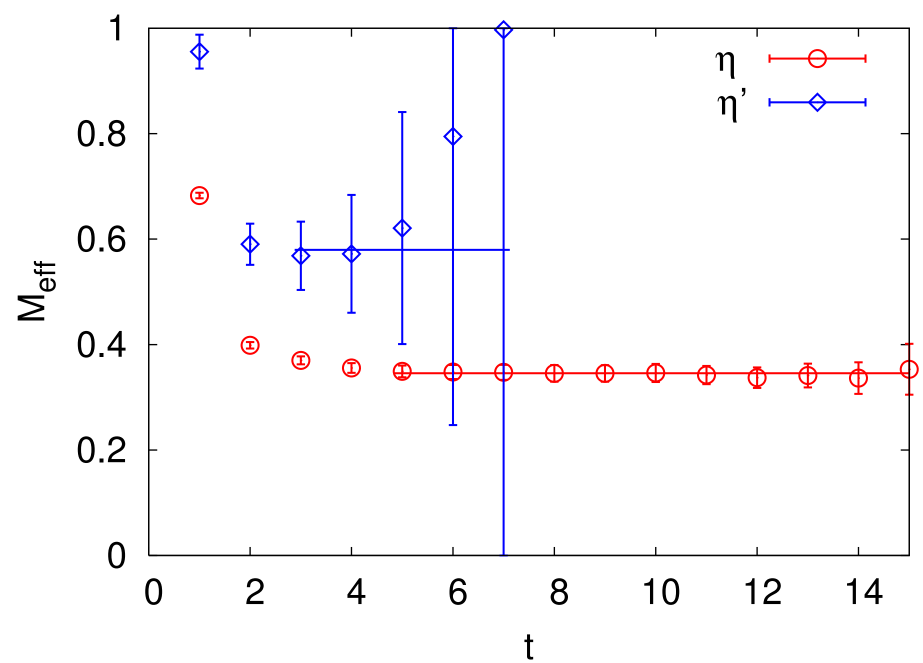
Spectra of unstable/mixing hadrons (2): mixing of $\eta - \eta'$

Connected versus disconnected contributions:

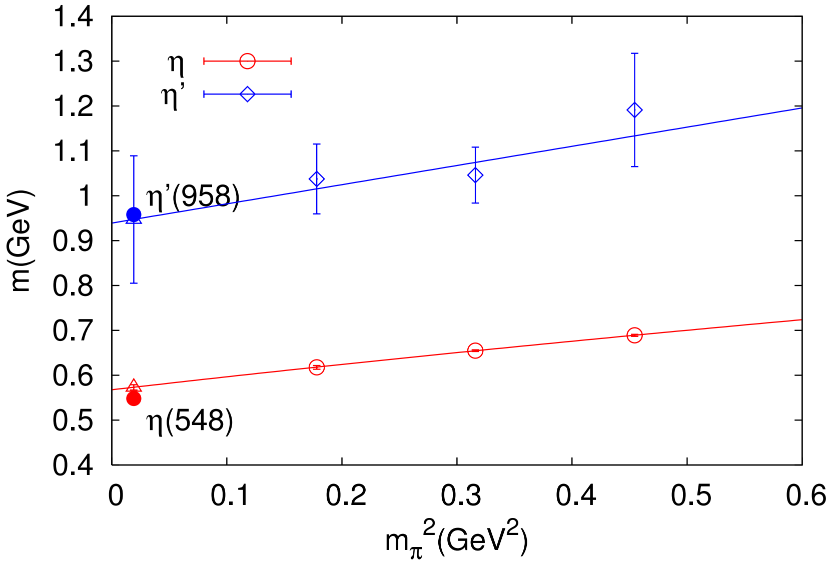


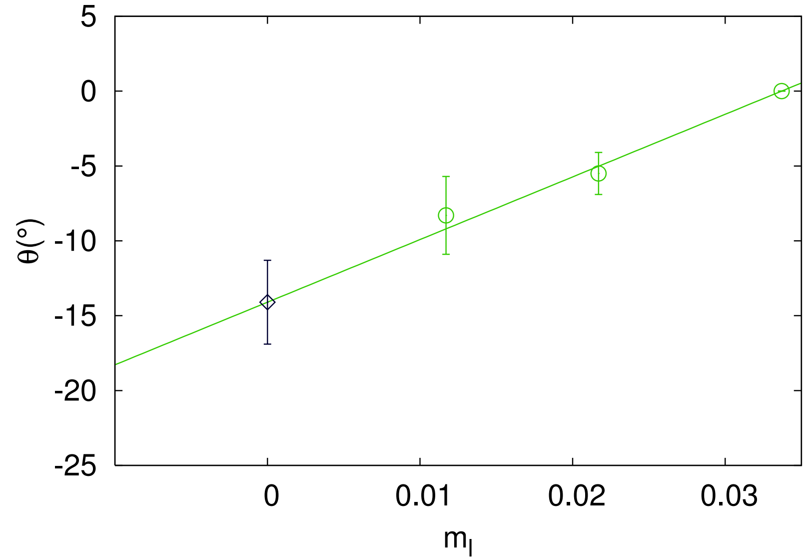
Matrix-valued correlator is rotated to mass eigenstates:



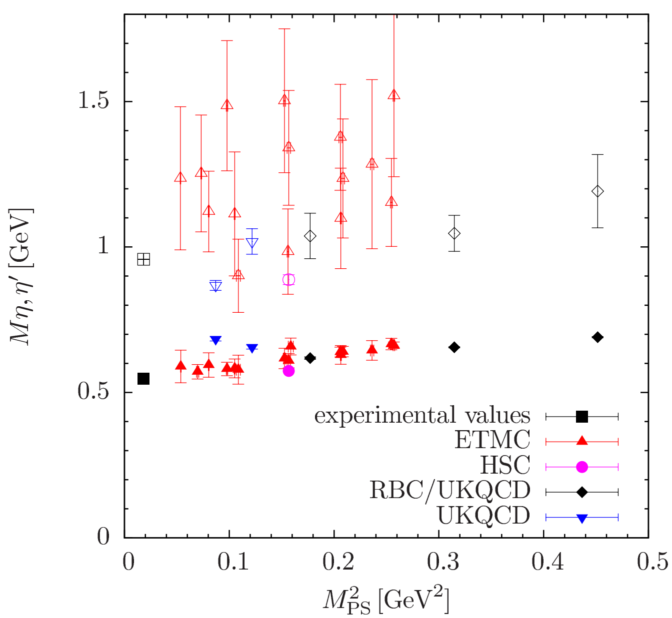


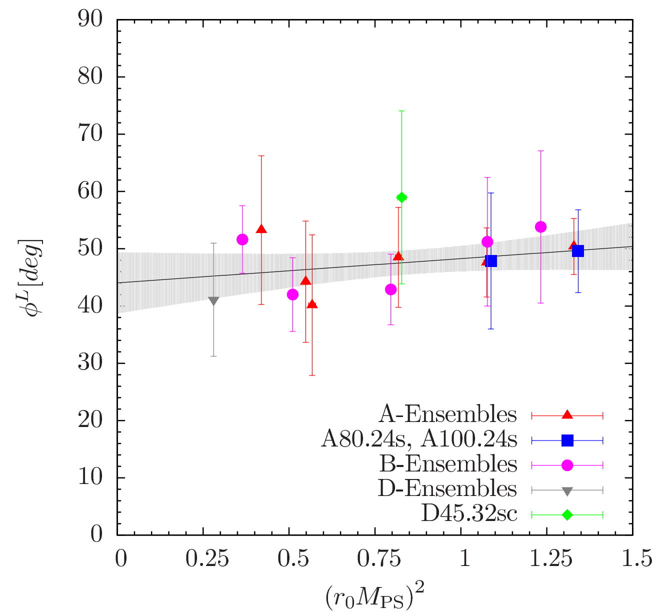
- **RBC/UKQCD** Christ et al, Phys. Rev. Lett. 105 (2010) 241601 [arXiv:1002.2999]



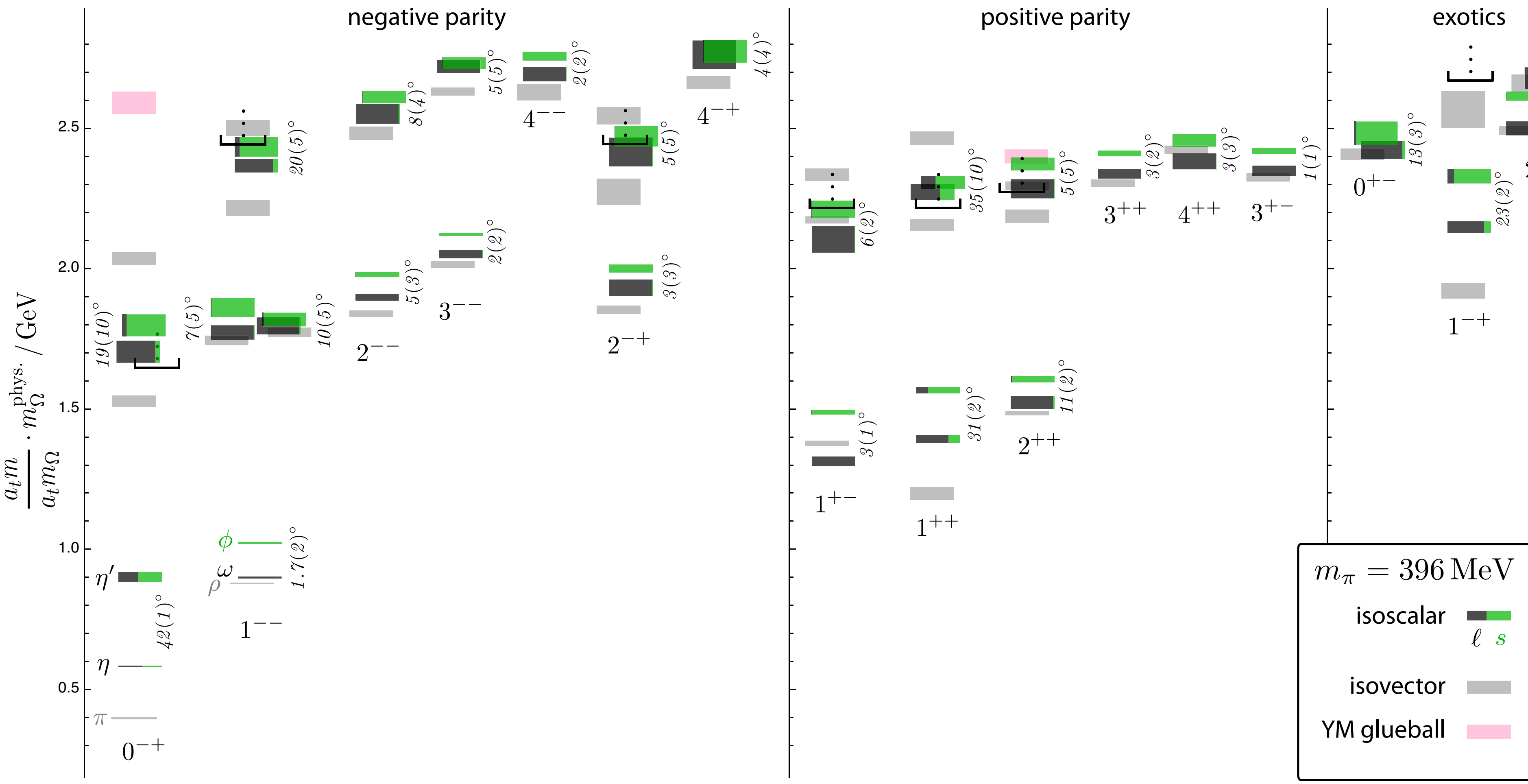


- **ETMC** Ottnad et al, arXiv:1206.6719





Spectra of unstable/mixing hadrons (3): more isoscalars

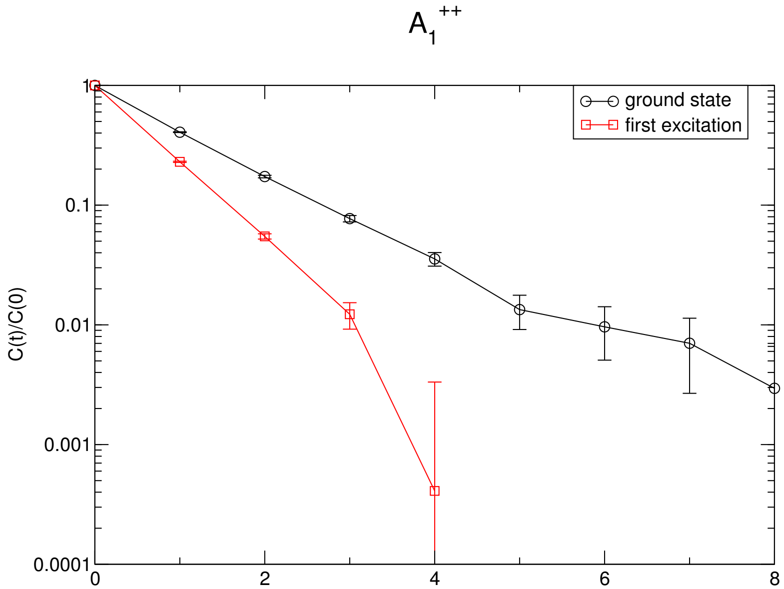


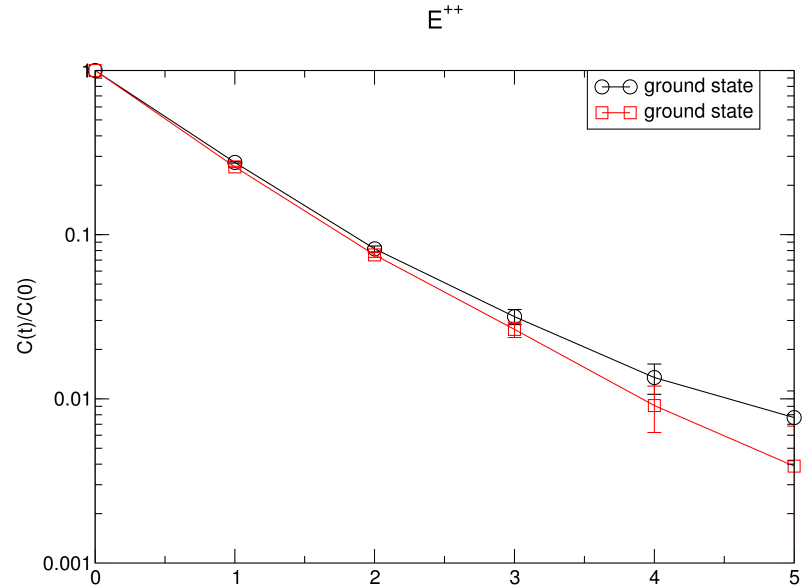
Lattices with $M_\pi = 396\,\text{MeV}$, strange-light mixing is $\theta_{\eta-\eta'} = 42(1)^\circ$, $\theta_{\omega\phi} = 1.7(2)^\circ$

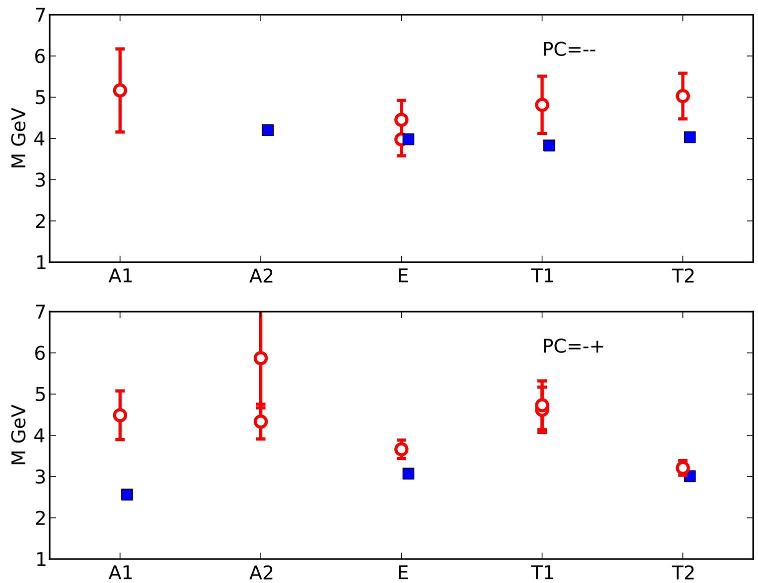
Dudek et al, Phys. Rev. D 83 (2011) 111502 [arXiv:1102.4299]

