

Particle Colliders and Concept of Luminosity


(or: explaining the jargon...)

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Werner Herr, CERN

http://cern.ch/Werner.Herr/CAS2012/lectures/Granada_luminosity.pdf

Werner Herr, Particle Colliders, CAS 2012, Granada




Particle Colliders and Concept of Luminosity

(or: explaining the jargon^{*)} ...)

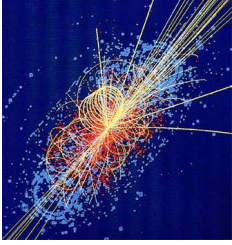
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^{*)} (beta*, squeeze, femtobarn, inverse femtobarn, lumi scan, crossing angle, filling schemes, pile-up, hour glass effect, crab crossing ...)

Werner Herr, Particle Colliders, CAS 2012, Granada



Particle colliders ?



- Used in particle physics
- Look for rare interactions
- Want highest energies
- Many interactions (events)

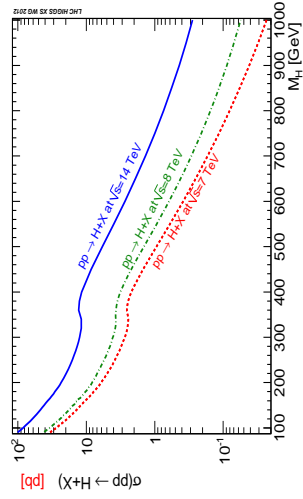
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■ Figures of merit for a collider:

➤ energy

➤ number of collisions

Rare interactions and high energy



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➤ Often seen: cross section σ for Higgs particle in LHC

➤ Two parameters: \sqrt{s} and pb

Energy: why colliding beams ?

- Two particles: $E_1, \vec{p}_1, E_2, \vec{p}_2, m_1 = m_2 = m$
- $E_{cm} = \sqrt{s} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$
- Collider versus fixed target:
 - Fixed target: $\vec{p}_2 = 0 \rightarrow \sqrt{s} = \sqrt{2m^2 + 2E_1m}$
 - Collider: $\vec{p}_1 = -\vec{p}_2 \rightarrow \sqrt{s} = E_1 + E_2$
- LHC (pp): 8000 GeV versus 87 GeV
- LEP (e⁺e⁻): 210 GeV versus ?

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Rare interactions and cross section

- Cross section σ measures how often a process occurs
- Characteristic for a given process
- Measured in: barn = b = $10^{-28}m^2$ (pb = $10^{-40}m^2$)
More common: barn = b = $10^{-24}cm^2$ (pb = $10^{-36}cm^2$)
- We have for the LHC energy:
 - $\sigma(pp \rightarrow X) \approx 0.1 \text{ b}$ and
 - $\sigma(pp \rightarrow X + H) \approx 1 \cdot 10^{-11} \text{ b}$
 - $\sigma(pp \rightarrow X + H \rightarrow \gamma\gamma) \approx 50 \cdot 10^{-15} \text{ b} = 50 \text{ fb}$ (femtobarn)
- VERY rare (one in $2 \cdot 10^{12}$), need many collisions ...

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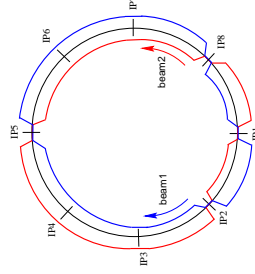
Types of particle colliders

- Basic requirement: (at least) two beams
- Can be realized as:
 - Double ring colliders
 - Single ring colliders
 - Linear colliders
- Some more exotic: gas jets, ...

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Remark: ring colliders are usually storage rings

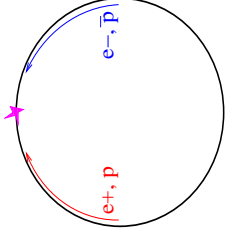
Double ring colliders (LHC, RHIC, ISR, HERA, ...)



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- Can accelerate and collide (in principle) any type of particles (p-p, p-Pb, e-p, ..)
- Usually requires crossing angle

Single ring colliders (ADA, LEP, PEP, SPS, Tevatron, ...)

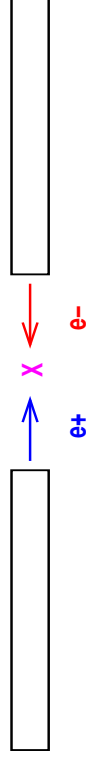


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→ Can accelerate and collide particle and anti-particle

→ Usually no crossing angle

Linear colliders (SLC, CLIC, ILC, ...)



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→ Mainly used (proposed) for leptons
(reduced synchrotron radiation)

→ With or without crossing angle

Collider challenges

- Circular colliders, beams are reused many time, efficient use for collisions
 - Requires long life time of beams (up to 10^9 turns)
- Linear colliders, beams are used once, inefficient use for collisions
 - Very small beam sizes for collisions, beam stabilization
 - Power consumption becomes an issue

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Collider performance issues

- Available energy
- Number of interactions per second (useful collisions)
- Total number of interactions
- Secondary issues:
 - Time structure of interactions (how often and how many at the same time: **pile-up**)
 - Space structure of interactions (size of interaction region: **vertex density**)
 - Quality of interactions (background, dead time etc.)

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Figure of merit: Luminosity

We want:

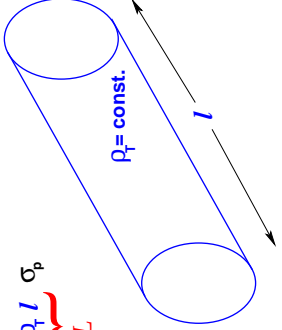
- Proportionality factor between cross section $\frac{dR}{dt}$ and number of interactions per second

$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p \quad (\rightarrow \text{units: cm}^{-2}\text{s}^{-1})$$

- Relativistic invariant
- Independent of the physical reaction
- Reliable procedures to compute and measure

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Fixed target luminosity

$$\frac{dR}{dt} = \underbrace{\phi \rho_t l}_{\mathcal{L}} \sigma_p$$


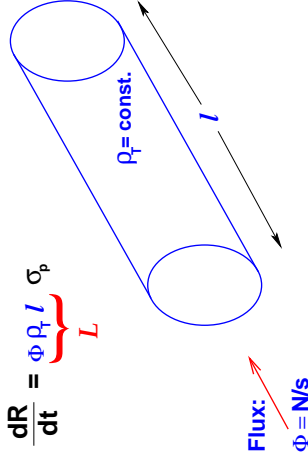
Flux: $\phi = N/s$

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Interaction rate from flux and target density and size

Fixed target luminosity

$$\frac{dR}{dt} = \underbrace{\phi \rho_t l}_{L} \phi_p$$

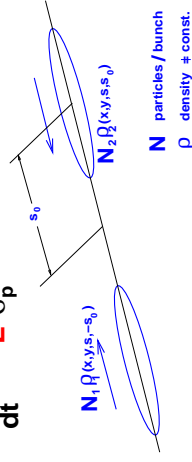


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In a collider: target is the other beam ... (and it is moving !)

