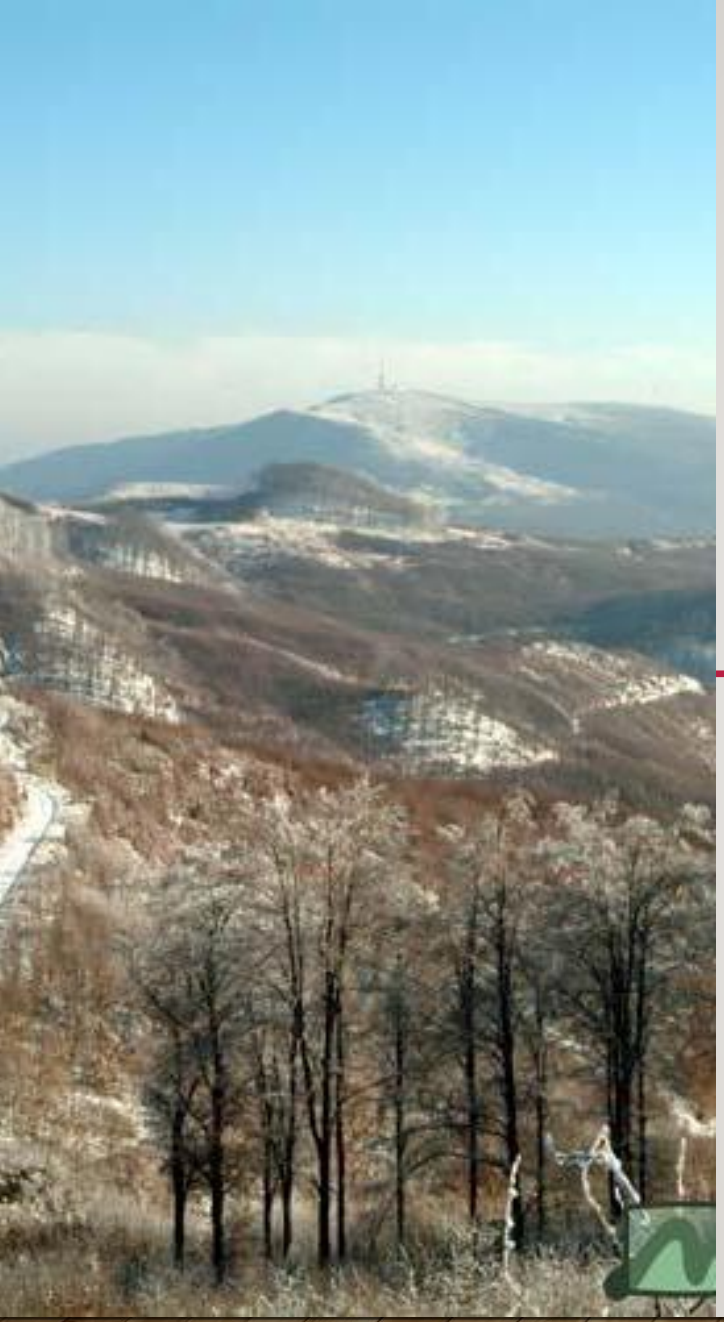

LÉVY-SHAPED SOURCES FROM EXACT SOLUTIONS OF RELATIVISTIC HYDRODYNAMICS

M. CSANÁD (EÖTVÖS LORÁND UNIVERSITY, BUDAPEST)

12TH DAYS OF FEMTOSCOPY 2026

DoF '26

12TH DAYS OF FEMTOSCOPY

MATE

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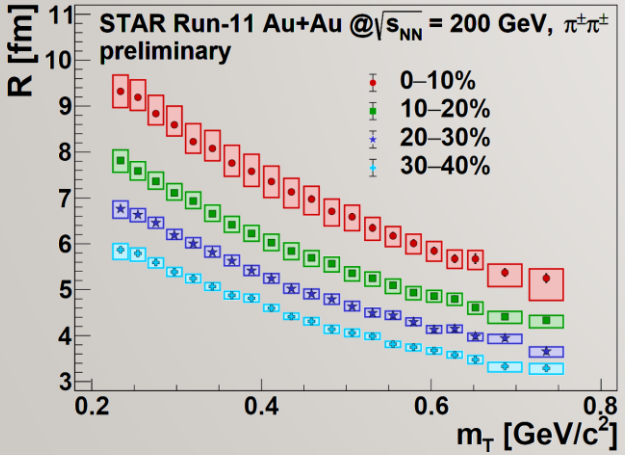
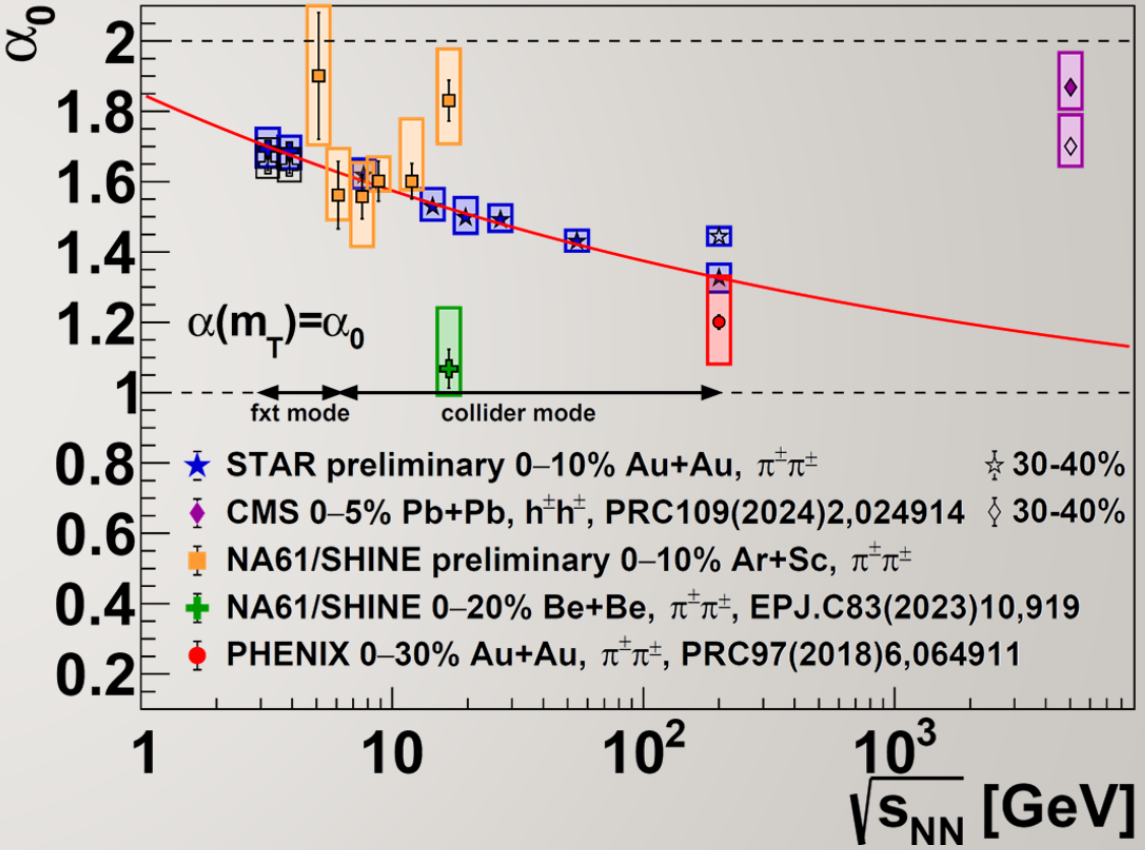
LÉVY-HYDRO PUZZLE: LÉVY SHAPE, HYDRO SCALING

- Measured pion sources: Lévy-stable, $\alpha < 2$
- SPS, RHIC, LHC: α between 1 and 2
- Lévy tail: $S(r) \propto r^{-(3+\alpha)}$
- Lévy scale: $R^{-2} \approx a + b\, m_{\mathrm{T}}$, hydro-like scaling
- Hydro expectation: Gaussian-like sources
- Puzzle: **Lévy shape + hydro scaling together?**



