CERN, 14/15 May 2012

Approaching Higgs production

from an effective-theory point of view

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THE CASE OF A LARGE-MASS HIGGS; 14/15 MAY 2012; CERN

outline

Part 1

inroduction

- $i_1 \, i_2 \rightarrow H \rightarrow f_1 \, f_2$
- a diagrammatic point of view

Part 2

with M. Beneke, S. Chapovsky, G. Zanderighi

- toy model
- an effective-theory approach

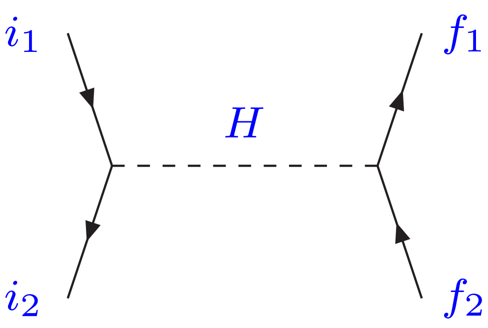
Part 3

with C. Anastasiou, F. Dulat, B. Mistlberger, Z. Kunszt

- from toy model towards the Standard Model
- under construction

part 1

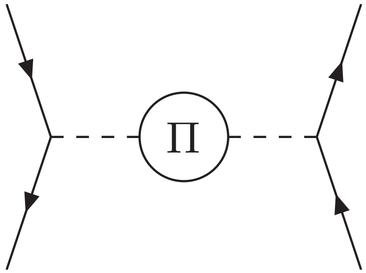
- Consider process $i_1\, i_2 \to H \to f_1\, f_2$ with additional constraint $s \equiv (p_{f_1} + p_{f_2})^2 \sim M_H^2$.
- observable is not H, but $f_1\, f_2$ pair with invariant mass $s \sim M_H^2$.
- two small parameters: α and $\delta \equiv \dfrac{s - M_H^2}{M_H^2}$
- hierarchy of scales $(s - M_H^2) \ll s \sim M_H^2$ is the feature, not gauge invariance (gauge invariance has to be automatic)
- systematically (double) expand in $\delta \sim \alpha \ll 1$ and do not worry about gauge invariance
- start with the tree-level diagram



$$\sim \frac{1}{s - M_H^2} \sim \frac{1}{\delta} \quad \Rightarrow \quad \text{LO}$$

resummation of self-energy $\Pi(s, M_H^2, m_X^2)$

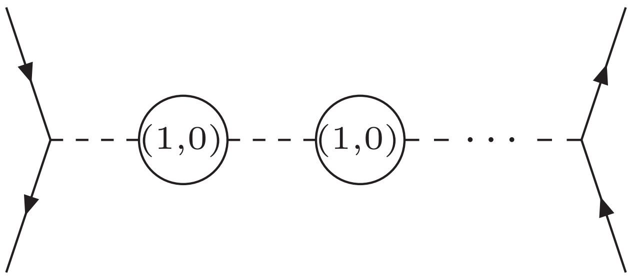
usual problem:



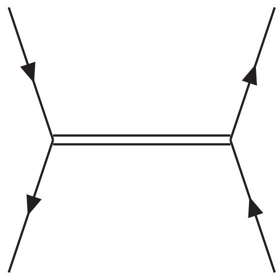
$$\sim \frac{1}{s - M_H^2}\,\Pi\,\frac{1}{s - M_H^2} \sim \frac{1}{\delta}\,\alpha\,\frac{1}{\delta} \sim \frac{1}{\delta} \quad \Rightarrow \text{LO}$$

expansion in α and δ: $$\Pi(s, M_H^2, m_X^2) = \sum_{n=1} \alpha^n \sum_{m=0} \delta^m\, \Pi^{(n,m)}(M_H^2, m_X^2)$$

only leading part $\alpha\Pi^{(1,0)}(M_H^2, m_X^2)$ needs to be resummed



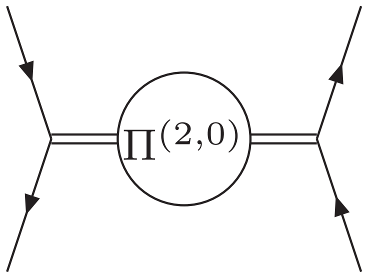
$\Rightarrow$



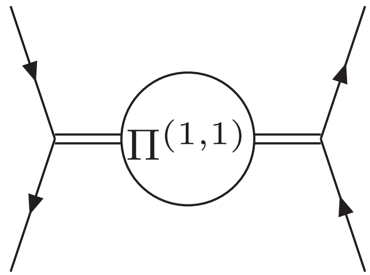
$\Rightarrow$ LO

$$\frac{1}{s - M_H^2}\,\alpha\Pi^{(1,0)}\,\frac{1}{s - M_H^2}\,\alpha\Pi^{(1,0)} \ldots \quad \Rightarrow \quad \frac{1}{s - M_H^2 - \alpha\Pi^{(1,0)}}$$ (gauge invariant)

