Data-driven bounds on QCDF amplitudes,

Gilberto Tetlalmatzi-Xolocotzi

Based on:

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with T. Huber

and 2604.19612 [hep-ph]

with T. Huber, X.-Q. Li, W.-S. Fang and E. Malami

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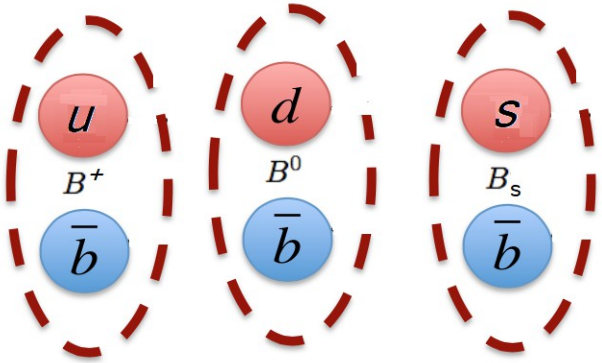
Jahrestreffen der deutschen LHCb-Gruppen

Aachen, October 2026

Non-leptonic B meson decays

In this talk, we will focus on the description of *B* mesons decays into pairs of light pseudoscalar mesons

$$B \to \pi\pi, \qquad B \to K\pi, \qquad B \to KK, \qquad B \to \eta\eta$$



$$q_i \otimes \overline{q}_j \to 3 \otimes \bar{3} = 8 \oplus 1$$

$$i,\ j \in [u, d, s]$$

$$B = (B^+, B_d^0, B_s^0)$$

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta_q'}{\sqrt{2}} & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_q}{\sqrt{2}} + \frac{\eta_q'}{\sqrt{2}} & \bar{K}^0 \\ K^+ & K^0 & \eta_s + \eta_s' \end{pmatrix},$$

Non-leptonic B meson decays

Anatomy of the decay amplitude

$$\mathcal{A}^{\mathrm{TDA}} = i\,\frac{G_F}{\sqrt{2}}\left[\mathcal{T}^{\mathrm{TDA}} + \mathcal{P}^{\mathrm{TDA}}\right],$$

$\lambda_u^{(D)}$ $\lambda_t^{(D)}$

The decay amplitudes can be decomposed in *two sectors depending on the CKM structure*

$$\lambda_u^{(D)} = V_{ub}\,V_{uD}^*, \qquad \lambda_t^{(D)} = V_{tb}\,V_{tD}^* \qquad D = d, s$$

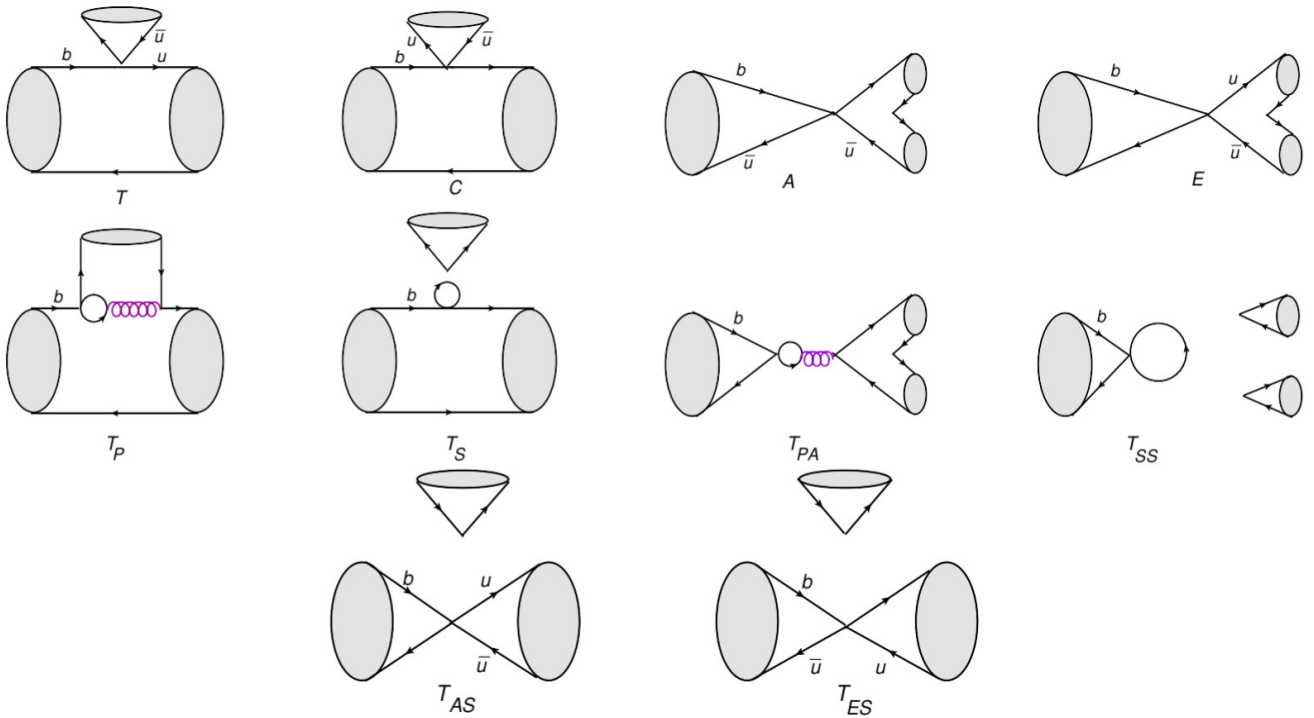
$$\lambda_u^{(D)} + \lambda_c^{(D)} + \lambda_t^{(D)} = 0$$

Topological description

The flavour structure allows a systematic decomposition of the physical decay amplitudes.

$$\begin{aligned}\mathcal{T}^{\rm TDA} = & \underline{T}\, B_i (M_1)^i_j\, \bar{H}^j_k (M_2)^k_l + \underline{C}\, B_i (M_1)^i_j\, \bar{H}^l_k (M_2)^k_l + \underline{A}\, B_i\, \bar{H}^l_j (M_1)^j_k (M_2)^k_i \\ & + \underline{E}\, B_i\, \bar{H}^{li}_j (M_1)^j_k (M_2)^k_l + \underline{T_{ES}}\, B_i\, \bar{H}^{ij}_l (M_1)^l_j (M_2)^k_k + \underline{T_{AS}}\, B_i\, \bar{H}^{ji}_l (M_1)^l_j (M_2)^k_k \\ & + \underline{T_S}\, B_i (M_1)^i_j\, \bar{H}^{lj}_l (M_2)^k_k + \underline{T_{PA}}\, B_i\, \bar{H}^{li}_l (M_1)^j_k (M_2)^k_j + \underline{T_P}\, B_i (M_1)^i_j (M_2)^j_k\, \bar{H}^{lk}_l \\ & + \underline{T_{SS}}\, B_i\, \bar{H}^{li}_l (M_1)^j_j (M_2)^k_k + (M_1 \leftrightarrow M_2)\,,\end{aligned}$$

There is a diagrammatic correspondence for each amplitude



Topological description

$$\bar{H}_1^{12} = \lambda_u^{(d)}, \quad \bar{H}_1^{13} = \lambda_u^{(s)}, \qquad \bar{H}^2 = \lambda_u^{(d)}, \quad \bar{H}^3 = \lambda_u^{(s)},$$

$$\tilde{H}_1^{12} = -\lambda_t^{(d)}, \quad \tilde{H}_1^{13} = -\lambda_t^{(s)}, \qquad \tilde{H}^2 = -\lambda_t^{(d)}, \quad \tilde{H}^3 = -\lambda_t^{(s)},$$