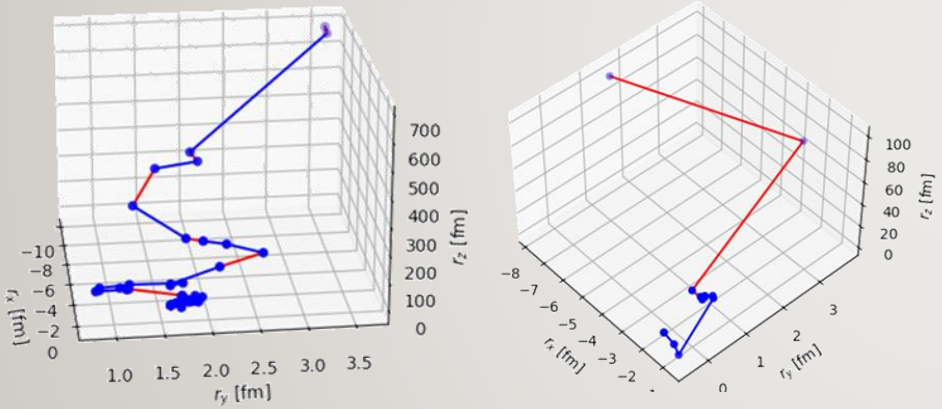
$$C(Q) = 1 + \lambda\, e^{-|QR|^{\alpha}}$$

STAR: R falling with m_{T}, similarly to Gaussian radii



LÉVY WALK: NATURAL TAILS, BUT SCALING?

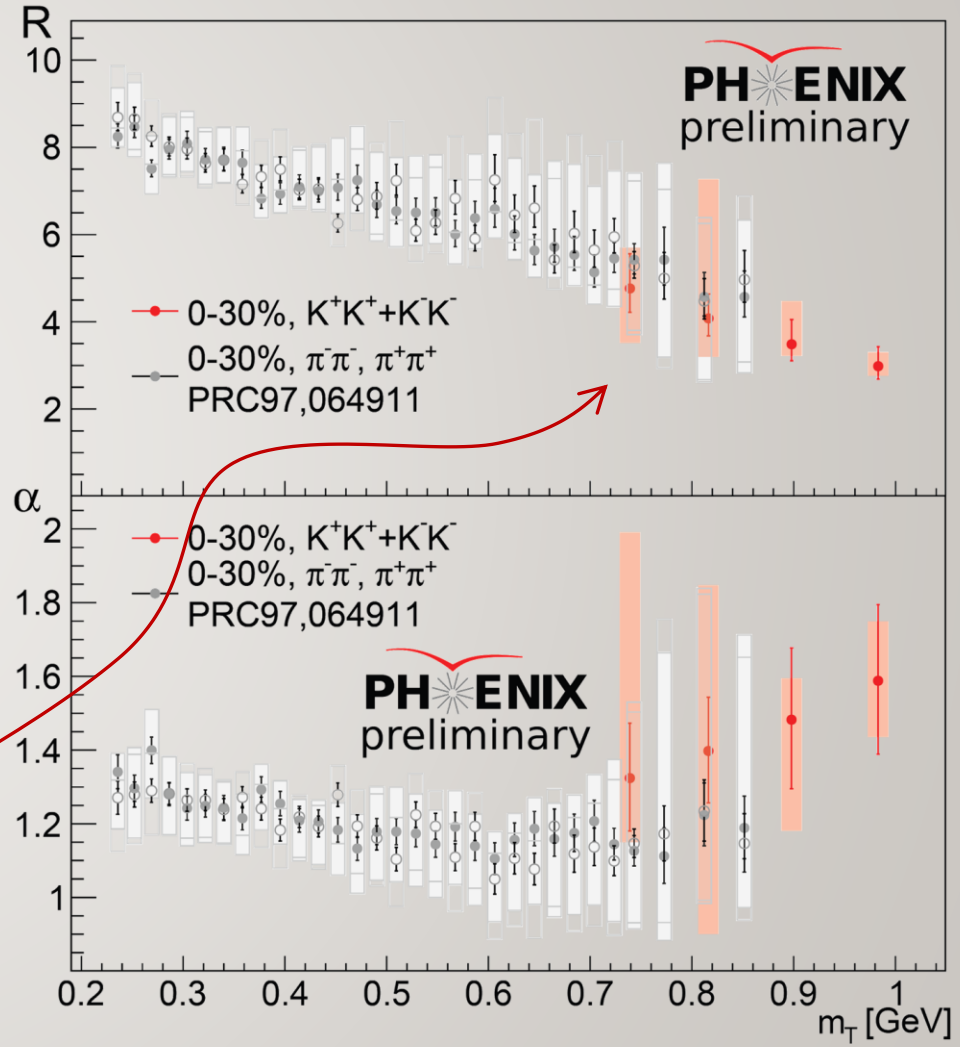
- What if Lévy tails created after hydrodynamics?
- Lévy flight & walk: ecology, climatology, solid state
  - Step sizes without finite width: Lévy-stable limit
  - Heavy-ion collisions: growing mean free path
  - UrQMD, EPOS: scatterings, decays → Lévy tails



Lévy walk of pions in UrQMD

Commun.Phys.8(2025)55

- Hadron species: different cross sections, decays
  - Yet data: **same $R(m_T)$ trend for π and K**
- Question: **Lévy tails from hydrodynamics itself?**



EXACT SOLUTIONS: WHAT IS STILL MISSING?

- Exact solutions:
  insight, transparency, code tests
- Landau, Bjorken:
  1+1D; only Landau accelerating
- Csörgő-Nagy-Csanád:
  accelerating, 1+1D
- Hubble-type class (Csörgő et al.):
  1+3D, realistic, **no acceleration**
- Gubser flow:
  conformal, boost-invariant, transverse
- Goal: **relativistic & accelerating & 1+3D**
  - **…and realistic observables, Lévy-like source**

Solution	relat.	accel.	1+3D	observ.	Lévy
Landau (1953)	✓	✓	✗	~	✗
Bjorken (1983)	✓	✗	✗	~	✗
Csörgő-Nagy-Csanád	✓	✓	✗	✓	✗
Hubble / Buda-Lund	✓	✗	✓	✓	✗
Gubser (2010)	✓	✓	~	~	✗