Propagator insertions beyond LO

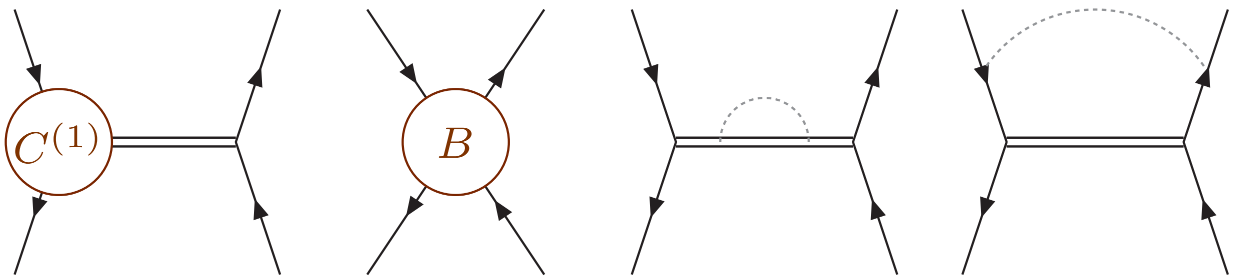


$\sim \frac{1}{\delta}\,\alpha^2\,\frac{1}{\delta} \sim \frac{\alpha}{\delta}$



$\sim \frac{1}{\delta}\,\alpha\delta\,\frac{1}{\delta} \sim \frac{\alpha}{\delta}$ $\Rightarrow$ NLO

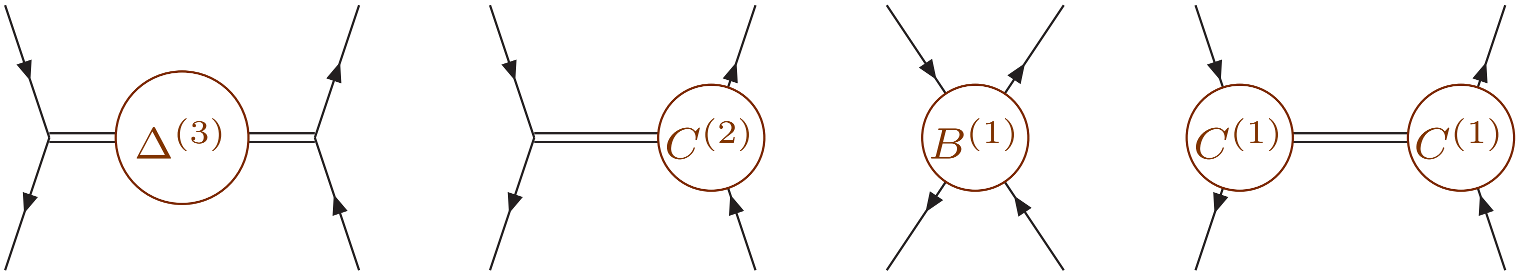
There are additional NLO corrections, have to distinguish between hard $k \sim M$ and soft $k \ll M$



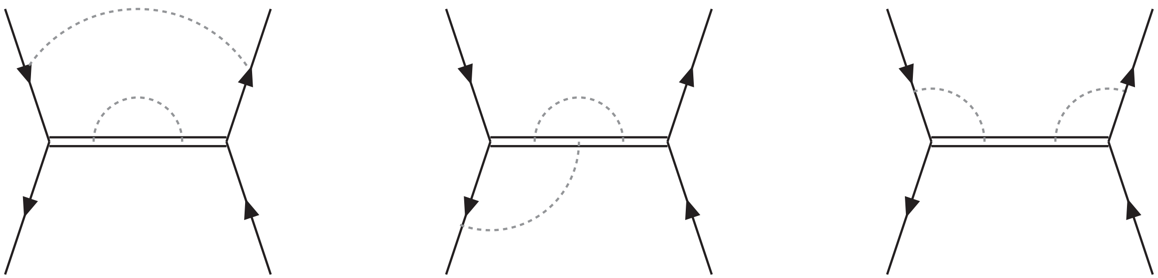
$\sim \frac{1}{\delta}\,\alpha$ $\Rightarrow$ NLO