Collider luminosity (per bunch crossing)

$$\frac{dR}{dt} = L \sigma_p$$



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$$\mathcal{L} \propto K N_1 N_2 \int \int \int_{-\infty}^{+\infty} \rho_1(x, y, s, -s_0) \rho_2(x, y, s, s_0) dx dy ds s_0$$

s_0 is "time"-variable: $s_0 = c \cdot t$

Kinematic factor: $K = \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2} / c^2$

Collider luminosity (per beam)

- Assume uncorrelated densities in all planes
- factorize: $\rho(x, y, s, s_0) = \rho_x(x) \cdot \rho_y(y) \cdot \rho_s(s \pm s_0)$
- For head-on collisions ($\vec{v}_1 = -\vec{v}_2$) we get:
$$\mathcal{L} = 2 \cdot N_1 N_2 \cdot f \cdot n_b \cdot \int \int \int_{-\infty}^{+\infty} dx dy ds ds_0$$
$$\rho_{1x}(x) \rho_{1y}(y) \rho_{1s}(s - s_0) \cdot \rho_{2x}(x) \rho_{2y}(y) \rho_{2s}(s + s_0)$$
- In principle: should know all distributions
- Mostly use Gaussian ρ for analytic calculation (in general: it is a good approximation)

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Gaussian distribution functions

Transverse:

$$\rho_z(u) = \frac{1}{\sigma_u \sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right) \quad u = x, y$$

Longitudinal:

$$\rho_s(s \pm s_0) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp\left(-\frac{(s \pm s_0)^2}{2\sigma_s^2}\right)$$

For non-Gaussian profiles not always possible to find analytic form, need a numerical integration

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Luminosity for two beams (1 and 2)

- Simplest case : equal beams
 - $\sigma_{1x} = \sigma_{2x}, \quad \sigma_{1y} = \sigma_{2y}, \quad \sigma_{1s} = \sigma_{2s}$
 - but: $\sigma_{1x} \neq \sigma_{1y}, \quad \sigma_{2x} \neq \sigma_{2y}$ is allowed
- Further: no dispersion at collision point

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Integration (head-on)

for beams of equal size: $\sigma_1 = \sigma_2 \rightarrow \rho_1 \rho_2 = \rho^2$:

$$\mathcal{L} = \frac{2N_1 N_2 f n_b}{(\sqrt{2\pi})^4 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{y^2}{\sigma_x^2} - \frac{s^2}{\sigma_y^2}} e^{-\frac{x^2}{\sigma_s^2} - \frac{s^2}{\sigma_s^2}} dx dy ds_0$$

integrating over s and s_0 , using:

$$\int_{-\infty}^{+\infty} e^{-at^2} dt = \sqrt{\pi/a}$$
$$\mathcal{L} = \frac{2N_1 N_2 f n_b}{8(\sqrt{\pi})^4 \sigma_x^2 \sigma_y^2} \int \int e^{-\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2}} dx dy$$

finally after integration over x and y : $\Rightarrow \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y}$

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Luminosity for two (equal) beams (1 and 2)

Simplest case : $\sigma_{1x} = \sigma_{2x}, \sigma_{1y} = \sigma_{2y}, \sigma_{1s} = \sigma_{2s}$

or: $\sigma_{1x} \neq \sigma_{2x} \neq \sigma_{1y} \neq \sigma_{2y}$, but : $\sigma_{1s} \approx \sigma_{2s}$

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$$\Rightarrow \mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \left(\mathcal{L} = \frac{N_1 N_2 f n_b}{2\pi \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}} \right)$$

Here comes β^* : $\sigma_{x,y} = \sqrt{\epsilon \cdot \beta_{x,y}^*}$

β^* is the β -function at the collision point !

Special optics for colliders

We had in the simple case: $\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y}$

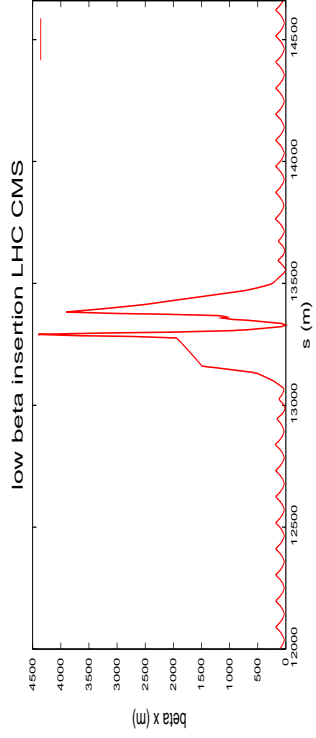
and:

$$\sigma_{x,y} = \sqrt{\epsilon \cdot \beta_{x,y}^*}$$

- For high luminosity need small $\beta_{x,y}^*$
- Done with special regions
- Special arrangement of quadrupoles etc.
- So-called low β insertions

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Example low β insertion (LHC)



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- In regular lattice: ≈ 180 m
- At collision point (squeezed optics) : ≈ 0.5 m
- ➡ Must be integrated into regular optics

Examples: some circular colliders

	Energy (GeV)	\mathcal{L}_{max} $\text{cm}^{-2}\text{s}^{-1}$	rate s^{-1}	σ_x/σ_y $\mu\text{m}/\mu\text{m}$	Particles per bunch
SPS ($p\bar{p}$)	315x315	$6 \cdot 10^{30}$	$4 \cdot 10^5$	60/30	$\approx 10 \cdot 10^{10}$
Tevatron ($p\bar{p}$)	1000x1000	$100 \cdot 10^{30}$	$7 \cdot 10^6$	30/30	$\approx 30/8 \cdot 10^{10}$
HERA (e^+p)	30x920	$40 \cdot 10^{30}$	40	250/50	$\approx 3/7 \cdot 10^{10}$
LHC (pp)	7000x7000	$10000 \cdot 10^{30}$	10^9	17/17	$\approx 16 \cdot 10^{10}$
LEP (e^+e^-)	105x105	$100 \cdot 10^{30}$	≤ 1	200/2	$\approx 50 \cdot 10^{10}$
PEP (e^+e^-)	9x3	$8000 \cdot 10^{30}$	NA	150/5	$\approx 2/6 \cdot 10^{10}$

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What else ?

What about linear colliders ?

→ See later ...

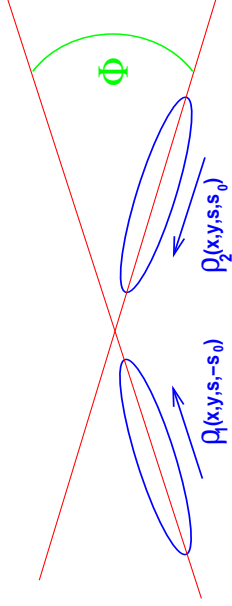
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Complications

- Crossing angle
- Hour glass effect
- Collision offset (wanted or unwanted)
- Non-Gaussian profiles
- Dispersion at collision point
- Displaced waist ($\partial\beta^*/\partial s = \alpha^* \neq 0$)
- Strong coupling
- etc.

