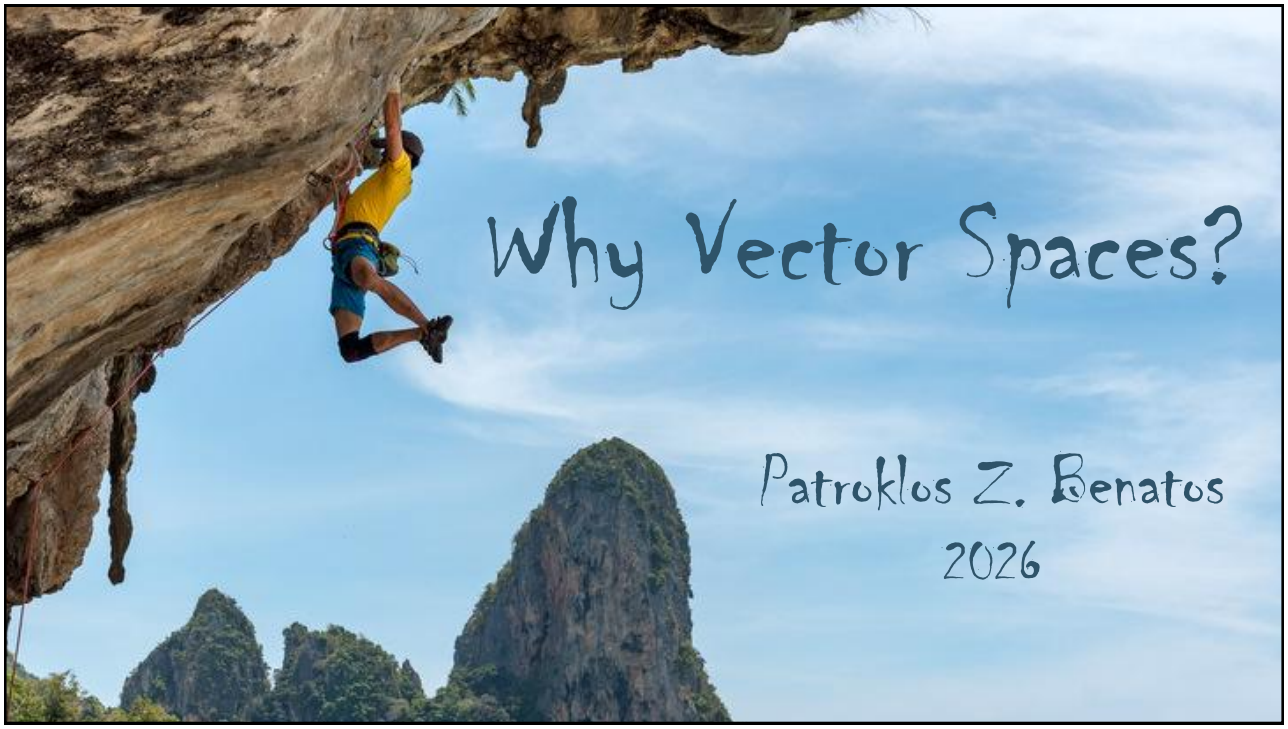
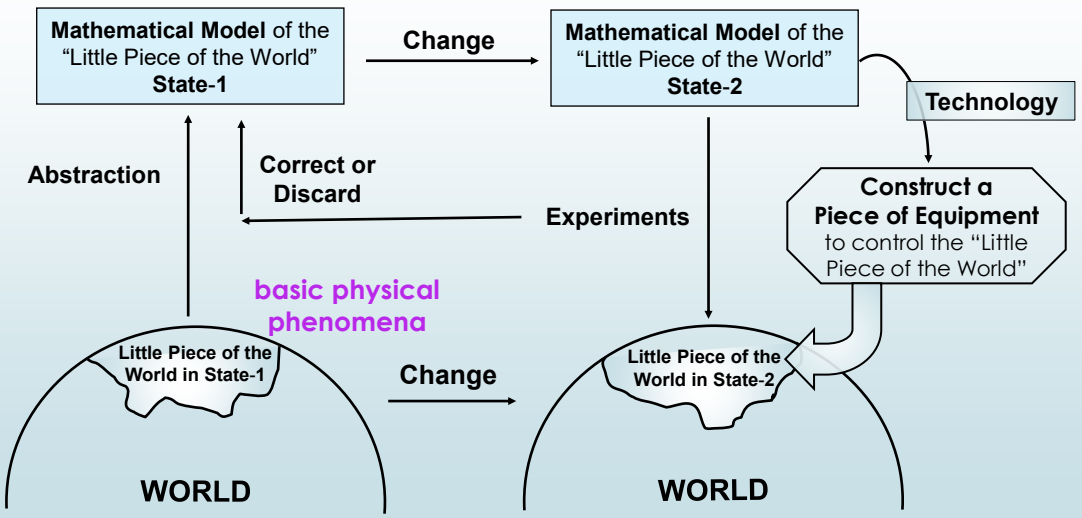
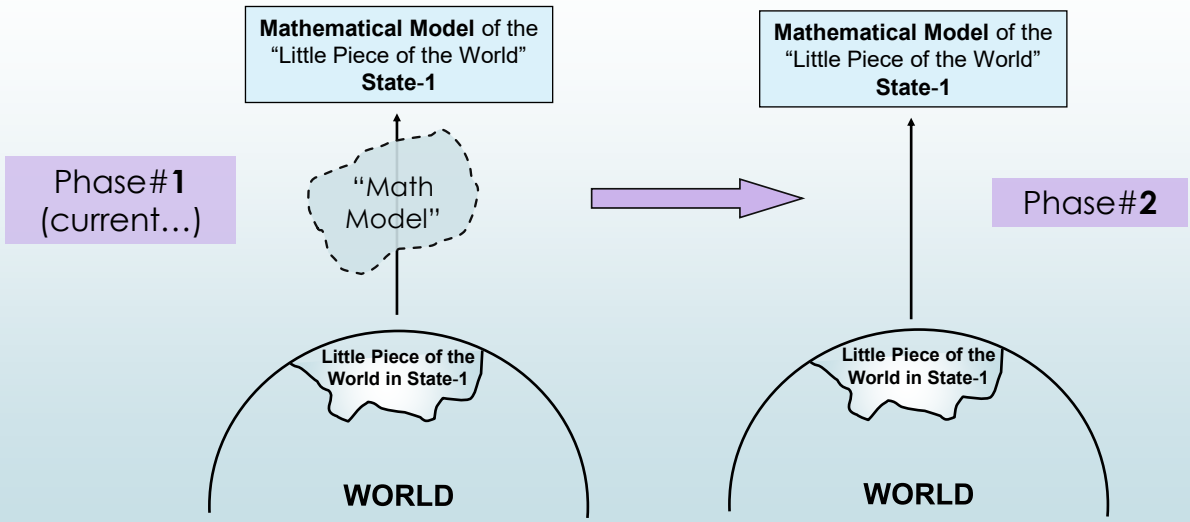