- the hard contributions are separately gauge invariant
- the sum of all soft contributions is also gauge invarian
- this is not a concidence, but is due to an underlying stucture $\mathcal{L}_{\rm eff} = \sum c_i\, O_i$

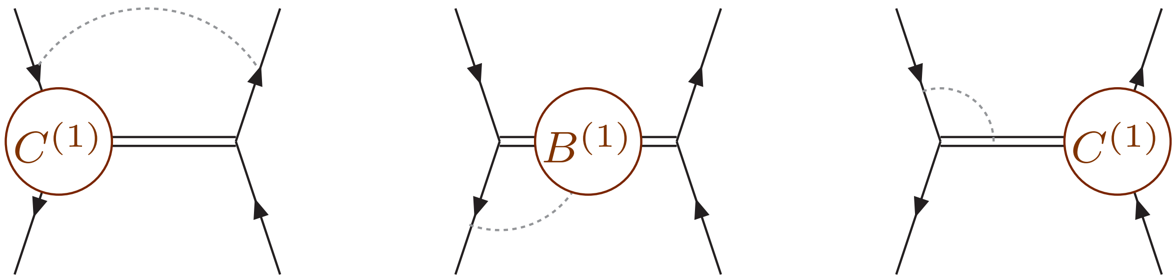
At NNLO there are purely hard contributions . . .



. . . purely soft contributions . . .



. . . and mixed hard and soft contributions.



- each hard coefficient is separately gauge independentt factorizable corrections
- gauge dependence of soft contributions cancel in the sum of all soft diagrams non-factorizable corrections

- split into hard and soft cannot be done on a diagrammatic level, certain Feynman diagrams contribute to hard and soft
- use method of regions to split into hard and soft [Beneke, Smirnov] and follow usual effective-theory procedure (HQET, NRQCD, SCET)
- use effective theory methods to systematically expand in $\alpha \sim \delta \sim \Gamma/m$ [Chapovsky, Khoze, AS, Stirling]
- identify relevant modes (usually more than simply hard and soft, depends on details of observable) $\to$ asymptotic expansion [Beneke, Chapovsky, AS, Zanderighi]
- integrate out ‘unwanted’ modes $\to$ tower of effective theories (Unstable Particle Effective Theory)
- hard effects correspond to factorizable corrections
- non-factorizable corrections due to still dynamical modes
- this is neither a “quick-fix” nor a “free lunch”, it is a method to identify the minimal amount of calculation to be done for a systematic expansion in the small parameters
- gauge invariance is automatic since the split into the various contributions respects gauge invariance

Toy model with charged (under massless $U(1)$) Higgs, massless electron and massless neutrino [Beneke, Chapovsky, AS, Zanderighi]

- Lagrangian:

$$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - \hat{M}^2\phi^\dagger\phi + \overline{\psi} i \not{D}\psi + \overline{\chi} i \not{\partial}\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + y\phi\overline{\psi}\chi + y^*\phi^\dagger\overline{\chi}\psi - \frac{\lambda}{4}\left(\phi^\dagger\phi\right)^2 - \mathcal{L}_{\rm ct}$$

- Process:

$$\bar{\nu}(p_1)e^-(p_2) \to \phi \to X$$

with $s - \hat{M}^2 \sim M\Gamma$. Use optical theorem and compute $\mathrm{Im}\,\mathcal{T}$

- scales: decay time $1/M$, lifetime $1/\Gamma \gg 1/M$
- counting: $\alpha_g = \frac{g^2}{4\pi} \sim \alpha_y = \frac{yy^*}{4\pi} \sim \delta \equiv \frac{s-\hat{M}^2}{\hat{M}^2}$ and $\frac{\alpha_\lambda}{4\pi} = \frac{\lambda}{(4\pi)^2} \sim \frac{\alpha_g^2}{4\pi}$
- expand in $\alpha \sim \alpha_g \sim \alpha_y$ and $\delta \sim \Gamma/M \sim \alpha$ “at Lagrangian level”
- fermions: SCET; scalar (higgs): H”Q”ET

underlying theory

$\mathcal{L}(\phi_h, \phi_c, \phi_s)$

dynamical modes:

