MC modeling for Heavy Ions

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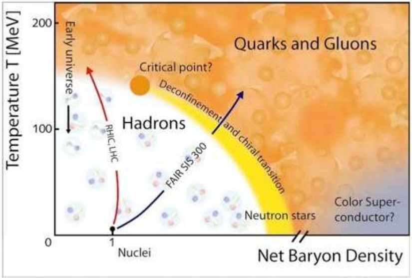
Outline

- The motivation: exploring the QCD phase diagram
- Virtual experiment: lattice-QCD simulations
- Real experiments: heavy-ion collisions
  - Collision geometry (Glauber model)
  - Evolution of the produced medium (hydrodynamics)
- "External" probes of the medium:
  - Heavy flavor: relaxation to thermal equilibrium
  - Jet quenching: *medium-induced* parton branchings

Throughout my lecture I will try to stress the role of numerical simulations and Monte Carlo tools, emphasizing – when possible – analogies/differences with *pp* collisions

Heavy-ion collisions: exploring the QCD phase-diagram



QCD phases identified through the *order parameters*

- Polyakov loop $\langle L\rangle \sim$ energy cost to add an isolated color charge
- Chiral condensate $\langle\overline{q}q\rangle \sim$ effective mass of a "dressed" quark in a hadron

Region explored at LHC: *high-T/low-density* (early universe, $n_B/n_\gamma \sim 10^{-9}$)

- From QGP (color deconfinement, chiral symmetry restored)
- to hadronic phase (confined, chiral symmetry breaking[1])

NB $\langle\overline{q}q\rangle \neq 0$ responsible for most of the baryonic mass of the universe: *only* $\sim$*35 MeV of the proton mass from* $m_{u/d} \neq 0$

[1] V. Koch, *Aspects of chiral symmetry*, Int.J.Mod.Phys. E6 (1997)

Virtual experiments: lattice-QCD simulations

- The best (unique?) tool to study QCD in the non-perturbative regime
- Limited to the study of equilibrium quantities

QCD on the lattice

The QCD partition function

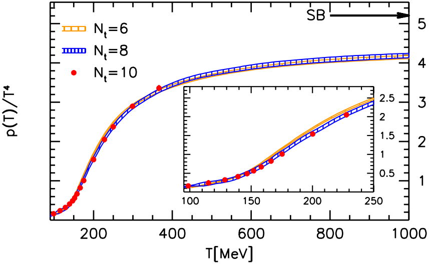
$$\mathcal{Z} = \int [dU] \exp\left[-\beta S_g(U)\right] \prod_q \det\left[M(U, m_q)\right]$$

is evaluated on the lattice through a MC sampling of the field configurations, where

- $\beta = 6/g^2$
- S_g is the gauge action, weighting the different field configurations;
- $U \in SU(3)$ is the link variable connecting two lattice sites;
- $M \equiv \gamma_\mu D_\mu + m_q$ is the Dirac operator

QCD on the lattice: results

From the partition function on gets all the thermodynamical quantities[2]:



- Pressure: $P = (T/V)\ \ln \mathcal{Z}$;

[2]Wuppertal group, JHEP 1011 (2010) 077

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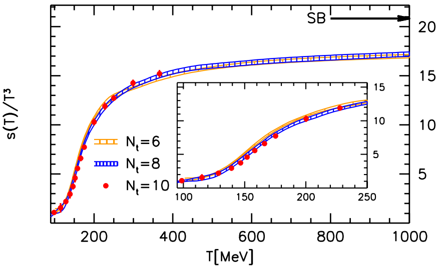

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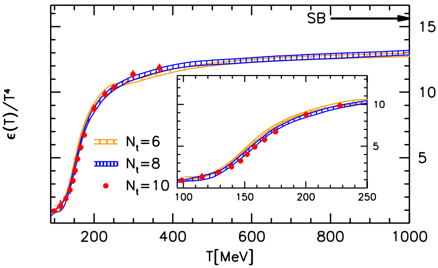

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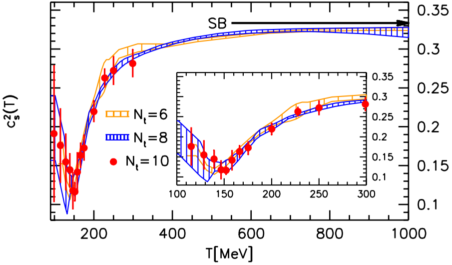