✓: yes ~: partly ✗: no

NEW SOLUTIONS VIA THE DRESSING THEOREM

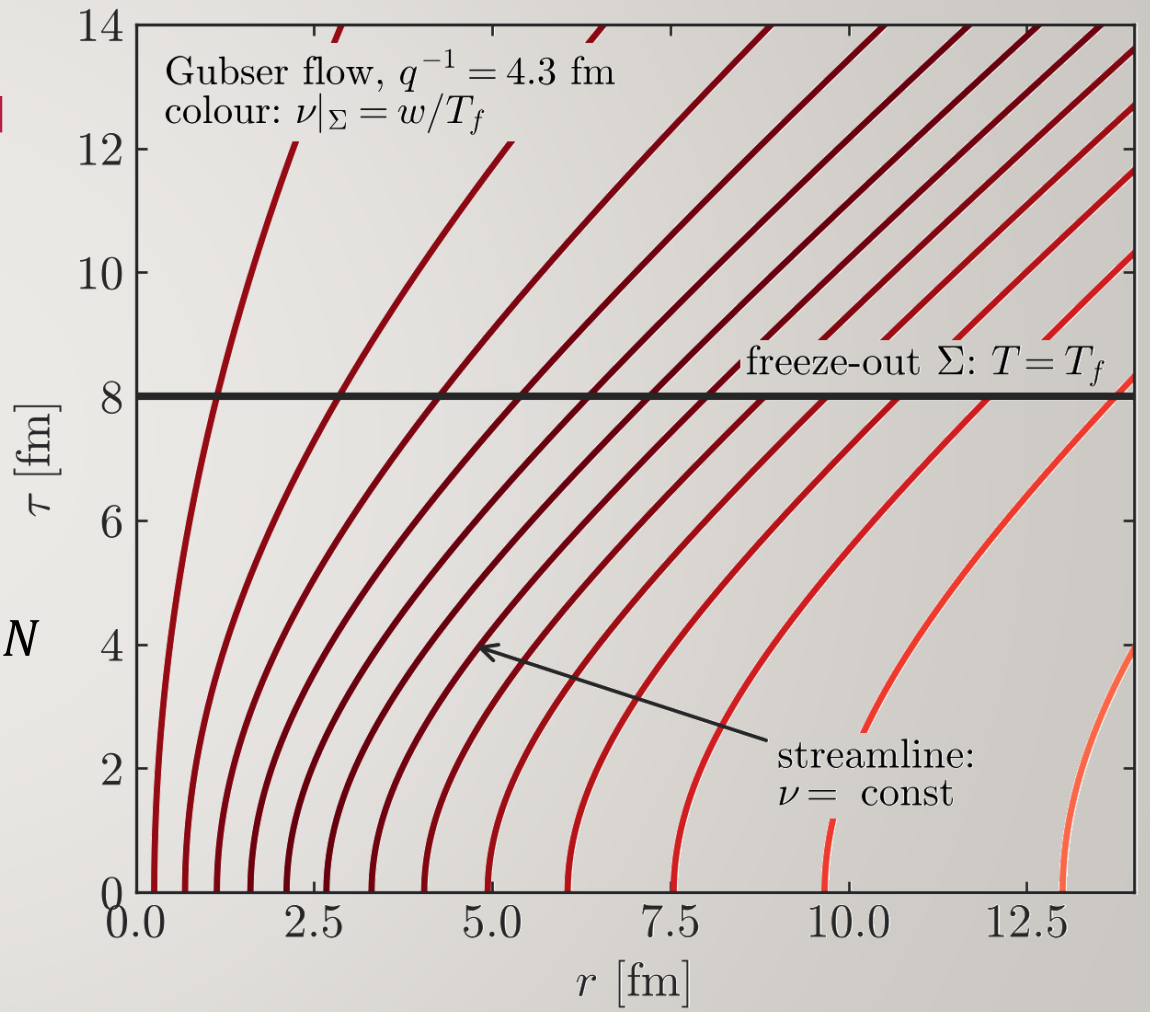
- EoS: $\varepsilon = \kappa\, p, p = nT$, speed of sound $c_s^2 = 1/\kappa$
- Projected hydro equations: u^μ **and** p **alone, closed**

$$\partial_\mu\left(p^{\kappa/(\kappa+1)}u^\mu\right) = 0, \qquad (\kappa+1)\, p\, u^\nu \partial_\nu u^\mu = \Delta^{\mu\nu}\partial_\nu p$$

- Dressing: any comoving ν, with $w \equiv p^{1/(\kappa+1)}$

$$n = w^\kappa\, \nu, \qquad T = \frac{w}{\nu}, \qquad u^\mu \partial_\mu \nu = 0$$

- $\nu \propto e^{-\sigma/(\kappa+1)}$, encodes entropy per particle $\sigma = S/N$
  - Both are constant on streamlines
- Isothermal dressing: $T = T_\mathrm{f}$ **on freeze-out**, $n \propto p$
  - Alternative: isentropic ⇒ homentropic (σ = const.)
- Hubble-type class: special case ($n \propto \nu, T \propto 1/\nu$)

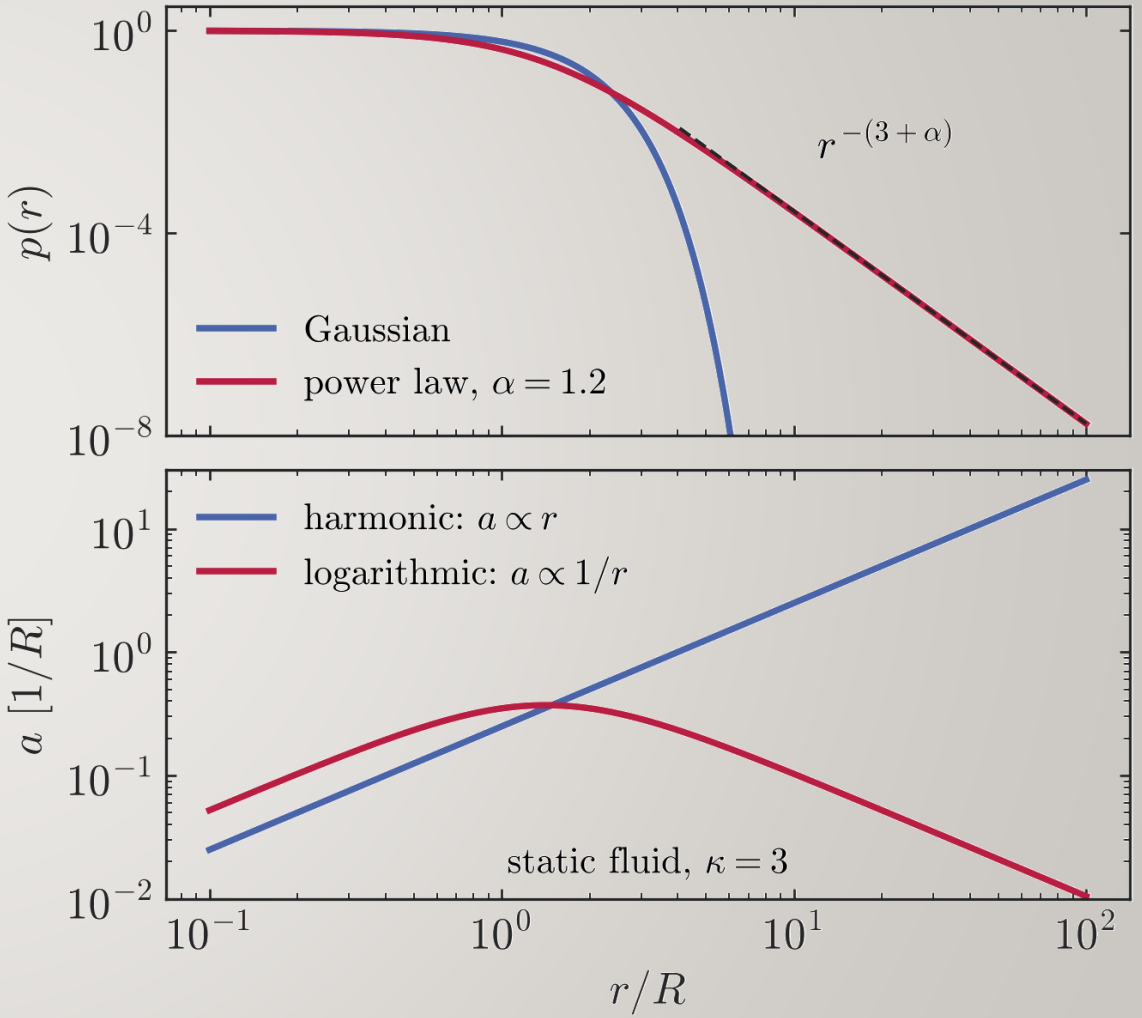


TWO ROUTES TO POWER-LAW TAILS: FAST OR SLOW

- Cooper–Frye: $S \propto \left(n/n_{\mathrm{eq}}\right) e^{-K\cdot u/T} K \cdot d\Sigma$
  - Flux: $K \cdot d\Sigma/d^3r = E - \boldsymbol{kr}/t$, bounded ($t/\tau$ corr.)
  - Relativistic outer flow: $K \cdot u \propto r \rightarrow$ cutoff
- **Fast tail**: T rising at least linearly, fugacity tames it
- **Slow tail**: slow flow, uniform T on F.O., $n \propto p$

$$a^\mu \equiv u^\nu \partial_\nu u^\mu = \frac{\Delta^{\mu\nu}\partial_\nu \ln p}{\kappa + 1}$$