hard, collinear, soft

integrate out hard modes

factorizable corrections

non-factorizable corrections

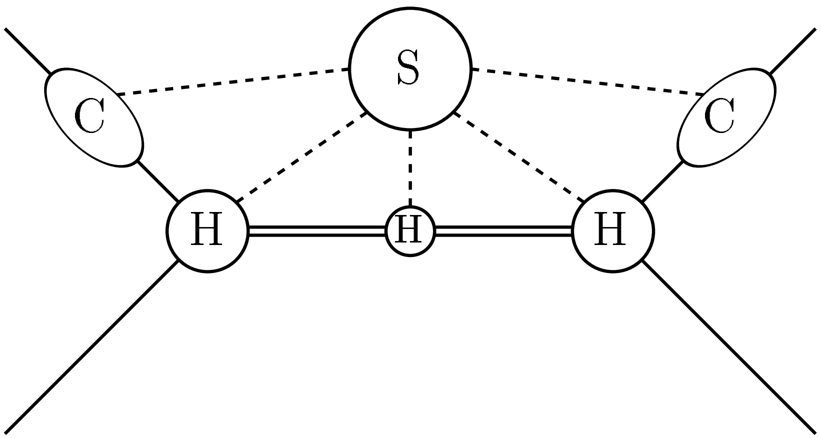
effective theory

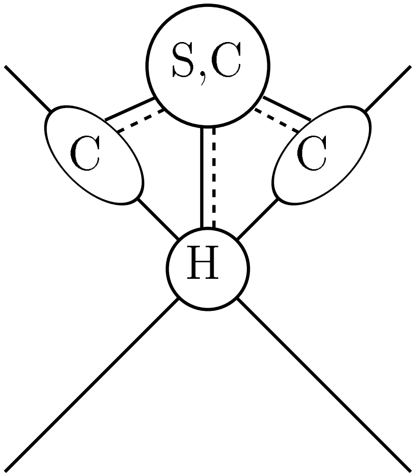
$\mathcal{L} = \sum_n c_n(\text{h})\, O_n(\phi_c, \phi_s)$

dynamical modes:

collinear, soft

Soft-Collinear Effective Theory	+	Heavy "Quark" Effective Theory
fermions		higgs
$p^\mu = (n_+p)\frac{n_-}{2} + (n_-p)\frac{n_+}{2} + p_\perp$		$q^\mu = Mv^\mu + k^\mu; \quad q_\top = q^\mu - v^\mu(qv)$
$n_\pm^2 = 0, \quad n_+n_- = 2$		$v^\mu \equiv (q_1^\mu + q_2^\mu)/\sqrt{s}, \quad v^2 = 1$
hard: $p \sim M$		hard: $k^\mu \sim M$
(u)soft: $p \sim M\delta$		soft: $k^\mu \sim \delta$
collinear: $p_\perp \sim M\delta^{1/2};\ n_+p \sim M;\ n_-p \sim M\delta$		





The effective Lagrangian for the NLO line shape:

$$\begin{aligned}\mathcal{L}_{\rm eff} \;&=\; -\frac{1}{4}F_{\rm s}^{\mu\nu}F_{{\rm s}\mu\nu} + 2\hat{M}\,\phi_v^\dagger\left(i(vD_{\rm s}) - \frac{\Delta}{2}\right)\phi_v + 2\hat{M}\,\phi_v^\dagger\left(\frac{iD_{{\rm s}\top}^2}{2\hat{M}} + \frac{\Delta^2}{8\hat{M}}\right)\phi_v \\ &+\; \overline{\psi}_{\rm s} i\not{D}_{\rm s}\psi_{\rm s} + \overline{\chi}_{\rm s} i\not{\partial}\chi_{\rm s} + \overline{\psi}_{n-}\left(in_-D + \not{D}_{{\rm c}\top}\frac{i}{n_+D_{\rm c}}\not{D}_{{\rm c}\top}\right)\psi_{n-} \\ &+\; C\left(y\,\phi_v\bar{\psi}_{n-}\chi_{n+} + y^*\,\phi_v^\dagger\bar{\chi}_{n+}\psi_{n-}\right) + \frac{yy^*B}{4\hat{M}^2}\left(\bar{\psi}_{n-}\chi_{n+}\right)\left(\bar{\chi}_{n+}\psi_{n-}\right) + \ldots\end{aligned}$$

Matching coefficients (contain hard effects)

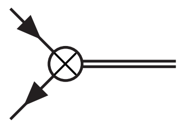
- $\Delta \equiv (\bar{s} - \hat{M}^2)/\hat{M} = \sum_i \Delta^{(i)}$

  $= \hat{M}\,\Pi^{(1,0)} + \hat{M}\left(\Pi^{(2,0)} + \Pi^{(1,1)}\Pi^{(1,0)}\right) + \ldots$