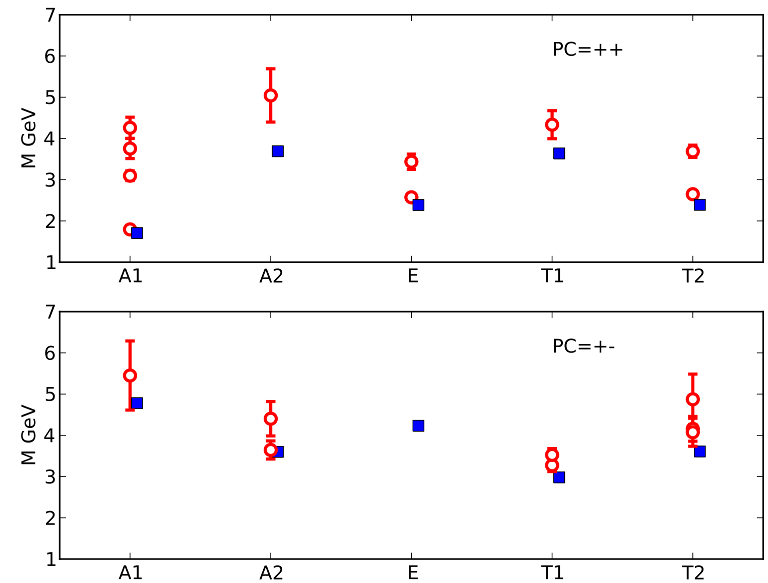
Similar results for charmonium: Bali, Collins, Ehmann, Phys. Rev. D 84 (2011) 094506

Spectra of unstable/mixing hadrons (4): glueballs





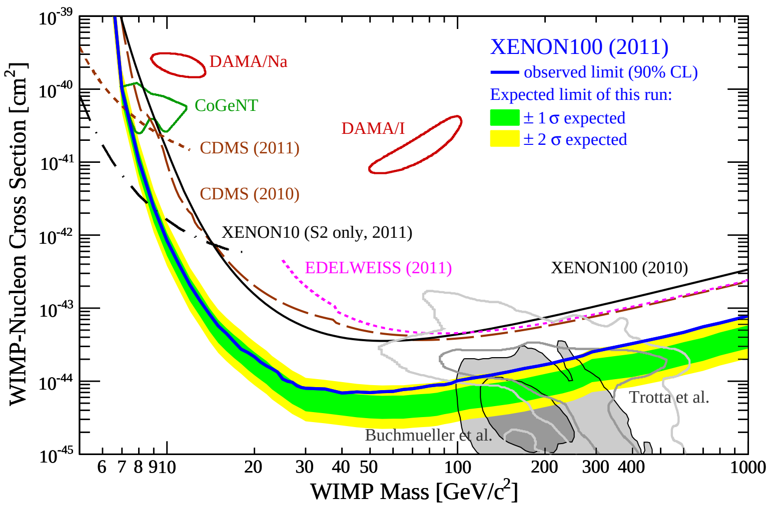




Gregory et al, arXiv:1208.1858

Nucleon sigma terms and dark matter

Composition of the universe: 73% dark energy, 23% dark matter, 4% baryons



XENON 100, PRL 107 (2011) 131302

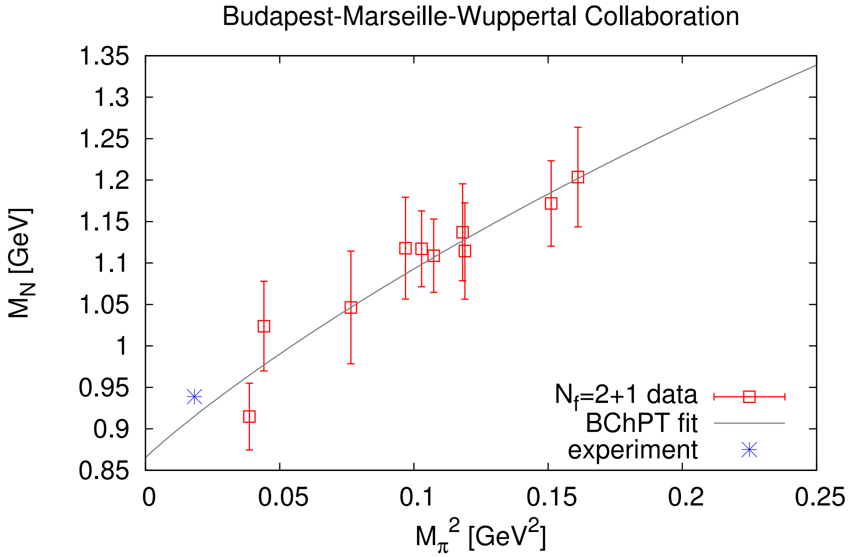
Dark matter stays dark, unless WIMP-Nucleon scattering can be probed down to tiny cross-sections.

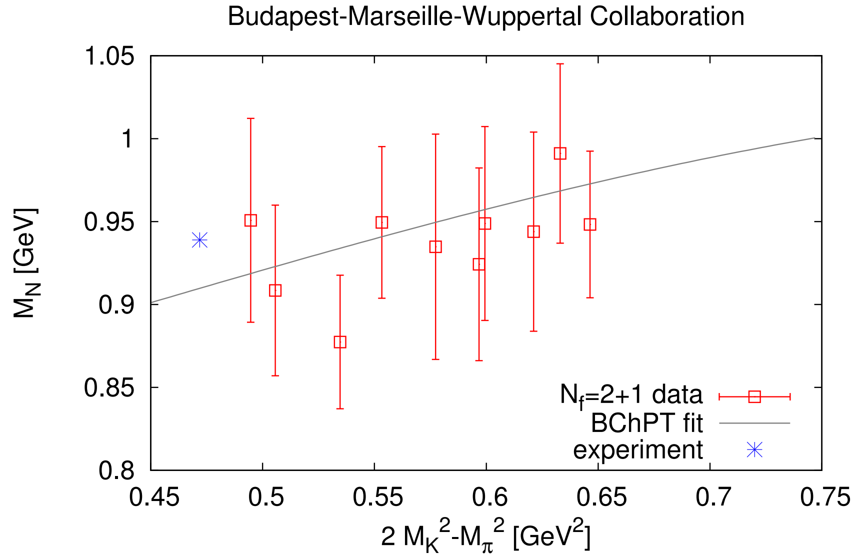
Significant uncertainty from the matrix elements [RGI, dimension of mass]
$\sigma_{ud} = m_{ud}\langle N|u\bar{u}+d\bar{d}|N\rangle$ and $\sigma_s = 2m_s\langle N|s\bar{s}|N\rangle$ [be aware of factor 2].

σ_{ud} can be determined from πN scattering and Chiral Perturbation Theory (ChPT).
σ_s obtained from $\sigma_0 - \sigma_{ud}$, where $\sigma_0 = m_{ud}\langle N|u\bar{u}+d\bar{d}-2s\bar{s}|N\rangle$ has large uncertainty.

Lattice can compute σ_{ud} and σ_s from 3-pt function or via Feynman-Hellman theorem,
$\sigma_{ud} = m_{ud}\frac{\partial M_N}{\partial m_{ud}} = M_\pi^2\frac{\partial M_N}{\partial M_\pi^2}$ and $\sigma_s = 2m_s\frac{\partial M_N}{\partial m_s} = (4M_K^2 - 2M_\pi^2)\frac{\partial M_N}{\partial(2M_K^2 - M_\pi^2)}$.

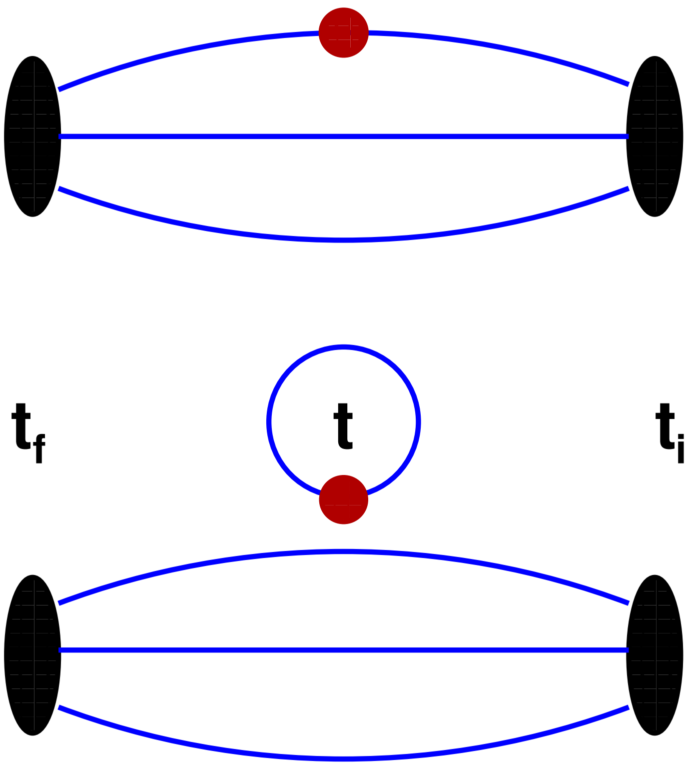
- **Feynman-Hellman: measure slope in M_N versus M_π^2**

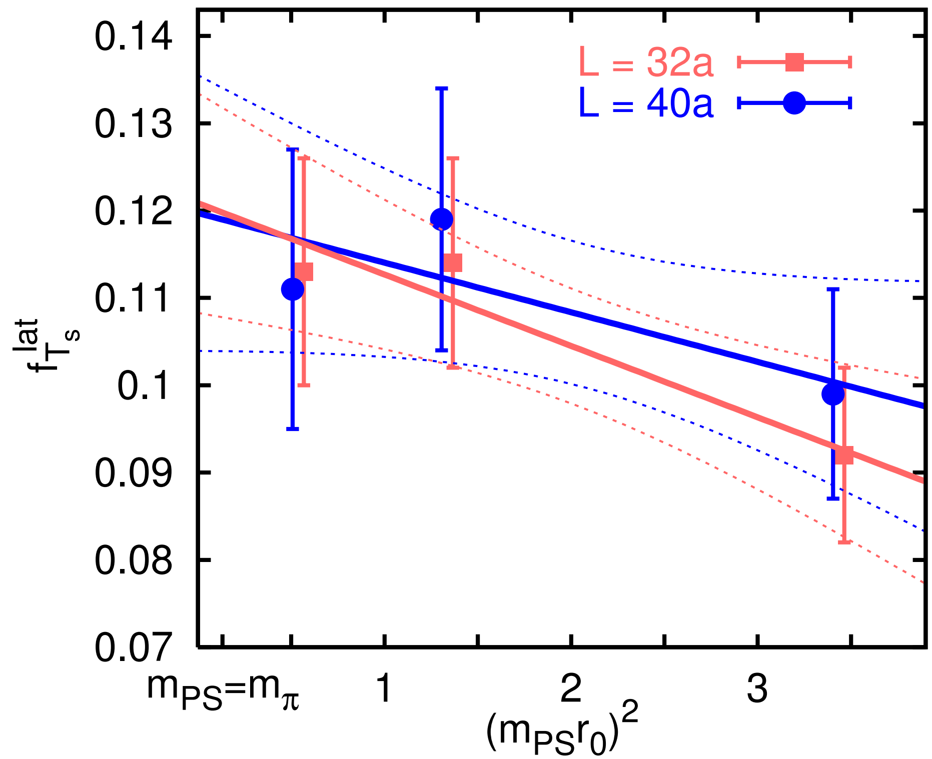




$66.7(1.3)(?)$	$\times$	Alexandrou et al, PRD 78 (2008) 014509, [0803.3190]
$84(17)(20)$	$\times$	Walker-Loud et al, PRD 79 (2009) 054502, [0806.4549]
$47(9)(3)$	$62(30)(8)$	Young et al, PRD 81 (2010) 014503, [0901.3310]
$75(15)(?)$	$\times$	Ishikawa et al, PRD 80 (2009) 054502, [0905.0962]
$59(2)(17)$	$-8(46)(50)$	Camalich et al, PRD 82 (2010) 074504, [1003.1929]
$39(4)(^{+18}_{-7})$	$67(27)(^{+55}_{-47})$	Durr et al [BMW], PRD 85 (2012) 014509, [1109.4265]
$\times$	$79(14)(9)$	Freeman et al [MILC], arXiv:1204.3866
$45(6)(5)$	$44(12)(0)$	Shanahan et al, arXiv:1205.5365
$37(8)(6)$	$\times$	Bali et al [QCDSF], arXiv:1206.7034

- **Nuclear 3-pt function $\langle N|q\bar{q}|N\rangle$ with disconnected contributions**

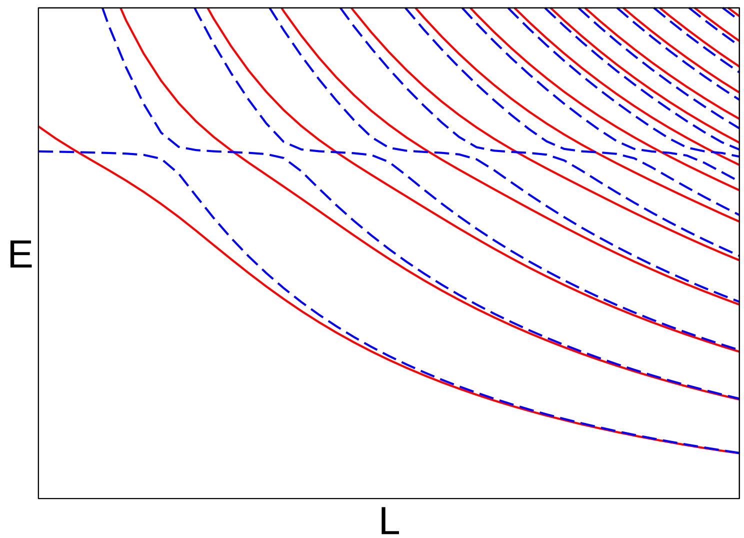




38(9)(8)	$22(26)(^{+19}_{-6})$	Bali et al [QCDSF], Phys.Rev. D85 (2012) 054502
$\times$	17(28)(30)	Ohki et al [JLQCD], arXiv:1208.4185

A straight (unweighted) average of all central values and total errors would suggest that $\sigma_{ud} = 54(13)\,\mathrm{MeV}$ and $\sigma_s = 40(36)\,\mathrm{MeV}$ [with my factor 2].

Scattering of $\pi\pi$, πK, KK, πN, NN



Scattering length and phase-shift can be determined in Euclidean space from tower of states in finite volume [Lüscher 1991].