An analogous decomposition holds for $\mathcal{P}^{TDA}$

The sub-amplitudes $\{T, C, A, E, T_{ES}, T_{AS}, T_S, T_{PA}, T_P, T_{SS}\}$

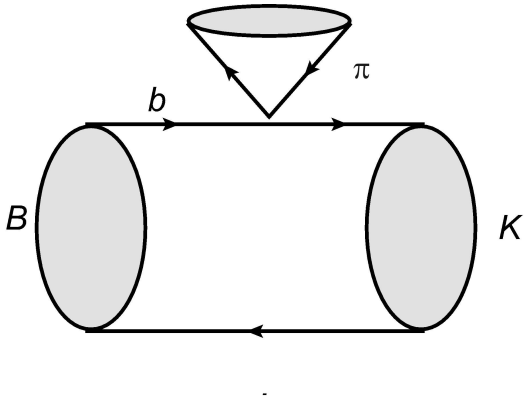
$$\{P,\ P_T,\ S,\ P_C,\ P_{TA},\ P_A,\ P_{TE},\ P_{AS},\ P_{SS},\ P_{ES}\}$$

can then be obtained from fits to data

He and Wang 1803.04227

Dynamical approach

Consider $B \to K\pi$



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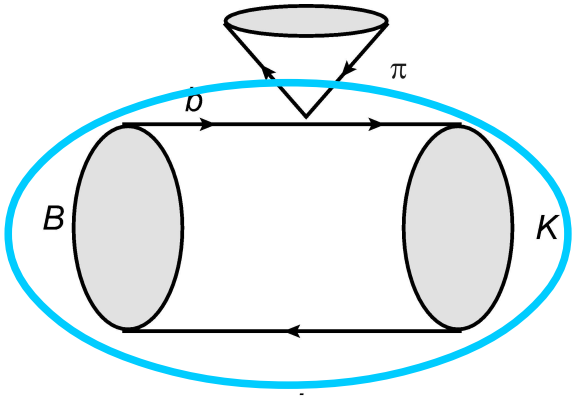
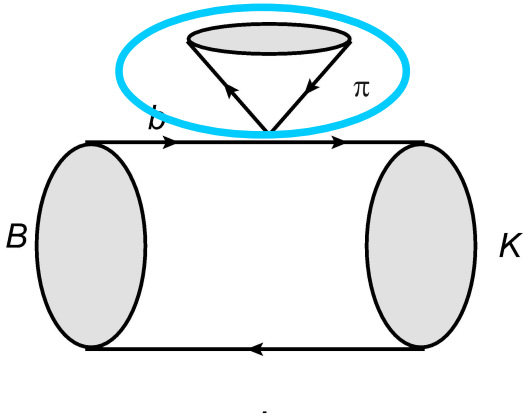


Naive factorization

$$\langle K\pi | Q | B \rangle \sim F_{B\to K} f_{\pi}$$

Dynamical approach

Consider $B \to K\pi$



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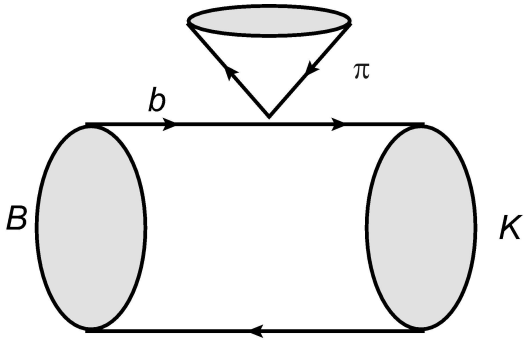

$$\langle K\pi | Q | B \rangle \sim F_{B\to K} f_{\pi}$$

A well accepted formalism to describe these processes theoretically is

QCD-factorization:

Beneke, Buchalla, Neubert, Sachrajda, 9905312

Dynamical approach

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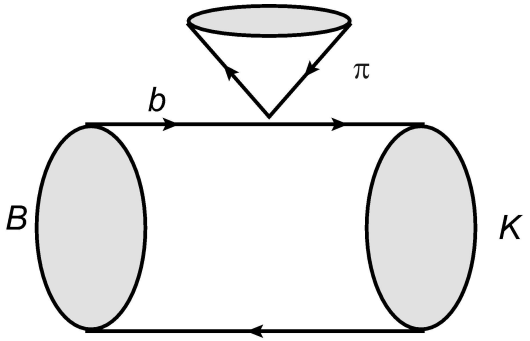

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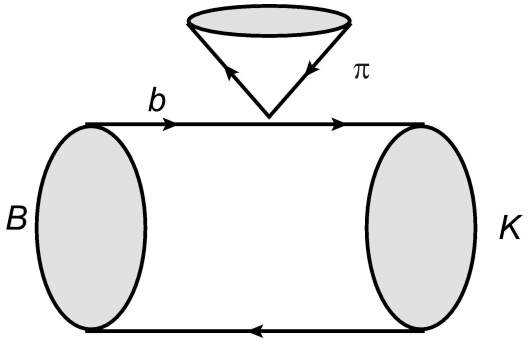
$$\langle M_1 M_2 | \hat{Q}_i | B \rangle = \sum_j F_j^{B\to M_1}(0) \int_0^1 du T_{ij}^I(u) \Phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$
$$+ \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(v) \Phi_{M_2}(u).$$

Dynamical approach

Consider $B \to K\pi$



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Beneke, Buchalla, Neubert, Sachrajda, 9905312

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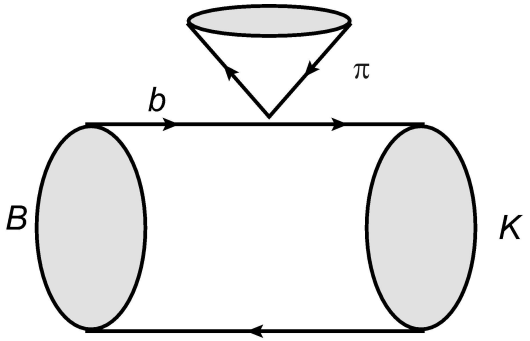

$$\langle K\pi | Q | B \rangle \sim F_{B\to K} . f_{\pi}$$

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Dynamical approach

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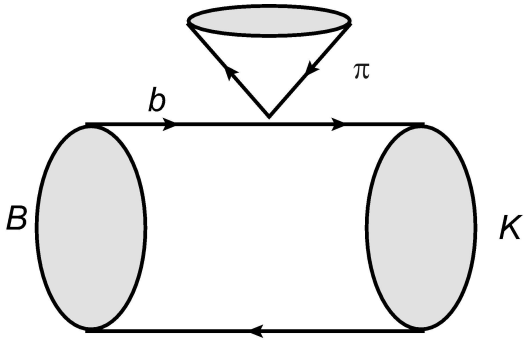
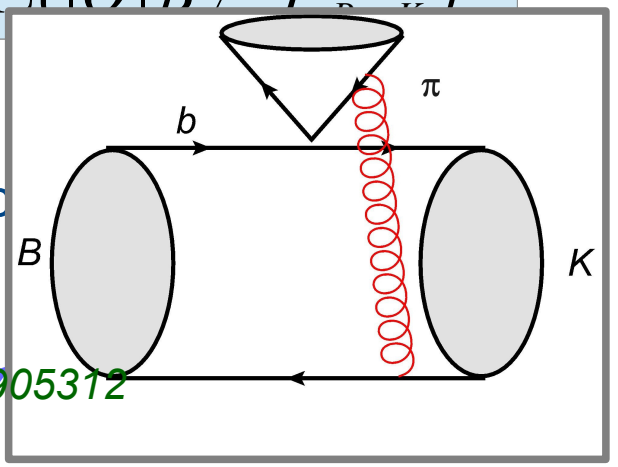


$\langle K\pi|Q|B\rangle \sim F \cdot f$



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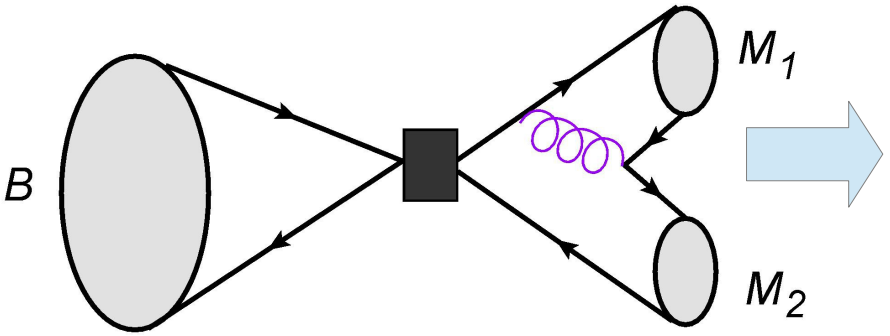
$$+ \int_0^1 d\xi du dv T_i^{II}(\xi,u,v)\Phi_B(\xi)\Phi_{M_1}(v)\Phi_{M_2}(u).$$

Dynamical approach

Unfortunately, most decay channels involve (power suppressed) contributions which lead to large uncertainties.