- Quasi-static tail:
  $p \propto r^{-(3+\alpha)} \rightarrow a \approx (3+\alpha)/[(\kappa+1)r]$
  - Recall for later, possibility of a shockwave
- Gauss: $a \propto r$; **Lévy: $a \propto 1/r$ acceleration**

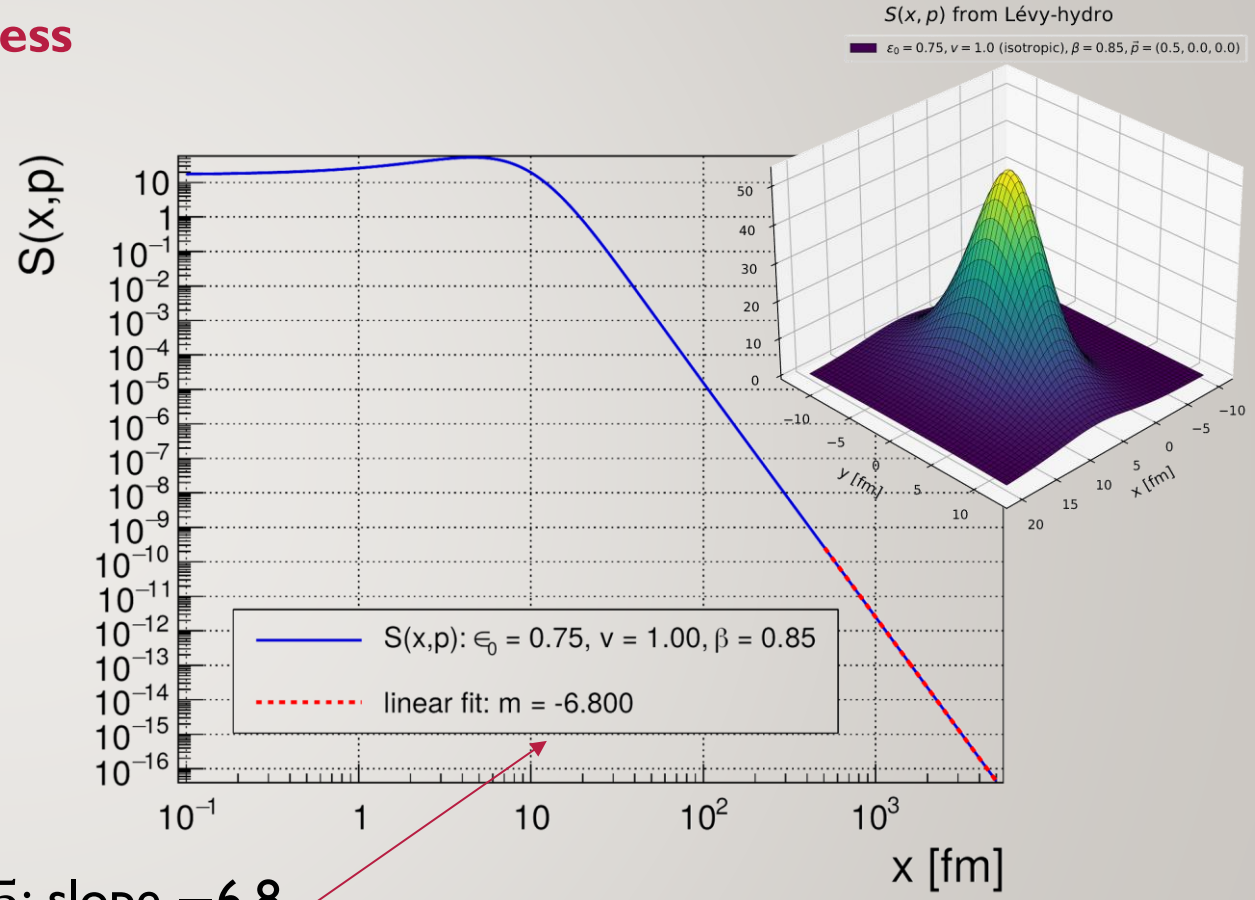


HUBBLE-LÉVY SOLUTION: HOT TAIL, TAMED

- Hubble flow $u^\mu = x^\mu/\tau$: 1+3D, **accelerationless**
- Scaling variable: ellipsoidal $\bar{s}$, regulator ε_0
- Scaling function: $v = (s_0/\bar{s})^\beta \rightarrow n \propto r^{-2\beta}$
- Uniform p: $T \propto r^{2\beta}$, often seen as pathology
- Fugacity tames source for $T \gg m$: $n/n_{\mathrm{eq}} \propto v^4$

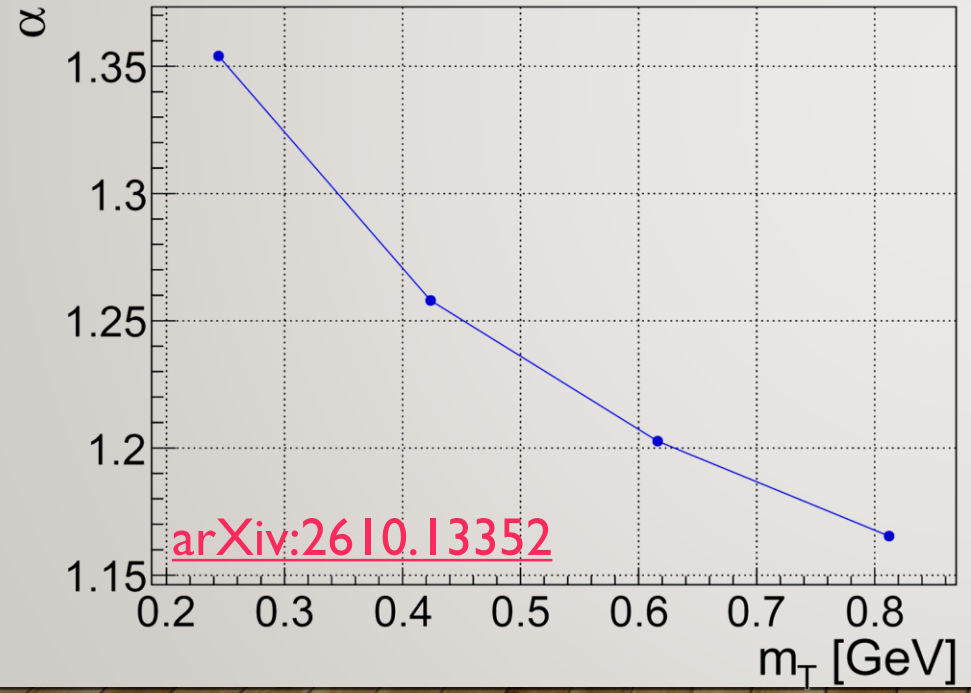
$$S \propto \frac{n}{n_{\mathrm{eq}}(T)} e^{-K\cdot u/T} \propto v^4 \propto r^{-8\beta}$$