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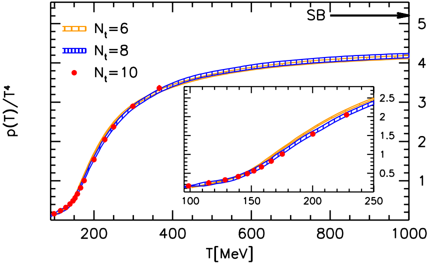

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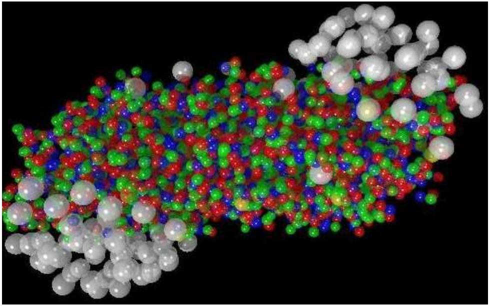
- Rapid rise in thermodynamical quantities suggesting a change in the *number of active degrees of freedom* (hadrons $\longrightarrow$ partons);
- One observes a systematic $\sim$20% deviation from the Stephan-Boltzmann limit even at large T: how to interpret it?

[2] Wuppertal group, JHEP 1011 (2010) 077

Real experiments: heavy-ion collisions

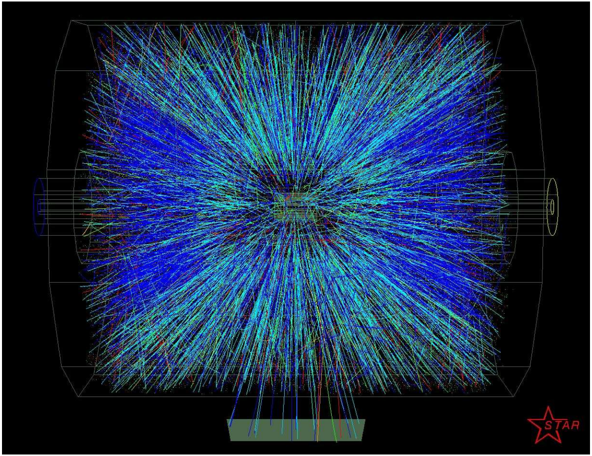
Heavy-ion collisions: a typical event



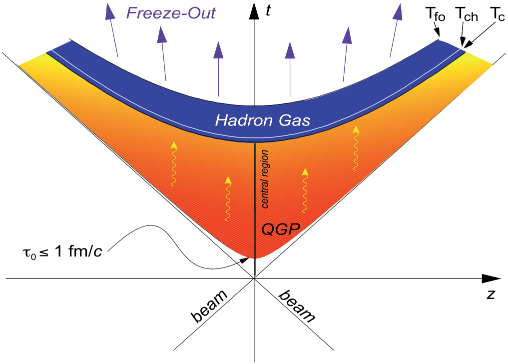
- Valence quarks of participant nucleons act as sources of strong color fields giving rise to *particle production*
- Spectator nucleons don't participate to the collision;

Almost all the energy and baryon number carried away by the remnants

Heavy-ion collisions: a typical event



Heavy-ion collisions: a cartoon of space-time evolution

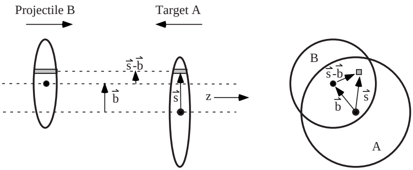


- Soft probes (low-p_T hadrons): collective behavior of the *medium*;
- Hard probes (high-p_T particles, heavy quarks, quarkonia): produced in *hard pQCD processes* in the initial stage, allow to perform a tomography of the medium

Collision Geometry: the Glauber Model

- For a nice overview: M.L. Miller *et al.*, nucl-ex/0701025;
- For some references to *pp* physics: T. Sjöstrand and M. van ZijL, PRD 36, 2019 (1987)

Glauber Model: outline



a) Side View b) Beam-line View

- Nuclei are extended/composite objects: they can cross at different impact parameter b and with a different number of elementary binary collisions $N_{\rm coll}$;
- the Glauber Model (optical or MC) is used to describe the geometry of the collision

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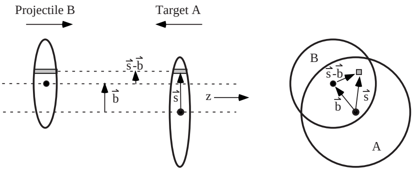

