Inclusive and "Exclusive" Cross Sections in the regime of MPI

When MPI are an important feature of the inelastic event, it makes sense to measure both the ***inclusive*** and the ***exclusive MPI cross sections***

***Inclusive* cross section**:

In the inclusive cross section one counts with weight 1 each partonic collisions (in a given interval of rapidity and transverse momenta).
If a single event produces **n** partonic collisions, the event contributes **n** times to the inclusive cross section. The inclusive cross section is thus an average quantity with respect to the distribution in multiplicity of the MPI.

The single scattering inclusive cross section, which has the well known expression of the QCD parton model, thus gives the most basic information on the distribution in the number of collisions, namely the average number.
Analogously the K^{th} scattering inclusive cross section gives the K^{th} moment of the distribution in the number of collisions and is related directly to the K-partons distribution of the hadron structure.

***Exclusive* cross section**:

To measure an exclusive cross section one needs to search the hadronic events where a given Multi-Parton Interaction is present. The hadronic events, where the MPI of interest is present, ***are counted with weight 1***, the others ***are counted with weight 0***.

Inclusive and exclusive cross sections are thus measured independently. The two sets of cross sections are however linked by the following relations:

$$\sigma_H \equiv \sum_{N=1}^{\infty} \tilde{\sigma}_N, \qquad \sigma_K \equiv \sum_{N=K}^{\infty} \frac{N(N-1)\dots(N-K+1)}{K!}\tilde{\sigma}_N$$

$\tilde{\sigma}_N$ N^{th} scattering "exclusive" cross sections (where one selects the events where only N partonic collisions are present)

σ_K K^{th} scattering inclusive cross section

Notice that the relations above represents also a set of ***sum rules connecting the inclusive and the "exclusive" cross sections***

While the inclusive cross sections are given in pQCD by the convolution of the multi-parton distributions with the elementary partonic cross sections (at any order in the number of partonic collisions), the exclusive cross sections need (at least in principle an infinite non-perturbative input to be evaluated).
The number of hard partonic collisions which can be observed directly is limited, the ***exclusive cross sections*** may hence be expressed in terms of known quantities after ***expanding*** the expressions above ***in the number of elementary interactions.***
In other words the sum rules may be saturated by few terms. ***Suppose that in a given phase space interval only triple parton collisions are important.*** On has:

$$\begin{aligned} \sigma_1 &= \tilde{\sigma}_1 + 2\tilde{\sigma}_2 + 3\tilde{\sigma}_3 \\ \sigma_2 &= \tilde{\sigma}_2 + 3\tilde{\sigma}_3 \\ \sigma_3 &= \tilde{\sigma}_3 \end{aligned}$$

$$\begin{aligned} \tilde{\sigma}_1 &= \sigma_1 - 2\sigma_2 + 3\sigma_3 \\ \tilde{\sigma}_2 &= \sigma_2 - 3\sigma_3 \\ \tilde{\sigma}_3 &= \sigma_3 \end{aligned}$$

By checking the number of terms needed to saturate the sum rules, in a given phase space interval, one thus finds the correspondingly relevant number of hard partonic collisions.

Suppose that, in a given phase space interval, only single and double collisions give sizable contributions. In such a case one obtains:

$$\frac{d\sigma_1}{d\mathbf{p}_\perp dy} - \frac{d\tilde{\sigma}_1}{d\mathbf{p}_\perp dy} = \frac{d\sigma_1}{d\mathbf{p}_\perp dy}\frac{\sigma_1}{\sigma_{eff}}$$

One thus obtains the value of the effective cross section by measuring the difference between the single scattering inclusive and "exclusive" cross sections. By comparing the behavior, as a function of the fractional momenta, of the difference on the left hand side of the equation with the right hand side one obtains information on the dependence of the effective cross section on y and p_t