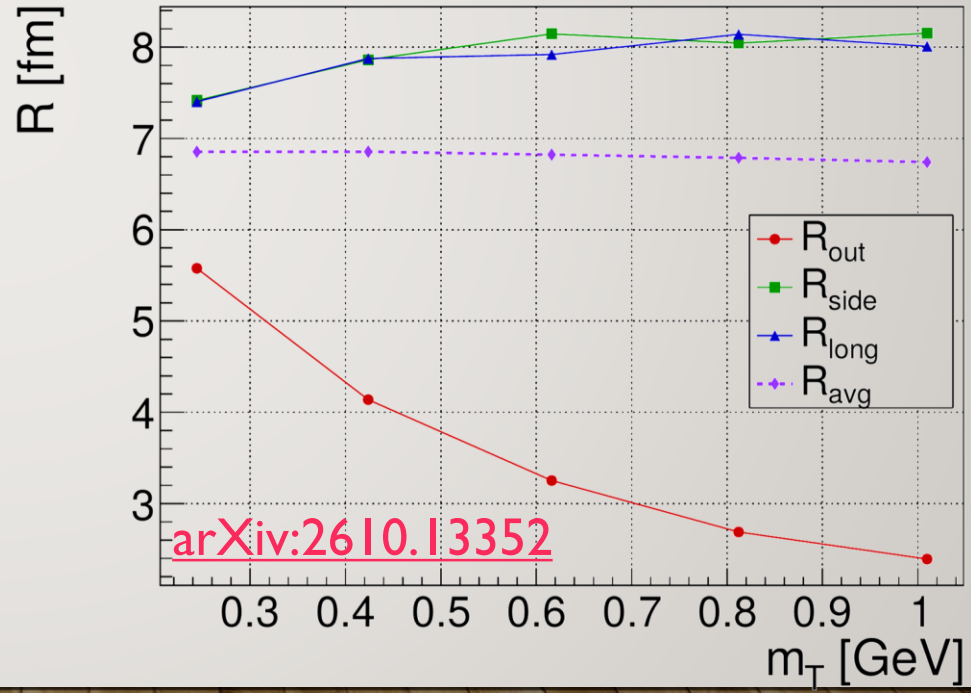
- **Power-law source**: $\alpha = 8\beta - 3$
- **Lévy-stable** for $1/2 \leq \beta < 5/8$
- **Lévy-like in finite range** otherwise: $\beta = 0.85$: slope -6.8
- Paper: B. Szentkuti, M. Csanád, arXiv:2610.13352



HUBBLE-LÉVY: LÉVY-LIKE SOURCE, BUT SCALING?

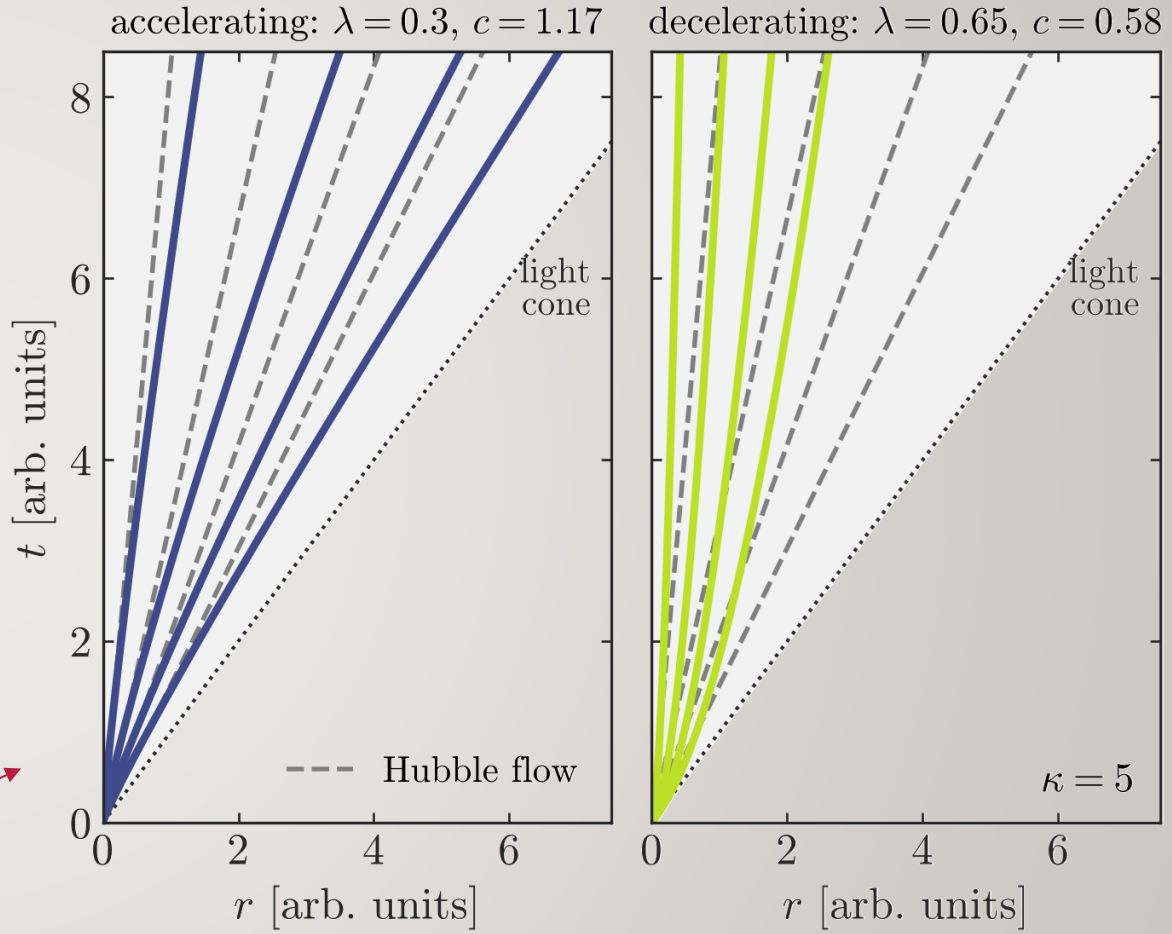
- Correlation functions analyzed: FFT of the pair source, 3D Lévy fits in LCMS
- Lévy index: $\alpha < 2$, Lévy-like source
- R_{out} falling as $R^{-2} \propto m_{\mathrm{T}}$, but R_{side}, R_{long} close to constant
- Would acceleration help? Let's search for new accelerating solutions!





WHAT CAN ACCELERATION DO?

- Hubble class: $Du^\mu = 0$, uniform p on τ = const
- Wanted: accelerating family, Hubble as special case
- Q1: **how much acceleration possible?**
- Q2: acceleration → **Lévy tail?**
- Q3: accelerating core for **Hubble-Lévy source?**
- Short answers:
  - Q1: bounded for smooth flows · Q2: no · Q3: yes
- Self-similar potential flow, one parameter λ
  - Acceleration controlled by $c = \kappa(1-\lambda)/3$
  - Hubble: $c = 1$
- Let's call this λ potential flow: λ-PF [manuscript under prep.]

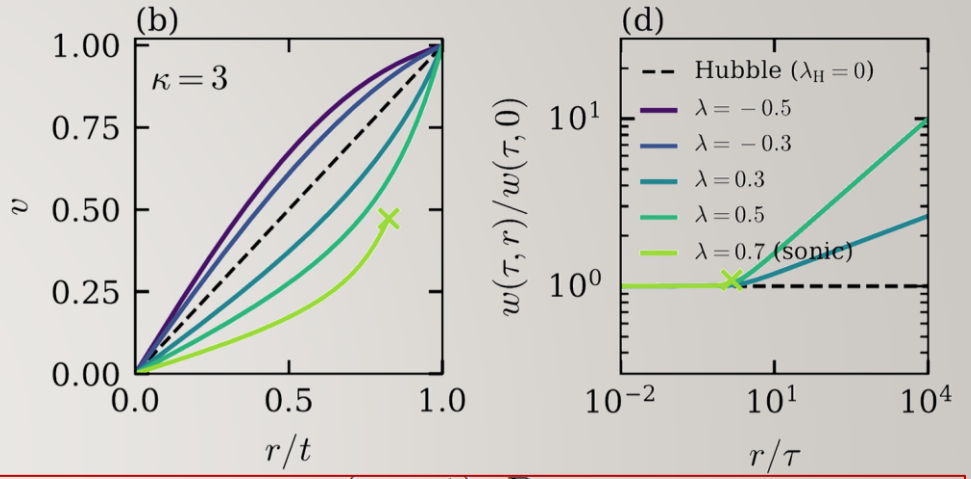


SELF-SIMILAR POTENTIAL FLOW: λ-PF

- Let's start with an irrotational (potential) flow
- Then: $wu_\mu = \partial_\mu \Phi, \mathrm{w} = \mathrm{p}^{1/(\kappa+1)}$
- Euler equation: **satisfied identically**

$$\partial_\mu(w^{\kappa-1}\partial^\mu\Phi) = 0, \qquad \Phi = t^\lambda F(s), \qquad \mathrm{s} = \frac{\mathrm{r}^2}{\mathrm{t}^2}$$