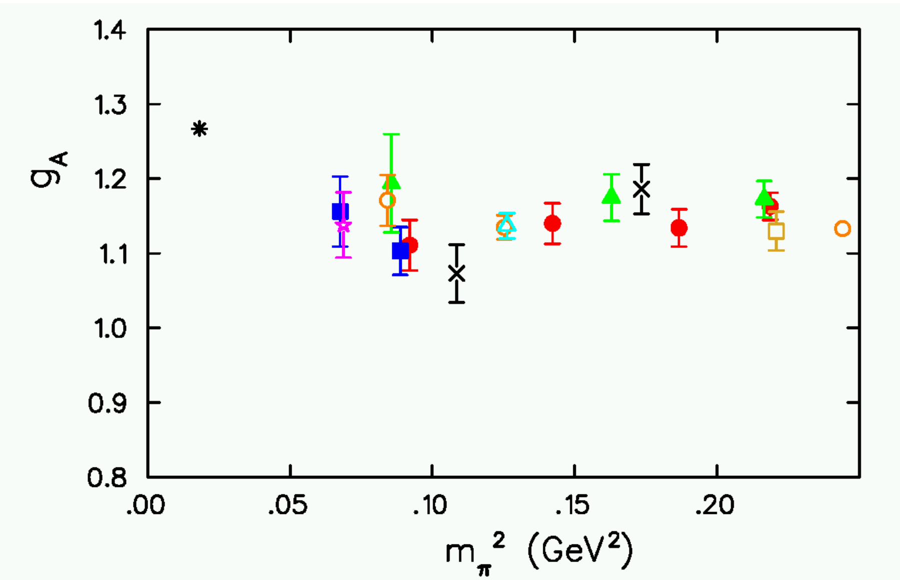
Example: L-dependence of states with $\pi\pi$ or ρ quantum numbers is different for small (dashed blue) versus large (full red) $g_{\pi\pi\rho}$.

Original framework by Lüscher refined in many respects [Rummukainen and Gottlieb, Rusetsky et al] and successfully applied to a variety of systems.

Method in practice rather demanding, since limited number of L values available, and extraction of high-lying states remains a challenge.

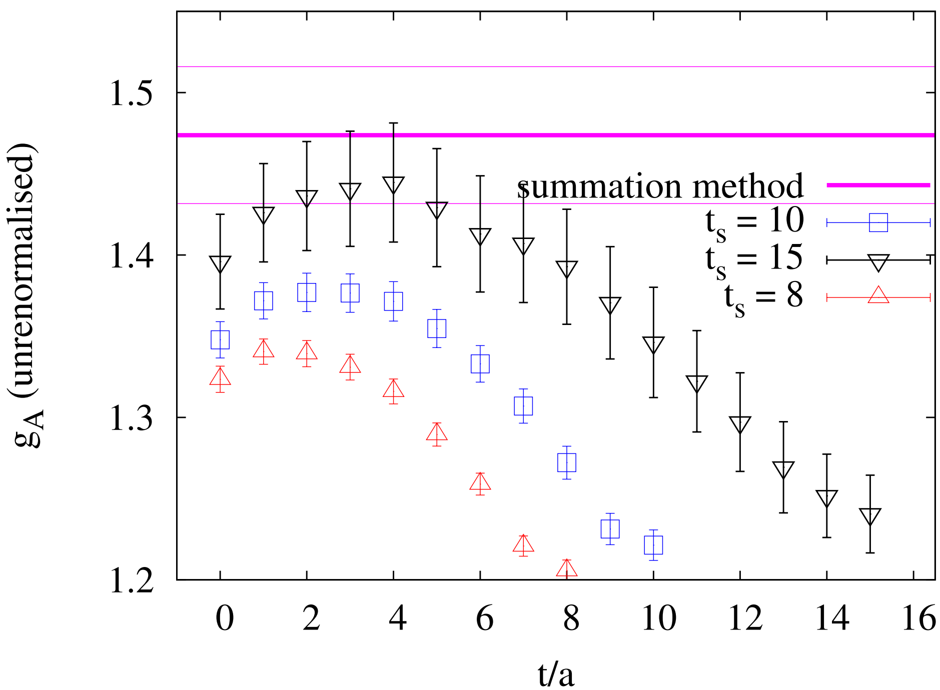
Results on $\pi\pi, \pi K, KK, \pi D, \pi N, NN, \ldots$ from various groups, e.g. Beane/Savage et al [NPLQCD], Dudek et al [HSC], Lang et al, Mohler et al, Aoki et al [HAL-QCD], ...

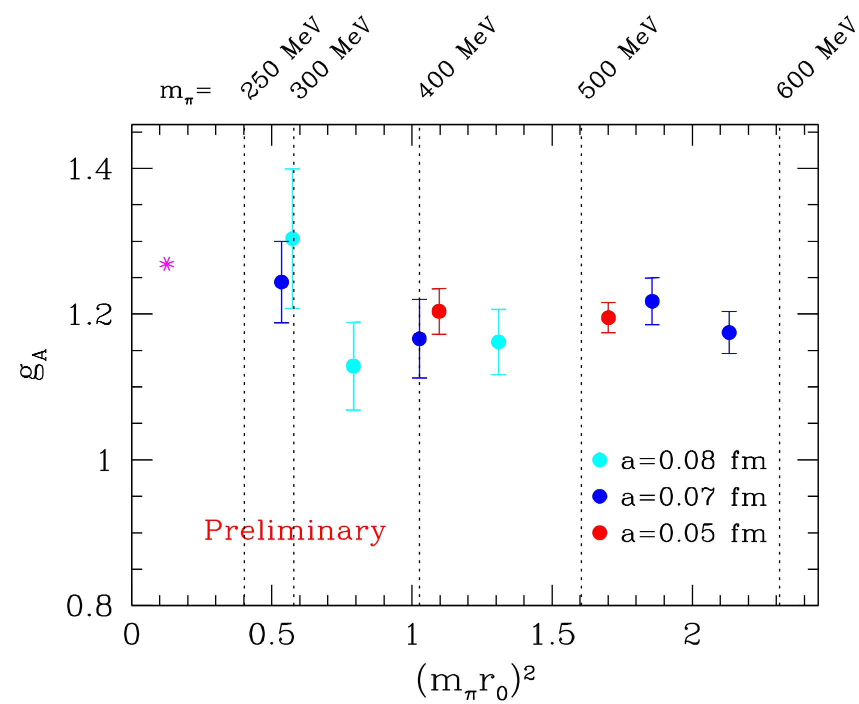
New hope for g_A on the lattice



Among "nuclear structure" quantities g_A has been particularly difficult to postdict correctly [Alexandrou, Lat'2010].

Now, there is new hope from "summation method" by CLS / Mainz-group.





Flavor physics and FLAG

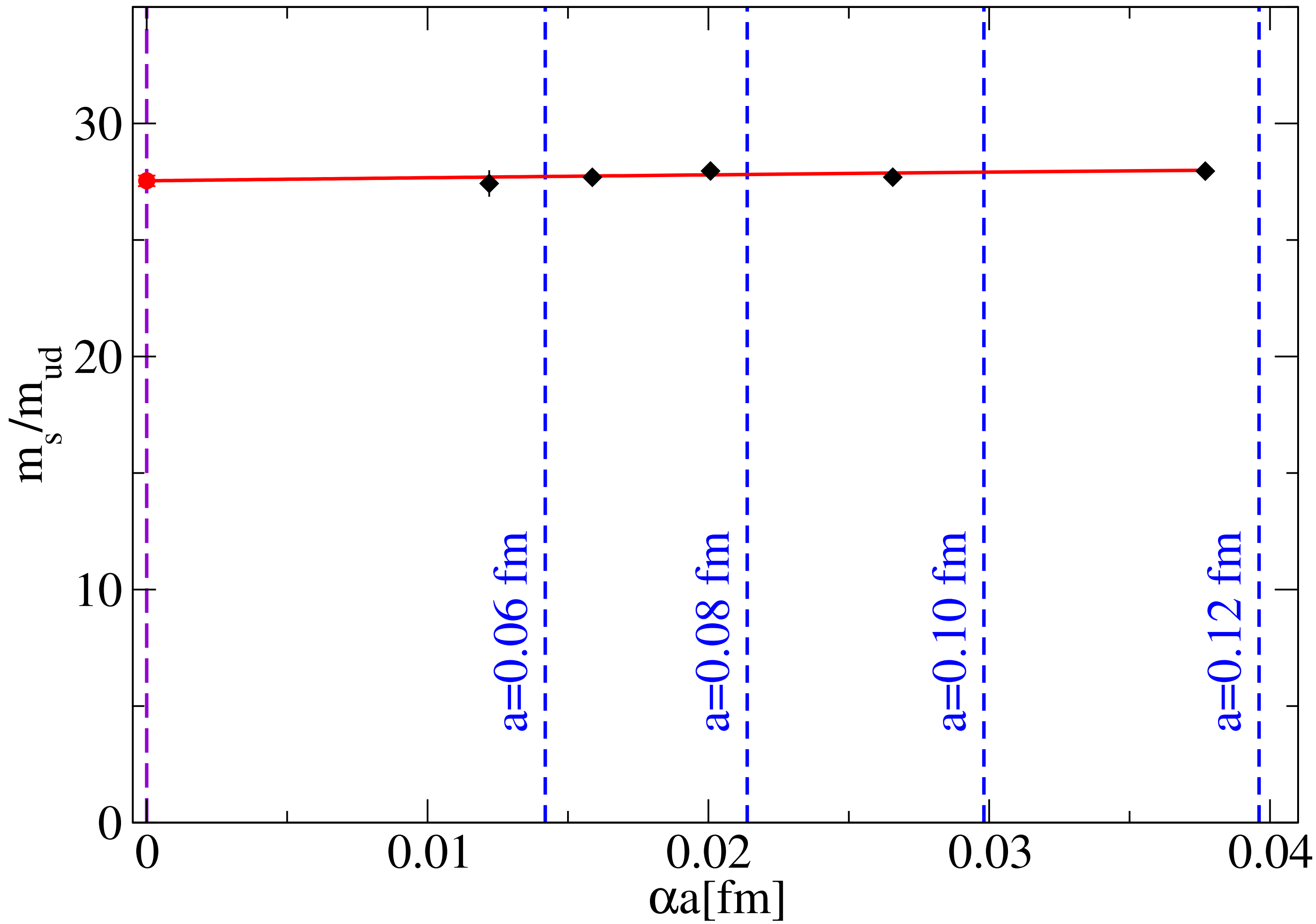
Quark masses (1): anatomy of $N_f = 2 + 1$ computation

1. Choose observables to be "burned", e.g. M_π, M_K, M_Ω in $N_f\!=\!2\!+\!1$ QCD, and get "polished" experimental values, e.g. $M_\pi\!=\!134.8(3)\,\mathrm{MeV}$, $M_K\!=\!494.2(5)\,\mathrm{MeV}$ in a world without isospin splitting and without electromagnetism [arXiv:1011.4408].
2. For a given bare coupling β (yields a) tune bare masses $1/\kappa_{ud,s}$ such that the ratios M_π/M_Ω, M_K/M_Ω assume their physical values (in practice: inter-/extrapolation).
3. Read off $1/\kappa_{ud,s}$ or determine bare $am_{ud,s}$ via AWI and convert them (perturbatively or non-perturbatively) to the scheme of your choice (e.g. $\overline{\mathrm{MS}}$ at $\mu\!=\!3\,\mathrm{GeV}$).
4. Repeat steps 2 and 3 for at least 3 different lattice spacings and extrapolate the (finite-volume corrected) result to the continuum via Symanzik scaling.

Depending on details, step 3 can be rather demanding [RI/MOM, SF renormalization]. Below, guided tour using plots from BMW-collaboration [arXiv:1011.2403,1011.2711].

Quark masses (2): Final result for ratio m_s/m_{ud}

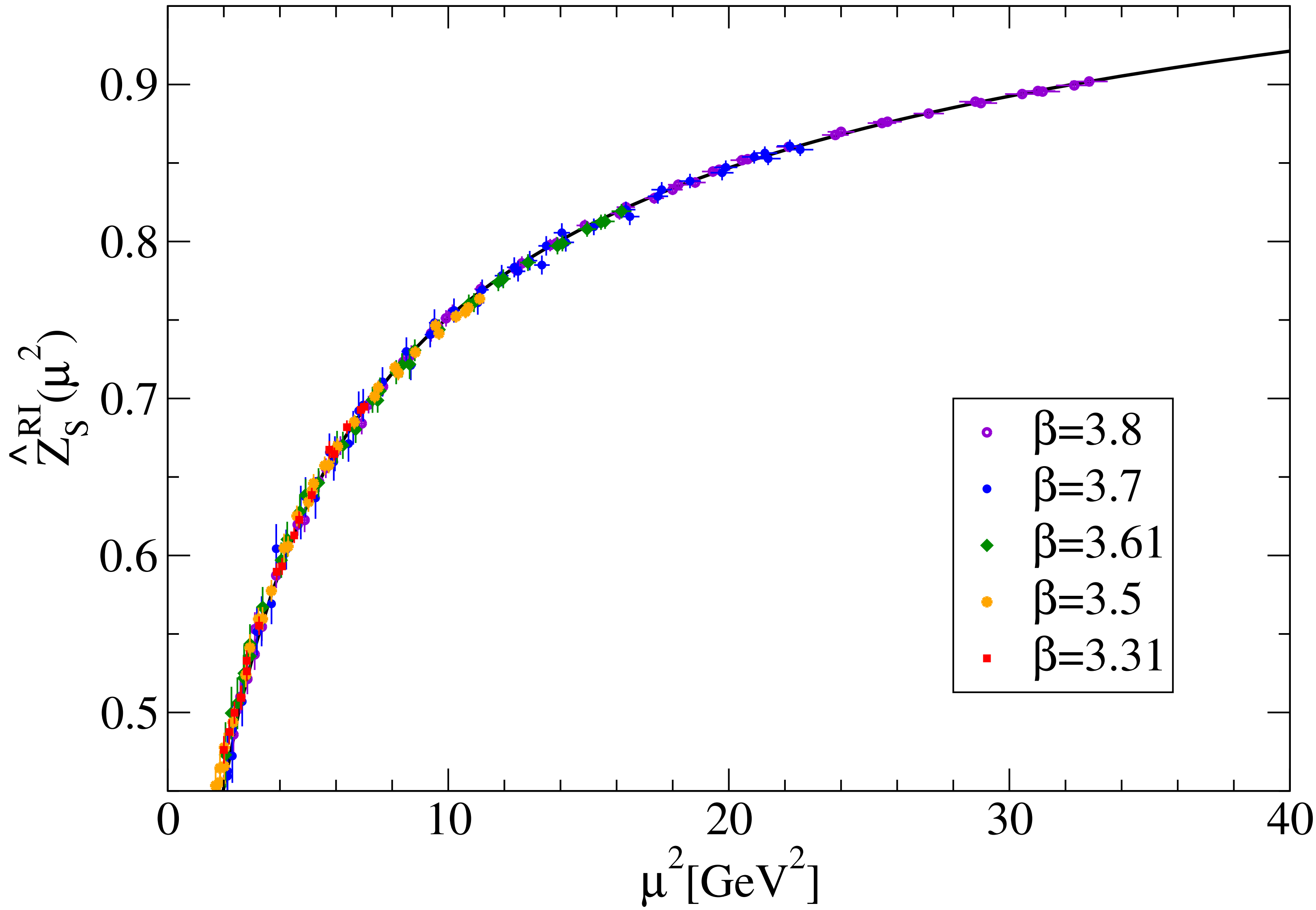
In QCD ratios like m_s/m_{ud} are renormalization group invariant (RGI), hence step 3 in this list is skipped (detail: we invoke αa and a^2 scaling).



Final result $m_s/m_{ud} = 27.53(20)(08)$ amounts to 0.78% precision.