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Collisions at crossing angle

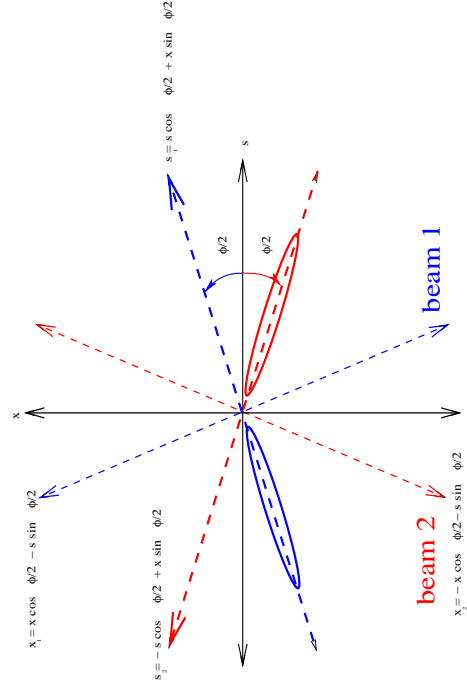


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- Needed to avoid unwanted collisions
- For colliders with many bunches: LHC, CESR, KEKB
- For colliders with coasting beams



Collisions angle geometry (horizontal plane)



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Crossing angle

Assume crossing in **horizontal (x, s)**- plane.
Transform to new coordinates:

$$\begin{cases} x_1 = x \cos \frac{\phi}{2} - s \sin \frac{\phi}{2}, & s_1 = s \cos \frac{\phi}{2} + x \sin \frac{\phi}{2}, \\ x_2 = x \cos \frac{\phi}{2} + s \sin \frac{\phi}{2}, & s_2 = s \cos \frac{\phi}{2} - x \sin \frac{\phi}{2} \end{cases}$$

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$$\mathcal{L} = 2 \cos^2 \frac{\phi}{2} N_1 N_2 f n_b \int \int \int_{-\infty}^{+\infty} dx dy ds s_0 \rho_{1x}(x_1) \rho_{1y}(y_1) \rho_{1s}(s_1 - s_0) \rho_{2x}(x_2) \rho_{2y}(y_2) \rho_{2s}(s_2 + s_0)$$

Integration (crossing angle)

use as before:

$$\int_{-\infty}^{+\infty} e^{-at^2} dt = \sqrt{\pi/a}$$

and:

$$\int_{-\infty}^{+\infty} e^{-(at^2+bt+c)} dt = \sqrt{\pi/a} \cdot e^{\frac{b^2-ac}{4a}}$$

Further: since σ_x , x and $\sin(\phi/2)$ are small:

- drop all terms $\sigma_x^k \sin^l(\phi/2)$ or $x^k \sin^l(\phi/2)$ for all:
 $k+l \geq 4$
- approximate: $\sin(\phi/2) \approx \tan(\phi/2) \approx \phi/2$

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Crossing angle

■ Crossing Angle \Rightarrow
$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot S$$

■ S is the geometric factor

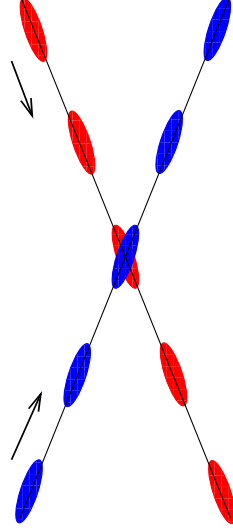
■ For small crossing angles and $\sigma_s \gg \sigma_{x,y}$
 $\Rightarrow S = \frac{1}{\sqrt{1 + \left(\frac{\sigma_s}{\sigma_x} \tan \frac{\phi}{2}\right)^2}} \approx \frac{1}{\sqrt{1 + \left(\frac{\sigma_s \phi}{2\sigma_x}\right)^2}}$

Example LHC (at 7 TeV):

$\phi = 285 \mu\text{rad}$, $\sigma_x \approx 17 \mu\text{m}$, $\sigma_s = 7.5 \text{ cm}$, $S = 0.84$

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Large crossing angle

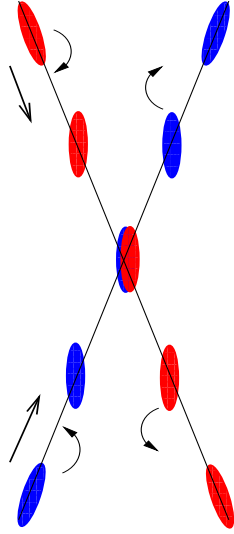


↑ Large crossing angle: large loss of luminosity

↑ "crab" crossing can recover geometric factor

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"crab" crossing scheme

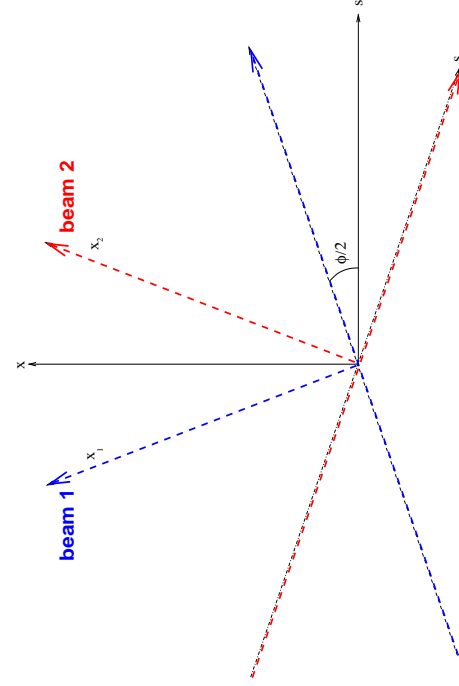


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- Done with transversely deflecting cavities (if you wondered what they can be used for)
- Feasibility needs to be demonstrated

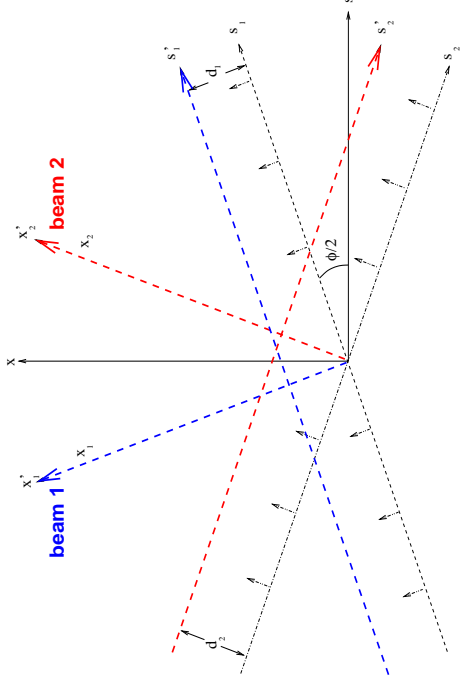


Offset and crossing angle



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Offset and crossing angle



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Offset and crossing angle

■ Transformations with offsets in crossing plane:

$$\begin{cases} x_1 = d_1 + x \cos \frac{\phi}{2} - s \sin \frac{\phi}{2}, & s_1 = s \cos \frac{\phi}{2} + x \sin \frac{\phi}{2}, \\ x_2 = d_2 + x \cos \frac{\phi}{2} + s \sin \frac{\phi}{2}, & s_2 = s \cos \frac{\phi}{2} - x \sin \frac{\phi}{2} \end{cases}$$