- Potential ansatz → ODE for $F(s)$
- From regularity at the centre: **λ the only parameter**



$$F'' = \frac{[2s(\lambda-2)+6]F' - \kappa(\lambda-1)A + \dfrac{(\kappa-1)sB}{H^2}\left[2A(\lambda-2)F' - 4F'^2\right]}{4s\left[(s-1)+(\kappa-1)sB^2/H^2\right]}.$$

with

$$A = \lambda F - 2sF',$$
$$B = \lambda F + 2(1-s)F' = A + 2F',$$
$$H^2 = A^2 - 4sF'^2.$$

- Velocity: $v = -\frac{2\sqrt{s}F'}{A} \simeq c\, r/t$ in the core, with $c = \kappa(1-\lambda)/3$
- Hubble case: $c = 1,\ \lambda_H = 1 - \frac{3}{\kappa},\ F = (1-s)^{\lambda_H/2}$
- Sonic point when F'' denominator vanishes ('x' sign on the top plots)

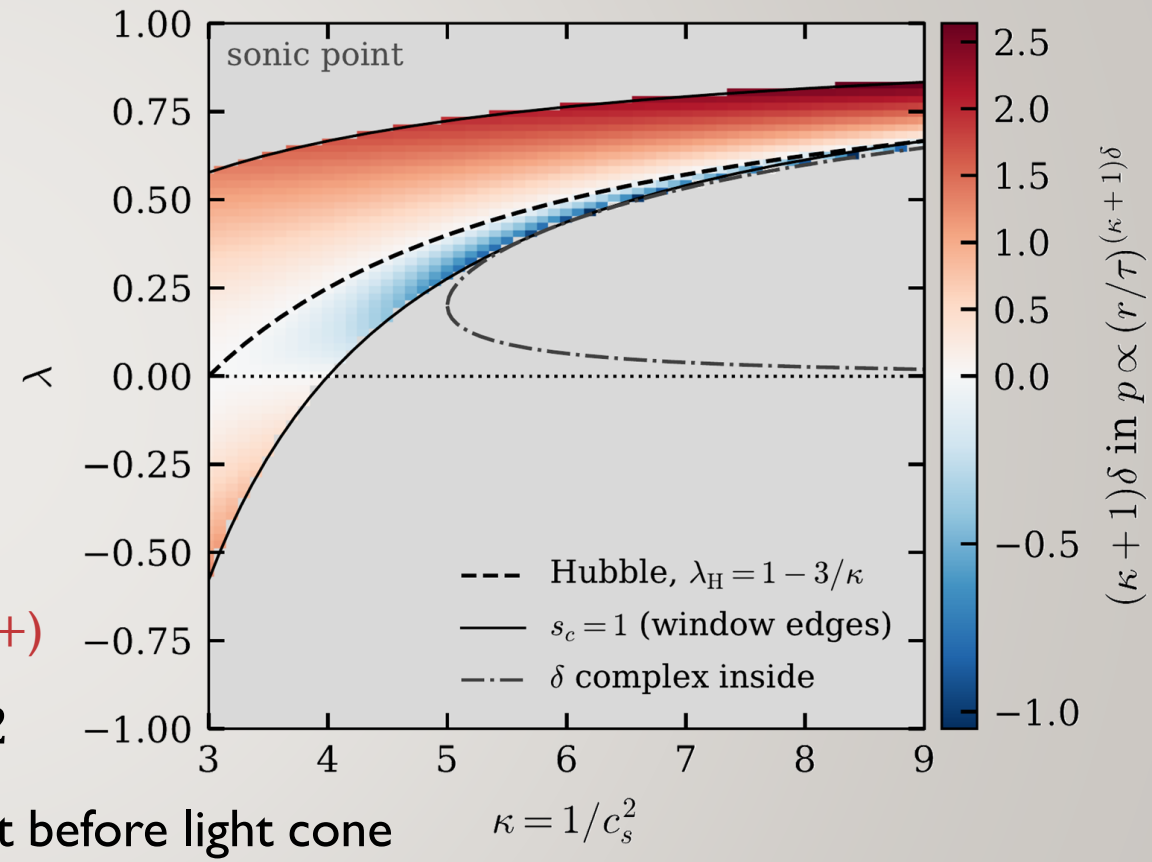
SONIC LINE: SUBSONIC WINDOW TO LIGHT CONE

- Singular ODE: $v_{\mathrm{rel}} = c_s$ relative to pattern $r = \xi\, t$
- Origin: sound characteristic along similarity line
- Analogues, e.g. Bondi accretion in astrophysics
- Single critical point:

$$s_c = \frac{4}{\kappa[(\kappa-1)\lambda-(\kappa-3)]^2}$$

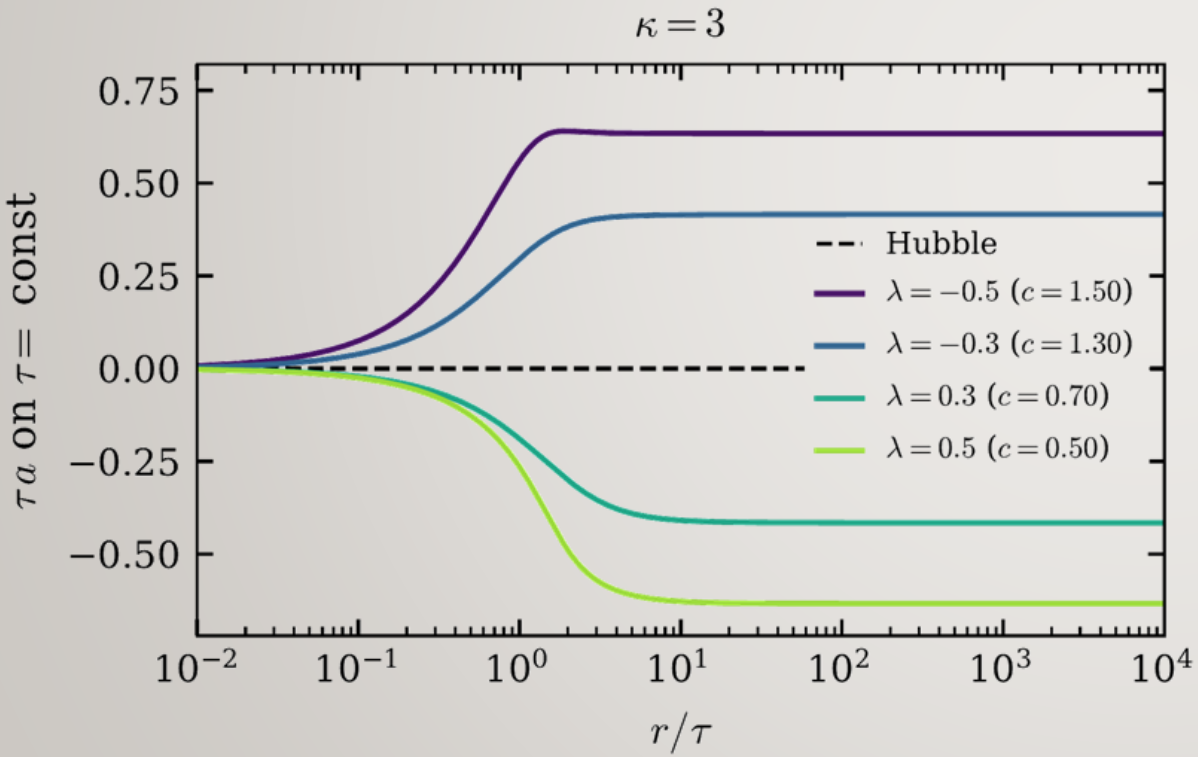
- **Physical window:** $\sqrt{\kappa}\cdot|(\kappa-1)\lambda-(\kappa-3)|<2$
  - $\lambda > 0$: sign of $(\kappa+1)\delta$: accelerate (−) / decelerate (+)
- For $\kappa = 3$: $|\lambda| < 0.58$; for $\kappa = 5$: $0.28 < \lambda < 0.72$
- Outside (grey area): smooth solution hits sonic point before light cone
  - Matter outside light cone (transonic) or sonic shock