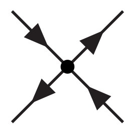
  

  In the pole scheme: $\Delta = -i\Gamma$

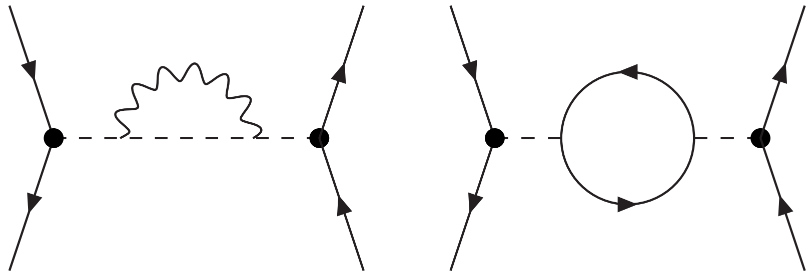
- $C = 1 + \alpha\, C^{(1)} + \ldots$

- $B = 1 + \alpha B^{(1)} + \ldots$

part 2

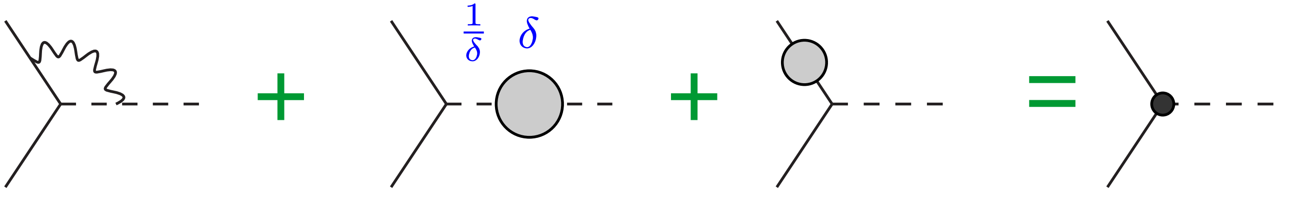
Consider self-energy diagrams  + higher orders

split self-energy into hard and soft part $\Pi(s) = \Pi_h(s) + \Pi_s(s)$ and expand the hard part of the self energy $\Pi_h(s) = \hat{M}^2 \sum \alpha^k \delta^l \Pi^{(k,l)}$

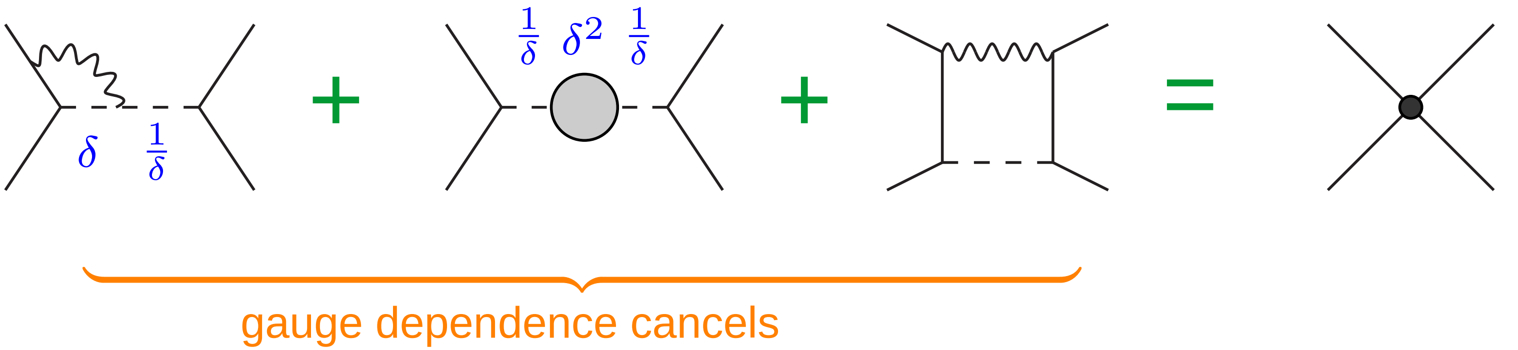
- $\Pi^{(1,0)}$ (gauge independent) $\rightarrow$ $\Delta^{(1)}$ (LO, Propagator)
- $\Pi^{(1,1)}$ (gauge dependent) $\rightarrow$ $C^{(1)}$ (NLO)
- $\Pi^{(1,2)}$ (gauge dependent) $\rightarrow$ $B^{(1)}$ (NNLO)
- $\Pi^{(2,0)}$ and $\Pi^{(1,0)}\Pi^{(1,1)}$ (separately gauge dependent) $\rightarrow$ $\Delta^{(2)}$ (NLO, gauge independent)
- Π_s (gauge dependent) $\rightarrow$ diagram in effective theory (NLO)