A very well known example of this are annihilation topologies which are affected by LCDA end point singularities



$$\beta, b \quad \int_0^1 \frac{dx}{x} \to \int_{\Lambda_h/m_B}^1 \frac{dx}{x} = \ln \frac{m_B}{\Lambda_h},$$

the annihilation sub-amplitudes are ubiquitous to most of the QCDF amplitudes, consider for instance

$$\mathcal{A}_{\bar{B}_s \to \pi^+\pi^-} = B_{\pi\pi}\left[b_4^p - \tfrac{1}{2} b_{4,\mathrm{EW}}^p\right] + B_{\pi\pi}\left[\delta_{pu} b_1 + b_4^p + b_{4,\mathrm{EW}}^p\right]$$

Uncertainties can be large, of order 100%

$$\mathcal{B}(B_s \to \pi^+\pi^-) = (0.024^{+0.160}_{-0.024}) \times 10^{-6}$$

Dynamical/Topological approach

Decay amplitudes in QCDF can be obtained with the ***magic formula***

$$
\begin{aligned}
\mathcal{A}^{\rm QCDF} =& i\frac{G_F}{\sqrt{2}}\sum_{p=u,c} A_{M_1M_2}\Big\{ BM_1\left(\alpha_1\delta_{pu}\hat{U}_p+\alpha_4^p\hat{I}+\alpha_{4\,\rm EW}^p\hat{Q}\right)M_2\Lambda_p \\
&+ BM_1\Lambda_p\cdot{\rm Tr}\left[\left(\alpha_2\delta_{pu}\hat{U}_p+\alpha_3^p\hat{I}+\alpha_{3,\rm EW}^p\hat{Q}\right)M_2\right] \\
&+ B\left(\beta_2\delta_{pu}\hat{U}_p+\beta_3^p\hat{I}+\beta_{3,\rm EW}^p\hat{Q}\right)M_1M_2\Lambda_p \\
&+ B\Lambda_p\cdot{\rm Tr}\left[\left(\beta_1\delta_{pu}\hat{U}_p+\beta_4^p\hat{I}+\beta_{4,\rm EW}^p\hat{Q}\right)M_1M_2\right] \\
&+ B\left(\beta_{S2}\delta_{pu}\hat{U}_p+\beta_{S3}^p\hat{I}+\beta_{S3,\rm EW}^p\hat{Q}\right)M_1\Lambda_p\cdot{\rm Tr}\,M_2 \\
&+ B\Lambda_p\cdot{\rm Tr}\left[\left(\beta_{S1}\delta_{pu}\hat{U}_p+\beta_{S4}^p\hat{I}+\beta_{S4,\rm EW}^p\hat{Q}\right)M_1\right]\cdot{\rm Tr}\,M_2\Big\}+(M_1\leftrightarrow M_2)\,,
\end{aligned}
$$

$$
\Lambda_p=\begin{pmatrix}0\\ \lambda_p^{(d)}\\ \lambda_p^{(s)}\end{pmatrix},\qquad
\hat{U}=\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix},
$$

$$
\hat{Q}=\frac{3}{2}Q=\begin{pmatrix}1&0&0\\0&-\frac{1}{2}&0\\0&0&-\frac{1}{2}\end{pmatrix},\qquad
\hat{I}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},
$$

Dynamical/Topological approach

Decay amplitudes in QCDF can be obtained with the ***magic formula***

$$\begin{aligned}\mathcal{A}^{\rm QCDF} =& i\frac{G_F}{\sqrt{2}}\sum_{p=u,c} A_{M_1M_2}\Big\{BM_1\left(\alpha_1\delta_{pu}\hat{U}_p+\alpha_4^p\hat{I}+\alpha_{4\,\rm EW}^p\hat{Q}\right)M_2\Lambda_p\\ &+BM_1\Lambda_p\cdot{\rm Tr}\left[\left(\alpha_2\delta_{pu}\hat{U}_p+\alpha_3^p\hat{I}+\alpha_{3,\rm EW}^p\hat{Q}\right)M_2\right]\\ &+B\left(\beta_2\delta_{pu}\hat{U}_p+\beta_3^p\hat{I}+\beta_{3,\rm EW}^p\hat{Q}\right)M_1M_2\Lambda_p\\ &+B\Lambda_p\cdot{\rm Tr}\left[\left(\beta_1\delta_{pu}\hat{U}_p+\beta_4^p\hat{I}+\beta_{4,\rm EW}^p\hat{Q}\right)M_1M_2\right]\\ &+B\left(\beta_{S2}\delta_{pu}\hat{U}_p+\beta_{S3}^p\hat{I}+\beta_{S3,\rm EW}^p\hat{Q}\right)M_1\Lambda_p\cdot{\rm Tr}\,M_2\\ &+B\Lambda_p\cdot{\rm Tr}\left[\left(\beta_{S1}\delta_{pu}\hat{U}_p+\beta_{S4}^p\hat{I}+\beta_{S4,\rm EW}^p\hat{Q}\right)M_1\right]\cdot{\rm Tr}\,M_2\Big\}+(M_1\leftrightarrow M_2)\,,\end{aligned}$$

The α_i sub-amplitudes correspond to factorizable contributions, some of them have been estimated up to NNLO in QCD at leading order in Λ_{QCD}/m_b

The β_i sub-amplitudes correspond to annihilation contributions which are non-factorizable

They are normally addressed using educated ansätze.

Dynamical/Topological approach

Due to the current limitations of our first-principles techniques we aim to get bounds on the different QCDF subamplitudes using data

In principle each one of the subamplitudes α_i and β_i is channel dependent, however there is not enough experimental information to fit all of them in a channel dependent fashion.

The idea is to decompose the **magic formula** considering the different **flavour structures**.

This naturally leads to a natural equivalence between the topological decomposition (a parametrization of the physical amplitudes) and the dynamical formalism of QCDF

Even though there is a systematic decomposition using flavour structures this does not imply that the flavour symmetry SU(3) has to be exact.

Λ_u and Λ_t parameterization

$$T = A_{M_1 M_2}\left[\alpha_1 + \frac{3}{2}\alpha^u_{4,\mathrm{EW}} - \frac{3}{2}\alpha^c_{4,\mathrm{EW}}\right], \qquad C = A_{M_1 M_2}\left[\alpha_2 + \frac{3}{2}\alpha^u_{3,\mathrm{EW}} - \frac{3}{2}\alpha^c_{3,\mathrm{EW}}\right],$$

$$E = A_{M_1 M_2}\left[\beta_1 + \frac{3}{2}\beta^u_{4,\mathrm{EW}} - \frac{3}{2}\beta^c_{4,\mathrm{EW}}\right], \qquad A = A_{M_1 M_2}\left[\beta_2 + \frac{3}{2}\beta^u_{3,\mathrm{EW}} - \frac{3}{2}\beta^c_{3,\mathrm{EW}}\right],$$

$$T_{AS} = A_{M_1 M_2}\left[\beta_{S1} + \frac{3}{2}\beta^u_{S4,\mathrm{EW}} - \frac{3}{2}\beta^c_{S4,\mathrm{EW}}\right],$$

$$T_{ES} = A_{M_1 M_2}\left[\beta_{S2} + \frac{3}{2}\beta^u_{S3,\mathrm{EW}} - \frac{3}{2}\beta^c_{S3,\mathrm{EW}}\right],$$

$$T_{PA} = A_{M_1 M_2}\left[\beta^u_4 - \beta^c_4 - \left(\frac{\beta^u_{4,\mathrm{EW}}}{2} - \frac{\beta^c_{4,\mathrm{EW}}}{2}\right)\right],$$

$$T_{SS} = A_{M_1 M_2}\left[\beta^u_{S4} - \beta^c_{S4} - \left(\frac{\beta^u_{S4,\mathrm{EW}}}{2} - \frac{\beta^c_{S4,\mathrm{EW}}}{2}\right)\right],$$

$$T_S = A_{M_1 M_2}\left[\alpha^u_3 - \alpha^c_3 - \left(\frac{\alpha^u_{3,\mathrm{EW}}}{2} - \frac{\alpha^c_{3,\mathrm{EW}}}{2}\right) + (\beta^u_{S3} - \beta^c_{S3}) - \left(\frac{\beta^u_{S3,\mathrm{EW}}}{2} - \frac{\beta^c_{S3,\mathrm{EW}}}{2}\right)\right],$$

$$T_P = A_{M_1 M_2}\left[\alpha^u_4 - \alpha^c_4 - \left(\frac{\alpha^u_{4,\mathrm{EW}}}{2} - \frac{\alpha^c_{4,\mathrm{EW}}}{2}\right) + (\beta^u_3 - \beta^c_3) - \left(\frac{\beta^u_{3,\mathrm{EW}}}{2} - \frac{\beta^c_{3,\mathrm{EW}}}{2}\right)\right],$$