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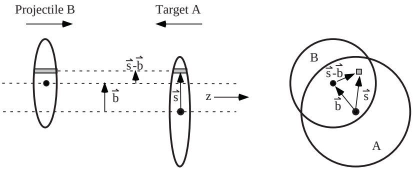
Modeling collision geometry important to interpret the data

- Thicker/denser medium going *from peripheral to central collisions* (higher particle multiplicity, larger jet quenching...);
- *Initial eccentricity and fluctuations* leave their fingerprints in final hadronic observables

Analogies with modeling of UE and MPI in *pp* collisions

Glauber Model: the optical limit



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- Nuclear "thickness function" [Area^{-1}]:

$$\widehat{T}_A(\mathbf{s}) \equiv \int dz_A \, \rho_A(\mathbf{s}, z_A)$$

- Nuclear "overlap function" [Area^{-1}]:

$$\widehat{T}_{AB}(\mathbf{b}) \equiv \int d\mathbf{s} \, \widehat{T}_A(\mathbf{s}) \, \widehat{T}_B(\mathbf{s} - \mathbf{b})$$

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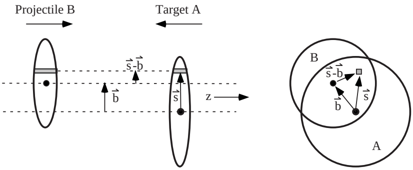

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- Probability of elementary inelastic collision: $p_{\rm coll}^{NN}(b) = \sigma_{\rm in}^{NN}\, \widehat{T}_{AB}(b)$
- Collisions *at a given impact parameter b* is described by a *binomial distribution*:

$$P(n,b) = \binom{AB}{n} [p_{\rm coll}^{NN}(b)]^n [1 - p_{\rm coll}^{NN}(b)]^{AB-n}$$

Glauber Model: results in the optical limit

- Number of binary collisions (per $A-B$ crossing, $\sum_{n=0}^{AB} P(n,b)=1$):

$$N_{\rm coll}(b)=\sum_{n=1}^{AB} n\,P(n,b)=AB\,\widehat{T}_{AB}(b)\,\sigma_{\rm in}^{NN}$$

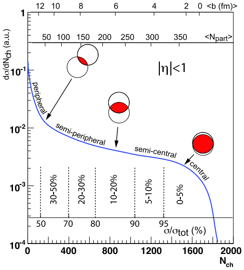
- Number of participants:

$$N_{\rm part}(b)=A\int d\mathbf{s}\widehat{T}_A(\mathbf{s})\left\{1-[1-\widehat{T}_B(\mathbf{s}-\mathbf{b})\sigma_{\rm in}^{NN}]^B\right\}$$
$$+B\int d\mathbf{s}\widehat{T}_B(\mathbf{s}-\mathbf{b})\left\{1-[1-\widehat{T}_A(\mathbf{s})\sigma_{\rm in}^{NN}]^A\right\}$$

- Total inelastic cross section $\sigma_{\rm in}^{AB}=\int_0^\infty 2\pi\,bdb\,p_{\rm in}^{AB}(b)$ obtained integrating the *probability of having at least one inelastic interaction*

$$p_{\rm in}^{AB}(b)=\sum_{n=1}^{AB} P(n,b)=1-[1-\widehat{T}_{AB}(b)\sigma_{\rm in}^{NN}]^{AB}$$

Glauber Model: centrality classes



- *Centrality classes* defined from measured $dN_{\mathrm{evt}}/dN_{\mathrm{ch}}$, dividing total inelastic cross-section in percentiles;

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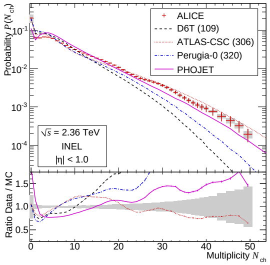

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- Analogous observable considered in UE studies in *pp* collisions, and used for MC-tunes

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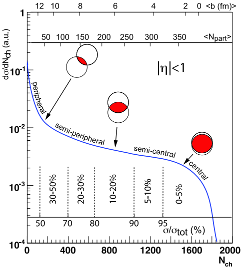

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- Which is the range of impact parameters (*to use in a theory calculation*!) corresponding to a given centrality class?
- A simple geometrical picture arises from the Glauber Model, e.g.