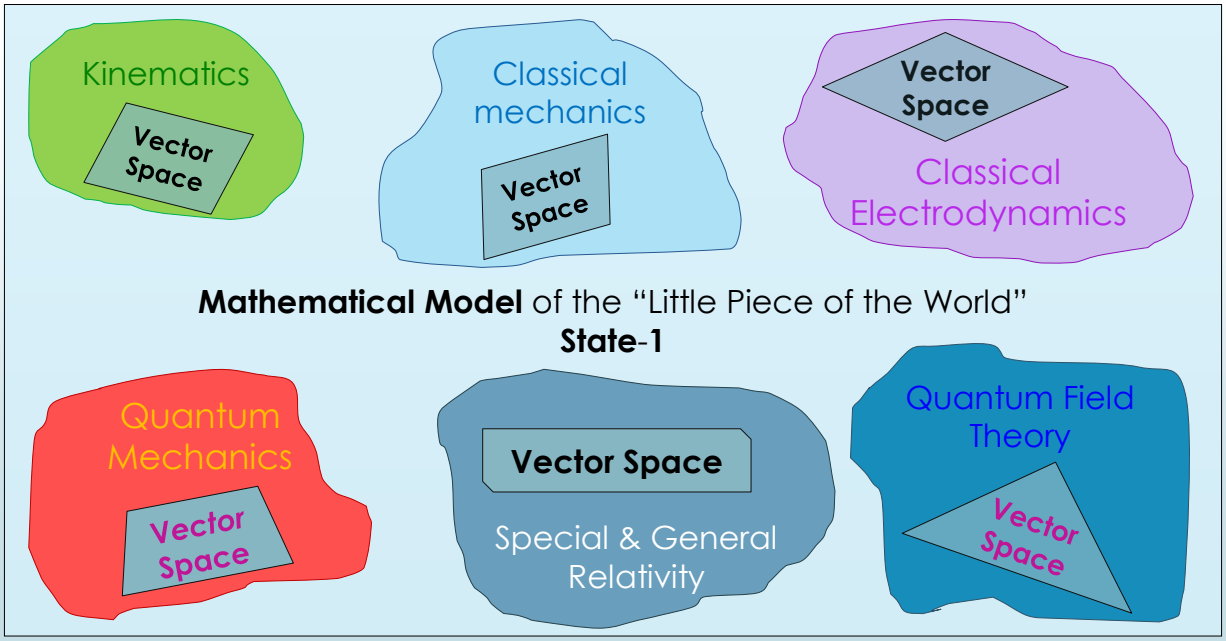
The Setup of Science & Technology



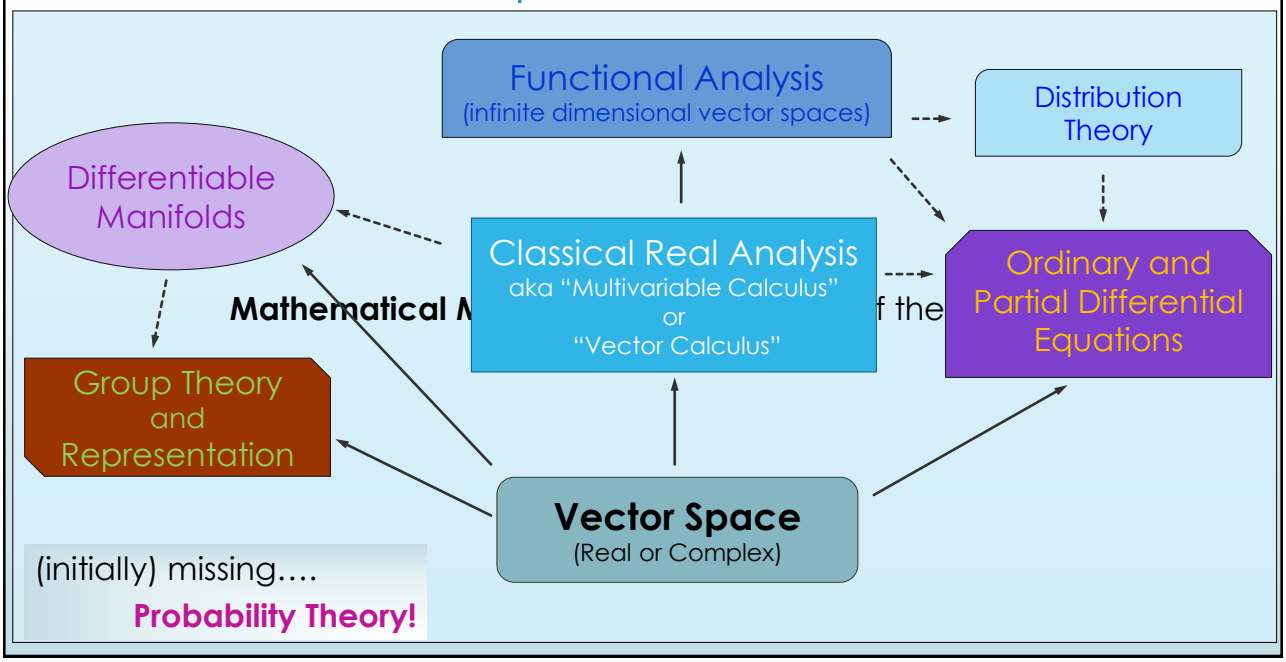
Short Digress: Abstraction – Really?