Matching of C (in $\overline{\mathrm{MS}}$ scheme)

$$C =$$
$$1 + \frac{\alpha_y}{4\pi}\left[\ln\frac{M^2}{\mu^2} - \frac{1}{4} - \frac{i\pi}{2}\right] + \frac{\alpha_g}{4\pi}\left[-\frac{1}{\bar\epsilon^2} + \frac{1}{\bar\epsilon}\left(\ln\frac{M^2}{\mu^2} - \frac{5}{2}\right) - \frac{1}{2}\ln^2\frac{M^2}{\mu^2} + \frac{7}{4}\ln\frac{M^2}{\mu^2} - \frac{15}{4} - \frac{\pi^2}{12}\right]$$



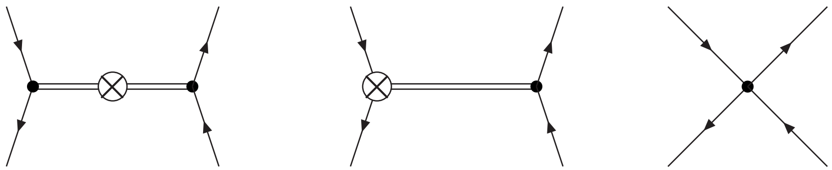
Matching of B at order α (contributes at NNLO)



Explicit results in $\overline{\rm MS}$ scheme

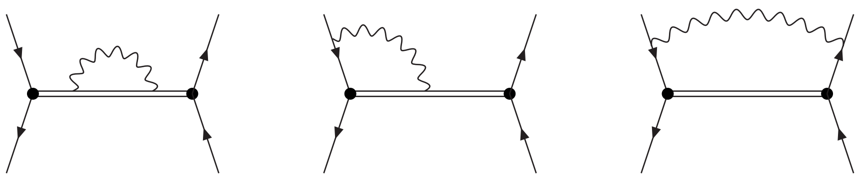
$$
\begin{aligned}
\frac{\Delta^{(1)}}{\hat{M}} &= a_g \left(-3\ln\frac{\hat{M}^2}{\mu^2} + 7\right) + a_y \left(2\ln\frac{\hat{M}^2}{\mu^2} - 4 - 2\,i\pi\right) \\
\frac{\Delta^{(2)}}{\hat{M}} &= a_g^2 \left(8\,\ln^2\frac{\hat{M}^2}{\mu^2} + \frac{16}{3}\ln\frac{\hat{M}^2}{\mu^2} - \frac{193}{4} + \frac{40\pi^2}{3} - 16\pi^2\log(2) + 24\zeta(3)\right) \\
&+ a_y^2 \left(\ln^2\frac{\hat{M}^2}{\mu^2} - \left(11 + 10\,i\pi\right)\ln\frac{\hat{M}^2}{\mu^2} + \frac{89}{4} - \frac{23\pi^2}{3} + 13\,i\pi\right) \\
&+ a_g a_y \left(-9\ln^2\frac{\hat{M}^2}{\mu^2} + \left(31 + 12\,i\pi\right)\ln\frac{\hat{M}^2}{\mu^2} - \frac{115}{4} + 5\pi^2 - 24\zeta(3) - 41\,i\pi + \frac{8\,i\pi^3}{3}\right) \\
&+ a_\lambda \left(\ln\frac{\hat{M}^2}{\mu^2} - 1\right) \\
C^{(1)} &= a_y \left(\log\frac{\hat{M}^2}{\mu^2} - \frac{1}{4} - \frac{i\pi}{2}\right) \\
&+ a_g \left(-\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\ln\frac{\hat{M}^2}{\mu^2} - \frac{5}{2}\right) - \frac{1}{2}\ln^2\frac{\hat{M}^2}{\mu^2} + \frac{7}{4}\ln\frac{\hat{M}^2}{\mu^2} - \frac{15}{4} - \frac{\pi^2}{12}\right)
\end{aligned}
$$

Forward scattering amplitude at NLO:



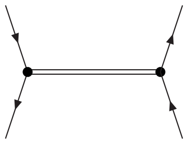
$$i\,\mathcal{T}_h^{(1)} = i\,\mathcal{T}^{(0)} \times$$

$$\left(2\,C^{(1)} - \frac{[\Delta^{(1)}]^2}{8\mathcal{D}\hat{M}} + \frac{\Delta^{(2)}}{2\mathcal{D}} - \frac{\mathcal{D}}{2\hat{M}} \right)$$