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Λ_u and Λ_t parameterization

$$S = A_{M_1M_2}\left[\alpha_3^c + \beta_{S3}^c - \frac{\alpha_{3,\mathrm{EW}}^c}{2} - \frac{\beta_{S3,\mathrm{EW}}^c}{2}\right],$$

$$P = A_{M_1M_2}\left[\alpha_4^c + \beta_3^c - \frac{\alpha_{4,\mathrm{EW}}^c}{2} - \frac{\beta_{3,\mathrm{EW}}^c}{2}\right],$$

$$P_A = A_{M_1M_2}\left[\beta_4^c - \frac{\beta_{4,\mathrm{EW}}^c}{2}\right], \qquad P_{SS} = A_{M_1M_2}\left[\beta_{S4}^c - \frac{\beta_{S4,\mathrm{EW}}^c}{2}\right],$$

$$P_C = \frac{3}{2}A_{M_1M_2}\alpha_{3,\mathrm{EW}}^c, \qquad P_T = \frac{3}{2}A_{M_1M_2}\alpha_{4,\mathrm{EW}}^c,$$

$$P_{TA} = \frac{3}{2}A_{M_1M_2}\beta_{3,\mathrm{EW}}^c, \qquad P_{TE} = \frac{3}{2}A_{M_1M_2}\beta_{4,\mathrm{EW}}^c,$$

$$P_{AS} = \frac{3}{2}A_{M_1M_2}\beta_{S4,\mathrm{EW}}^c, \qquad P_{ES} = \frac{3}{2}A_{M_1M_2}\beta_{S3,\mathrm{EW}}^c,$$

These relationships are exact

To disentangle individual QCDF subamplitudes further assumptions are required.

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Λ_u and Λ_t parameterization

Assumptions based on QCDF at NLO and NNLO

$$\alpha_3^u = \alpha_3^c = \alpha_3, \qquad \alpha_{3,\mathrm{EW}}^u = \alpha_{3,\mathrm{EW}}^c = \alpha_{3,\mathrm{EW}},$$

$$\beta_i^u = \beta_i^c = \beta_i, \qquad b_i^u = b_i^c = b_i,$$

The following differences are small $\quad |\alpha_{4,\mathrm{EW}}^c - \alpha_{4,\mathrm{EW}}^u|$ NLO $\qquad |a_4^c - a_4^u|$ NNLO

We can disentangle several individual amplitudes

$$T = A_{M_1 M_2}\,\alpha_1, \qquad C = A_{M_1 M_2}\,\alpha_2, \qquad E = B_{M_1 M_2}\,b_1,$$

$$A = B_{M_1 M_2}\,b_2, \qquad T_{AS} = B_{M_1 M_2}\,b_{S1}, \qquad T_{ES} = B_{M_1 M_2}\,b_{S2},$$

$$T_{PA} = 0, \qquad T_{SS} = 0, \qquad T_S = 0, \qquad |T_P| < 0.02,$$

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Λ_u and Λ_t parameterization

Assumptions based on QCDF at NLO and NNLO

$$\alpha_3^u = \alpha_3^c = \alpha_3, \qquad \alpha_{3,\mathrm{EW}}^u = \alpha_{3,\mathrm{EW}}^c = \alpha_{3,\mathrm{EW}},$$
$$\beta_i^u = \beta_i^c = \beta_i, \qquad b_i^u = b_i^c = b_i,$$

The following differences are small $|\alpha_{4,\mathrm{EW}}^c - \alpha_{4,\mathrm{EW}}^u|$ NLO $|a_4^c - a_4^u|$ NNLO

We can disentangle several individual amplitudes

$$S = A_{M_1M_2}\left[\alpha_3 + \beta_{S3} - \frac{\alpha_{3,\mathrm{EW}}}{2} - \frac{\beta_{S3,\mathrm{EW}}}{2}\right], \quad P = A_{M_1M_2}\left[\alpha_4^c + \beta_3 - \frac{\alpha_{4,\mathrm{EW}}^c}{2} - \frac{\beta_{3,\mathrm{EW}}}{2}\right],$$

$$P_A = B_{M_1M_2}\left(b_4 - \frac{b_{4,\mathrm{EW}}}{2}\right), \qquad P_{SS} = B_{M_1M_2}\left(b_{S4} - \frac{b_{S4,\mathrm{EW}}}{2}\right),$$

$$P_C = \frac{3}{2}A_{M_1M_2}\,\alpha_{3,\mathrm{EW}}, \qquad P_T = \frac{3}{2}A_{M_1M_2}\,\alpha_{4,\mathrm{EW}}^c,$$

$$P_{TA} = \frac{3}{2}B_{M_1M_2}\,b_{3,\mathrm{EW}}, \qquad P_{TE} = \frac{3}{2}B_{M_1M_2}\,b_{4,\mathrm{EW}},$$

$$P_{AS} = \frac{3}{2}B_{M_1M_2}\,b_{S4,\mathrm{EW}}, \qquad P_{ES} = \frac{3}{2}B_{M_1M_2}\,b_{S3,\mathrm{EW}},$$

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χ^2 -fit

We extract the QCDF subamplitudes performing a χ^2 fit

To find the minimum of the χ^2 function we use

random sampling together with the minimization algorithm

Sequential-Least-Squares (Python SciPy optimize library)

Sequential Least Squares allows us to account for bounds on the observables

Our analysis includes:

Branching fractions, direct CP asymmetries and mixing induced CP asymmetries

χ^2-fit

Best fit point

$\tilde{T}$	$1.073 + 0.044\,i$
$\tilde{C}$	$0.334 - 0.689\,i$
$\tilde{A}$	$4.782 + 8.524\,i$
$\tilde{E}$	$-7.655 + 12.782\,i$
$\tilde{T}_{ES}$	$-80.703 + 8.656\,i$
$\tilde{T}_{AS}$	$-8.747 + 20.809\,i$
$\tilde{T}_P$	$-7.421 \times 10^{-5} - 0.0200\,i$

$\tilde{P}_T$	$-0.245 - 0.133\,i$
$\tilde{P}_C$	$0.179 + 0.154\,i$
$\tilde{P}_{TA}$	$-57.273 - 40.248\,i$
$\tilde{P}$	$0.081 + 0.186\,i$
$\tilde{P}_{TE}$	$35.058 + 28.547\,i$
$\tilde{P}_A$	$-18.150 - 10.960\,i$
$\tilde{P}_{AS}$	$-24.555 - 41.077\,i$
$\tilde{P}_{ES}$	$77.683 + 54.308\,i$
$\tilde{P}_{SS}$	$19.785 + 18.715\,i$
$\tilde{S}$	$-0.199 - 0.112\,i$

$$\chi^2/dof = 1.67 \qquad p_{value} = 0.072$$

The data is compatible with the NNLO QCDF calculation

$$\alpha_1 = \tilde{T}, \qquad \alpha_1(\pi\pi) = 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i\,,$$

$$\alpha_2 = \tilde{C}, \qquad \alpha_2(\pi\pi) = 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i$$

Beneke, Hubert, Xin-Qiang Li, 0911.3655

χ^2-fit

Best fit point

$\tilde{T}$	$1.073 + 0.044\,i$	$\tilde{P}_T$	$-0.245 - 0.133\,i$
$\tilde{C}$	$0.334 - 0.689\,i$	$\tilde{P}_C$	$0.179 + 0.154\,i$
$\tilde{A}$	$4.782 + 8.524\,i$	$\tilde{P}_{TA}$	$-57.273 - 40.248\,i$
$\tilde{E}$	$-7.655 + 12.782\,i$	$\tilde{P}$	$0.081 + 0.186\,i$
$\tilde{T}_{ES}$	$-80.703 + 8.656\,i$	$\tilde{P}_{TE}$	$35.058 + 28.547\,i$
$\tilde{T}_{AS}$	$-8.747 + 20.809\,i$	$\tilde{P}_A$	$-18.150 - 10.960\,i$
$\tilde{T}_P$	$-7.421 \times 10^{-5} - 0.0200\,i$	$\tilde{P}_{AS}$	$-24.555 - 41.077\,i$
		$\tilde{P}_{ES}$	$77.683 + 54.308\,i$
		$\tilde{P}_{SS}$	$19.785 + 18.715\,i$
		$\tilde{S}$	$-0.199 - 0.112\,i$

$$\chi^2/dof = 1.67 \qquad p_{value} = 0.072$$

$$\alpha^c_{4,\mathrm{EW}} = \frac{2}{3}\tilde{P}_T, \qquad \alpha^c_{4,\mathrm{EW}} = \left(-0.163^{+0.001}_{-0.001}\right) + \left(-0.089^{+0.002}_{-0.002}\right) i$$