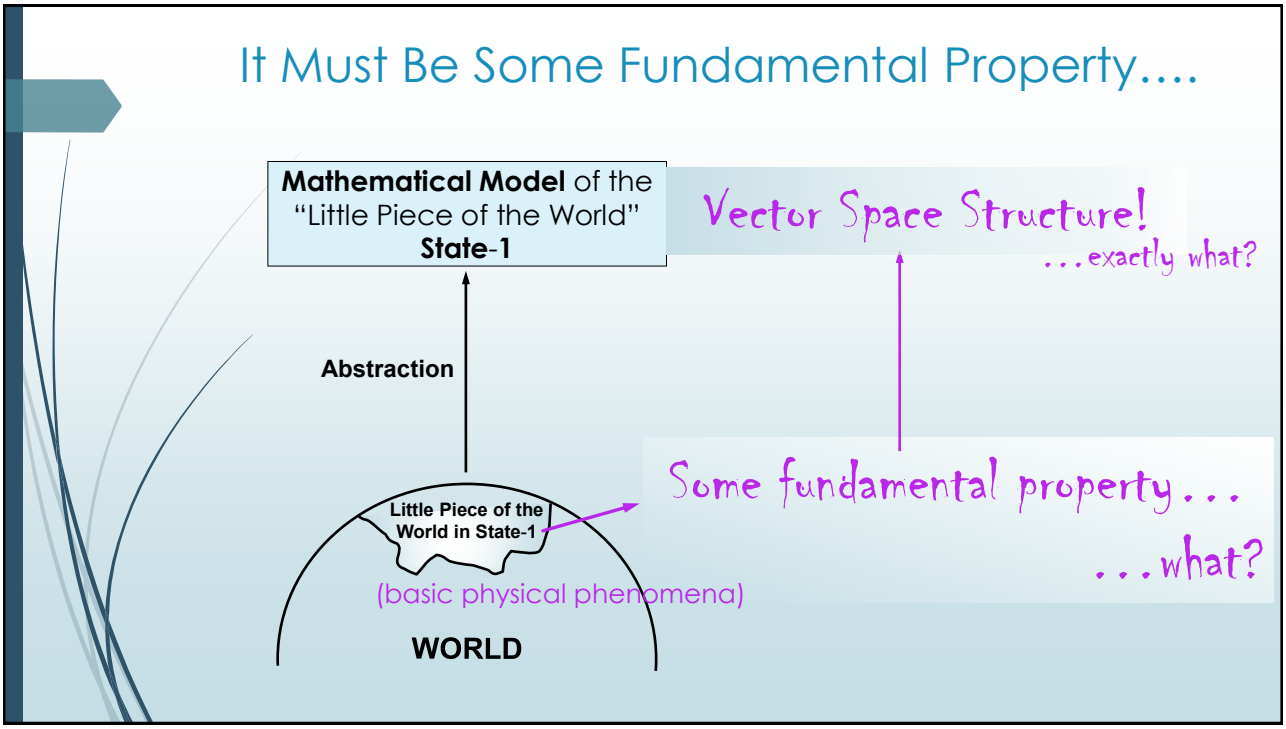
Vector Spaces – Everywhere!



Vector Space in Mathematics



It Must Be Some Fundamental Property....