■ Gives after integration over y and s_0 :

$$\mathcal{L} = \frac{\mathcal{L}_0}{2\pi\sigma_s\sigma_x} 2 \cos^2 \frac{\phi}{2} \iint e^{-\frac{x^2 \cos^2(\phi/2) + s^2 \sin^2(\phi/2)}{\sigma_x^2}} e^{-\frac{x^2 \sin^2(\phi/2) + s^2 \cos^2(\phi/2)}{\sigma_s^2}} \times e^{-\frac{d_1^2 + d_2^2 + 2(d_1 + d_2)x \cos(\phi/2) - 2(d_2 - d_1)s \sin(\phi/2)}{2\sigma_x^2}} dx ds.$$

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Offset and crossing angle

After integration over x:

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{8\pi^{\frac{3}{2}} \sigma_s} \cdot 2 \cos \frac{\phi}{2} \int_{-\infty}^{+\infty} W \cdot \frac{e^{-(As^2+2Bs)}}{\sigma_x \sigma_y} ds$$

with:

$$A = \frac{\sin^2 \frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2 \frac{\phi}{2}}{\sigma_s^2} \quad B = \frac{(d_2 - d_1) \sin(\phi/2)}{2\sigma_x^2}$$

and $W = e^{-\frac{1}{4\sigma_x^2}(d_2-d_1)^2}$

⇒ After integration: Luminosity with correction factors

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Luminosity with correction factors

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi \sigma_x \sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S$$

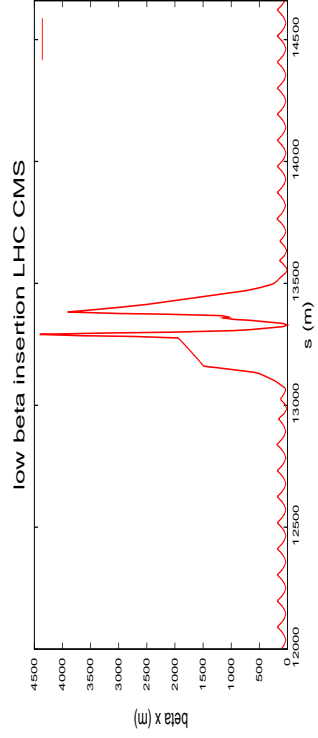
→ W : correction for beam offset (one per plane)

→ S : correction for crossing angle

→ $e^{\frac{B^2}{A}}$: correction for crossing angle **and** offset
(if in the same plane)

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Hour glass effect

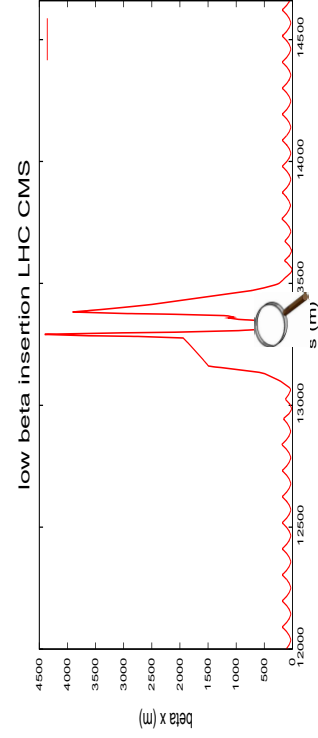


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Remember the insertion: β -functions depends on position s



Hour glass effect

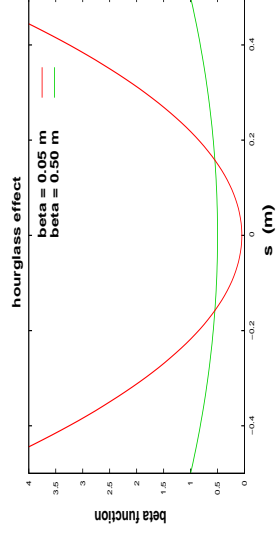


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Remember the insertion: β -functions depends on position s



Hour glass effect

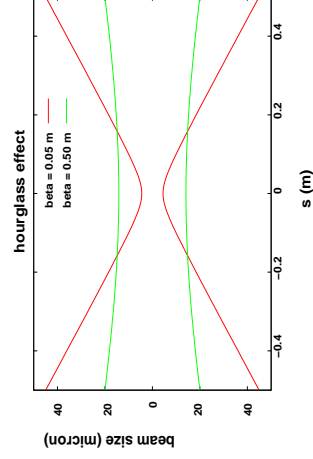


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- In our low β insertion we have: $\beta(s) \approx \beta^*(1 + \left(\frac{s}{\beta^*}\right)^2)$
- For small β^* the beam size grows very fast !



Hour glass effect



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Beam size σ ($\propto \sqrt{\beta^*(s)}$) depends on position s



Hour glass effect

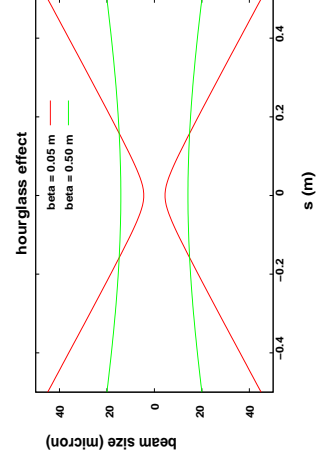


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Beam size has shape of an Hour Glass



Hour glass effect

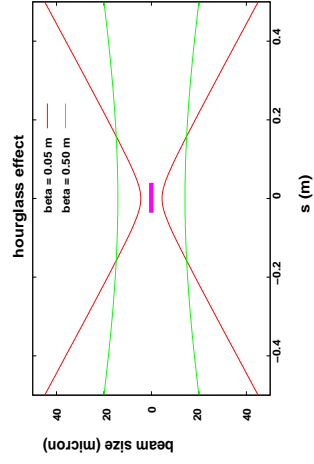


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Beam size σ ($\propto \sqrt{\beta^*(s)}$) depends on position s



Hour glass effect - short bunches

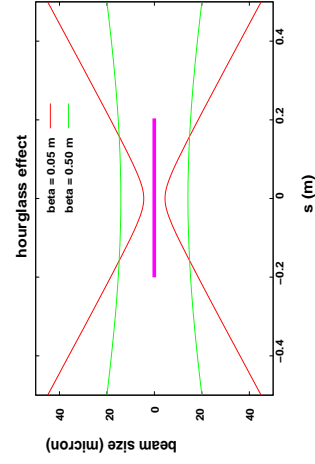


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Small variation of beam size along bunch



Hour glass effect - long bunches



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Significant effect for long bunches and small β^*



Hour glass effect

- β -functions depends on position s
- Need modification to overlap integral
- Usually: $\beta(s) = \beta^* \left(1 + \left(\frac{s}{\beta^*}\right)^2\right)$
 - i.e. $\sigma \implies \sigma(s) \neq \text{const.}$
 - $\sigma(s) = \sigma^* \sqrt{1 + \left(\frac{s}{\beta^*}\right)^2}$
- Important when β^* comparable to the r.m.s. bunch length σ_s (or smaller !)