Compare against the NLO results which are expected to be of $\mathcal{O}(10^{-3})$

This suggests that the power corrections may play a leading role which are expected to be of O(20%)

χ^2-fit

Best fit point

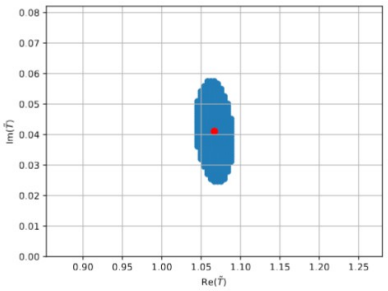
Annihilation contributions

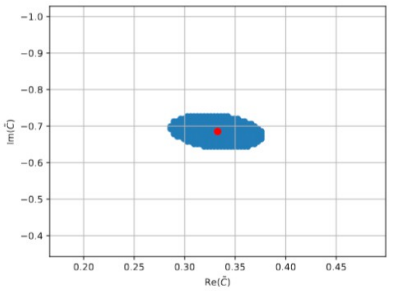
Channel	β_1	β_2	β_{S1}	β_{S2}
$B \to \pi\pi$	$(-3.60^{+3.50}_{-3.70}) + (5.90^{+5.10}_{-4.80})\,i$	$(2.22^{+1.84}_{-1.84}) + (3.91^{+0.72}_{-0.71})\,i$		
$B_s \to \pi\pi$	$(-3.00^{+3.00}_{-3.00}) + (5.00^{+4.10}_{-4.10})\,i$			
$B \to \pi K$		$(2.22^{+1.84}_{-1.84}) + (3.91^{+0.72}_{-0.71})\,i$		
$B_s \to KK$	$(-4.70^{+4.70}_{-4.80}) + (8.00^{+6.30}_{-6.40})\,i$			
$B \to KK$	$(-3.50^{+3.50}_{-3.40}) + (5.90^{+4.90}_{-4.80})\,i$	$(1.60^{+2.90}_{-2.80}) + (2.80^{+2.60}_{-2.60})\,i$		
$B \to \pi\eta_q$	$(-3.60^{+3.50}_{-3.70}) + (5.90^{+5.10}_{-4.80})\,i$	$(2.22^{+1.84}_{-1.84}) + (3.91^{+0.72}_{-0.71})\,i$	$(-4.14^{+43.66}_{-43.65}) + (9.75^{+50.80}_{-50.77})\,i$	$(-37.73^{+5.84}_{-6.85}) + (4.02^{+13.45}_{-13.29})\,i$
$B \to \pi\eta'_q$	$(-3.60^{+3.50}_{-3.70}) + (5.90^{+5.10}_{-4.80})\,i$	$(2.22^{+1.84}_{-1.84}) + (3.91^{+0.72}_{-0.71})\,i$	$(-4.14^{+43.66}_{-43.65}) + (9.75^{+50.80}_{-50.77})\,i$	$(-37.73^{+5.84}_{-6.85}) + (4.02^{+13.45}_{-13.29})\,i$
$B \to \eta_q\pi$	$(-3.58^{+3.52}_{-3.64}) + (5.98^{+4.10}_{-4.62})\,i$			
$B \to \eta'_q\pi$	$(-3.56^{+3.52}_{-3.60}) + (6.02^{+5.07}_{-4.93})\,i$			

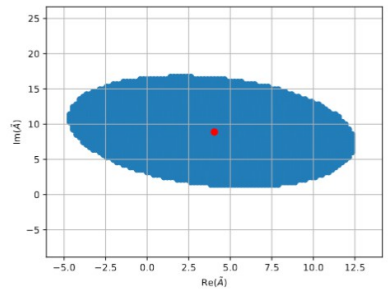
	$\beta_{3,\mathrm{EW}}$	$\beta_{4,\mathrm{EW}}$
$B \to \pi\pi$	$(-17.6^{+1.8}_{-2.3}) + (-12.4^{+1.3}_{-1.6})\,i$	$(10.8^{+1.6}_{-1.3}) + (8.8^{+1.2}_{-1.0})\,i$
$B \to KK$	$(-12.5^{+0.4}_{-0.4}) + (-8.8^{+0.3}_{-0.4})\,i$	$(10.8^{+0.6}_{-0.6}) + (8.8^{+0.3}_{-0.3})\,i$
$B \to \pi K$	$(-17.6^{+1.8}_{-2.3}) + (-12.4^{+1.3}_{-1.6})\,i$	
$B_s \to KK$		$(13.0^{+0.8}_{-0.8}) + (10.6^{+0.4}_{-0.4})\,i$
$B_s \to \pi\pi$		$(9.1^{+0.6}_{-0.6}) + (7.4^{+0.3}_{-0.3})\,i$

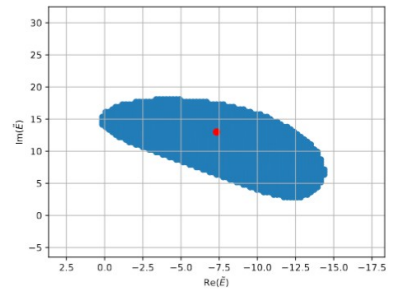
These values are within the expectations

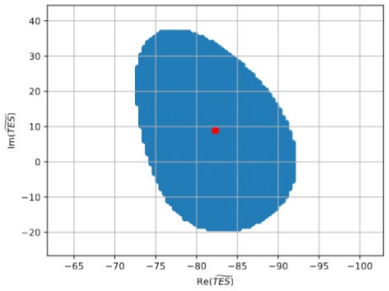
χ^2 -fit

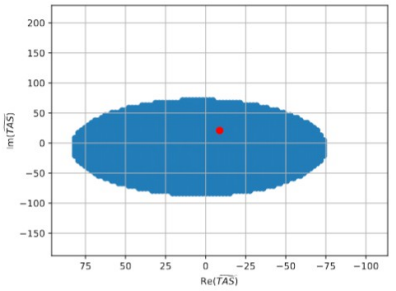

Im($\bar{T}$)
Re($\bar{T}$)

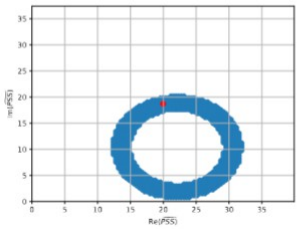

Im($\bar{C}$)
Re($\bar{C}$)

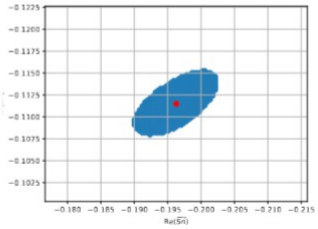

Im($\bar{A}$)
Re($\bar{A}$)

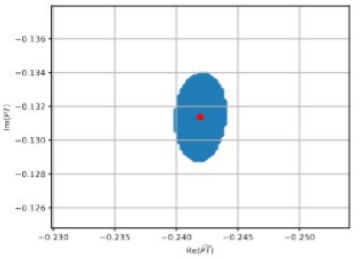

Im($\bar{E}$)
Re($\bar{E}$)

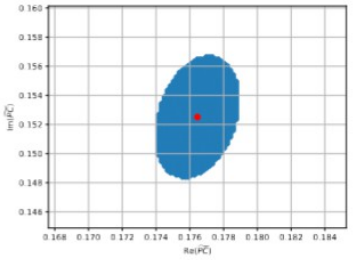

Im($\overline{TES}$)
Re($\overline{TES}$)

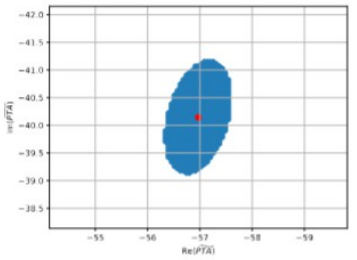

Im($\overline{TAS}$)
Re($\overline{TAS}$)