Simplest Physical Phenomenon:
Things Move

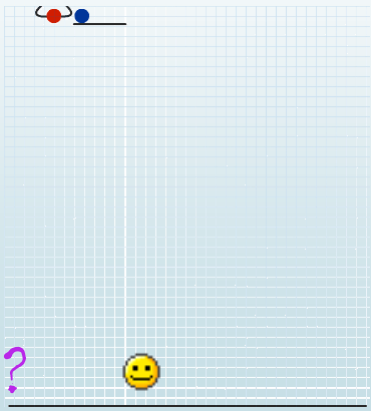
Step#1: Uniform linear motion ✓

Step#2: Uniformly accelerating motion ✓

Step#3: Compound (projectile) motion:

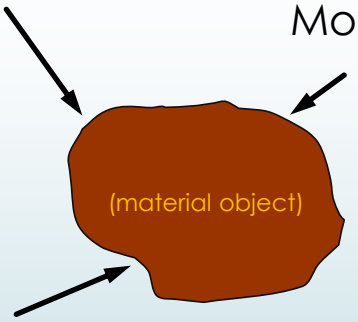
Can be "put together" using two motions

1. freefall
2. horizontal uniform motion



These do not interfere with each other?? 🙂

Forces – Combined Effect?



Moves as though a **single** force is acting on it!

What force?

Experiments…

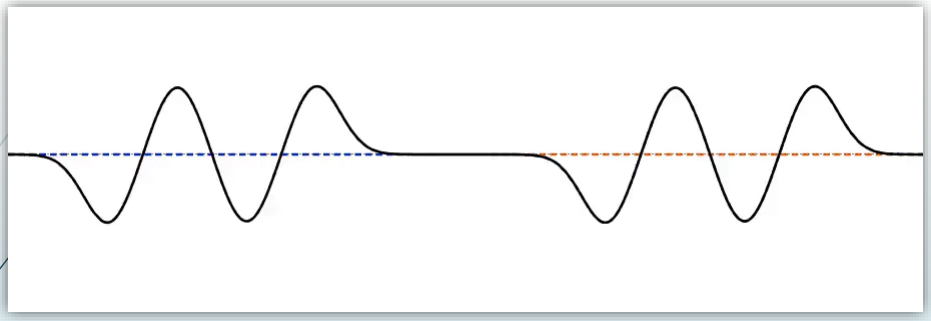
…resultant force:

Component-wise simple addition!

Their **joint effect** does **NOT** create anything "new" …

… they simply "stack up" on each other! 🙂

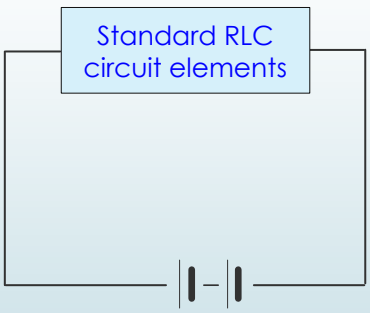
How do Waves "Interact"?



Interference:

"Simple Sum" of individual amplitudes!

Electric Circuits – Combined Effect?



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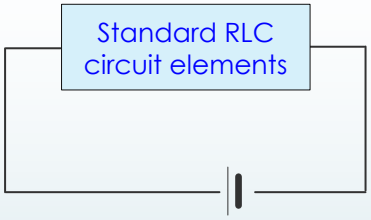
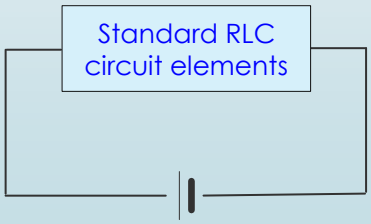
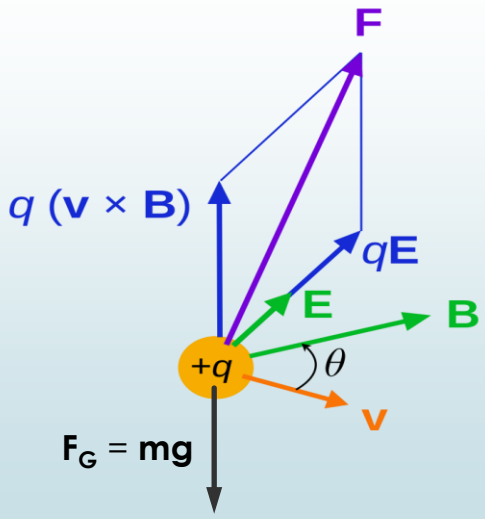


simple +



→ "Simple Sum"
of the two "electric effects"?

Charged Particle with Mass?