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Hour glass effect

Using the expression: $u_x = \beta^* / \sigma_s$

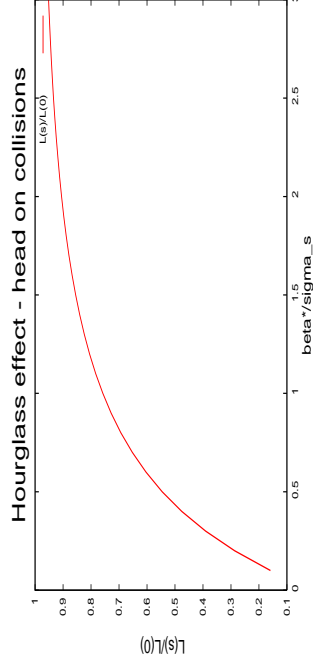
Without crossing angle and for symmetric, round Gaussian beams we get the relative luminosity reduction as:

$$\frac{\mathcal{L}(\sigma_s)}{\mathcal{L}(0)} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} \frac{e^{-u^2}}{\left[1 + \left(\frac{u}{u_x}\right)^2\right]} du = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \text{erfc}(u_x)$$

$$\mathcal{L}(\sigma_s) = \mathcal{L}(0) \cdot H \quad \text{with : } H = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \text{erfc}(u_x)$$

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Hour glass effect



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- Hourglass reduction factor as function of ratio β^*/σ_s .
- A lesson: small β^* does not always lead to high luminosity !

Luminosity with (more) correction factors

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x \sigma_y} \cdot W \cdot e^{\frac{B^2}{A}} \cdot S \cdot H$$

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- W : correction for beam offset
- S : correction for crossing angle
- $e^{\frac{B^2}{A}}$: correction for crossing angle and offset
- H : correction for hour glass effect

Calculations for the LHC

- $N_1 = N_2 = 1.15 \times 10^{11}$ particles/bunch
- $n_b = 2808$ bunches/beam
- $f = 11.2455$ kHz, $\phi = 285$ μ rad
- $\beta_x^* = \beta_y^* = 0.55$ m
- $\sigma_x^* = \sigma_y^* = 16.6$ μ m, $\sigma_s = 7.7$ cm

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- Simplest case (Head on collision):

$$\mathcal{L} = 1.200 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

- Effect of crossing angle:

$$\mathcal{L} = 0.973 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

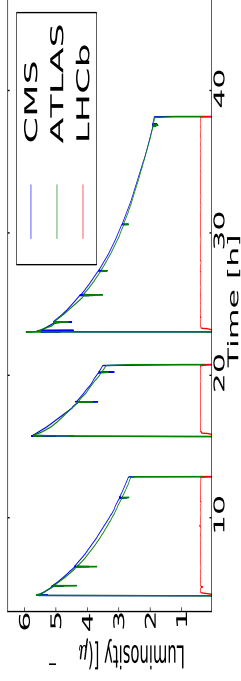
- Effect of crossing angle & Hourglass:

$$\mathcal{L} = 0.969 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$$

➔ Most important: effect of crossing angle

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Luminosity in LHC as function of time



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(Courtesy X. Buffat)

- Luminosity evolution in LHC during 2 typical days
- Run time up to 15 hour
- Preparation time 3 - 4 hours

What really counts: Integrated luminosity

- $\mathcal{L}_{\text{int}} = \int_0^T \mathcal{L}(t) dt$
- $\mathcal{L}_{\text{int}} \cdot \sigma_p = \int_0^T \mathcal{L}(t) dt \cdot \sigma_p =$ total number of events observed of process p
- Unit is: cm^{-2} , i.e. inverse cross-section
- Often expressed in inverse barn
- 1 fb^{-1} (inverse femto-barn) is $10^{39} cm^{-2}$
- for 1 fb^{-1} : requires $10^6 s$ at $L = 10^{33} cm^{-2} s^{-1}$

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What really counts: Integrated luminosity

- What does it mean ?
- Assume:
 - You have accumulated 20 fb^{-1} (inverse femto-barn)
 - You are interested in $\sigma(pp \rightarrow X + H \rightarrow \gamma\gamma) \approx 50 \text{ fb}$ (femtobarn)
 - You have $20 \text{ fb}^{-1} \cdot 50 \text{ fb} = 1000$
 - You have 1000 events of interest in your data !!

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Integrated luminosity

- Luminosity decays as function of time
- For studies: assume some life time behaviour.
 - E.g. $\mathcal{L}(t) \rightarrow \mathcal{L}_0 \exp\left(-\frac{t}{\tau}\right)$
- Contributions to life time from: intensity decay, emittance growth etc.
- Aim: optimize integrated luminosity, taken into account preparation time

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Integrated luminosity

Knowledge of preparation time and luminosity decay allows optimization of \mathcal{L}_{int}



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Integrated luminosity

- Typical run times LHC:
 $t_r \approx 8 - 15$ hours
 - For optimization:
 - ↑ Need to know preparation time t_p
 - ↑ very important: good model of luminosity evolution as function of time $\mathcal{L}(t)$
- depends on many parameters !!

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Interactions per crossing

- Luminosity/ $f n_b \propto N_1 N_2$
- In LHC: crossing every 25 ns
- Per crossing approximately 20 interactions
- May be undesirable (pile-up in detector)
- \Rightarrow more bunches n_b , or smaller N ??

Beware: maximum (peak) luminosity \mathcal{L}_{max} is not the whole story ... !