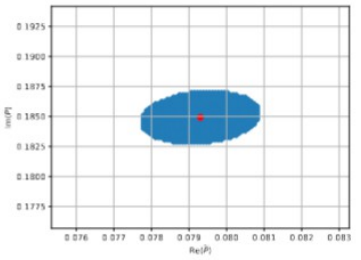

Im($\overline{PSS}$)
Re($\overline{PSS}$)

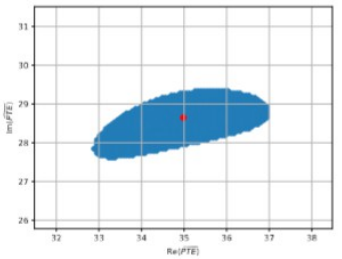

Re($\overline{Sn}$)

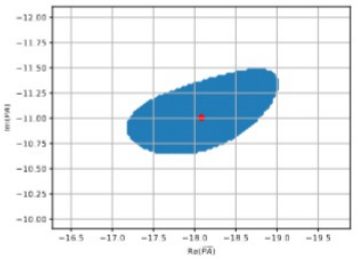

Im($\overline{PT}$)
Re($\overline{PT}$)

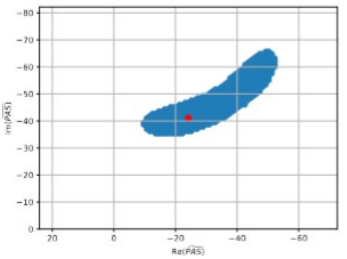

Im($\overline{PC}$)
Re($\overline{PC}$)

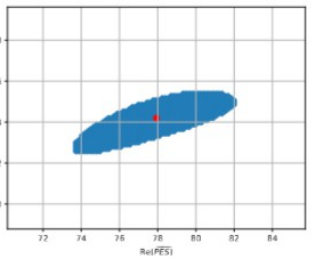

Im($\overline{PTA}$)
Re($\overline{PTA}$)


Im($\overline{P}$)
Re($\overline{P}$)


Im($\overline{PTE}$)
Re($\overline{PTE}$)


Im($\overline{PA}$)
Re($\overline{PA}$)


Im($\overline{PAS}$)
Re($\overline{PAS}$)


Im($\overline{PES}$)
Re($\overline{PES}$)

χ^2-fit

Branching Fractions I			
Channel	Fit result $[10^{-6}]$	Experiment $[10^{-6}]$	Pull $[\sigma]$
$B^- \to \pi^0\pi^-$	$5.3738^{+1.3656}_{-1.1793}$	5.31 ± 0.26	0.046
$B^- \to K^0K^-$	$1.3335^{+0.0996}_{-0.0921}$	1.32 ± 0.17	0.069
$\bar{B}^0 \to \pi^+\pi^-$	$5.3533^{+1.3631}_{-1.1854}$	5.37 ± 0.20	0.012
$\bar{B}^0 \to \pi^0\pi^0$	$1.5027^{+0.4190}_{-0.3509}$	1.55 ± 0.17	0.105
$\bar{B}^0 \to K^+K^-$	$0.0977^{+0.0649}_{-0.0394}$	0.078 ± 0.015	0.296
$\bar{B}^0 \to K^0\bar{K}^0$	$1.2245^{+0.1394}_{-0.1289}$	1.21 ± 0.16	0.068
$\bar{B}_s \to \pi^0K^0$	$1.3048^{+0.2707}_{-0.2336}$	–	–
$\bar{B}_s \to \pi^-K^+$	$5.7076^{+0.9206}_{-0.8239}$	5.90 ± 0.70	0.166
$B^- \to \pi^0K^-$	$13.2864^{+3.0063}_{-2.6580}$	13.20 ± 0.40	0.028
$B^- \to \pi^-\bar{K}^0$	$24.0024^{+2.9280}_{-2.3080}$	23.90 ± 0.60	0.034
$\bar{B}^0 \to \pi^+K^-$	$19.9871^{+4.7900}_{-4.2332}$	20.00 ± 0.40	0.003
$\bar{B}^0 \to \pi^0\bar{K}^0$	$10.2119^{+1.9219}_{-1.5219}$	10.10 ± 0.40	0.057
$\bar{B}_s \to \pi^+\pi^-$	$0.7952^{+0.3738}_{-0.2890}$	0.72 ± 0.10	0.194
$\bar{B}_s \to \pi^0\pi^0$	$0.3976^{+0.1869}_{-0.1445}$	< 7.70	–
$\bar{B}_s \to K^+K^-$	$27.6593^{+5.9902}_{-5.3600}$	27.20 ± 2.30	0.072
$\bar{B}_s \to K^0\bar{K}^0$	$18.0978^{+2.8146}_{-2.5834}$	17.60 ± 3.10	0.119

Branching Fractions II			
Channel	Fit result $[10^{-6}]$	Experiment $[10^{-6}]$	Pull $[\sigma]$
$B^- \to \eta\pi^-$	$3.9738^{+1.0222}_{-0.8559}$	4.02 ± 0.27	0.044
$\bar{B}^0 \to \eta\pi^0$	$0.3760^{+0.2314}_{-0.1529}$	0.41 ± 0.17	0.118
$\bar{B}_s \to \eta K^0$	$0.8322^{+0.1816}_{-0.1553}$	–	–
$B^- \to \eta K^-$	$2.5890^{+1.1184}_{-0.8719}$	2.40 ± 0.40	0.159
$\bar{B}^0 \to \eta\bar{K}^0$	$1.3418^{+0.9157}_{-0.6291}$	$1.23^{+0.27}_{-0.24}$	0.117
$\bar{B}_s \to \eta\pi^0$	$0.9244^{+1.5723}_{-0.6541}$	< 1000	–
$B^- \to \eta'\pi^-$	$2.5436^{+0.8560}_{-0.5528}$	2.70 ± 0.90	0.126
$\bar{B}^0 \to \eta'\pi^0$	$4.8107^{+4.5518}_{-2.7193}$	1.20 ± 0.60	0.786
$\bar{B}_s \to \eta' K^0$	$2.0434^{+0.3788}_{-0.3307}$	< 8.16 [131]	–
$B^- \to \eta' K^-$	$71.5236^{+13.1335}_{-11.3445}$	70.40 ± 2.50	0.084
$\bar{B}^0 \to \eta'\bar{K}^0$	$65.8116^{+9.7872}_{-8.6365}$	66.00 ± 4.00	0.018
$\bar{B}_s \to \eta'\pi^0$	$33.4106^{+54.7752}_{-24.9864}$	–	–

Branching Fractions III			
Channel	Fit result $[10^{-6}]$	Experiment $[10^{-6}]$	Pull $[\sigma]$
$\bar{B}^0 \to \eta\eta$	$0.6103^{+0.4590}_{-0.3000}$	< 1.0	–
$\bar{B}_s \to \eta\eta$	$6.2049^{+5.2315}_{-3.7727}$	< 143 [132]	–
$\bar{B}^0 \to \eta'\eta'$	$4.5537^{+4.2729}_{-2.6016}$	< 1.70	–
$\bar{B}_s \to \eta'\eta'$	$53.1927^{+74.8192}_{-38.6425}$	33.0 ± 7.0	–
$\bar{B}^0 \to \eta'\eta$	$3.3237^{+3.6718}_{-1.9904}$	< 1.2	–
$\bar{B}_s \to \eta'\eta$	$97.0778^{+101.0321}_{-62.2639}$	< 65 [133]	–