Quark masses (3): $N_f = 3$ RI-running extrapolation for Z_S

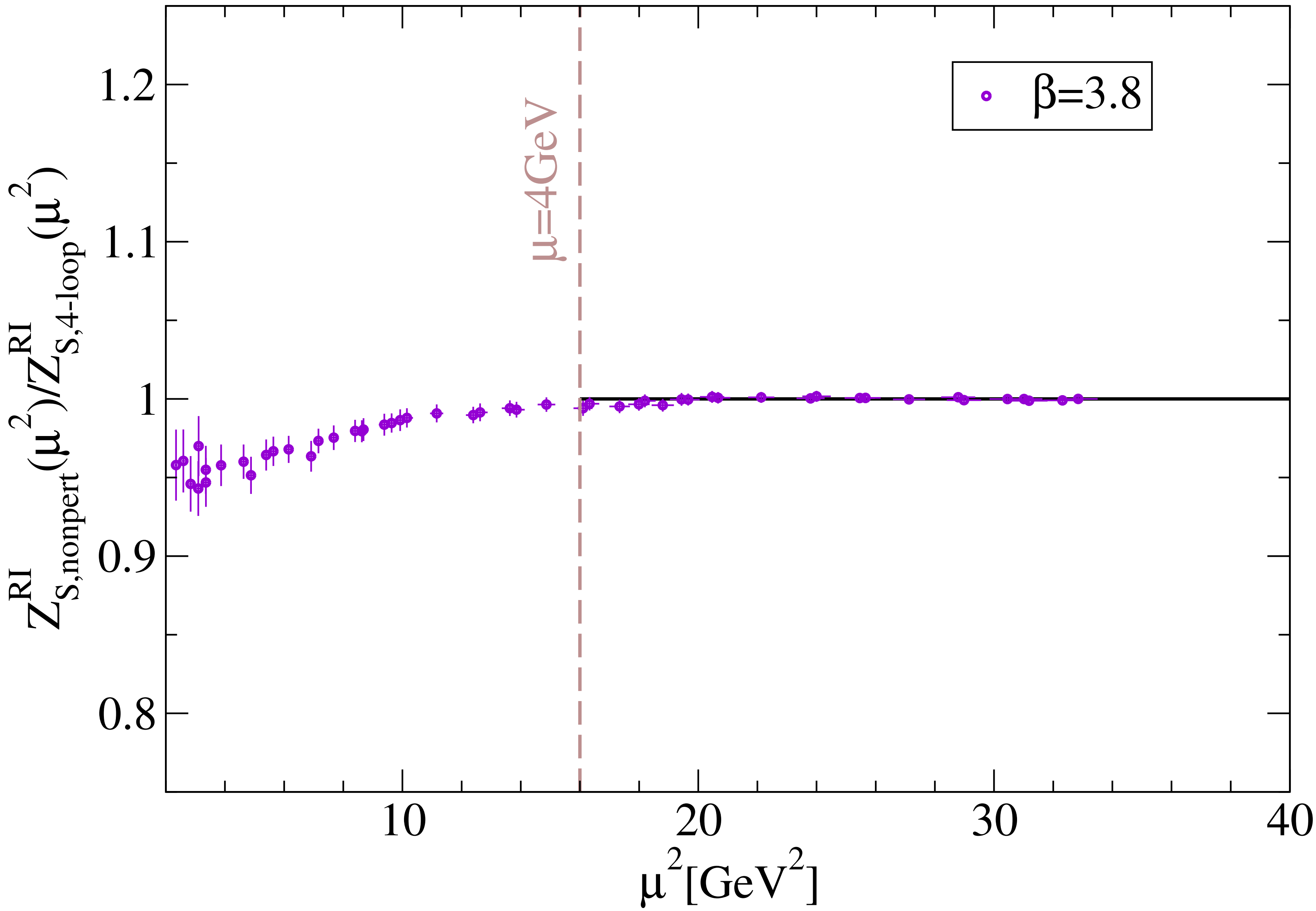
Evolution $Z_S^{\rm RI}(\mu)/Z_S^{\rm RI}(4\,{\rm GeV})$ has no visible cut-off effects among three finest lattices:



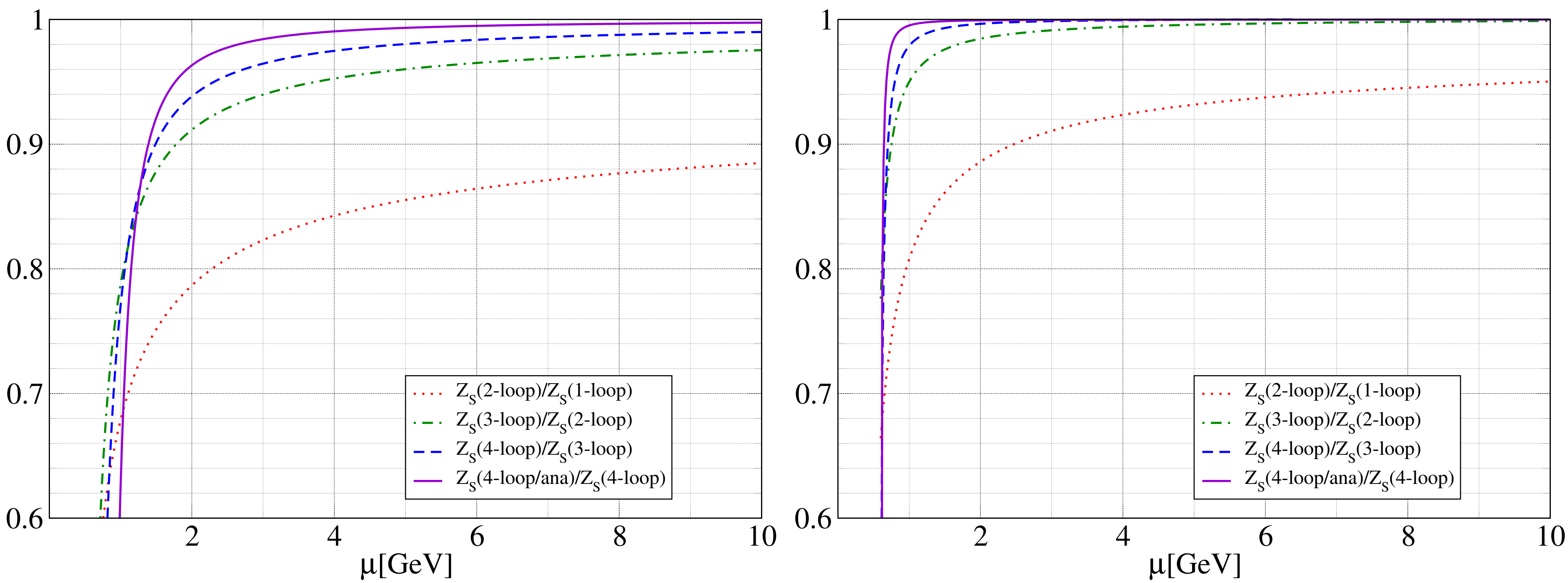
$\longrightarrow$ separate continuum limit with $R_S^{\rm RI}(\mu, 4\,{\rm GeV}) = \lim_{\beta\to\infty} Z_{S,\beta}^{\rm RI}(4\,{\rm GeV})/Z_{S,\beta}^{\rm RI}(\mu)$

Quark masses (4): $N_f = 3$ RI-scheme-running ratio for Z_S

On the finest lattice we make contact within errors to 4-loop PT for $\mu \geq 4\,\mathrm{GeV}$:



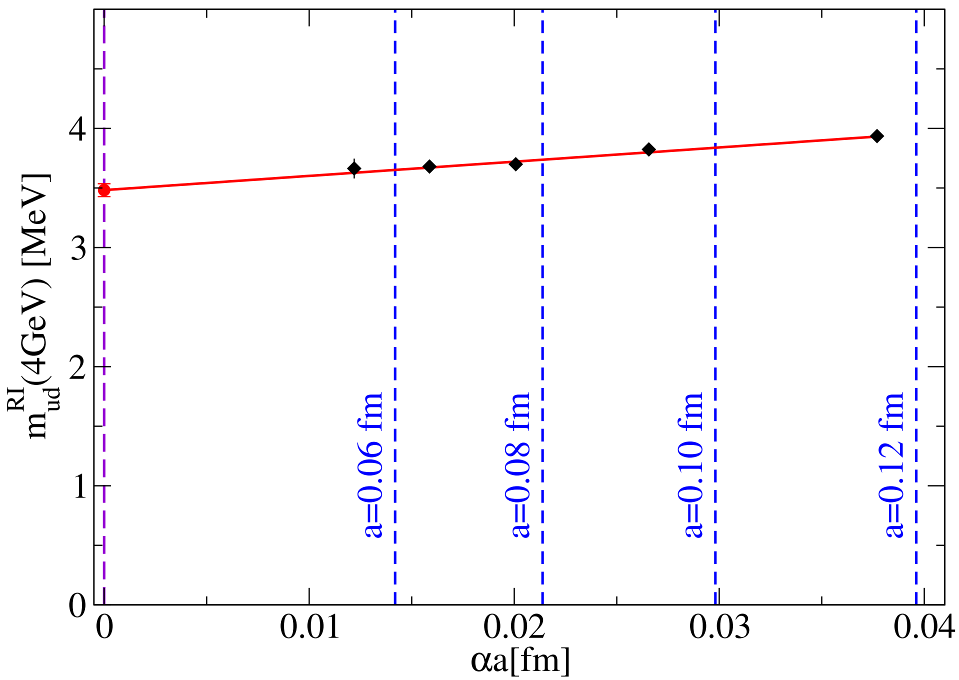
Quark masses (5): $N_f = 3$ RI and $\overline{\mathrm{MS}}$ perturbative series for Z_S

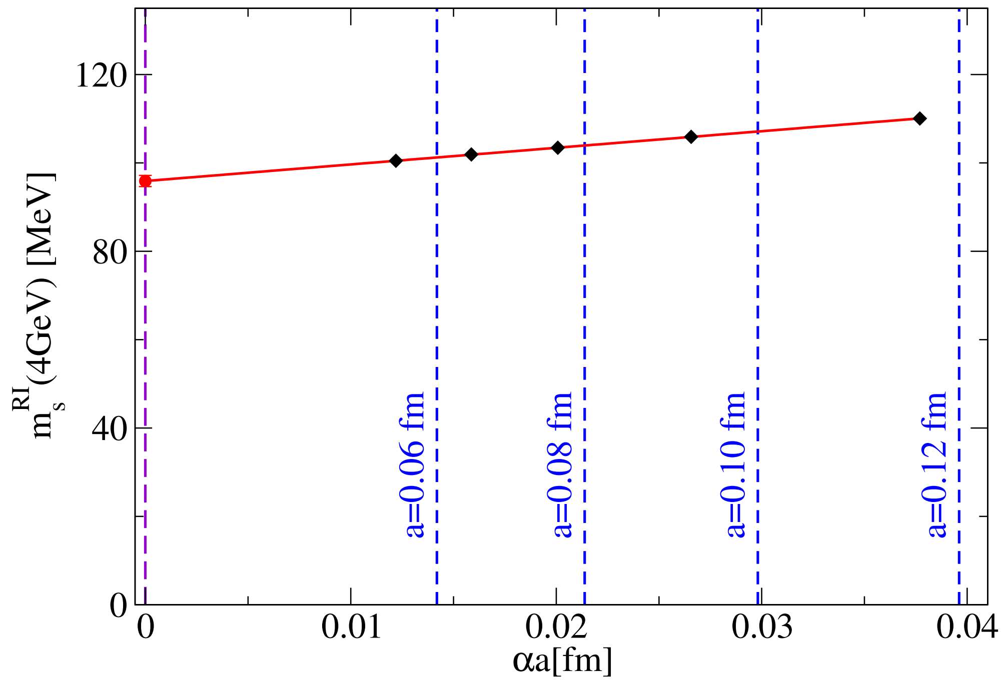


- RI series (left) converges less convincingly than $\overline{\mathrm{MS}}$ series (right)
- difference "4-loop" to "4-loop/ana" indicates size of 5-loop effects
- ratio suggests that higher-loop effects in RI are $<1\%$ at $\mu = 4\,\mathrm{GeV}$
- ratio suggests that higher-loop effects in $\overline{\mathrm{MS}}$ are negligible down to $\mu = 2\,\mathrm{GeV}$

Quark masses (6): Final results for m_s and m_{ud}

Good scaling of $m_{ud,s}^{\mathrm{RI}}(4\,\mathrm{GeV})$ out to the coarsest lattice ($a\!\sim\!0.116\,\mathrm{fm}$):





Conversion with analytical 4-loop formula at $4\,\mathrm{GeV}$ and downwards running in $\overline{\mathrm{MS}}$:

	m_{ud}	m_s
RI$(4\,\mathrm{GeV})$	3.503(48)(49)	96.4(1.1)(1.5)
RGI	4.624(63)(64)	127.3(1.5)(1.9)
$\overline{\mathrm{MS}}(2\,\mathrm{GeV})$	3.469(47)(48)	95.5(1.1)(1.5)

RGI$/\overline{\mathrm{MS}}$ results (table 1.9% prec.) need to be augmented by a $\sim\!1\%$ conversion error.

Quark masses (7): splitting m_{ud} with information from $\eta \to 3\pi$

The process $\eta \to 3\pi$ is highly sensitive to QCD isospin breaking (from $m_u \neq m_d$) but rather insensitive to QED isospin breaking (from $q_u \neq q_d$), and this is captured in Q.

Rewrite the Leutwyler ellipse in the form

$$\frac{1}{Q^2} = 4 \left(\frac{m_{ud}}{m_s}\right)^2 \frac{m_d - m_u}{m_d + m_u}$$

and use the conservative estimate $Q = 22.3(8)$ of [Leutwyler, Chiral Dynamics 09] together with our result $m_s/m_{ud} = 27.53(20)(08)$ to get the asymmetry parameter

$$\frac{m_d - m_u}{m_d + m_u} = 0.381(05)(27) \quad \longleftrightarrow \quad m_u/m_d = 0.448(06)(29)$$

from which we then obtain individual m_u, m_d values (note: $m_u = 0$ strongly disfavored)

	m_u	m_d	m_s
RI$(4\,\mathrm{GeV})$	2.17(04)(10)	4.84(07)(12)	96.4(1.1)(1.5)
RGI	2.86(05)(13)	6.39(09)(15)	127.3(1.5)(1.9)
$\overline{\mathrm{MS}}(2\,\mathrm{GeV})$	2.15(03)(10)	4.79(07)(12)	95.5(1.1)(1.5)

FLAG effort (1): collaborators and goal

FLAG = Flavianet Lattice Averaging Group

Members [as of 2010]:

Gilberto Colangelo (Bern)
Stephan Dürr (Wuppertal/Jülich, BMW)
Andreas Jüttner (Southampton→CERN, RBC/UKQCD)
Laurent Lellouch (Marseille, BMW)
Heiri Leutwyler (Bern)
Vittorio Lubicz (Rome 3, ETM)
Silvia Necco (CERN, Alpha)
Chris Sachrajda (Southampton, RBC/UKQCD)
Silvano Simula (Rome 3, ETM)
Tassos Vladikas (Rome 2, Alpha and ETM)
Urs Wenger (Bern, ETM)
Hartmut Wittig (Mainz, Alpha)

Goal:

Compile results from lattice calculations in a form useful to non-lattice experts.

FLAG effort (2): methodology and quantities covered

For each quantity FLAG provides:

- complete list of references
- summary of essential ingredients of each study [N_f, action, ...]
- averages for "mature" quantities
- *pressure* on reader to cite original papers !

Quantities covered in first edition [Eur.Phys.J. C71 (2011) 1695, arXiv:1011.4408]:

- light quark masses m_{ud}, m_s
- chiral low-energy constants (LECs)
- decay constants (of pions and kaons)
- form factors (of pions and kaons)
- kaon bag parameter B_K

In 2012 FLAG merged with "latticeaverages.org", and expanded with new structure [AB, EB]. Future updates of the report under `http://itpwiki.unibe.ch/flag` .

