ACCELERATOR SCHOOL

SPECIAL RELATIVITY

TUTORIAL PROBLEMS

In the following problems, the speed of light can be taken as $c = 3 \times 10^8\, m\, s^{-1}$, the rest energy of an electron is 511 KeV, and the rest energy of a proton is 938 MeV.

1. A spaceship is moving directly away from the Earth with uniform velocity. A radar pulse is transmitted from the earth at 12:00 h. The pulse reflected from the rear of the spaceship is received back at the radar base at 12:02 h. The pulse reflected from the front of the spaceship is received back at the radar base 2 μs later. A second radar pulse is transmitted at 12:04 h and, after reflection at the rear end of the spaceship, is received back at base at 12:18 h. Find the speed and proper length of the spaceship.

2. A clock C moves with speed v along the x-axis of an inertial frame F and sends out two light signals towards an observer at its origin O separated by an interval τ of proper time. What is the interval between the times when these signals are received by the observer at O?

3. A physicist is arrested for driving through the red lights at a road junction. At the trial, the physicist claims that the car was going so fast that the red light appeared green to the driver. "Plea accepted," said the judge, "but I fine you one euro for each kilometre per hour your speed exceeded the speed limit of 45 kilometres per hour." Calculate the fine, taking the wavelength of green light to be 530 nm and that of red light to be 630 nm.

4. Write down an equation connecting the rest mass m, the total energy E and the momentum $\mathbf{p}$ of a relativistic particle. Show that

$$m = \frac{p^2c^2 - T^2}{2Tc^2}$$

where T is its kinetic energy.

What type of particle has kinetic energy 1 GeV and momentum 2.18 GeV/c?

5. A beam is made up of particles travelling with speed $\frac{\sqrt{3}}{2}c$ with an uncertainty of $\pm 1\%$. Calculate the level of uncertainty in the momentum $(\Delta p/p)$, kinetic energy $(\Delta T/T)$, total energy $(\Delta E/E)$ and $\Delta\gamma/\gamma$.

6. A particle of rest mass m splits into two particles of rest masses m_1 and m_2, moving with speeds v_1 and v_2 with respect to a frame in which the original particle was at rest.

(a) Show that

$$m_1 = m\left(1 - \frac{v_1^2}{c^2}\right)^{\frac{1}{2}}\left(1 + \frac{v_1}{v_2}\right)^{-1}.$$

(b) Show that the energies E_1, E_2 of the parts are given by

$$E_1 = \frac{m^2 + m_1^2 - m_2^2}{2m}\,c^2, \qquad E_2 = \frac{m^2 - m_1^2 + m_2^2}{2m}\,c^2.$$

7. A particle of rest-mass m and charge e moves in an electromagnetic field given by $\mathbf{E} = \mathbf{0}$, $\mathbf{B} = (0, 0, B)$. Show from the Lorentz force law that the particle can move in a circle of radius a in a plane of constant z at an angular velocity ω given by

$$\omega^2 = \frac{\omega_0^2}{1 + \dfrac{a^2\omega_0^2}{c^2}}$$

where $\omega_0 = \dfrac{eB}{m}$.

8. A particle with rest mass m and charge e moves in a uniform, static, electric field $\mathbf{E}$.

(a) Solve for the velocity and position of the particle, assuming the initial velocity $\mathbf{v}_0$ was perpendicular to $\mathbf{E}$.

(b) Obtain the trajectory of the particle in space and discuss the shape of the path for short and long times.