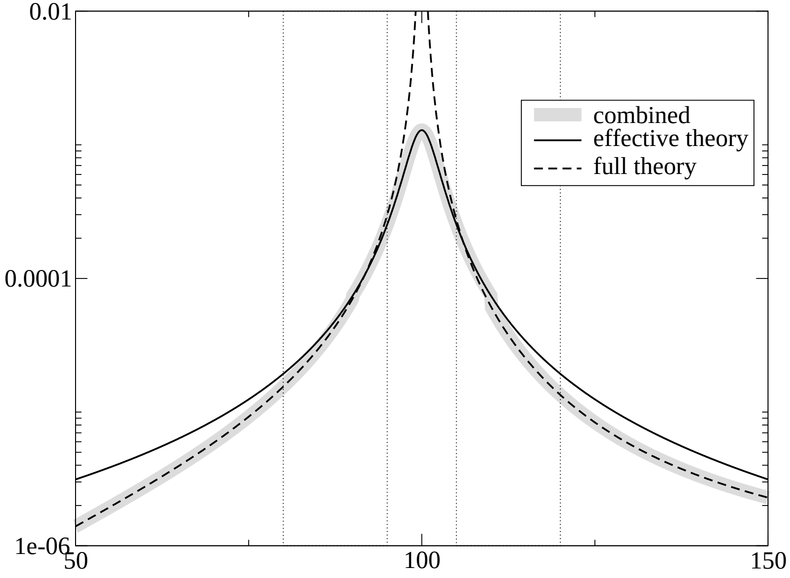
$$i\,\mathcal{T}_s^{(1)} = i\,\mathcal{T}^{(0)}\,\frac{\alpha_g}{4\pi}\left(\frac{-2\mathcal{D}}{\mu}\right)^{-2\epsilon} \times$$

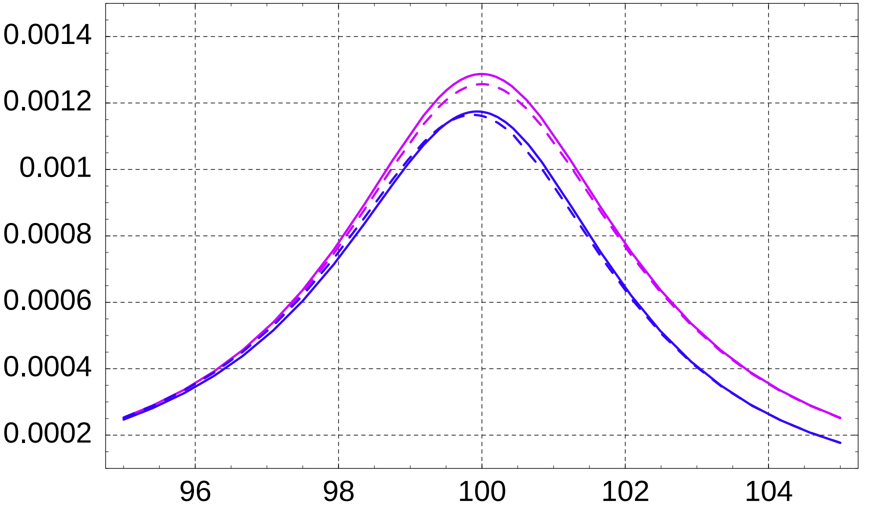
$$\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon} + 4 + \frac{5\pi^2}{6} \right)$$

where  $= i\,\mathcal{T}^{(0)} = \frac{-yy^* s}{4\hat{M}\mathcal{D}}$ with $\mathcal{D} \equiv \sqrt{s} - \hat{M} - \frac{\Delta^{(1)}}{2}$

poles $1/\epsilon$ cancel when adding soft and hard contributions (up to initial state collinear singularity)

Partonic cross section for $M = 100$ GeV as a function of $\sqrt{s}$.