"Simple Sum" of

- electric
- magnetic
- gravitational

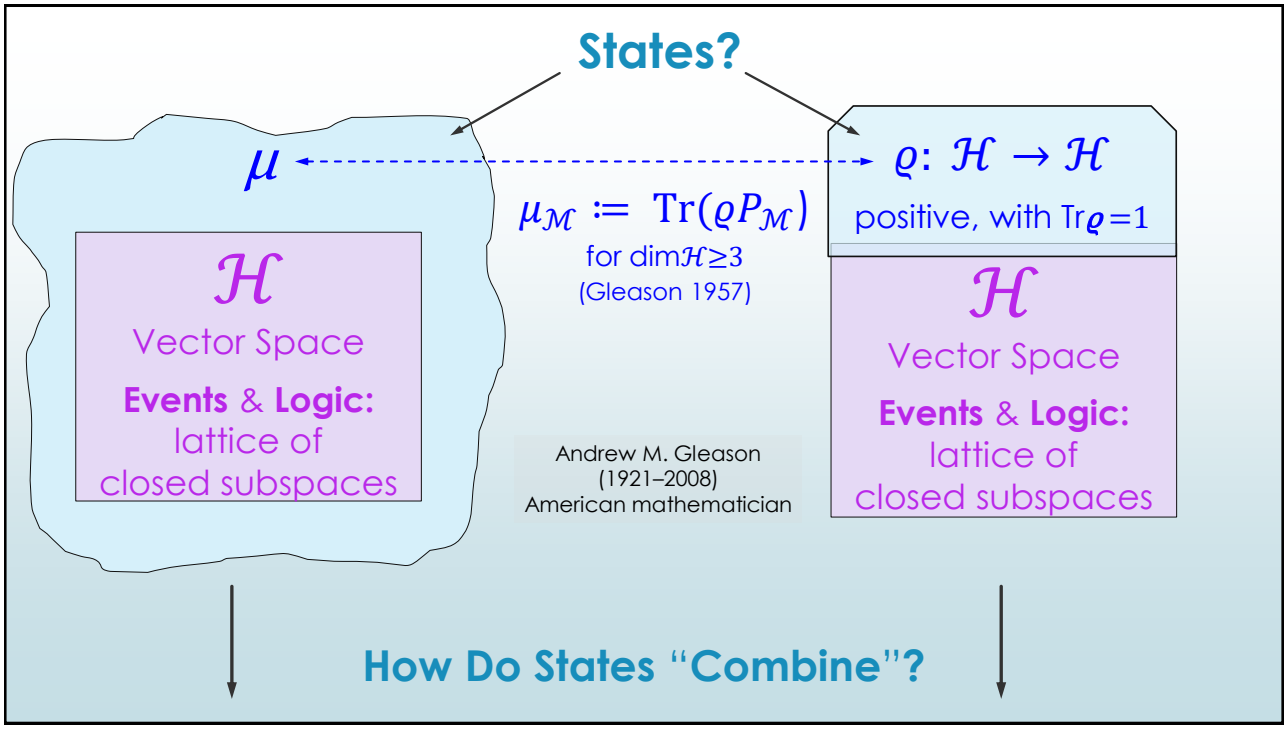
forces?

Quantum Mechanics: **States Combined?**

Ω
Event Space
Events & Logic:
σ-algebra of subsets
with set operations
("boolean logic")

→

$\mathcal{H}$
Vector Space
Events & Logic:
Lattice of closed subspaces
("quantum logic")



How Do States Combine?

$$\mu = \sum_{i\in I} \alpha_i \mu_i$$

s. t. $\forall i \in I$: $\alpha_i \geq 0$, $\Sigma_{i\in I}\alpha_i = 1$
("σ-convex combination")

$$\varrho = \sum_{i\in I} \beta_i (a_i \otimes a_i^*)$$

$$= \sum_{i\in I} \beta_i (|a_i\rangle \langle a_i|)$$ ("bra-ket" notation)

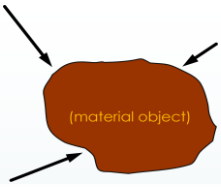
s. t. $\forall i \in I$: $a_i \in \mathcal{H}$ and $\beta_i \geq 0$, $\Sigma_{i\in I}\beta_i = 1$,

"**mixed**" **state**: such non-trivial decomposition exists.
"**pure**" **state**: only trivial decomposition exists.

"Mixed" state
=
"Simple Sum" of "Pure" states

Was A Question **Ever Asked**?

Yes!



Meg kell még vizsgálnunk, hogy mily
kor egyszerre több testtel van kölcsönhatá
Két eset lehetséges: 1. Mindegyik te
megállapitható erőtörvény szerinti gyorsul
nek. 2. Ez nem teljesül, az eredő mozgás
törvények által megszabott gyorsulásokból
tásokkal egyszerű összefüggésben nem lev
("Háromtest-erők".)

Hogy a kettő közül melyik valósul m
nak kell eldönteni. Vizsgáljunk meg tehát

KOSSUTH LAJOS TUDOMÁNYEGYETEM
TERMÉSZETTUDOMÁNYI KAR

Dede Miklós

KÍSÉRLETI FIZIKA

1. KÖTET

Punchline – ¾ way:

Mathematical Model of the "Little Piece of the World" State-1

Vector Space Structure

...exactly what?

Abstraction

Little Piece of the World in State-1

Superposition Principle!

(for basic physical phenomena)

WORLD

Core Structure of Vector Spaces

Vector Space Structure?

Infinite (continuum) "number" of vectors

=

"simple sum" of a "few vectors"!

"simple sum" ↓

"Linear combination":

(defining operation of vector spaces)

$\boldsymbol{v} = \alpha_1 \mathbf{x}_1 + \alpha_2 \mathbf{x}_2 + \alpha_3 \mathbf{x}_3 + \ldots + \alpha_n \mathbf{x}_n$

↓ "few vectors"

1. Linearly independent:

any one of them **cannot be replaced by the others**,
no matter how hard you try!

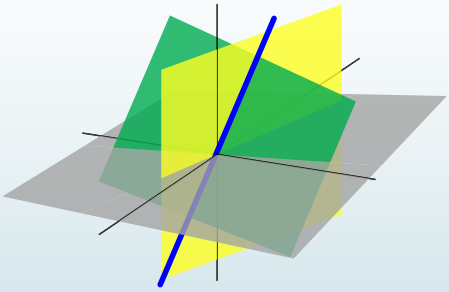
2. Span the whole vector space:

any one vector in the space can be reached
via linear combination of the "few".