χ^2-fit

Direct CP Asymmetries I			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$B^- \to \pi^0\pi^-$	$-2.4414^{+1.5688}_{-1.5706}$	-1.20 ± 3.10	0.357
$B^- \to K^0K^-$	$9.9042^{+20.4979}_{-21.1721}$	4.00 ± 14.0	0.233
$\bar{B}^0 \to \pi^+\pi^-$	$35.9107^{+4.6619}_{-4.3762}$	31.4 ± 3.0	0.814
$\bar{B}^0 \to \pi^0\pi^0$	$33.4359^{+7.1074}_{-7.2281}$	23.0 ± 18.0	0.538
$\bar{B}^0 \to K^+K^-$	$-30.9353^{+55.4875}_{-46.6351}$	–	–
$\bar{B}^0 \to K^0\bar{K}^0$	$-4.3341^{+3.7669}_{-3.9982}$	-60.0 ± 70.0	0.794
$\bar{B}_s \to \pi^0K^0$	$42.0670^{+4.0089}_{-3.8294}$	–	–
$\bar{B}_s \to \pi^-K^+$	$21.5213^{+2.0595}_{-2.0289}$	22.4 ± 1.2	0.369
$B^- \to \pi^0K^-$	$2.7391^{+2.8502}_{-2.7146}$	2.70 ± 1.20	0.013
$B^- \to \pi^-\bar{K}^0$	$-0.3975^{+0.8215}_{-0.8240}$	-2.30 ± 1.40	1.171
$\bar{B}^0 \to \pi^+K^-$	$-8.3535^{+0.7598}_{-0.7682}$	-8.31 ± 0.31	0.053
$\bar{B}^0 \to \pi^0\bar{K}^0$	$-20.4839^{+1.6261}_{-1.6508}$	-1.00 ± 13.0	1.487
$\bar{B}_s \to \pi^+\pi^-$	$2.5832^{+4.8704}_{-4.6516}$	–	–
$\bar{B}_s \to \pi^0\pi^0$	$2.5832^{+4.8704}_{-4.6516}$	–	–
$\bar{B}_s \to K^+K^-$	$-11.7899^{+2.0334}_{-2.3355}$	-16.2 ± 3.5	1.048
$\bar{B}_s \to K^0\bar{K}^0$	$0.4511^{+0.4045}_{-0.3964}$	–	–

Direct CP Asymmetries II			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$B^- \to \eta\pi^-$	$-11.1594^{+11.0944}_{-12.5383}$	-14.0 ± 7.0	0.198
$\bar{B}^0 \to \eta\pi^0$	$-23.9428^{+64.0877}_{-48.9835}$	–	–
$\bar{B}_s \to \eta K^0$	$-5.7331^{+4.3017}_{-4.2344}$	< 0.10	–
$B^- \to \eta K^-$	$-38.0846^{+11.7891}_{-16.0498}$	-37.0 ± 8.0	0.060
$\bar{B}^0 \to \eta\bar{K}^0$	$6.0055^{+12.3185}_{-16.9289}$	–	–
$\bar{B}_s \to \eta\pi^0$	$5.7625^{+36.9675}_{-46.0481}$	–	–
$B^- \to \eta'\pi^-$	$7.9717^{+28.2470}_{-35.1341}$	6.0 ± 16.0	0.051
$\bar{B}^0 \to \eta'\pi^0$	$-7.2495^{+72.2262}_{-64.7704}$	–	–
$\bar{B}_s \to \eta'K^0$	$-35.6340^{+7.2975}_{-6.7990}$	–	–
$B^- \to \eta'K^-$	$0.6529^{+1.7892}_{-1.7327}$	0.40 ± 1.10	0.120
$\bar{B}^0 \to \eta'\bar{K}^0$	$3.9775^{+0.6452}_{-0.6199}$	8.0 ± 4.0	0.993
$\bar{B}_s \to \eta'\pi^0$	$-0.3001^{+12.4792}_{-12.3931}$	–	–

Direct CP Asymmetries III			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$\bar{B}^0 \to \eta\eta$	$0.4^{+43.3}_{-44.5}$	–	–
$\bar{B}_s \to \eta\eta$	$-10.9^{+\ 7.6}_{-10.1}$	–	–
$\bar{B}^0 \to \eta'\eta'$	$1.5^{+68.0}_{-69.6}$	–	–
$\bar{B}_s \to \eta'\eta'$	$0.04^{+9.57}_{-9.48}$	–	–
$\bar{B}^0 \to \eta'\eta$	$-1.8^{+64.6}_{-63.8}$	–	–
$\bar{B}_s \to \eta'\eta$	$0.38^{+6.57}_{-6.40}$	–	–

χ^2-fit

Mixing-Induced CP Asymmetries I			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$\bar{B}^0 \to \pi^+\pi^-$	$-65.4123^{+5.2767}_{-4.9069}$	-66.6 ± 2.9	0.363
$\bar{B}^0 \to \pi^0\pi^0$	$48.3618^{+7.6002}_{-7.4294}$	–	–
$\bar{B}^0 \to K^+K^-$	$70.7086^{+21.4976}_{-53.3108}$	–	–
$\bar{B}^0 \to K^0\bar{K}^0$	$-13.1626^{+11.5952}_{-11.2942}$	-80.0 ± 50.0	1.336
$\bar{B}_s \to \pi^0 K^0$	$36.0626^{+6.3772}_{-6.7687}$	–	–
$\bar{B}_s \to \pi^- K^+$	$-97.5845^{+0.4910}_{-0.4304}$	–	–
$\bar{B}^0 \to \pi^+ K^-$	$-56.4665^{+1.1926}_{-1.1926}$	–	–
$\bar{B}^0 \to \pi^0\bar{K}^0$	$77.5335^{+2.4528}_{-2.2164}$	64.0 ± 13.0	1.039
$\bar{B}_s \to \pi^+\pi^-$	$-6.5202^{+5.1912}_{-5.5795}$	–	–
$\bar{B}_s \to \pi^0\pi^0$	$-6.5202^{+5.1912}_{-5.5795}$	–	–
$\bar{B}_s \to K^+K^-$	$16.0802^{+1.7755}_{-1.6590}$	14.0 ± 3.0	0.729
$\bar{B}_s \to K^0\bar{K}^0$	$0.1429^{+0.1732}_{-0.1284}$	–	–

Mixing-Induced CP Asymmetries II			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$\bar{B}^0 \to \eta\pi^0$	$-47.7436^{+61.6033}_{-37.2269}$	–	–
$\bar{B}_s \to \eta K^0$	$56.1392^{+7.8549}_{-8.8944}$	57.0 ± 10.0	0.085
$\bar{B}^0 \to \eta\bar{K}^0$	$95.3159^{+3.0308}_{-3.9466}$	–	–
$\bar{B}_s \to \eta\pi^0$	$19.0501^{+30.4357}_{-43.2701}$	–	–
$\bar{B}^0 \to \eta'\pi^0$	$-18.5105^{+75.7292}_{-57.7093}$	–	–
$\bar{B}_s \to \eta' K^0$	$31.9321^{+8.5757}_{-8.9752}$	–	–
$\bar{B}^0 \to \eta'\bar{K}^0$	$70.1850^{+1.1782}_{-1.1808}$	64.0 ± 5.0	1.239
$\bar{B}_s \to \eta'\pi^0$	$-2.0753^{+12.0910}_{-12.5058}$	–	–

Mixing-Induced CP Asymmetries III			
Channel	Fit result $[10^{-2}]$	Experiment $[10^{-2}]$	Pull $[\sigma]$
$\bar{B}^0 \to \eta\eta$	$-67.4530^{+34.0045}_{-21.4451}$	–	–
$\bar{B}_s \to \eta\eta$	$-14.6707^{+\ 7.3128}_{-10.6542}$	–	–
$\bar{B}^0 \to \eta'\eta'$	$1.0663^{+68.4755}_{-68.9586}$	–	–
$\bar{B}_s \to \eta'\eta'$	$-0.1777^{+9.5365}_{-9.4603}$	–	–
$\bar{B}^0 \to \eta'\eta$	$0.4957^{+63.2652}_{-64.0992}$	–	–
$\bar{B}_s \to \eta'\eta$	$0.4061^{+6.4154}_{-6.2142}$	–	–

χ^2 -fit

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a similar analysis is done in the

Λ_u and Λ_c basis

considering channels without singlets

The conclusions are similar concerning the fact that SU(3) breaking fits the data.