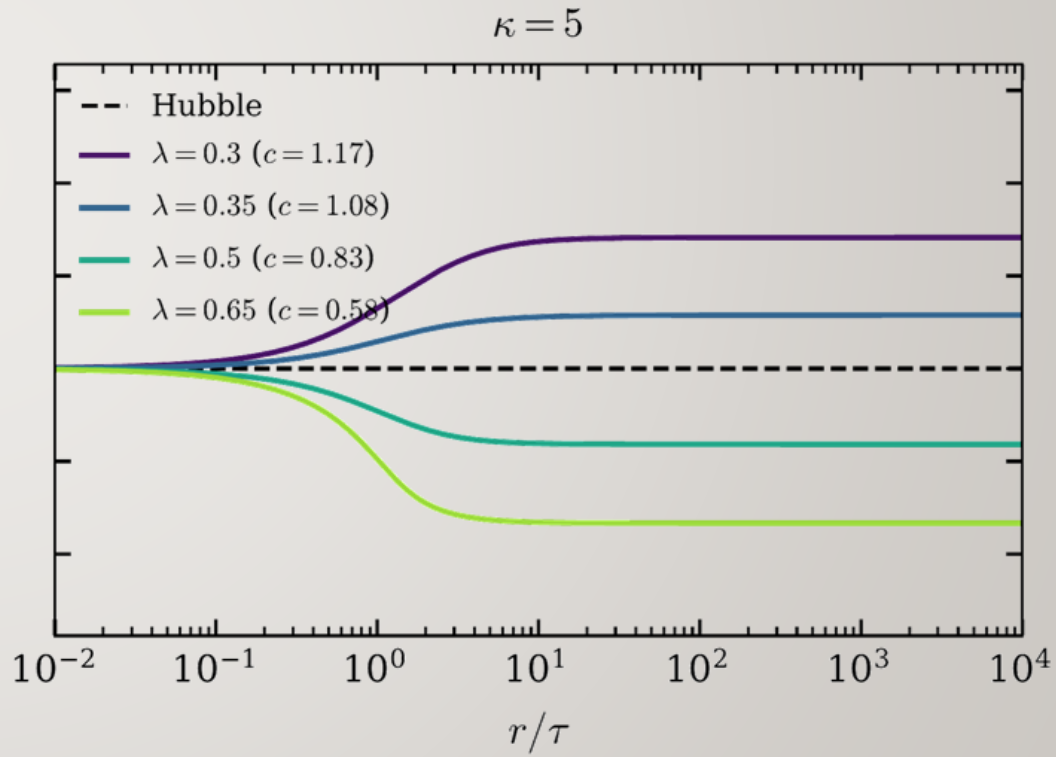
pressure falls, pressure rises (outward)



ACCELERATION: HARMONIC CORE, FLAT TAILS

- Core: $\tau\, a \simeq\ c(c-1)\, r/\tau$, harmonic acceleration
- Window: $0.42 < c < 1.58$ $(\kappa = 3)$
- Stiffer EoS: stronger acceleration allowed
- In the tail: $\tau a \rightarrow$ constant, set by λ
- Pressure decay: at most $r^{-(\kappa+1)/\kappa}$
- No $1/r$ fall-off: no Lévy tail from acceleration alone





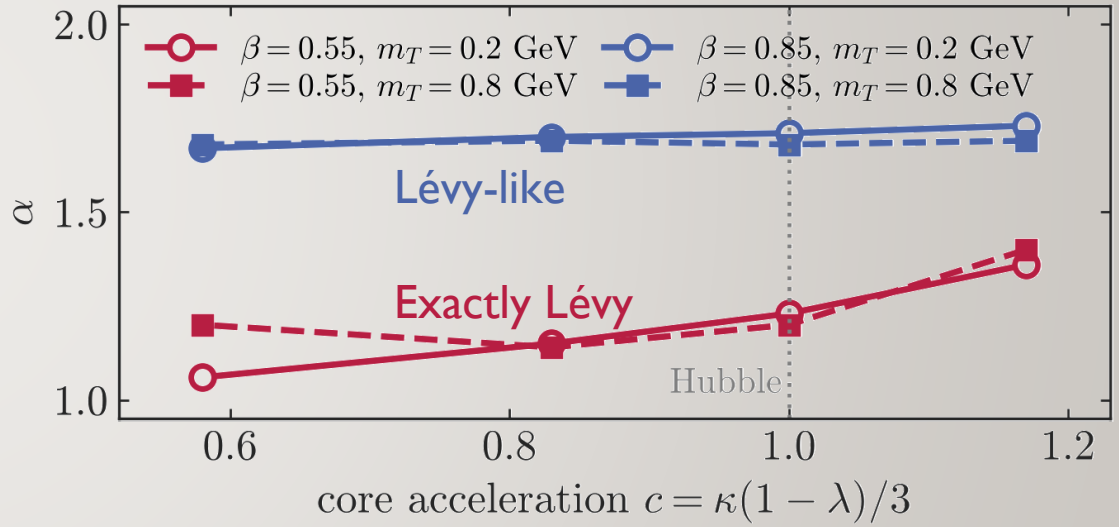
DRESSED λ-PF SOURCES: ACCELERATING HUBBLE-LÉVY

- Isothermal or isentropic (thus homentropic): cutoff, α close to 2.0
  - Finite second moment, not asymptotically Lévy
- For a power-law tail: **Hubble-type (fast-tail) dressing** needed
  - Not isothermal or isentropic, it is a third option
  - Achieved by fast-tail dressing, ν defined on τ_f as:

$$\nu|_{\tau_f} = \left(1 + \frac{r^2}{R_0^2}\right)^{-\beta}$$

- Then $n \propto r^{\kappa\delta-2\beta}, T \propto r^{\delta+2\beta}$, where $w \propto (r/\tau)^\delta$
  - Same mechanism as for Hubble case ($\delta = 0$)
- Tail: $S \propto r^{-[8\beta-(\kappa-3)\delta]}$ for $2\beta + \delta \geq 1$

$C(Q)$ fit result, finite range, not α_∞



- **Asymptotically Lévy-stable**: $\alpha_\infty = 8\beta - (\kappa - 3)\delta - 3 < 2$ for suitable β
- Lévy-like fits: $\alpha < 2$ also for accelerating core