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Luminosity measurement

- One needs to get a signal proportional to interaction rate \rightarrow **Beam diagnostics**
- Large dynamic range: $10^{27} \text{ cm}^{-2} \text{ s}^{-1}$ to $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
- Very fast, if possible for individual bunches
- Used for optimization
- For absolute luminosity need calibration

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Luminosity calibration

- Remember the basic definition:
$$\frac{dR}{dt} = \mathcal{L} \times \sigma_p$$
- For a well known and calculable process we know σ_p
- The experiments measure the counting rate $\frac{dR}{dt}$ for **this** process
- Get the absolute, calibrated luminosity

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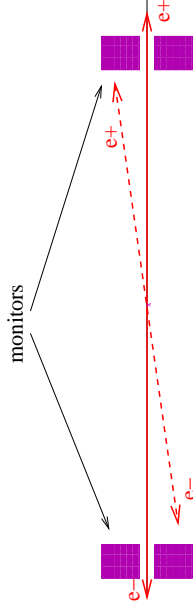
Luminosity calibration

(e^+e^-)

- Use well known and calculable process
- $e^+e^- \rightarrow e^+e^-$ elastic scattering (Bhabha scattering)
- Have to go to small angles ($\sigma_{el} \propto \Theta^{-3}$)
- Small rates at high energy ($\sigma_{el} \propto \frac{1}{E^2}$)

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Luminosity calibration



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- Measure coincidence at small angles
- Low counting rates, in particular for high energy !
- Background may be problematic

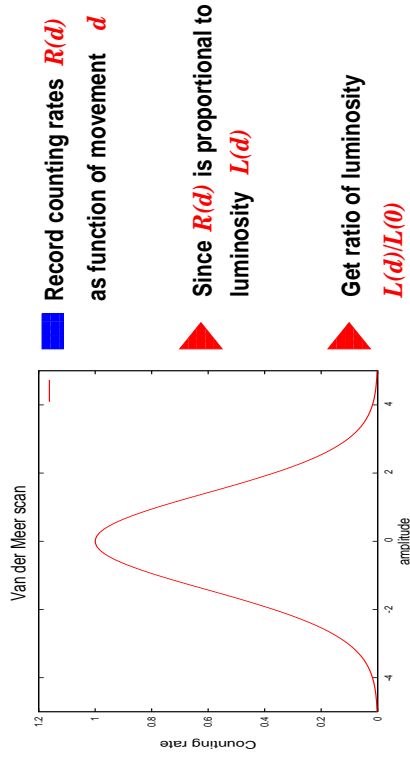
Luminosity calibration

(hadrons, e.g. pp or $p\bar{p}$)

- Must measure beam current and beam sizes
- Beam size measurement:
 - Wire scanner or synchrotron light monitors
 - Measurement with beam ... → remember luminosity with offset
 - Move the two beams against each other in transverse planes (van der Meer scan, ISR 1973 - LHC 2012)

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Luminosity optimization



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Luminosity optimization

- From ratio of luminosity $\mathcal{L}(d)/\mathcal{L}_0$
- Remember: $W = e^{-\frac{1}{4\sigma^2}(d_2 - d_1)^2}$
- Determines σ (lumi scan)
- ... and centres the beams !
- Others:
 - Beam-beam deflection scans **LEP**
 - Beam-beam excitation

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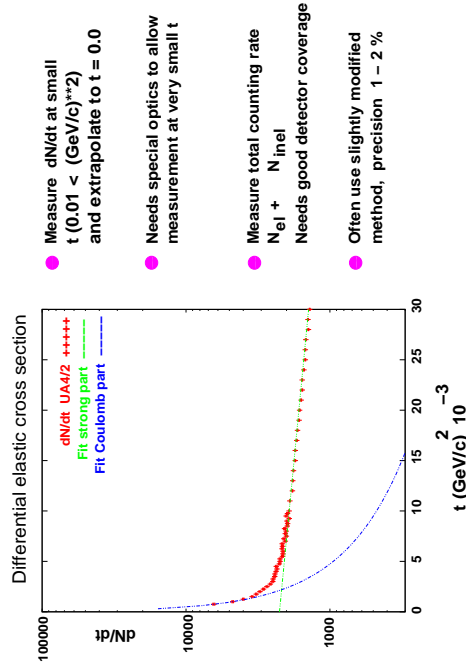
Absolute value of \mathcal{L} (pp or $p\bar{p}$)

- By Coulomb normalization:
- **Coulomb amplitude** exactly calculable:

$$\lim_{t \rightarrow 0} \frac{d\sigma_{el}}{dt} = \frac{1}{\mathcal{L}} \frac{dN_{el}}{dt} \Big|_{t=0} = \pi |f_C + f_N|^2$$

$$\simeq \pi \left| \frac{2\alpha_{em}}{-t} + \frac{\sigma_{tot}}{4\pi} (\rho + i) e^{b\frac{t}{2}} \right|^2 \simeq \frac{4\pi\alpha_{em}^2}{t^2} \Big|_{|t| \rightarrow 0}$$
- Fit gives: σ_{tot} , ρ , b and \mathcal{L}
- Can be done measuring elastic scattering at small angles

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- Measure dN/dt at small t ($0.01 < (GeV/c)^2$) and extrapolate to $t = 0.0$
- Needs special optics to allow measurement at very small t
- Measure total counting rate $N_{el} + N_{inel}$
Needs good detector coverage
- Often use slightly modified method, precision 1 – 2 %

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Luminosity in linear colliders

- Mainly (only) $e+e-$ colliders
- Past collider: SLC (SLAC)
- Under consideration: CLIC, ILC
- Special issues:
 - Particles collide only once (dynamics) !
 - Particles collide only once (beam power) !
- Must be taken into account

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Luminosity in linear colliders

- Basic formula:

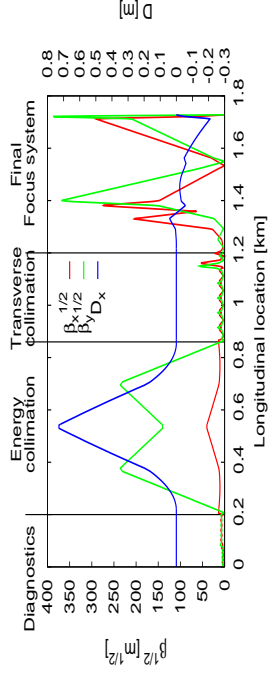
$$\text{From : } \mathcal{L} = \frac{N^2 f n_b}{4\pi\sigma_x\sigma_y} \quad \text{to : } \mathcal{L} = \frac{N^2 f_{rep} n_b}{4\pi\sigma_x\sigma_y}$$

- Replace frequency f by repetition rate f_{rep} .
- And introduce effective beam sizes $\overline{\sigma_x}, \overline{\sigma_y}$:

$$\mathcal{L} = \frac{N^2 f_{rep} n_b}{4\pi\overline{\sigma_x} \overline{\sigma_y}}$$

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Final focusing in linear colliders



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(Courtesy R. Tomas)

- "Final Focus" and "Beam delivery System"
- At the **end** of the beam line only !
- Smaller β^* in linear colliders (10 mm × 0.2 mm !!)