Basis...

...in more general form:

"Complementary System of Subspaces"



$V = U \oplus W$ "direct sum"

and

$U \cap W = \{0\}$ "independent".

In this case:

$\forall\, v \in V \;\; \exists$ unique $u \in U$ and $w \in W$ such that

$v = u + w$

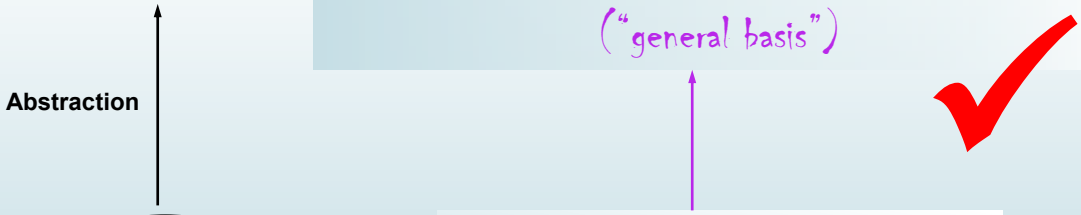
Big Punchline: Complete!

Mathematical Model of the "Little Piece of the World" State-1

Core Vector Space Structure:

Complementary System of Subspaces

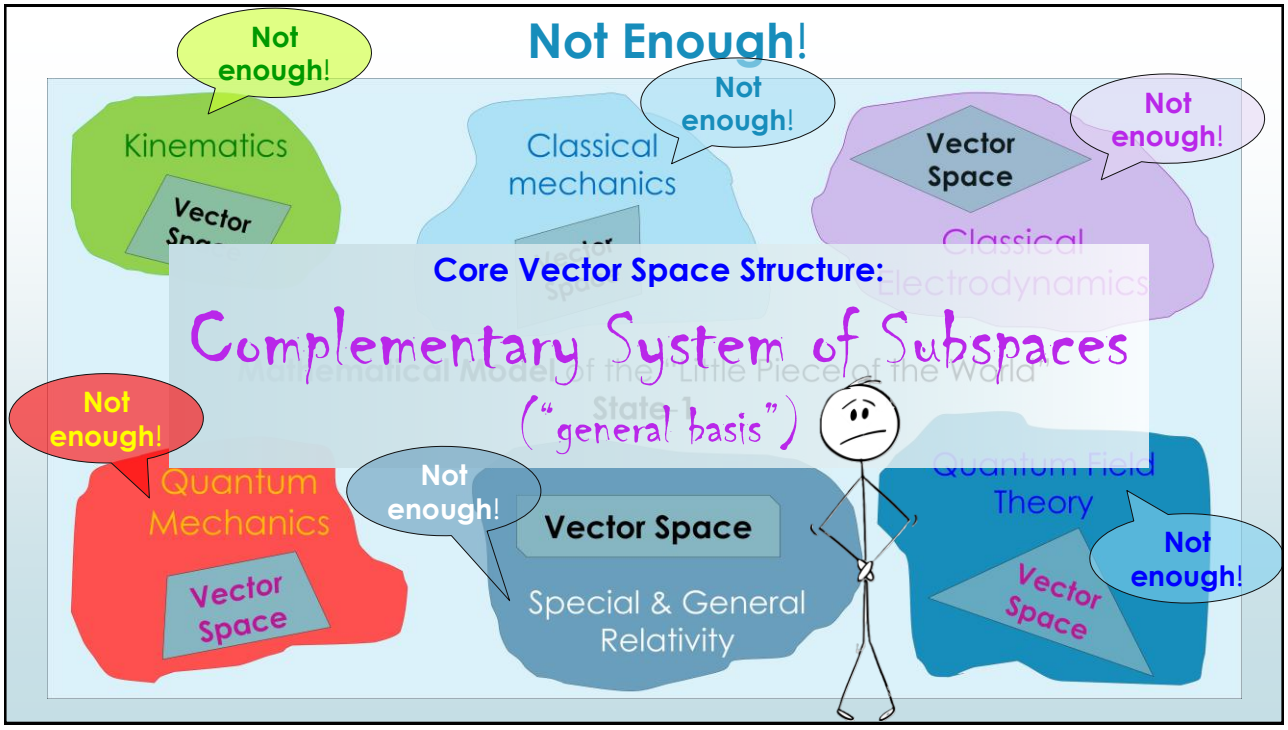
("general basis")



Abstraction

Superposition Principle!

(basic physical phenomena)


Not Enough!
Not enough!
Kinematics
Vector Space
Not enough!
Classical mechanics
Not enough!
Vector Space
Classical Electrodynamics
Core Vector Space Structure:
Complementary System of Subspaces
("general basis")
Not enough!
Quantum Mechanics
Vector Space
Not enough!
Vector Space
Special & General Relativity
Quantum Field Theory
Vector Space
Not enough!