FLAG effort (3): color coding

FLAG-1 definitions [will be subject to change with each new edition] as follows

Continuum extrapolation:
- ★ 3 or more lattice spacings *and* at least 2 points below $0.1\,\mathrm{fm}$
- ● 2 or more lattice spacings *and* at least 1 point below $0.1\,\mathrm{fm}$
- ■ otherwise

Finite-volume effects:
- ★ $(M_\pi L)_{\mathrm{min}} > 4$ *or* at least 3 volumes
- ● $(M_\pi L)_{\mathrm{min}} > 3$ *and* at least 2 volumes
- ■ otherwise

Chiral extrapolation:
- ★ $M_{\pi,\mathrm{min}} < 250$ MeV
- ● $250\,\mathrm{MeV} \leq M_{\pi,\mathrm{min}} \leq 400\,\mathrm{MeV}$
- ■ $M_{\pi,\mathrm{min}} > 400$ MeV

Renormalization (where applicable):
- ★ non-perturbatively
- ● 2-loop perturbation theory
- ■ otherwise

FLAG effort (4): compilation of quark masses

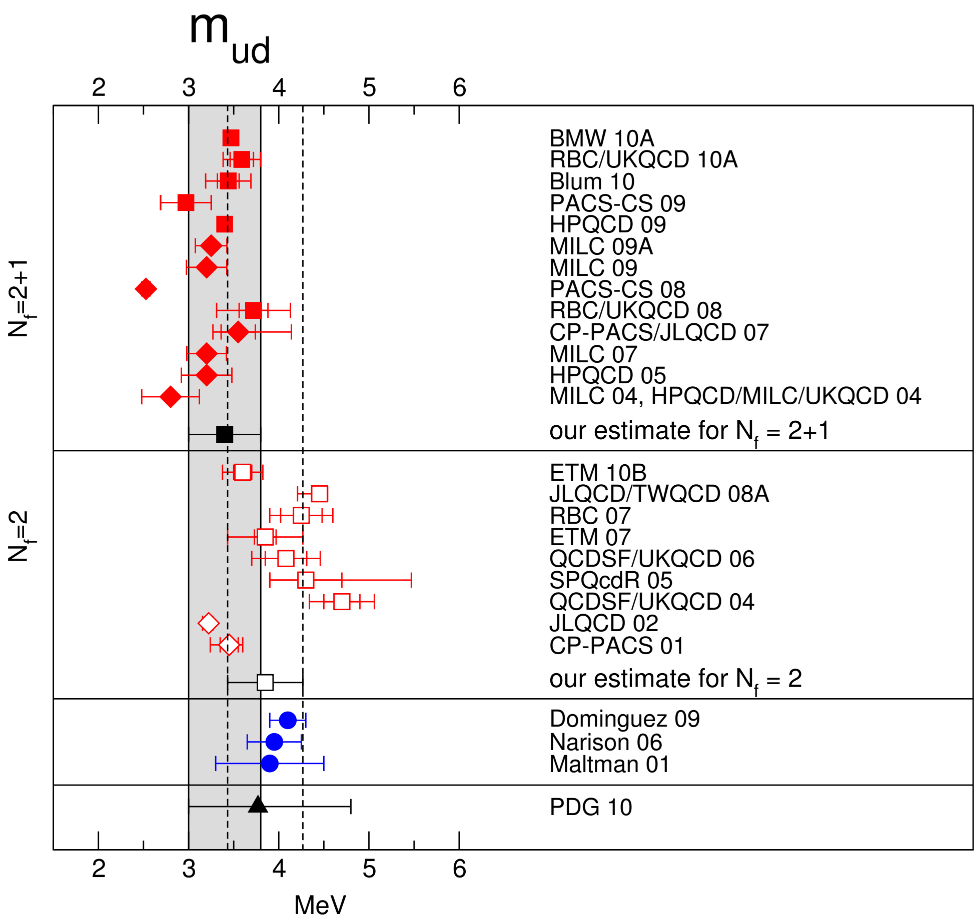
Collaboration	Ref.	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	m_{ud}	m_s
PACS-CS 10	[64]	P	★	■	■	★	a	2.78(27)	86.7(2.3)
MILC 10A	[103]	C	●	★	★	●	–	3.19(4)(5)(16)	–
HPQCD 10	[104]	A	●	★	★	★	–	3.39(6)*	92.2(1.3)
BMW 10A, 10B$^+$	[65, 105]	P	★	★	★	★	b	3.469(47)(48)	95.5(1.1)(1.5)
RBC/UKQCD 10A	[106]	P	●	●	★	★	c	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum 10†	[74]	P	●	■	●	★	–	3.44(12)(22)	97.6(2.9)(5.5)
PACS-CS 09	[42]	A	★	■	■	★	a	2.97(28)(3)	92.75(58)(95)
HPQCD 09	[107]	A	●	★	★	★	–	3.40(7)	92.4(1.5)
MILC 09A	[59]	C	●	★	★	●	–	3.25 (1)(7)(16)(0)	89.0(0.2)(1.6)(4.5)(0.1)
MILC 09	[6]	A	●	★	★	●	–	3.2(0)(1)(2)(0)	88(0)(3)(4)(0)
PACS-CS 08	[63]	A	★	■	■	■	–	2.527(47)	72.72(78)
RBC/UKQCD 08	[108]	A	●	■	★	★	–	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	[109]	A	■	★	★	■	–	$3.55(19)(^{+56}_{-20})$	$90.1(4.3)(^{+16.7}_{-4.3})$
HPQCD 05	[110]	A	●	●	●	●	–	3.2(0)(2)(2)(0)‡	87(0)(4)(4)(0)‡
MILC 04, HPQCD/ MILC/UKQCD 04	[77, 111]	A	●	●	●	■	–	2.8(0)(1)(3)(0)	76(0)(3)(7)(0)

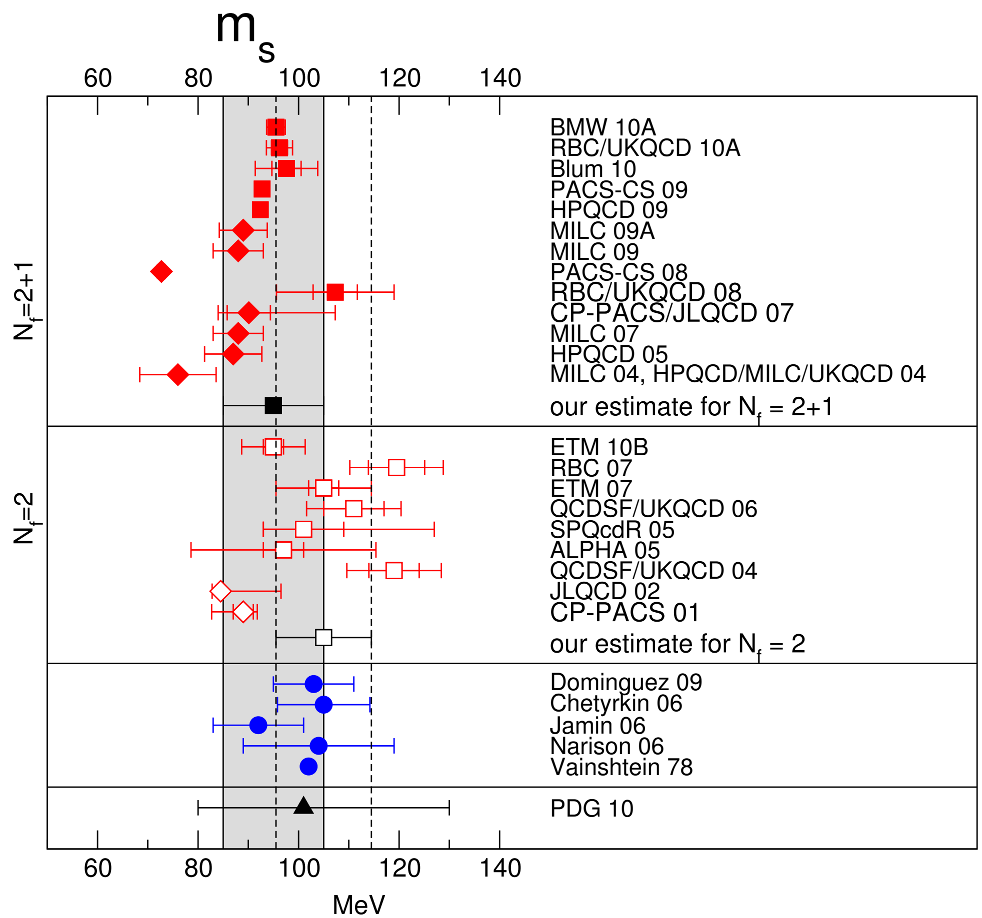
* Value obtained by combining the HPQCD 10 result for m_s with the MILC 09 result for m_s/m_{ud}.

$^+$ The fermion action used is tree-level improved.

† The calculation includes quenched e.m. effects.

FLAG effort (5): suggested values of m_u, m_d, m_s





$\longrightarrow$ apparent "tension" between $N_f=2$ (white band) and $N_f=2+1$ (grey band) likely due to better non-perturbative renormalization in the latter case.

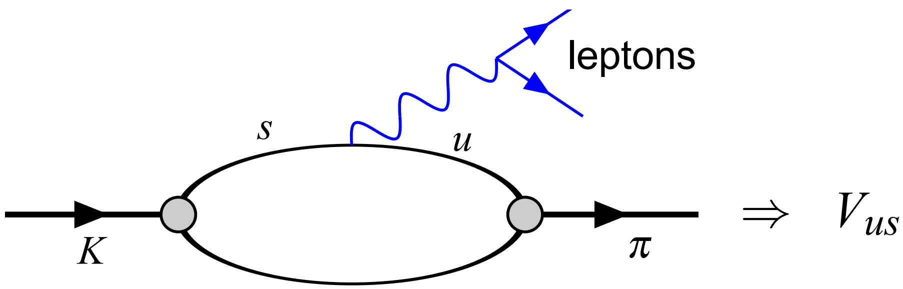
$\longrightarrow$ $N_f=2+1$ estimates: $m_u=2.19(15)\,\mathrm{MeV}$, $m_d=4.67(20)\,\mathrm{MeV}$, $m_s=94(3)\,\mathrm{MeV}$.

$\Longrightarrow$ FLAG estimates are *significantly more precise* than PDG estimates.

Decay constants, form factors and CKM-unitarity

- $|V_{\rm us}|$ **from** $K \to \pi$ **transition form factor** $f_+(0)$

Experiment can determine $|V_{\rm us}| f_+(0)$, lattice can determine $f_+(0)$.



$q \equiv p - p'$ momentum transfer

$$\langle \pi(p') | \bar{s} \gamma_\mu u | K(p) \rangle = f_0(q^2) \frac{M_K^2 - M_\pi^2}{q^2} q_\mu + f_+(q^2) \Big[(p+p')_\mu - \frac{M_K^2 - M_\pi^2}{q^2} q_\mu \Big]$$

chiral breakup: $f_+(0) = 1 + f_2 + f_4 + \ldots$, traditionally f_2 from ChPT, $f_4 + \ldots$ from models.

lattice flavor: $f_+(0) = 1$ for $m_{ud} = m_s$ means that $\Delta f_+(0) = f_2 + f_4 + \ldots$ is calculated with $\sim 20\%$ precision.

lattice momenta: with periodic boundary conditions, available (spatial) momenta have the form $p = 2\pi/L$, with $L = 2\,\mathrm{fm}$ one has $|p|_{\rm min} = 600\,\mathrm{MeV}$.

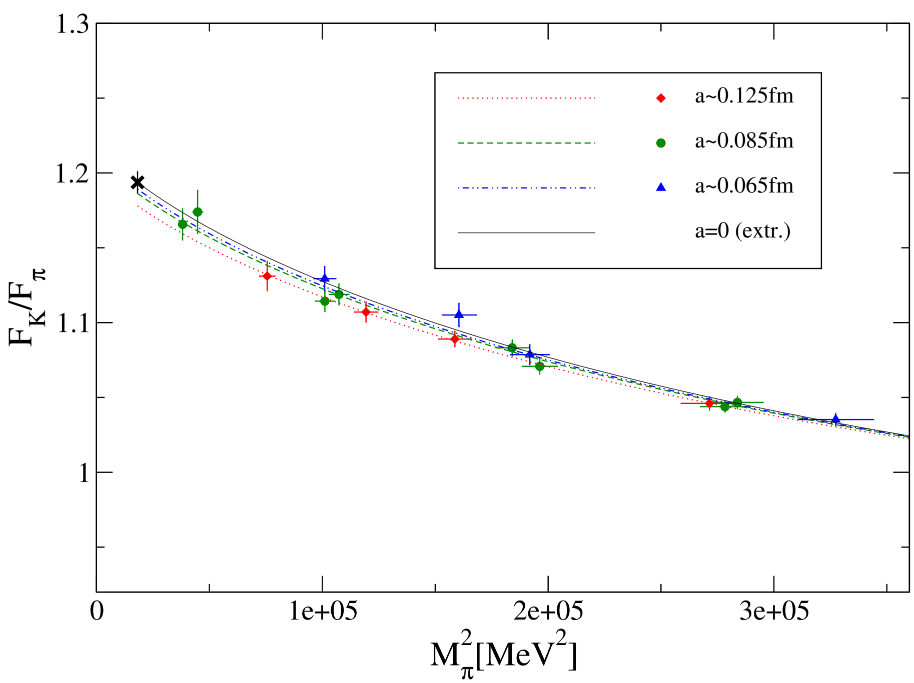
• $|V_{\rm us}|$ **from ratio** f_K/f_π **and Hardy-Towner**

Experiment can determine $|V_{\rm us}|f_K$, lattice can determine f_K.

This works, but there is a better way [Marciano, PRL 93 231803 (2004)]:

- $|V_{\rm ud}|$ is known, from nuclear β-decays, with 0.03% precision [Hardy Towner].
- $|V_{\rm us}|$ is much less precisely known, but can be linked to $|V_{\rm ud}|$ via a relation involving f_K/f_π, with everything else known rather accurately:

$$\frac{\Gamma(K \to l\bar{\nu}_l)}{\Gamma(\pi \to l\bar{\nu}_l)} = \frac{|V_{\rm us}|^2}{|V_{\rm ud}|^2}\frac{f_K^2}{f_\pi^2}\frac{M_K(1-m_l^2/M_K^2)^2}{M_\pi(1-m_l^2/M_\pi^2)^2}\{1+\frac{\alpha}{\pi}(C_K-C_\pi)\}$$



Plot from calculation by BMW-collaboration.

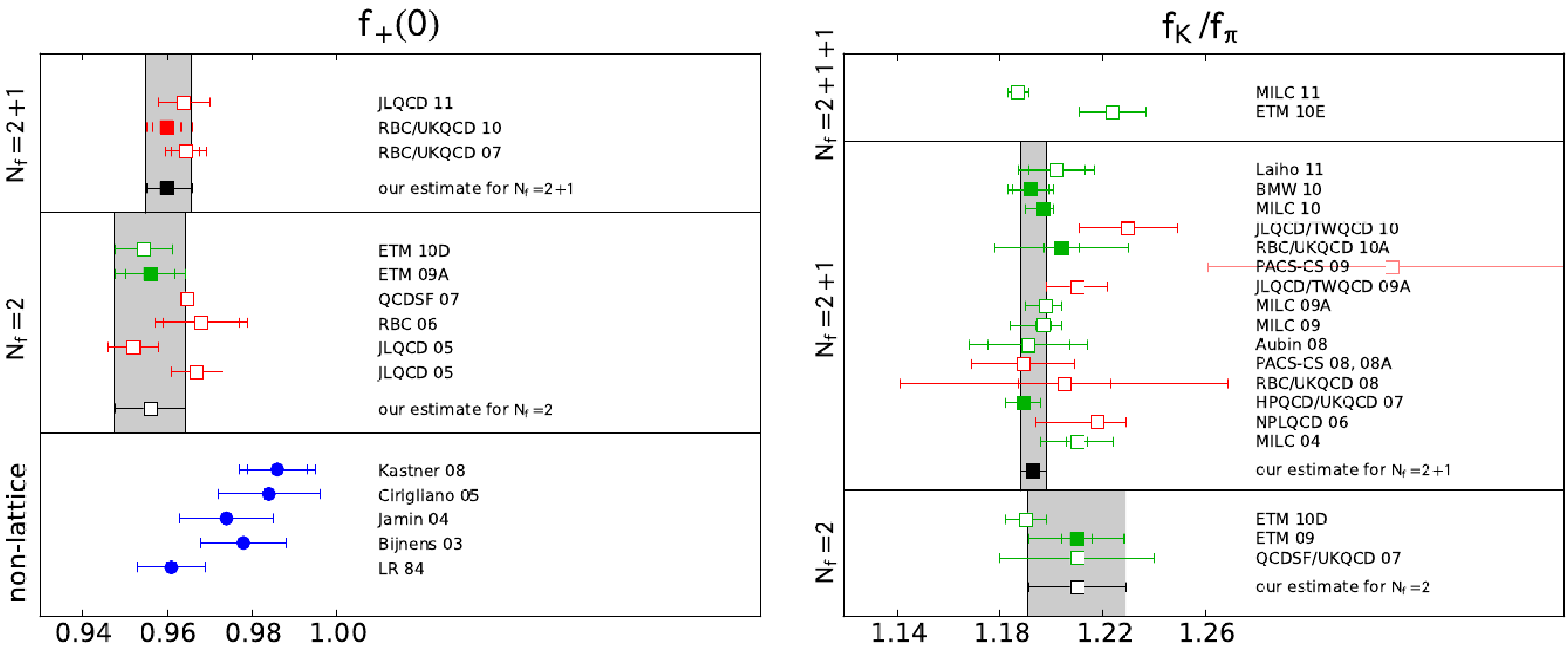
$\longrightarrow$ f_K/f_π has small cut-off effects; here $a^2/\mathrm{fm}^2 = 0.0042, 0.0072, 0.0156$.