Luminosity in linear colliders

- Using the enhancement factor H_D :

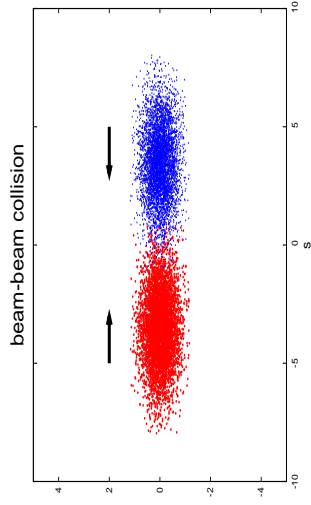
$$\mathcal{L} = \frac{N^2 f_{rep} \eta_b}{4\pi\bar{\sigma}_x \bar{\sigma}_y} \rightarrow \mathcal{L} = \frac{H_D \cdot N^2 f_{rep} \eta_b}{4\pi\sigma_x \sigma_y}$$

- Enhancement factor H_D takes into account reduction of nominal beam size by the disruptive field (**pinch effect**)
- Related to disruption parameter \mathcal{D} :

$$\mathcal{D}_{x,y} = \frac{2r_e N \sigma_z}{\gamma \sigma_{x,y} (\sigma_x + \sigma_y)}$$

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Pinch effect - disruption

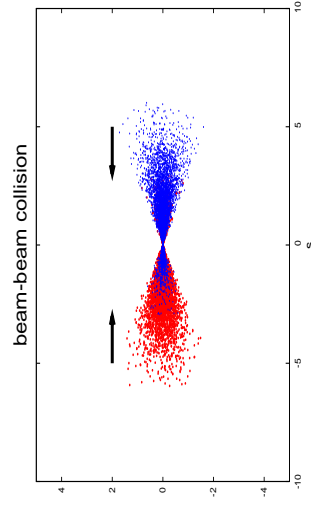


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➤ Additional focusing by opposing beams



Pinch effect - disruption



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➤ Additional focusing by opposing beams



Luminosity in linear colliders

- For weak disruption $\mathcal{D} \ll 1$ and round beams:

$$H_D = 1 + \frac{2}{3\sqrt{\pi}}\mathcal{D} + \mathcal{O}(\mathcal{D}^2)$$

- For strong disruption and flat beams: computer simulation necessary, maybe can get some scaling

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Beamstrahlung

- Disruption at interaction point is basically a strong "bending"
- Results in strong synchrotron radiation: beamstrahlung
- This causes (unwanted):
 - Spread of centre-of-mass energy
 - Pair creation and detector background
- Again: luminosity is not the only important parameter

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Beamstrahlung Parameter Y

Measure of the mean field strength in the rest frame normalized to critical field B_c :

$$Y = \frac{\langle E + B \rangle}{B_c} \approx \frac{5}{6} \frac{r_e^2 \gamma N}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

with:

$$B_c = \frac{m^2 c^3}{e \hbar} \approx 4.4 \times 10^{13} \text{G}$$

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Energy loss and power consumption

Average fractional energy loss δ_E :

$$\delta_E = 1.24 \frac{\alpha \sigma_z m_e}{\lambda_C E} \frac{Y}{(1 + (1.5Y)^{2/3})^{1/2}}$$

where E is beam energy at interaction point and λ_C the Compton wavelength.

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Luminosity in linear colliders

- Using the beam power P_b and beam energy E in the luminosity:

$$\mathcal{L} = \frac{H_D \cdot N^2 f_{rep} n_b}{4\pi\sigma_x \sigma_y} \quad \rightarrow \quad \mathcal{L} = \frac{H_D \cdot N \cdot P_b}{eE \cdot 4\pi\sigma_x \sigma_y}$$

- Beam power P_b related to AC power consumption P_{AC} via efficiency η_b^{AC}

$$P_b = \eta_b^{AC} \cdot P_{AC}$$

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Figure of merit in linear colliders

- Luminosity at given energy normalized to power consumption and momentum spread due to beamstrahlung:

$$M = \frac{\mathcal{L}E}{\sqrt{\delta_b} P_{AC}}$$

- With previous definition (and reasonably small beamstrahlung) this becomes:

$$M = \frac{\mathcal{L}E}{\sqrt{\delta_b} P_{AC}} \propto \frac{\eta_b^{AC}}{\sqrt{\epsilon_{xy}}}$$

- These are optimized in the linear collider design

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Not treated :

- Coasting beams (e.g. ISR)
- Asymmetric colliders (e.g. PEP, HERA, LHeC)
- All concepts can be formally extended ...

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How to cook high Luminosity ?

- Get high intensity
- Get small beam sizes (small ϵ and β^*)
- Get many bunches
- Get small crossing angle (if any)
- Get exact head-on collisions
- Get short bunches

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Luminosity in a nutshell

$$\mathcal{L} = \frac{N_1 N_2 f^{nb}}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{E^2}{A}} \cdot S \cdot H$$

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Luminosity in a nutshell

$$\mathcal{L} = \frac{N_1 N_2 f^{nb}}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{E^2}{A}} \cdot S \cdot H$$

■ Are there limits to what we can do ?

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Luminosity in a nutshell

$$\mathcal{L} = \frac{N_1 N_2 f_{nb}}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{E^2}{A}} \cdot S \cdot H$$

- Are there limits to what we can do ?
- Yes, there are **beam-beam effects**
- In LHC: $\approx 10^{11}$ collisions with the other beam per fill !!

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Luminosity in a nutshell

$$\mathcal{L} = \frac{N_1 N_2 f_{nb}}{4\pi\sigma_x\sigma_y} \cdot W \cdot e^{\frac{E^2}{A}} \cdot S \cdot H$$

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Summary

- Colliders are used exclusively for particle physics experiments
- Colliders are the only tools to get highest centre of mass energies
- Type of collider is decided by the type of particles and use
- Design and performance must take into account the needs of the experiments

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Energy: why colliding beams ?

- Two beams: $E_1, \vec{p}_1, E_2, \vec{p}_2, m_1 = m_2 = m$
- $E_{cm} = \sqrt{(E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$
- Collider versus fixed target:
 - Fixed target: $\vec{p}_2 = 0 \rightarrow E_{cm} = \sqrt{2m^2 + 2E_1m}$
 - Collider: $\vec{p}_1 = -\vec{p}_2 \rightarrow E_{cm} = E_1 + E_2$
- LHC (pp): 8000 GeV versus ≈ 87 GeV
- LEP (e^+e^-): 210 GeV versus ≈ 0.5 GeV !!!
- LC (e^+e^-): 2000 GeV versus ≈ 1.0 GeV !!!

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Bibliography

Some bibliography in the hand-out

Luminosity lectures and basics:

W. Herr, Concept of Luminosity, CERN Accelerator School, Zeuthen 2003, in: CERN 2006-002 (2006).

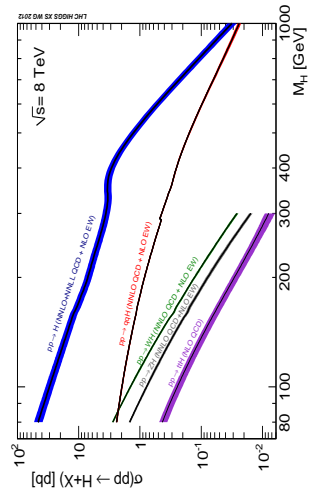
A. Chao and M. Tigner, Handbook of Accelerator Physics and Engineering, World Scientific, (1998).