Naked and Dressed Vector Spaces

Getting Dressed – **Phase-1**

Vector Space → Vector Space

Algebraic Structure:

- vector addition,
- multiplication by a scalar

→ Linear combination of vectors

→ Complementary system of subspaces

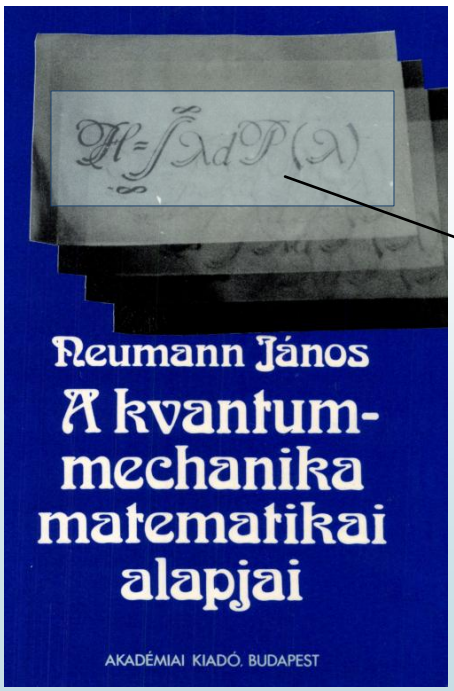
+

Topology:

- **unique** for **finite** dimension, **various** for **infinite** dimension (has to **fit the skeleton**)

Measure:

- measurability and (vector) measure (→ "integration")

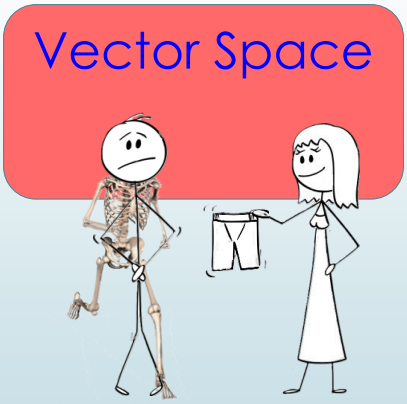


Measure:

- measurability and (vector) measure (→ "integration")

→ Projector-valued Measure

Getting Dressed – **Phase-2**



$f\colon V \times V \to \mathbb{R}$ or $\mathbb{C}$

- ~bilinear,
- ~symmetric,
- non-degenerate.

Only 3 different types,
which are...

Dress Type #**1**

Spacetime (nonrelativistic) and **Classical Mechanics**

$$\langle \cdot\ ,\ \cdot \rangle\ :\quad V \times V \to \mathbb{R}$$

bilinear, symmetric, positive definite

"scalar product", "inner product", "dot product"

length: $\|x\|^2 = \langle x, x\rangle$, **angle**: $\cos\alpha = \frac{\langle x,y\rangle}{\|x\|\cdot\|y\|}$

$$V = \mathbb{R}^n, \quad \langle x, y\rangle = \sum_{i=1}^{n} x_i\, y_i\ .$$

Cauchy-Schwarz inequality:

$$\langle x, y\rangle \leq \|x\| \cdot \|y\|$$

→ "Euclidean structure" ("triangle inequality").

Dress Type #2

Vector Space

Spacetime (special and general relativity):

$\boldsymbol{V}$ = 4-dimensional real vector space

$g\colon V \times V \to \mathbb{R}$, "Lorentz form"

bilinear, symmetric (1,3) signature, non-degenerate

$\boldsymbol{V} = \mathbb{R}^4, \quad \langle \boldsymbol{x}, \boldsymbol{y} \rangle = -x_0 y_0 + \sum_{i=1}^{3} x_i\, y_i\,.$

no length, no angle, no triangle-inequality

Model:

- Spacetime: Causal structure with universal vacuum light propagation,
- Space of an inertial observer: Euclidean structure.

Dress Type #3

Vector Space

Event space – Quantum Physics

$\langle \cdot\ ,\ \cdot \rangle\ :\quad \mathcal{H} \times \mathcal{H} \to \mathbb{C}$

(conjugate-bilinear, Hermitian, positive definite)

"scalar product", "inner product", "Hilbert space"

"**angle**": $|\langle \boldsymbol{x}, \boldsymbol{y} \rangle|$

"how much of $\boldsymbol{x}$ is in $\boldsymbol{y}$", in particular, orthogonality.

norm: $\|\boldsymbol{x}\|^2 = |\langle \boldsymbol{x}, \boldsymbol{x} \rangle|$

$\mathcal{H} = \mathbb{C}^n, \qquad \langle \boldsymbol{x}, \boldsymbol{y} \rangle = \sum_{i=1}^{n} x_i\, \overline{y_i}$

$\mathcal{H} = \mathcal{L}^2(\mathbb{R}^n), \quad \langle \boldsymbol{f}, \boldsymbol{g} \rangle = \int \boldsymbol{f}(x)\overline{\boldsymbol{g}(x)}\, dx$

Cauchy-Schwarz valid: $|\langle \boldsymbol{x}, \boldsymbol{y} \rangle| \leq \|\boldsymbol{x}\| \cdot \|\boldsymbol{y}\|$

Fully Dressed Up!

Algebraic structure:

Superposition principle!

+ Topology
+ Measurability & Measure
+ Bilinear form (***g*** or **< , >**)

Inside a Dressed-Up Vector Space:

"Commoners" and "Nobility"

Step#**1**:
Adjusted to the **Skeleton**!

Functions:

$f: V \to V$

Vector Space

subset of vectors / subspaces

Nobility – Step#1:

Adjusted to the **core algebraic structure**:

$$f(\alpha \boldsymbol{x} + \beta \boldsymbol{y}) = \alpha f(\boldsymbol{x}) + \beta f(\boldsymbol{y})$$

Linear function or **Operator**

Adjusted to the **subspace structure**:

Projector

Nobility – Step#1:

vectors: **Hamel basis**

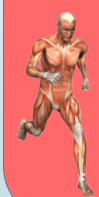
subspaces: $V = \oplus_{i \in I} M_i$

Step#**2**:
Adjusted to the **Skeleton** + **Musculature**!