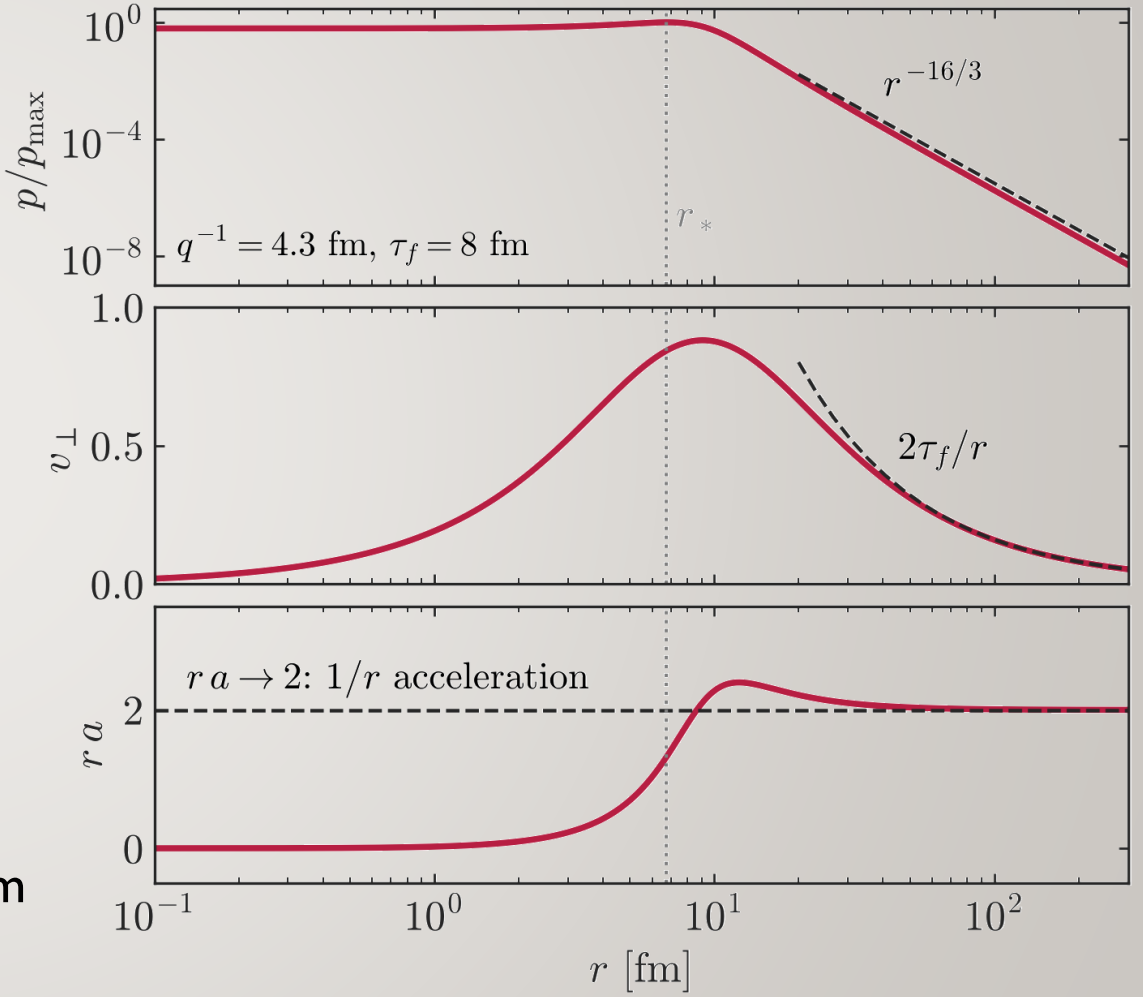
DRESSED GUBSER FLOW: EXACT $1/r$ ACCELERATION

- Gubser flow: conformal, boost-invariant, transverse

$$v_\perp = \tanh\chi = \frac{2q^2\tau r}{1+q^2\tau^2+q^2r^2}$$

Nucl. Phys. B 846 (2011) 469 [arXiv:1012.1314]
Phys. Rev. D 82 (2010) 085027 [arXiv:1006.0006]

- Isothermal dressing: $T = T_\mathrm{f}$ on $\tau_\mathrm{f}, n = p/T_\mathrm{f}$
- Conformal ($\kappa = 3$): $p \propto w^4, w \propto r^{-4/3}$ at fixed τ
- Pressure tail: $p \propto r^{-16/3}, v_\perp \to 2\tau/r$
- Proper acceleration: $a \to 2/r$, slow-tail type
- Ring at $r_* \approx 6.7$ fm, flow peak $v_\mathrm{max} \approx 0.88$ at $r \approx 9$ fm
  - For $q^{-1} = 4.3$ fm, $\tau_\mathrm{f} = 8$ fm



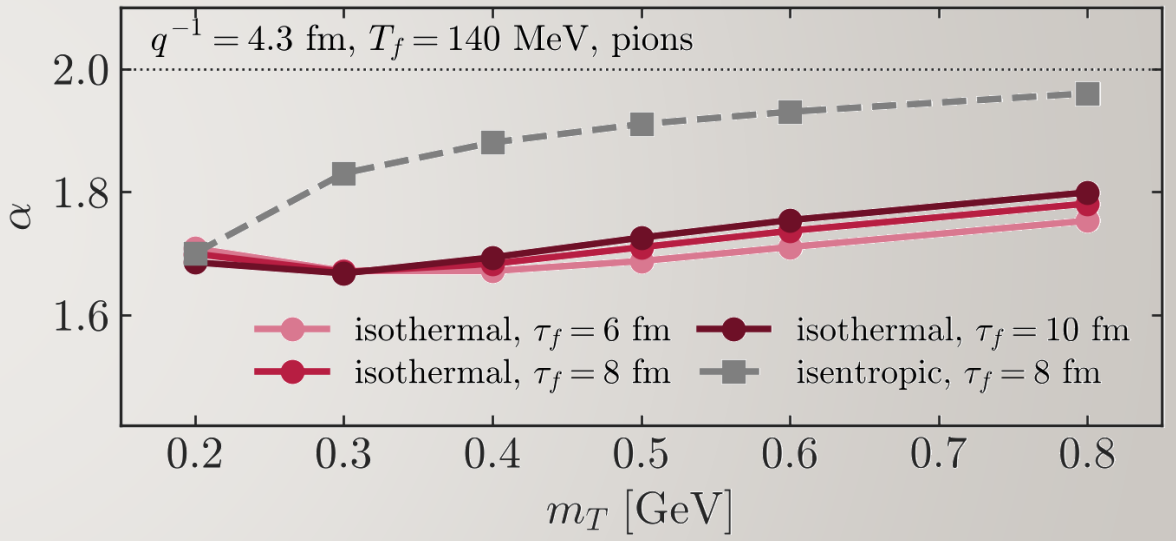
GUBSER SOURCE: POWER-LAW TAIL, LÉVY-LIKE

- Source in the transverse plane, at $y = 0$, integrated over η_s (space-time rapidity)

$$S_\perp \propto n\, K_1\left(\frac{m_T \cosh\chi}{T_f}\right) \exp\left[\frac{K_T \sinh\chi \cos(\varphi - \varphi_K)}{T_f}\right]$$

- Tail: $S_\perp \propto r^{-16/3}$, fixed by symmetry
- Lévy-stab. in d dim. needs $\alpha < 2$ in $S_\perp \propto r^{-(d+\alpha)}$
- Here ($d = 2$) we obtain $\alpha_\infty = 10/3$
- **Not Lévy-stable asymptotically**
- Isothermal fits: $\alpha \approx 1.7$, from core-to-tail crossover
- **Lévy-like over finite range: as probed by data**
- Isentropic: T falls outward, Boltzmann cutoff, stronger for large m_T, disappearing Lévy behavior

α from $C(Q_{\text{side}})$ fits where $1.1 < C < 1.9$



SUMMARY: WHICH SOLUTION IS PROMISING FOR LÉVY?

- Dressing: flow + p first; n, T free
- Tails: **fast (hot) or slow ($1/r$ acceleration)**
- Hubble-Lévy: **Lévy-stable tail** for suitable β [arXiv:2610.13352 with Bence Szentkuti]
- λ-PF: accelerating, same **Lévy-stable tail**
- Dressed Gubser: $1/r$ acceleration, **fixed tail**
  - Lévy-like in fit range, not Lévy-stable
- Open: regular core + tunable $1/r$ tail
- **Research ongoing**: observables, data, v_2
- New manuscript under preparation
  - With significant help from AI (Claude), and cross-checks done by Marci Nagy and my student Ágoston Benyó

Solution	relat.	accel.	1+3D	Lévy
Hubble / Buda–Lund	✓	✗	✓	✗
Gubser (2010)	✓	✓	~	✗
Hubble-Lévy (new)	✓	✗	✓	✓
λ-PF (new)	✓	✓	✓	✓
Dressed Gubser (new)	✓	✓	~	~

✓: yes ~: partly ✗: no