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- BACKUP SLIDES -

Rare interactions and high energy



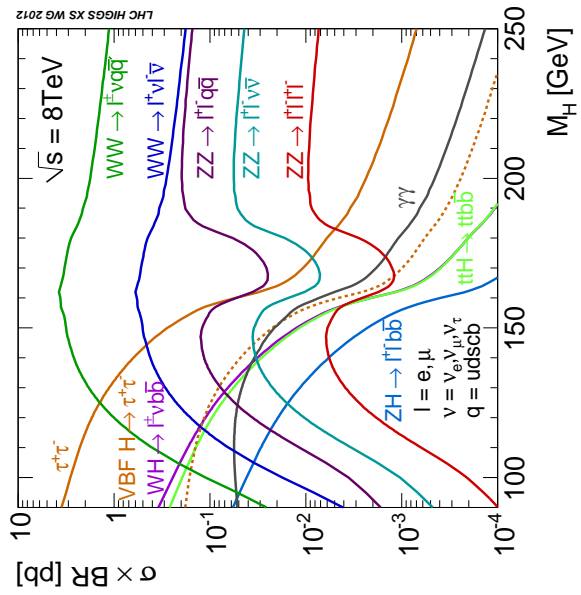
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- ↑ Often seen: cross section σ for Higgs particle
- ↑ Typical channels



Rare interactions and high energy

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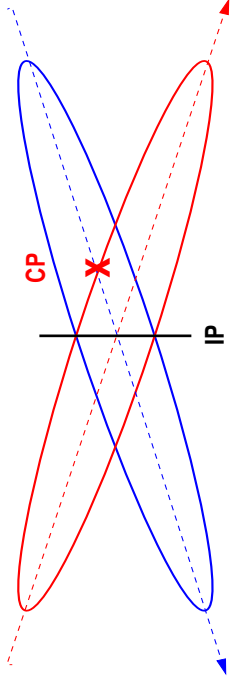
↑ Often seen: cross section σ for Higgs particle

↑ Typical channels

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Large crossing angle

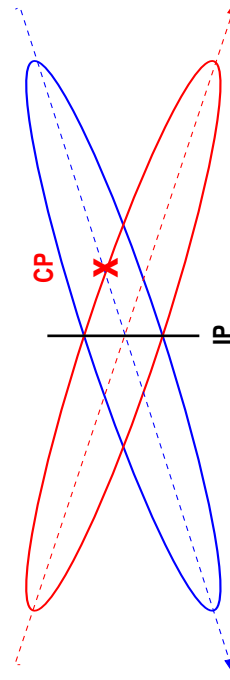


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- For large amplitude particles: collision point longitudinally displaced



Large crossing angle

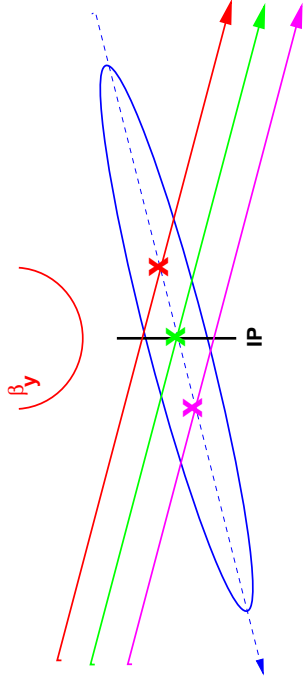


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- For large amplitude particles: collision point longitudinally displaced
- Can introduce coupling (transverse and synchro betatron, bad for flat beams)



Large crossing angle

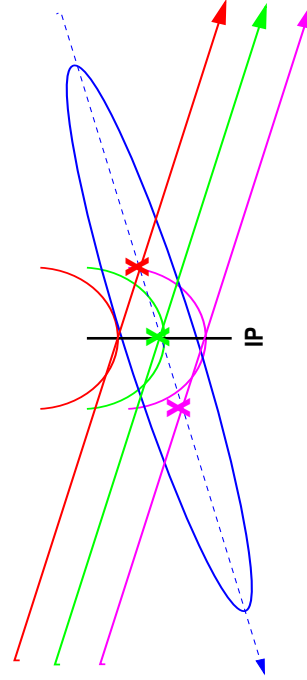


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- A particle's collision point amplitude dependent
- Different (vertical) β functions at collision points



Large crossing angle

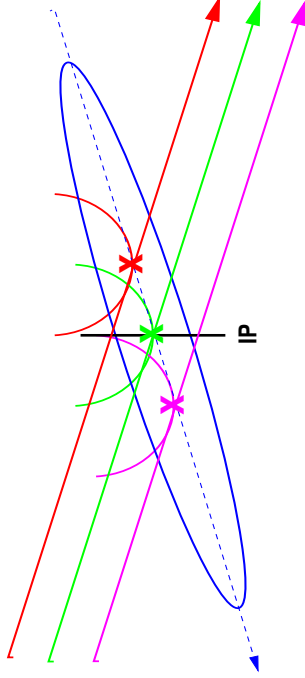


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- A particle's collision point amplitude dependent
- Different β functions at collision points (hour glass !)



"crab waist" scheme



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- Make vertical waist (β_y^{min}) amplitude (x) dependent
- All particles in both beams collide in minimum β_y region

"crab waist" scheme

- Make vertical waist (minimum of β) amplitude (x) dependent
- Without details: can be done with two sextupoles
- First tried at DAPHNE (Frascati) in 2008
- Geometrical gain small
- Smaller vertical tune shift as function of horizontal coordinate
- Less betatron and synchrotron coupling
- Good remedy for flat (i.e. lepton) beams with large crossing angle

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If the beams are not Gaussian ??

Exercise:

- Assume flat distributions (normalized to 1)

$$\rho_1 = \rho_2 = \frac{1}{2a}, \quad \text{for } [-a \leq z \leq a], \quad z = x, y$$

Calculate r.m.s. in x and y:

$$\langle (x, y)^2 \rangle = \int_{-\infty}^{+\infty} (x, y)^2 \cdot \rho(x, y) dx dy$$

and

$$\mathcal{L} = \int_{-\infty}^{+\infty} \rho_1(x, y) \rho_2(x, y) dx dy$$

- Compute: $\mathcal{L} \cdot \sqrt{\langle x^2 \rangle \cdot \langle y^2 \rangle}$
- Repeat for various distributions and compare

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Maximising Integrated Luminosity

- Assume exponential decay of luminosity

$$\mathcal{L}(t) = \mathcal{L}_0 \cdot e^{t/\tau}$$

- Average (integrated) luminosity $\langle \mathcal{L} \rangle$

$$\langle \mathcal{L} \rangle = \frac{\int_0^{t_r} dt \mathcal{L}(t)}{t_r + t_p} = \mathcal{L}_0 \cdot \tau \cdot \frac{1 - e^{-t_r/\tau}}{t_r + t_p}$$

- (Theoretical) maximum for:

$$t_r \approx \tau \cdot \ln(1 + \sqrt{2t_p/\tau} + t_p/\tau)$$

- Example LHC: $t_p \approx 10\text{h}$, $\tau \approx 15\text{h}$, $\Rightarrow t_r \approx 15\text{h}$

- Exercise: Would you improve τ (long t_r) or t_p ?

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