Operators:

$\boldsymbol{A}: V \to V$

Vector Space



basis / $V = \oplus_{i \in I} M_i$

Nobility – Step#2:

Adjusted to the **topological structure**:

Continuous ↔ **Bounded operators**

(many important ones not bounded)

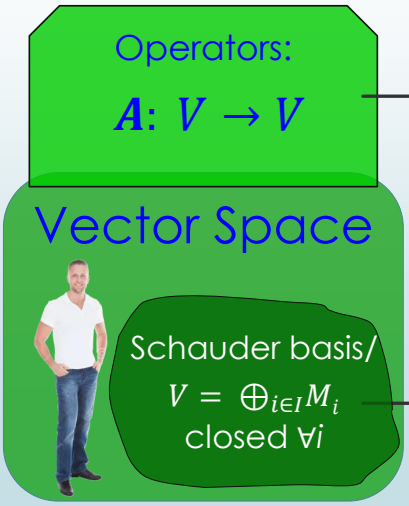
Nobility – Step#2:

basis: countable using **convergence**

"Schauder basis"

subspaces: in $V = \oplus_{i \in I} M_i$: **M_i closed ∀i**

Step#3
Adjusted to the **Skeleton** + **Musculature** + **Dress**!



Nobility – Step#3:

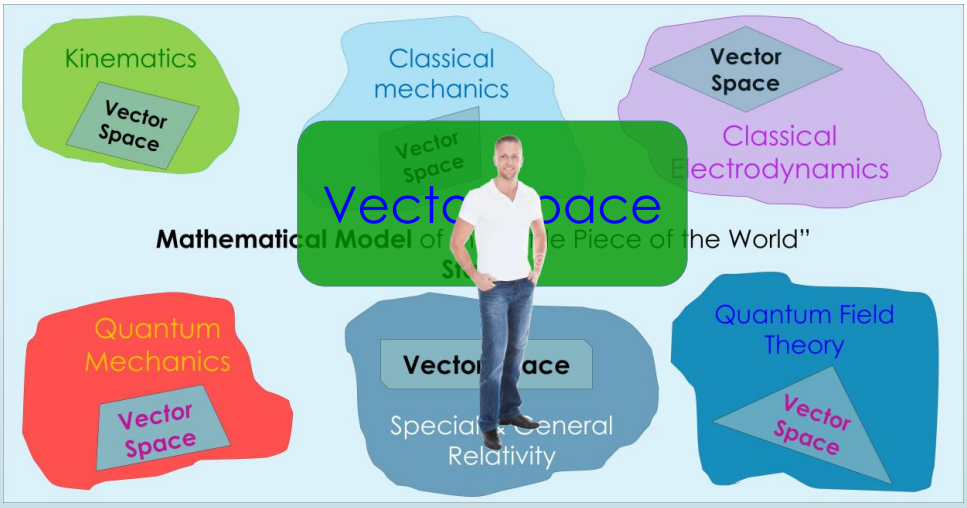
Adjusted to the dress on the vector space

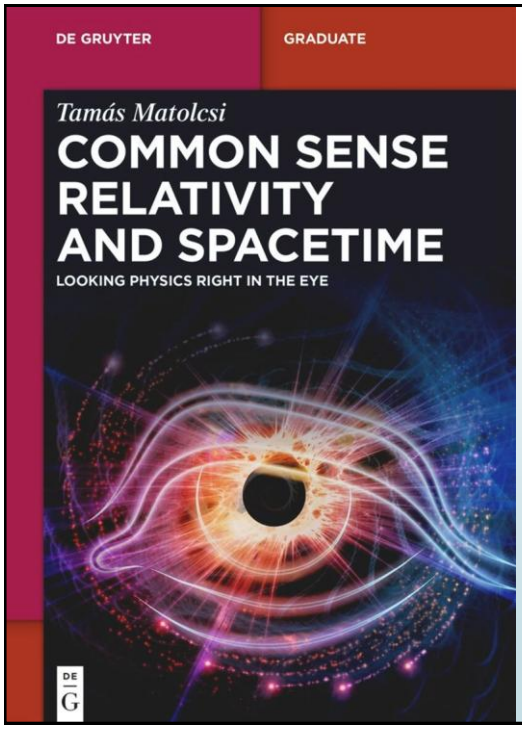
1. $\langle x, y \rangle = \langle Ux, Uy \rangle$
   "**Unitary**" or "**Orthogonal**"
2. $\langle Ax, y \rangle = \langle x, A^* y \rangle$ and $A = A^*$
   "**Self-Adjoint**" or "**Symmetric**"
3. $P_{\mathcal{N}} x = x_{\| \mathcal{N}}$, $where\ x = x_{\| \mathcal{N}} + x_{\perp \mathcal{N}}$;
   "**Orthogonal Projectors**"

Nobility – Step#3:

Basis: Schauder + **orthonormal**
Subspaces: **orthogonal**

Dressed Up Vector Spaces: **Usable**!





Book – 2026

Foreword by three former students:

How to look Physics right in the eye?

P. Benatos – Á. Szabó – P. Ván

Book link at De Gruyter