$\Longrightarrow$ $f_K/f_\pi = 1.192(7)(6)$ at physical m_{ud}, in continuum, and infinite volume.

• Summary on $f_+(0)$ and f_K/f_π

FLAG-1 estimates: $f_+(0) = 0.956(8)$ and $f_K/f_\pi = 1.193(5)$

Artist's impression of forthcoming FLAG-2 compilation (ignore gray bands):



- **Implication on 1st-row CKM unitarity**

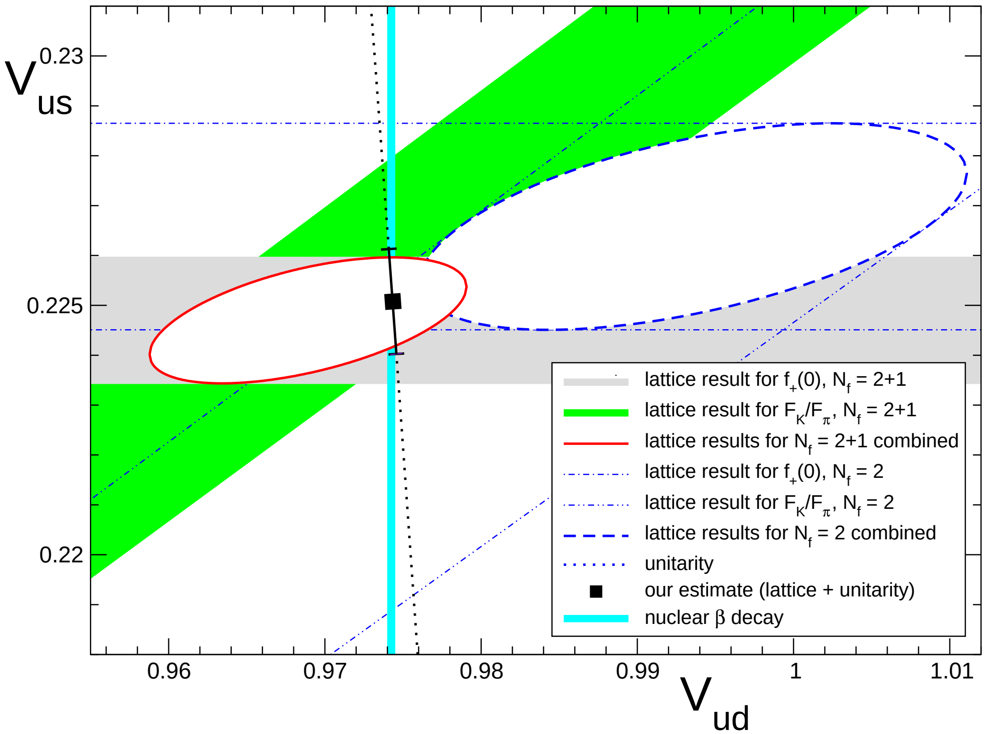
$$|V_{\rm ud}|^2+|V_{\rm us}|^2+|V_{\rm ub}|^2=1 \qquad \text{[SM]}$$

$$|V_{\rm us}|f_+(0)=0.2163(5) \qquad \text{[exp, FlavianetKaon 10]}$$

$$\frac{|V_{\rm us}|f_K}{|V_{\rm ud}|f_\pi}=0.2758(5) \qquad \text{[exp, FlavianetKaon 10]}$$

$\longrightarrow$ 3 relations for 4 unknowns, since $|V_{\rm ub}|=4.15(49)10^{-3}$ [PDG 12] is known/tiny

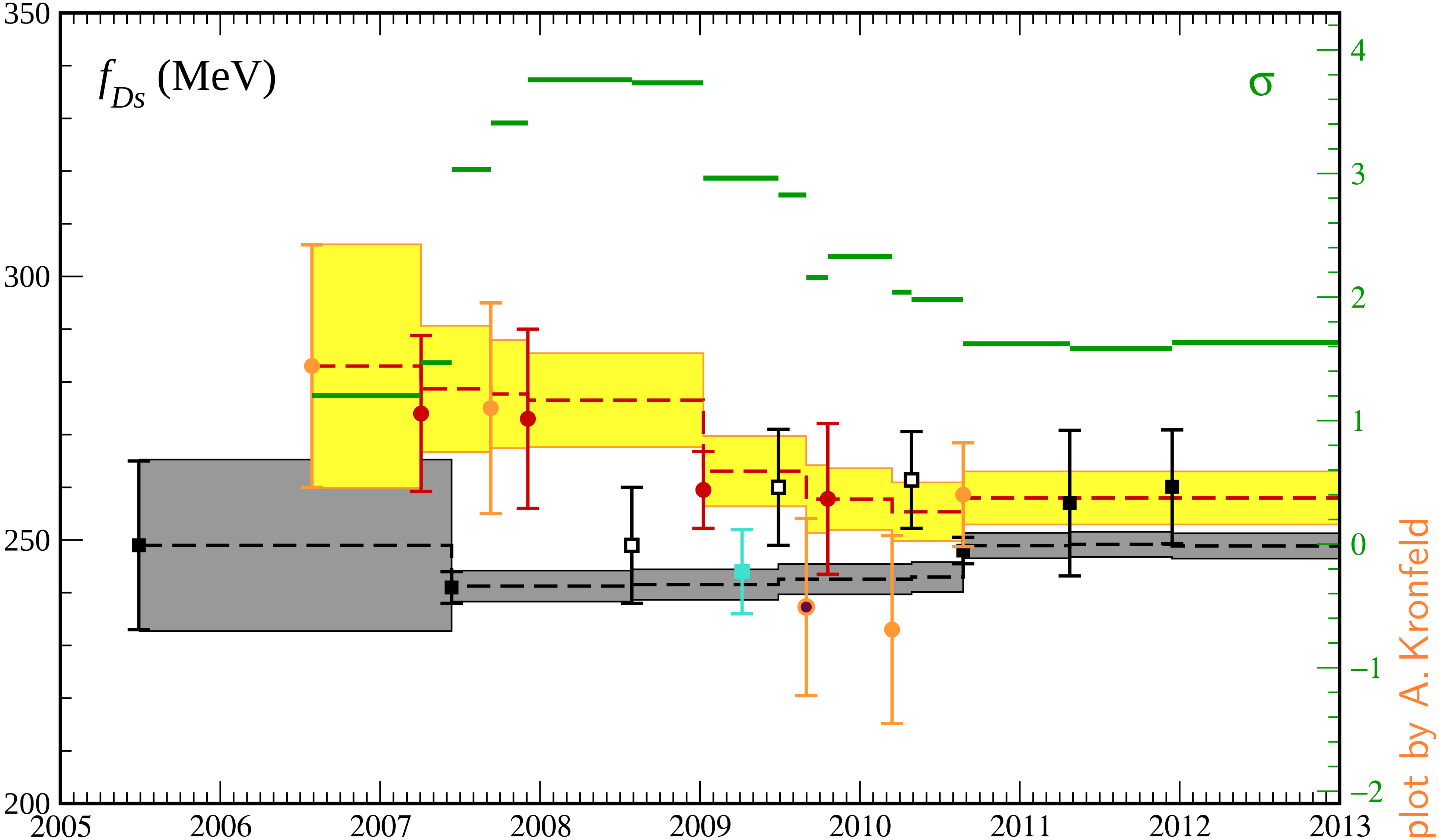
$\longrightarrow$ determine any one of $\underbrace{|V_{\rm ud}|,\ |V_{\rm us}|}_{\text{nucl/tau data}},\ \underbrace{f_+(0),\ f_K/f_\pi}_{\text{lattice QCD}}$ and get remaining three in SM



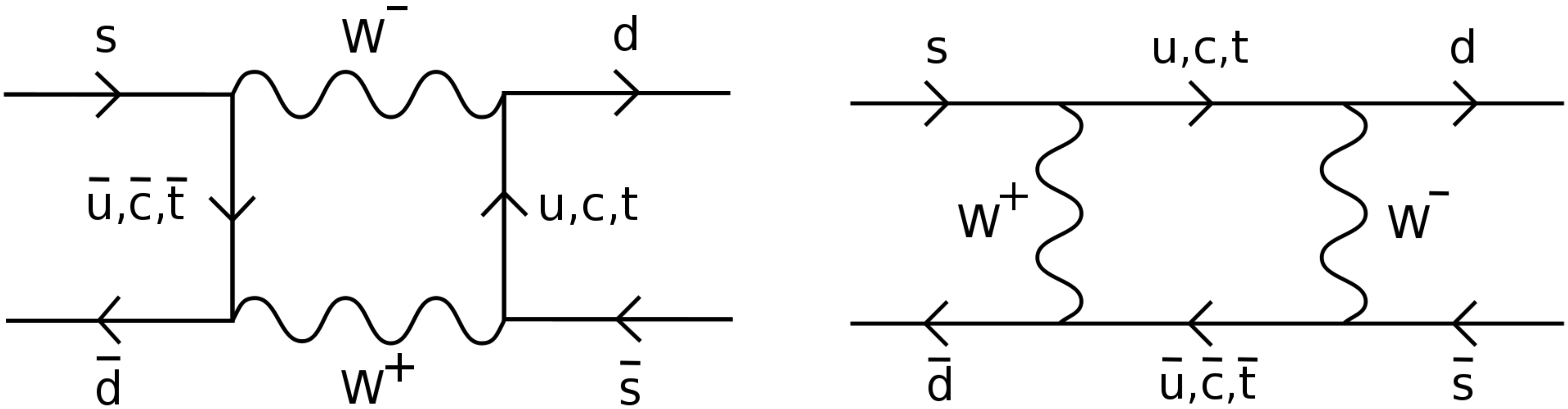
- drop unitary constraint, get (almost) *model-independent* test of BSM phys.
- compare to $|V_{\rm ud}|=0.97425(22)$ [HT]

- **Disappearance of "new physics" from f_{D_s}**

Red/Orange: running experimental average, based on CLEO-c, Babar, Belle.
Gray: running lattice average, based on Fermilab/MILC, HPQCD, CP-PACS.



Kaon mixing: B_K, B_{BSM} and $K \to 2\pi$ amplitude



Leading term $(d=6)$ in OPE is

$$B_K = \frac{\langle \bar{K}^0 | O_{VV+AA} | K^0 \rangle}{\frac{8}{3} \langle \bar{K}^0 | A_\mu | 0 \rangle \langle 0 | A_\mu | K^0 \rangle} = \frac{\langle \bar{K}^0 | O_{VV+AA} | K^0 \rangle}{\frac{8}{3} M_K^2 f_K^2}$$

and early estimates include $B_K=1$ ("VSA") and $B_K=3/4$ ("large N_c").

Note: ϵ_K and hence B_K quantify amount of *indirect* (via mixing) CP violation.

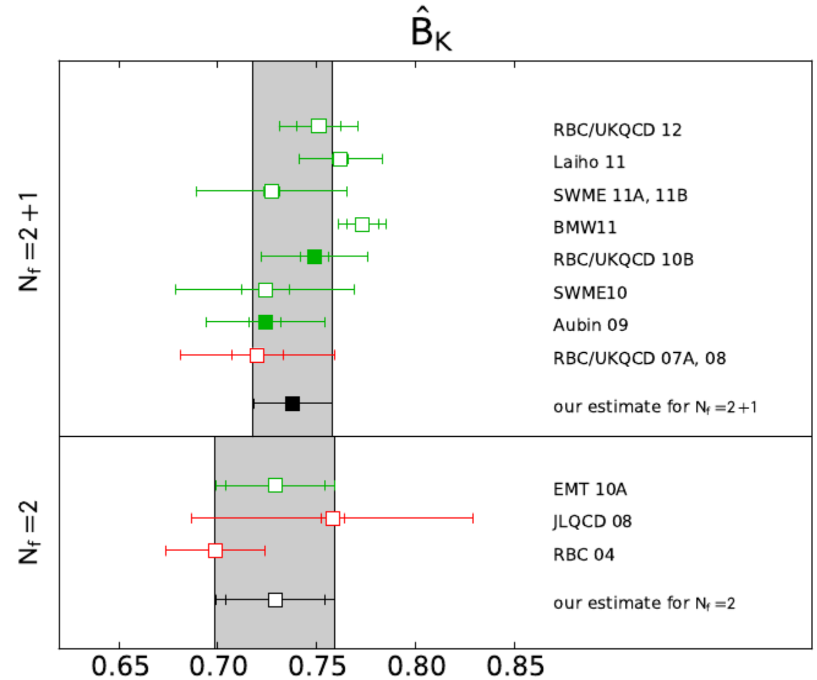
Note: Direct (in decay) CP violation significantly smaller: $\text{Re}(\epsilon'/\epsilon)=1.67(23)\,10^{-3}$.

• Kaon mixing parameter B_K

Most recent computations:

$$B_K^{\mathrm{RGI}} = 0.7727(81)(84) \text{ [BMW-c]}$$
$$0.727(04)(38) \text{ [SWME]}$$
$$0.766(04)(21) \text{ [LV]}$$
$$0.758(11)(19) \text{ [RBC/UKQCD]}$$

Artist's impression of forthcoming FLAG-2 compilation (ignore gray bands):



• Kaon mixing parameter B_{BSM}

Analogous definition, but with $O^{\Delta S=2}_{VV-AA}$ and $O^{\Delta S=2}_{SS\mp PP}$ and $O^{\Delta S=2}_{TT}$ inside; relevant in BSM theories whose low-energy EFT has other than $V-A$ structure.

Two recent computations:
arXiv:1206.5737 RBC/UKQCD: O_{2-5} from $N_f = 2+1$ overlap simulations
arXiv:1207.1287 ETMC: O_{2-5} from $N_f = 2$ twisted-mass simulations

Consequences for various BSM scenarios: arXiv:1207.3016, arXiv:1208.0534, ...

• First determination of $K \to (\pi\pi)_{I=2}$ amplitude

Blum et al [RBC/UKQCD], Phys.Rev.Lett. 108 (2012) 141601 [arXiv:1111.1699]
Blum et al [RBC/UKQCD], arXiv:1206.5142

After the lattice has struggled for decades with soft-pion theorems, this is the first direct computation of the $K \to \pi\pi$ amplitude with $\Delta I = 3/2$. They find: $\mathrm{Re}A_2 = 1.381(46)(258)10^{-8}\,\mathrm{GeV}$, $\mathrm{Im}A_2 = -6.54(46)(120)10^{-13}\,\mathrm{GeV}$.

$\mathrm{Re}A_2$ is in good agreement with the experiment, whereas $\mathrm{Im}A_2$ was hitherto unknown. Within the SM their result for $\mathrm{Im}A_2$ can be combined with the experimental results for $\mathrm{Re}A_0$, $\mathrm{Re}A_2$ and ϵ'/ϵ to give $\mathrm{Im}A_0/\mathrm{Re}A_0 = -1.61(28)10^{-4}$.