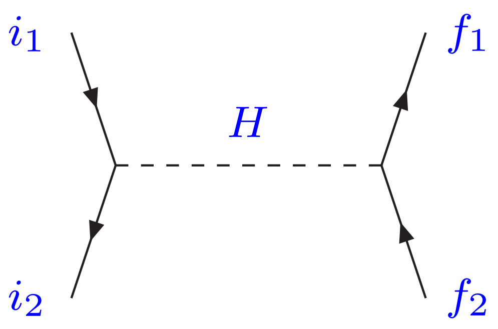
full range of $\sqrt{s}$: matching of resonant to off-resonant cross section

resonant region: LO vs. NLO for pole and $\overline{\mathrm{MS}}$ scheme

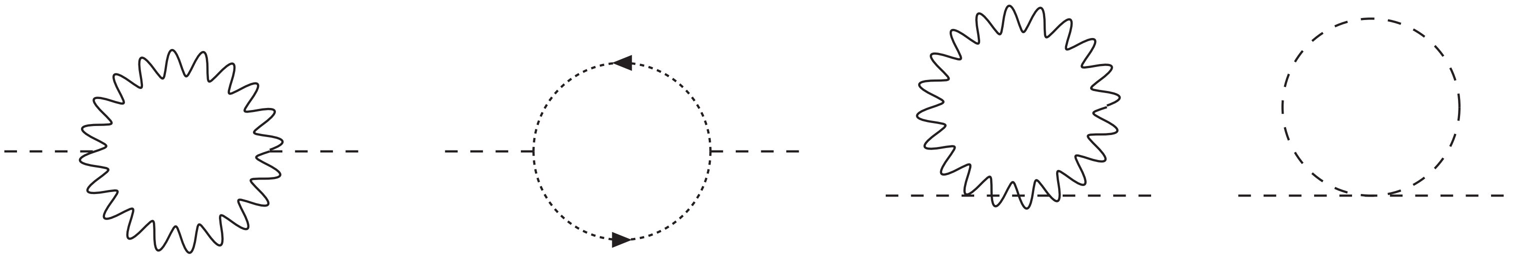
From toy model to Standard Model

- effective theory relies on the fact that all scales are explicit
- main issue in Standard Model: apart from M_H (hard) and $\delta \cdot M_H = (s - M_H)^2/M_H$ (soft) there are additional scales $M_W, M_Z, \xi M_W, \xi M_Z, M_t \ldots$
- external particles not necessarily massless
- structure of effective theory and method of region has to be adapted
- consider: $i_1\, i_2 \to H \to f_1\, f_2$

  with $(i_1, i_2) \in \{(g,g), (b,\bar{b})\}$ and $(f_1, f_2) \in \{(b,\bar{b}), (t,\bar{t}), (Z,Z), (W^+, W^-) \ldots\}$



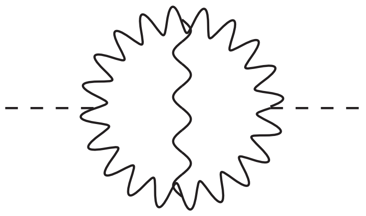
At LO we need hard part of one-loop self energy:



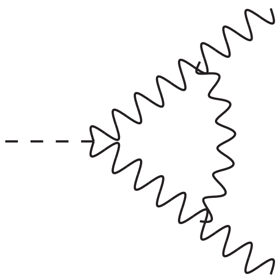
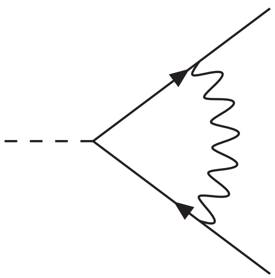
$+ \dots$

- expansion of tadpole and bubble integrals into hard $(k \sim M_H)$ and soft $(k \ll M_H)$, assuming $M_Z \sim \xi M_Z M_W \sim \xi M_W \sim\ll M_H$ reproduces full integrals
- these contrbutions, i.e. $\Delta^{(1)}$ are process independent (and obviously gauge independent) and have to be resummed $\Rightarrow$ effective-theory propagator
- the only (trivial) dependece on the process is in the tree-level vertices $H \to X\,Y$

At NLO we need hard part of two-loop self energy for process independent $\Delta^{(2)}$ ...

for example:  involves scales: M_H, $(\xi)M_W$ and $m_\gamma = 0$

... hard part of one-loop vertices for process dependent $C^{(1)}$ ...

for example:  

... and process dependent soft contributions

- effective theories are a proven tool for processes with kinematic constraints such as $s \sim M^2$
  - systematic expansion in all small quantities
  - resummation always through renormalization-group equations
- this offers a complementary approach, useful for comparison and cross checks
- application of effective-theory methods to the full Standard Model in the case of Higgs production has a number of additional complications
  - additional scales
  - massive external particles
  - more involved structure of effective theory
- goal: description of most relevant processes at NLO (in effective theory counting) this requires:
  - two-loop self-energy process independent
  - one-loop vertices process dependent
  - four-point vertices process dependent
  - soft (non-hard) contributions process dependent