Addressing non-leptonic Puzzles

Our setup leads to the following determinations

$$\Delta A_{CP} = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar{B}^0 \to \pi^+ K^-), \qquad R = \frac{\mathrm{Br}(\bar{B}^0 \to \pi^+ K^-)}{\mathrm{Br}(B^- \to \pi^- \bar{K}^0)} \cdot \frac{\tau_{B^-}}{\tau_{B^0}}, \tag{12}$$

$$R_c = \frac{2\,\mathrm{Br}(B^- \to \pi^0 K^-)}{\mathrm{Br}(B^- \to \pi^- \bar{K}^0)}, \qquad R_n = \frac{1}{2}\frac{\mathrm{Br}(\bar{B}^0 \to \pi^+ K^-)}{\mathrm{Br}(\bar{B}^0 \to \pi^0 \bar{K}^0)}, \tag{13}$$

$$R^{\pi\pi}_{+-} = \frac{2\,\mathrm{Br}(B^- \to \pi^- \pi^0)}{\mathrm{Br}(\bar{B}^0 \to \pi^+ \pi^-)} \cdot \frac{\tau_{B^0}}{\tau_{B^-}}, \qquad R^{\pi\pi}_{00} = \frac{2\,\mathrm{Br}(\bar{B}^0 \to \pi^0 \pi^0)}{\mathrm{Br}(\bar{B}^0 \to \pi^+ \pi^-)}, \tag{14}$$

$$R^{ss}_{KK} = \frac{\mathrm{Br}(\bar{B}_s \to K^0 \bar{K}^0)}{\mathrm{Br}(B_s \to K^+ K^-)}, \qquad R^{sd}_{KK} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{\mathrm{Br}(\bar{B}_s \to K^0 \bar{K}^0)}{\mathrm{Br}(\bar{B}^0 \to K^0 \bar{K}^0)} \cdot \frac{\tau_{B^0}}{\tau_{B_s}}.$$

Observable	Theory (Fit)	Experiment (PDG, HFLAV)
ΔA_{CP} [%]	$11.1^{+2.9}_{-2.9}$	11.0 ± 1.2
R	$0.90^{+0.24}_{-0.22}$	0.89 ± 0.03
R_c	$1.11^{+0.28}_{-0.25}$	1.10 ± 0.04
R_n	$0.98^{+0.31}_{-0.25}$	0.97 ± 0.04
$R^{\pi\pi}_{+-}$	$1.86^{+0.76}_{-0.54}$	1.83 ± 0.11
$R^{\pi\pi}_{00}$	$0.56^{+0.24}_{-0.17}$	0.58 ± 0.07
R^{ss}_{KK}	$0.65^{+0.20}_{-0.14}$	0.65 ± 0.13
R^{sd}_{KK}	$0.62^{+0.12}_{-0.11}$	0.61 ± 0.13

Conclusions

- We have performed a data-driven analysis to extract bounds on QCDF amplitudes for B mesons decaying into pairs of pseudoscalar light mesons.

- We find that it is possible to explain data (including final states with singlets) incorporating SU(3) flavour breaking effects.

- Our analysis finds that data is compatible with the QCDF values of the amplitudes α_1 and the real part of α_2.

- We also find that most of the annihilation contributions are bounded within the limits of 20%

- Nevertheless we find that data requires large strong phases and sizeable electroweak penguin values in comparison with QCDF which however are compatible with the expected size of power suppressed contributions.

Topological description

$$\mathcal{P}^{\rm TDA} = P\, B_i (M_1)^i_j (M_2)^j_k\, \tilde{H}^k + P_T\, B_i (M_1)^i_j\, \tilde{H}^{jl}_k (M_2)^k_l + S\, B_i (M_1)^i_j\, \tilde{H}^j (M_2)^k_k$$
$$+ P_C\, B_i (M_1)^i_j\, \tilde{H}^{lj}_k (M_2)^k_l + P_{TA}\, B_i\, \tilde{H}^{il}_j (M_1)^j_k (M_2)^k_l + P_A\, B_i\, \tilde{H}^i (M_1)^j_k (M_2)^k_j$$
$$+ P_{TE}\, B_i\, \tilde{H}^{ji}_k (M_1)^k_l (M_2)^l_j + P_{AS}\, B_i\, \tilde{H}^{ji}_l (M_1)^l_j (M_2)^k_k + P_{SS}\, B_i\, \tilde{H}^i (M_1)^j_j (M_2)^k_k$$
$$+ P_{ES}\, B_i\, \tilde{H}^{ij}_l (M_1)^l_j (M_2)^k_k + (M_1 \leftrightarrow M_2)$$

$$\bar{H}_1^{12} = \lambda_u^{(d)}, \quad \bar{H}_1^{13} = \lambda_u^{(s)}, \qquad \bar{H}^2 = \lambda_u^{(d)}, \quad \bar{H}^3 = \lambda_u^{(s)},$$
$$\tilde{H}_1^{12} = -\lambda_t^{(d)}, \quad \tilde{H}_1^{13} = -\lambda_t^{(s)}, \qquad \tilde{H}^2 = -\lambda_t^{(d)}, \quad \tilde{H}^3 = -\lambda_t^{(s)},$$

The flavour structure allows a systematic decomposition of the physical decay amplitudes.

$$\{T, C, A, E, T_{ES}, T_{AS}, T_S, T_{PA}, T_P, T_{SS}\}$$

The subamplitudes

$$\{P,\ P_T,\ S,\ P_C,\ P_{TA},\ P_A,\ P_{TE},\ P_{AS},\ P_{SS},\ P_{ES}\}$$

can then be obtained from fits to data

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Dynamical/Topological approach

We notice that the magic formula can be written as

$$\mathcal{A}^{\rm QCDF} = i\frac{G_F}{\sqrt{2}}A_{M_1M_2}\Big\{B_iM_j^i(\tilde{\hat{C}}_1)_k^{jl}M_l^k + B_iM_j^i(\tilde{\hat{C}}_2)_k^{lj}M_l^k + B_i(\tilde{\hat{C}}_3)_k^{ij}M_l^kM_j^l$$
$$+B_i(\tilde{\hat{C}}_4)_k^{li}M_r^kM_l^r + B_i(\tilde{\hat{C}}_5)_k^{ij}M_i^kM_l^l + B_i(\tilde{\hat{C}}_6)_k^{ji})M_j^kM_l^l\Big\},$$

where

$$(\tilde{\hat{C}}_r)_k^{ij} = \sum_{p=u,c}\Big[(\tilde{T}_r\delta_{pu}\hat{U} + \tilde{P}_r^{(1),p}\hat{I} + \tilde{P}_r^{(2),p}\hat{Q})_k^i\Lambda_p^j\Big]$$

and

$$\tilde{T}_k \in \{\alpha_1, \alpha_2, \beta_2, \beta_1, \beta_{S2}, \beta_{S1}\},$$
$$\tilde{P}_k^{(1),p} \in \{\alpha_4^p, \alpha_3^p, \beta_3^p, \beta_4^p, \beta_{S3}^p, \beta_{S4}^p\},$$
$$\tilde{P}_k^{(2),p} \in \{\alpha_{4,EW}^p, \alpha_{3,EW}^p, \beta_{3,EW}^p, b_{4,EW}^p, \beta_{S3,EW}^p, b_{S4,EW}^p\}$$

Dynamical/Topological approach

Now $\hat{Q} = \frac{3}{2}\hat{U} - \frac{1}{2}\hat{I}$ ⇒ all the terms can be written in terms of $\hat{U}$ and $\hat{I}$

The CKM unitarity allows two possibilities $\Lambda_t = -\Lambda_u - \Lambda_c$.

1) Λ_u and Λ_t decomposition

$$\tilde{\hat{C}}_r = \left[\tilde{T} + \frac{3}{2}\tilde{P}_2^u - \frac{3}{2}\tilde{P}_2^c\right]\hat{U}\otimes\Lambda_u + \left[\tilde{P}_1^u - \tilde{P}_1^c - \frac{1}{2}\left\{\tilde{P}_2^u - \tilde{P}_2^c\right\}\right]\hat{I}\otimes\Lambda_u$$

$$-\frac{3}{2}\tilde{P}_2^c\hat{U}\otimes\Lambda_t - \left[\tilde{P}_1^c - \frac{\tilde{P}_2^c}{2}\right]\hat{I}\otimes\Lambda_t,$$

Direct connection with the topological parametrization

$$U_k^i(\Lambda_u)^j = \bar{H}_k^{ij}, \qquad U_k^i(\Lambda_t)^j = \tilde{H}_k^{ij}, \qquad (\Lambda_t)^i = \tilde{H}^i$$

2) Λ_u and Λ_c decomposition

$$\tilde{\hat{C}}_r = \left[\tilde{T}_r + \frac{3}{2}\tilde{P}_r^{(2),u}\right]\hat{U}\otimes\Lambda_u + \left[\tilde{P}_r^{(1),u} - \frac{\tilde{P}_r^{(2),u}}{2}\right]\hat{I}\otimes\Lambda_u$$

$$+\frac{3}{2}\tilde{P}_r^{(2),c}\hat{U}\otimes\Lambda_c + \left[\tilde{P}_r^{(1),c} - \frac{\tilde{P}_r^{(2),c}}{2}\right]\hat{I}\otimes\Lambda_c.$$