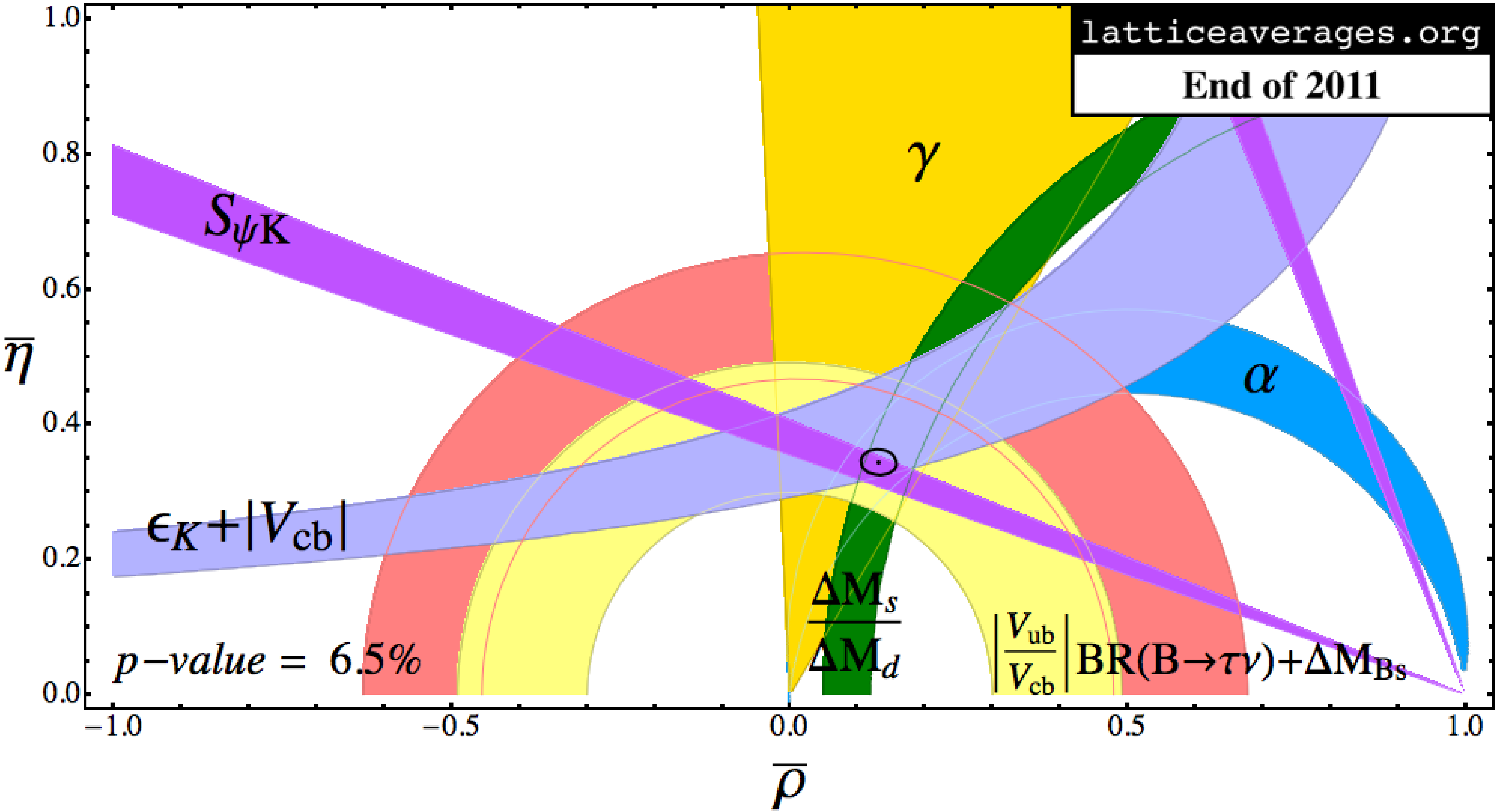
Their result for $\mathrm{Im}A_2$ implies that the electroweak penguin contribution to ϵ'/ϵ is $\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{EWP}}10^4 = -6.25 \pm 0.44 \pm 1.19$.
Still, direct computation of A_0 ($\Delta I = 1/2$, ϵ'/ϵ) remains "holy grail" for LQCD ...

• Recent computations of $B\bar{B}$-mixing

	$\xi = 1.268(63)$	arXiv:1205.7013	Fermilab/MILC
$f_{B_s}/f_{B_d} = 1.15(12)$	$\xi = 1.13(12)$	arXiv:1001.2023	RBC/UKQCD
$f_{B_s}/f_{B_d} = 1.226(26)$	$\xi = 1.258(33)$	arXiv:0902.1815	HPQCD

See also 1107.1441 [ETM], 1112.3051 [MILC], 1202.4914 [HPQCD] for f_{B_s}/f_{B_d}.

Unitarity fits with lattice input



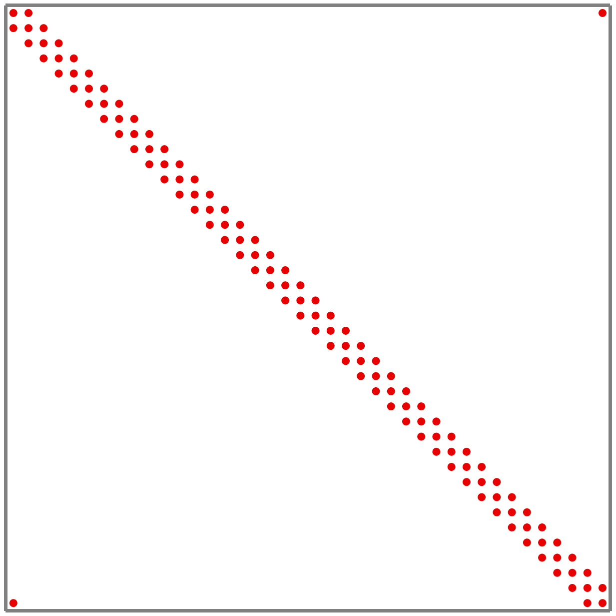
Discussion at `http://latticeaverages.org` [Lunghi, Laiho, Van de Water].
Like FLAG, they beg the user to cite original papers (to which they provide links).

Algorithms and machines

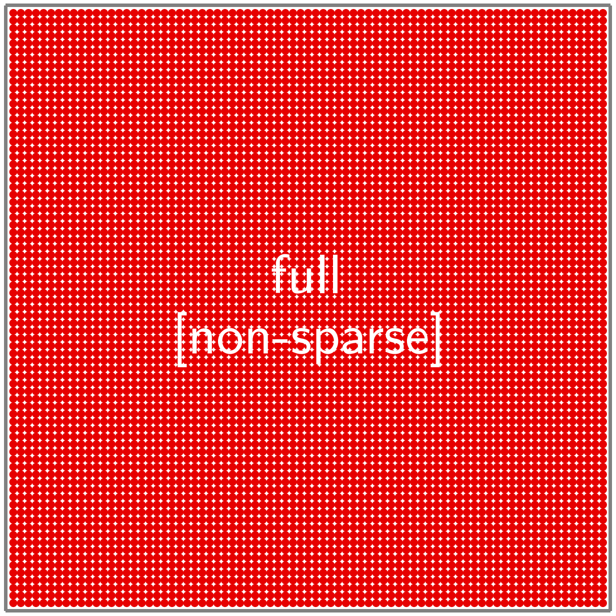
Sparse iterative solvers

$$D_\mathrm{W}(x,y) = \frac{1}{2}\sum_\mu \left\{(\gamma_\mu - I)U_\mu(x)\delta_{x+\hat\mu,y} - (\gamma_\mu + I)U_\mu^\dagger(x-\hat\mu)\delta_{x-\hat\mu,y}\right\} + (4+m_0)\delta_{x,y}$$

Wilson:



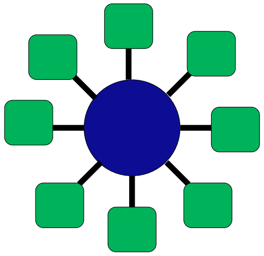
overlap:

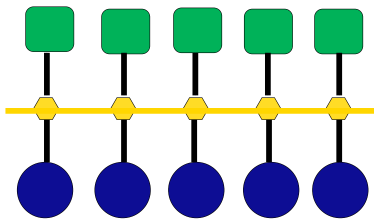


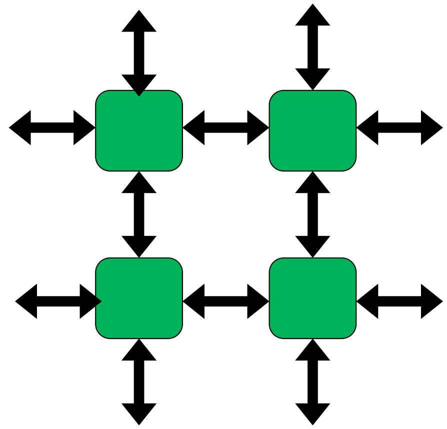
- D is $12N\times 12N$ complex sparse matrix, for $N=64^3\times 128$ this is $402\,10^6 \times 402\,10^6$
- each line/column contains only $1+3\cdot 2\cdot 8 = 49$ non-zero entries
- inverse is full [non-sparse], example above would require $2.4\,10^6$ TB of memory
- CG solver yields $D^{-1}\eta \simeq c_0\eta + c_1 D\eta + \ldots + c_n D^n\eta$ with $n^2 \propto \mathrm{cond}(D^\dagger D) = \frac{\lambda_\mathrm{max}}{\lambda_\mathrm{min}}$

New CPU packing strategies

SMP versus SIMD:

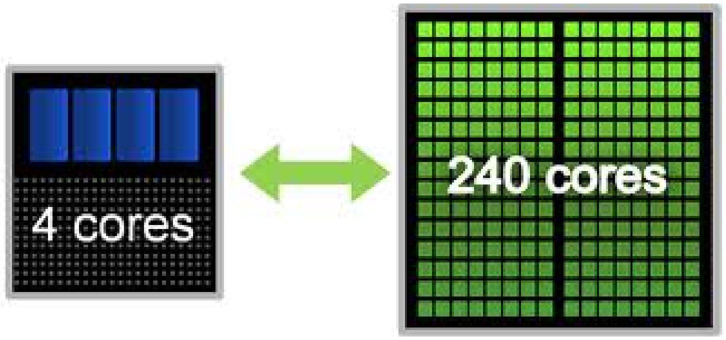






JUQUEEN [IBM BG/Q]	06/2012 - 10/2012	01/2013 - ...
processor type	64-bit PowerPC A2 1.6 GHz (205 Gflops each)	
compute node	16-way SMP processor (water cooled)	
racks, nodes, cores	8, 8'192, 131'072	...
memory	16 GB per node, aggregate 131 TB	...
performance (double)	1678/1380 Teraflops peak/Linpack	...
power consumption	$<$100 kW/rack, aggregate 0.8 MW	...
network topology	5D torus among compute nodes (incl. global barriers)	
network bandwidth	40 Gigabyte/s	
network latency	2.5 μsec (light travels 750 meters)	

New GPU programming models



GPUs originally designed for tasks in computer graphics (e.g. rendering).

GPUs nowadays frequently used for OpenMP-parallelizable scientific computations.

Hardware connection via PCI bus (overhead from data transfer before/after computation).

```
void transform_10000by10000grid(float in[10000][10000], float *out[10000][10000]){
   for(int x=0; x<10000; x++){
      for(int y=0; y<10000; y++){
         *out[x][y] = do_something(in[x][y]); // local operation !!!
      }
   }
}
```

Popular programming languages: CUDA, OpenCL, ...

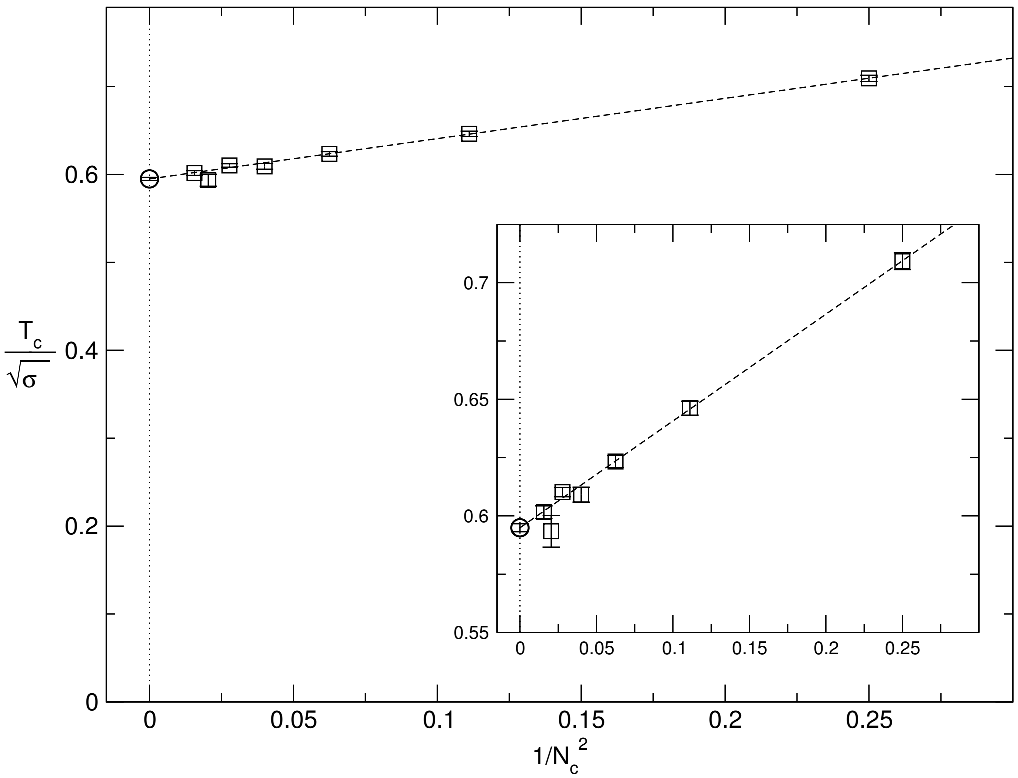
Issues of single (32bit) versus double (64bit) precision ...

Excellent price/performance ratio paid for by human work ...