$$\frac{\int_0^{b_{0.1}} bdb\{1-[1-\widehat{T}_{AB}(b)\sigma_{\rm in}^{NN}]^{AB}\}}{\int_0^{\infty} bdb\{1-[1-\widehat{T}_{AB}(b)\sigma_{\rm in}^{NN}]^{AB}\}} = 0.1$$

defines the 0-10% centrality class

Glauber model for hard processes

Hard pQCD processes ($c\bar{c}$ production, high-p_T scattering...) scale with $N_{\rm coll}$, hence the interest of estimating $\langle N_{\rm coll} \rangle$ in a given centrality class

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- Binary collisions *per inelastic event at given b*:

$$N_{\rm coll}^{\rm in.evt}(b) = N_{\rm coll}(b)/p_{\rm in}^{AB}(b)$$

(distinction relevant only for very peripheral events)

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- Average over all inelastic events at different b:

$$\langle N_{\rm coll}\rangle_{b_1-b_2} \equiv \frac{\int_{b_1}^{b_2} bdb\, N_{\rm coll}^{\rm in.evt}(b)\, p_{\rm in}^{AB}(b)}{\int_{b_1}^{b_2} bdb\, p_{\rm in}^{AB}(b)} = \frac{\int_{b_1}^{b_2} bdb\, N_{\rm coll}(b)}{\int_{b_1}^{b_2} bdb\, p_{\rm in}^{AB}(b)}$$

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One can then compare hard observables in *AA* collisions with a proper *rescaled pp* benchmark

Modeling of MPI in *pp*: some similarities

In QCD $\sigma_{\text{hard}}(p_T^{\text{min}}) > \sigma_{\text{tot}}^{\text{pp}}$ for small p_T^{min};
paradox solved by multiple interactions: $\langle n(p_T^{\text{min}})\rangle = \sigma_{\text{hard}}(p_T^{\text{min}})/\sigma_{\text{ND}}$

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- Interactions at given b assumed to follow a Poisson distribution

$$P_n(b) = \frac{[\overline{n}(b)]^n}{n!}\exp[-\overline{n}(b)], \quad \text{with} \quad \overline{n}(b) = k\,\underbrace{\mathcal{O}(b)}_{\text{overlap}}$$

NB: Poisson vs Binomial distribution in AB collisions

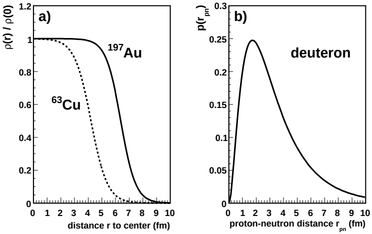
- Number of interactions *per inelastic event at given b*:

$$\langle n(b)\rangle = \frac{\overline{n}(b)}{p_{\rm in}(b)} = \frac{k\mathcal{O}(b)}{1-\exp[-k\mathcal{O}(b)]}$$

- Average number of interactions *per inelastic event*:

$$\langle n\rangle = \frac{\int bdb\,\langle n(b)\rangle\,p_{\rm in}(b)}{\int bdb\,p_{\rm in}(b)} = \frac{\int bdb\,\overline{n}(b)}{\int bdb\,p_{\rm in}(b)} = \frac{\sigma_{\rm hard}}{\sigma_{\rm ND}}$$

Glauber Model: Monte Carlo implementation



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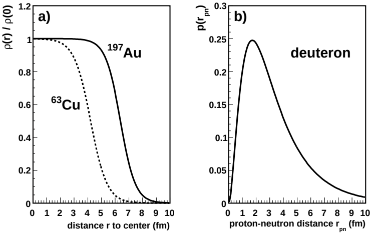

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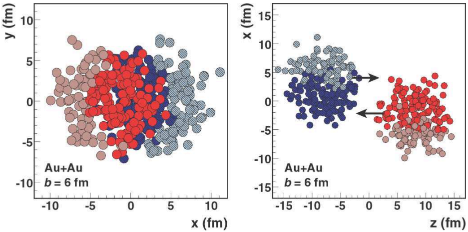

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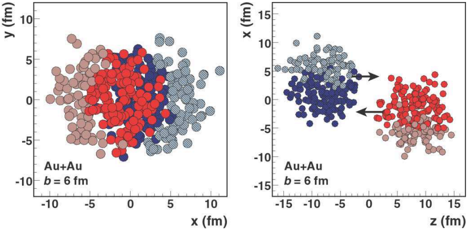

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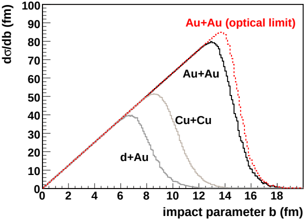
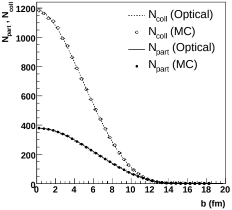
- Overall agreement except for most peripheral collisions;
- MC-Glauber provides *more granular* initial conditions





Glauber model provides the initial for...
...medium evolution: hydrodynamics

Some references...