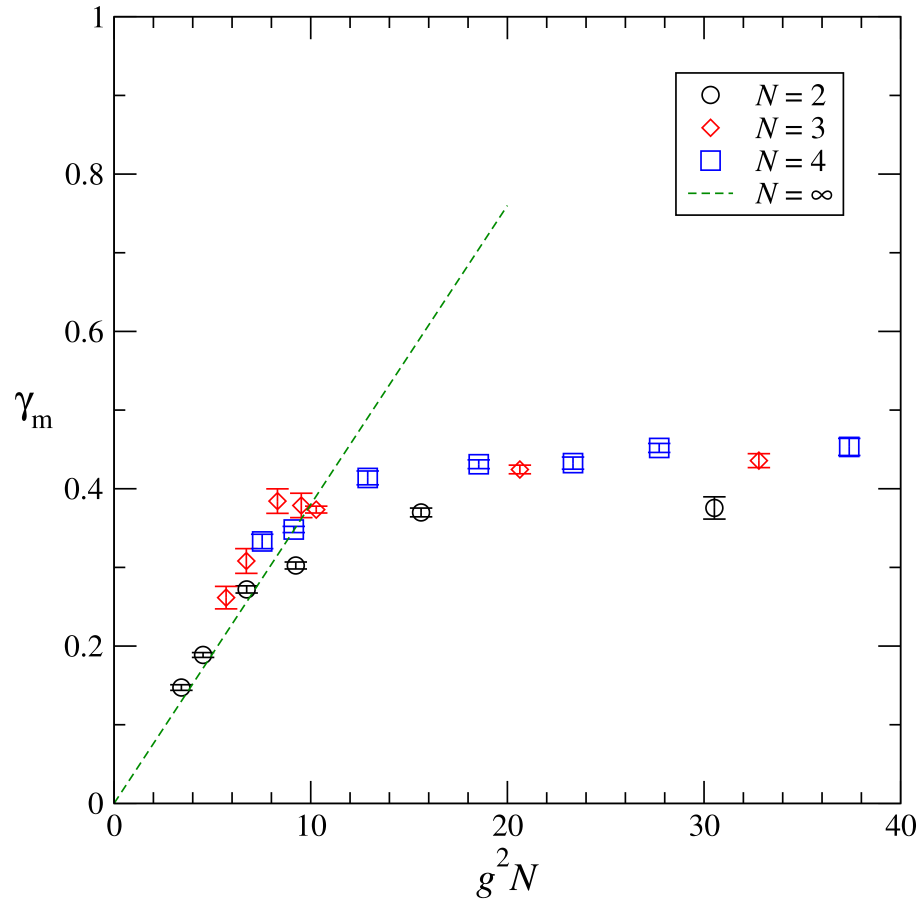
Other topics

Beyond QCD: Large N_c, larger N_f, different representations

QCD with $N_c \to \infty$ and fixed $\lambda = g^2 N_c$ gets much simpler [weakly coupled hadrons, OZI exact, chiral loops $\sim 1/N$, axial anomaly $\sim 1/N$]; lattice is almost unnecessary ;-)



$T_c/\sqrt{\sigma}$ for $N_c \to \infty$
Lucini, Rago, Rinaldi, 1202.6684



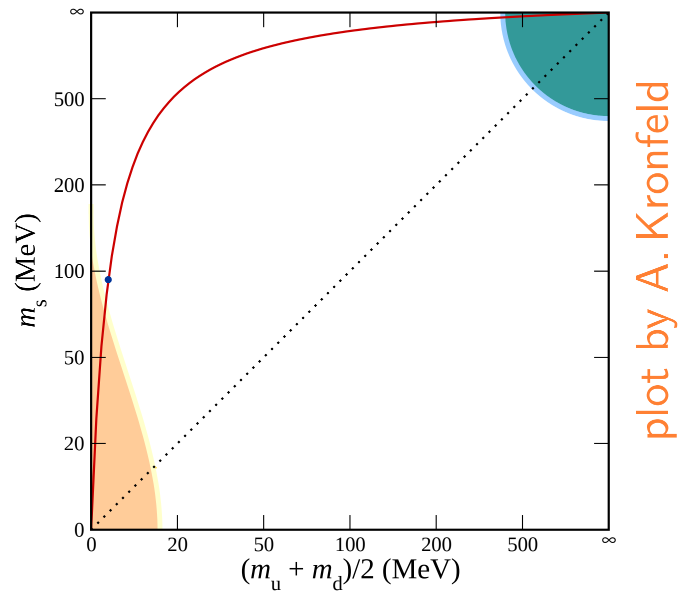
Anomalous mass dimension
2-index symm-representation, $N_f = 2$
DeGrand, Shamir, Svetitsky, 1202.2675

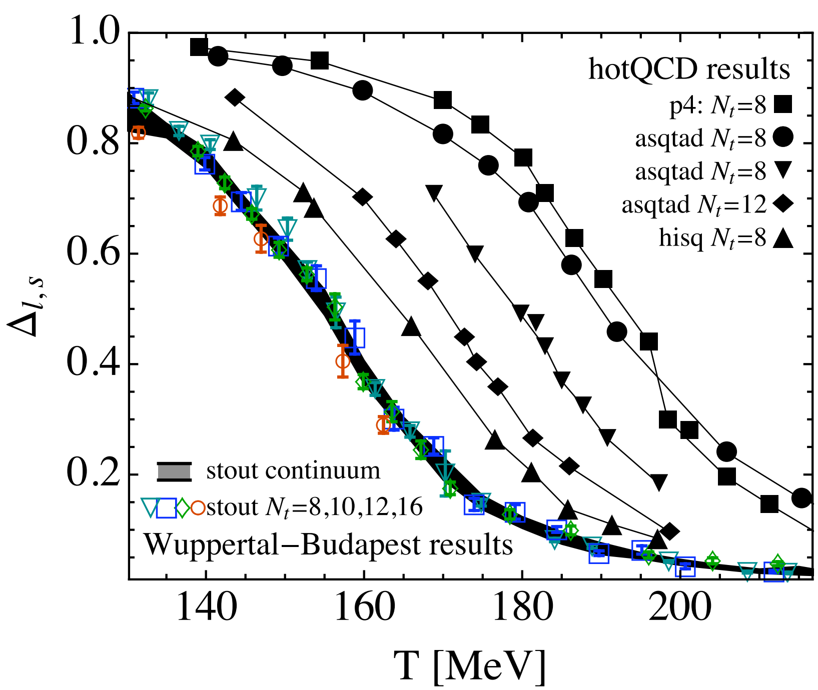
QCD thermodynamics at $\mu = 0$

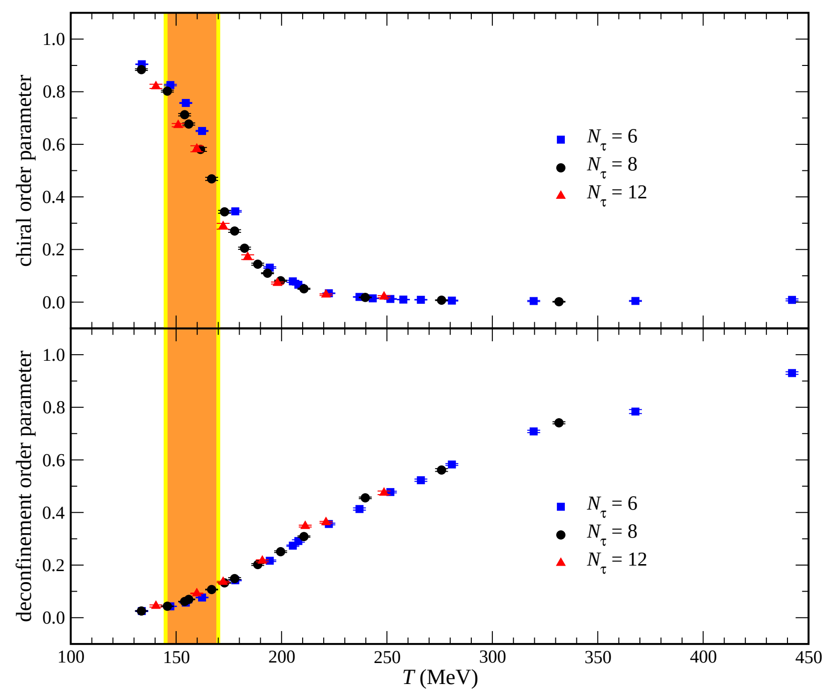
Established: QCD with physical m_{ud}, m_s at zero chemical potential (as relevant in early universe) shows *crossover*.

Different definitions of "transition temperature" T_c yield different values [P, $\langle\bar{\psi}\psi\rangle$, ...], but for one definition everyone should agree in the continuum.

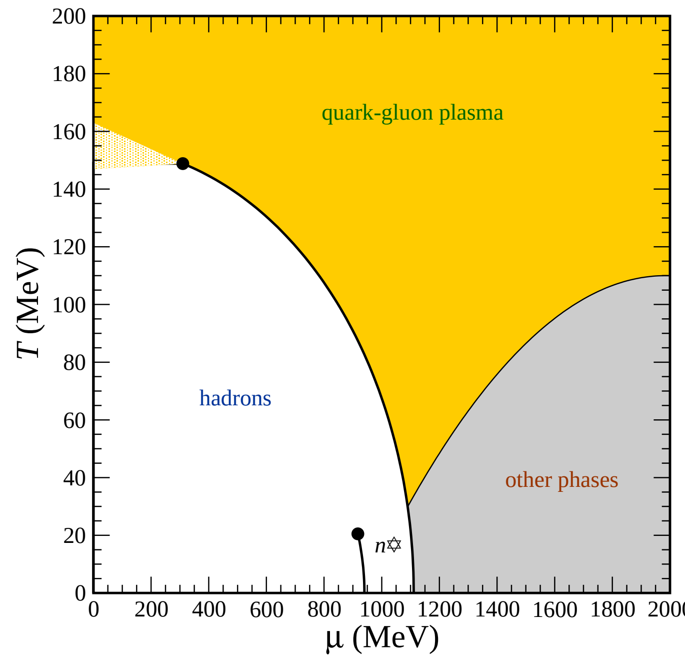
Long standing discrepancy between Wuppertal-Budapest (left) and HotQCD (right) now resolved.







QCD thermodynamics at $\mu > 0$



plot by A. Kronfeld

At non-zero baryon density (equivalent: chemical potential $\mu \neq 0$) the fermion determinant becomes complex, which creates a major difficulty to the concept of importance sampling.

A clear establishment of a second-order endpoint would be a major leap forward.

In QCD many approaches to solve the sign problem have been tried:

- absorb phase in observable [ancient]
- two-parameter reweighting from $\mu = 0$ [Fodor Katz]
- work at imaginary μ and continue [Philipsen deForcrand]
- compute Taylor coefficients at $\mu = 0$

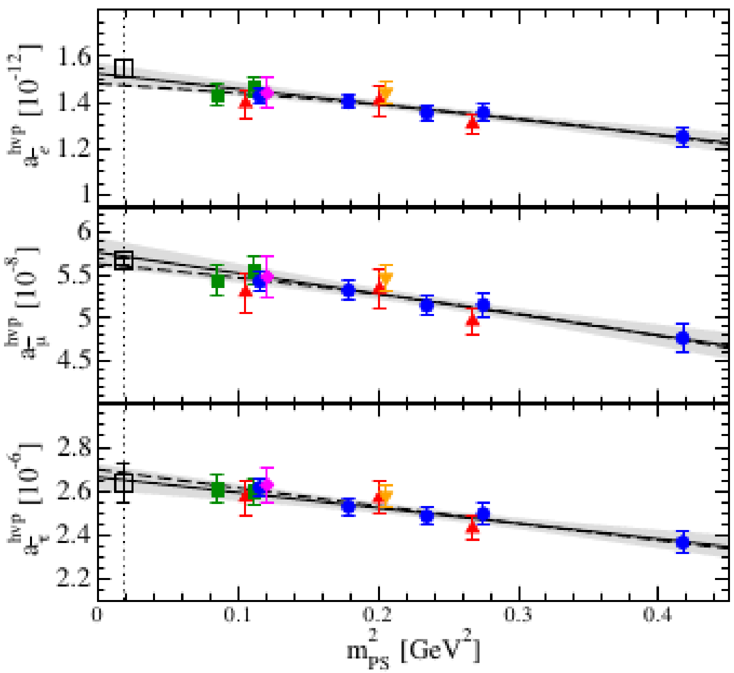
In QCD-inspired models many tricks/reformulations become possible.

Hadronic contributions to $g-2$ of the muon

Hadronic contributions to vacuum polarization provide one of the major sources of systematic uncertainty in the computation of $a_\mu = (g-2)_\mu$. Can the lattice help ?

$$a_\ell^{\rm HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 \; f(Q^2)\; \bar{\Pi}(Q^2)$$

with known f and $\bar{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$ and $\Pi_{\mu\nu}(q) = (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$ can be computed as the Fourier transformed 2-point function of the electromagnetic current.



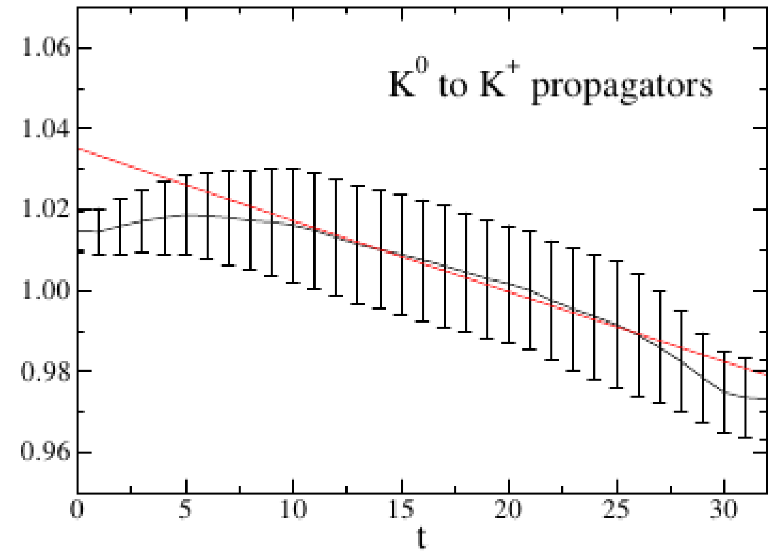
Recent computations include:
Feng et al, Phys.Rev.Lett. 107 (2011) 081802 [arXiv:1103.4818]
Della Morte et al, JHEP 1203 (2012) 055 [arXiv:1112.2894]
Kerrane et al, Phys.Rev. D85 (2012) 074504 [arXiv:1107.1497]

QCD with isospin splitting and/or electromagnetism

In standard $N_f = 2+1$ lattice studies two sources of isospin breaking are ignored (up-down mass difference, electromagnetic). Since they are both small, it would appear reasonable to include both of them a posteriori, by reweighting the configurations.

PACS-CS has long experience with reweighting in the quark mass; they used reweighting in m_{ud} to shift M_π from $156\,\mathrm{MeV}$ to $135\,\mathrm{MeV}$.

In arXiv:1205.2961 they extend this approach to account for QED effects and the up-down quark mass difference. They find $M_{K^0} > M_K^{\pm}$.



Pioneering publication for QCD+QED on the lattice is Duncan et al, Phys. Rev. Lett. 76 (1996) 3894-3897 [hep-lat/9602005].
Continuation by RBC/UKQCD Phys.Rev. D76 (2007), Phys.Rev. D82 (2010) 094508.

Still, there remain issues relating to finite-volume corrections, see e.g. Hayakawa Uno, Prog.Theor.Phys. 120 (2008) 413 and Portelli et al, PoS LATTICE2011 (2011) 136.

Outlook: $N_f = 1{+}1{+}1{+}1$ simulations with electromagnetism

- 2002-20??:

  $N_f = 2{+}1$ QCD requires 3 polished input values [e.g. M_π, M_K, M_Ω in theory with $m_u, m_d \to (m_u{+}m_d)/2$ and $e \to 0$]

  $\longrightarrow$ analysis suggests $M_\pi{=}134.8(3)\mathrm{MeV}, M_K{=}494.2(5)\mathrm{MeV}$ [see FLAG report]

- 2010-????:

  $N_f = 2{+}1{+}1$ QCD requires 4 polished input values [ditto and M_{D_s} in theory with $m_u, m_d \to (m_u{+}m_d)/2$ and $e \to 0$]

  $\longrightarrow$ charm unquenched, but no conceptual change on isospin issue

- 2014-????:

  $N_f = 1{+}1{+}1{+}1$ QCD requires 5 input variables [e.g. $M_{\pi^\pm}, M_{K^\pm}, M_{K^0}, M_{D_s}, M_\Omega$]

  $\longrightarrow$ requires disconnected contribution to flavor-singlet quantities

  $\longrightarrow$ analysis of π^0-η-η'-γ mixing mandatory to extract physical masses

  $\longrightarrow$ QED and QCD renormalization intertwined (m_s/m_d is RGI, m_u/m_d is not)

  $\longrightarrow$ final word on $m_u \overset{?}{=} 0$ [in QCD+QED] will be possible

List of topics not covered

- improved actions, matching with perturbation theory
- chiral symmetry in vector-like gauge theories
- chiral gauge theories and CP violation
- chiral symmetry and chemical potential
- sign problem at non-zero chemical potential
- supersymmetry on the lattice
- staggered fourth-root trick
- non-standard staggered mass terms
- large autocorrelation times
- new algorithmic developments
- new machine concepts
- ...

Summary

1. Lattice QCD is an intermediate step in the *definition* of QCD
2. Spectroscopy of stable hadrons with $N_f = 2+1$ is a mature field
3. Spectroscopy of mixing/unstable states is developing fast
4. Lattice yields vital input in CKM analysis and BSM bounds
5. FLAG/latticeaverages ask you to cite original papers !!!
6. Rapid progress on nuclear issues (strangeness, scattering, ...)
7. Rapid progress on QCD thermodynamics ($\mu = 0$ and $\mu > 0$)