- J.Y. Ollitrault, "*Phenomenology of the little bang*", J.Phys.Conf.Ser. 312 (2011) 012002;
- J.Y. Ollitrault, "*Relativistic hydrodynamics for heavy-ion collisions*", Eur.J.Phys. 29 (2008) 275-302
- U.W. Heinz, "Hydrodynamic description of ultrarelativistic heavy ion collisions", in *Hwa, R.C. (ed.) et al.: Quark gluon plasma* 634-714

Hydrodynamics and heavy-ion collisions

The success of hydrodynamics in describing particle spectra in heavy-ion collisions measured *at RHIC came as a surprise*!

- The general setup and its implications
- Predictions
  - Radial flow
  - Elliptic flow
- What can we learn?
  - Initial conditions
  - Event-by-event fluctuations and consequences

Hydrodynamics: the general setup

- Hydrodynamics is applicable in a situation in which $\lambda_{\rm mfp} \ll L$
- In this limit the behavior of the system is entirely governed by the *conservation laws*

$$\underbrace{\partial_\mu T^{\mu\nu} = 0}_{\rm four-momentum}\;,\qquad \underbrace{\partial_\mu j^\mu_B = 0}_{\rm baryon\ number}\;,$$

where

$$T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu}, \quad j^\mu_B = n_B u^\mu \quad {\rm and} \quad u^\mu = \gamma(1, \vec{v})$$

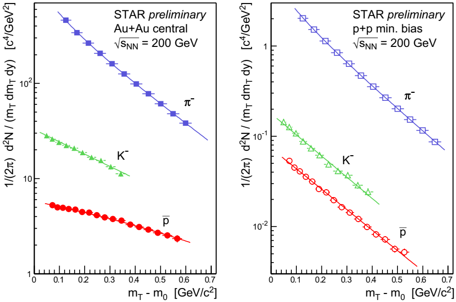
- Information on the medium is *entirely encoded* into the EOS

$$P = P(\epsilon)$$

- The transition from fluid to particles occurs at the *freeze-out hypersuface* $\Sigma^{\rm fo}$ (e.g. at $T = T_{\rm fo}$)

$$E(dN/d\vec{p}) = \int_{\Sigma^{\rm fo}} p^\mu d\Sigma_\mu \, \exp[-(p \cdot u)/T]$$

Hydro predictions: radial flow (I)



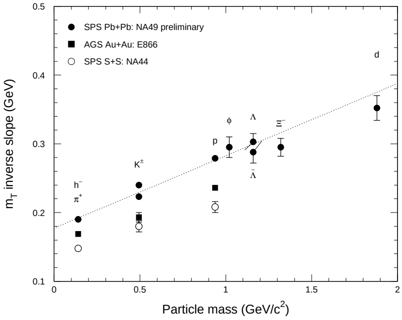
$$\frac{dN}{m_T dm_T} \sim e^{-m_T/\mathrm{T_{slope}}} \equiv e^{-\sqrt{p_T^2+m^2}/\mathrm{T_{slope}}}$$

- $T_{\mathrm{slope}}(\sim 167\,\mathrm{MeV})$ *universal* in pp collisions;
- T_{slope} *growing with m* in AA collisions: spectrum gets harder!

Hydro predictions: radial flow (II)

Physical interpretation:

Thermal emission on top of a collective flow



$$\frac{1}{2} m \langle \mathbf{v}_\perp^2 \rangle = \frac{1}{2} m \left\langle \left(\mathbf{v}_{\perp th} + \mathbf{v}_{\perp flow} \right)^2 \right\rangle$$

$$= \frac{1}{2} m \langle \mathbf{v}_{\perp th}^2 \rangle + \frac{1}{2} m \mathbf{v}_{\perp flow}^2$$

$$\Longrightarrow \quad T_{\rm slope} = T_{\rm fo} + \frac{1}{2} m \mathbf{v}_{\perp flow}^2$$

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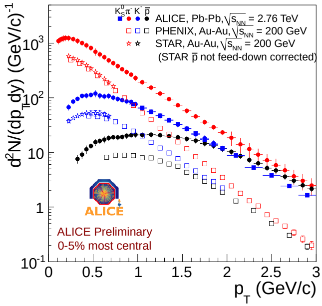
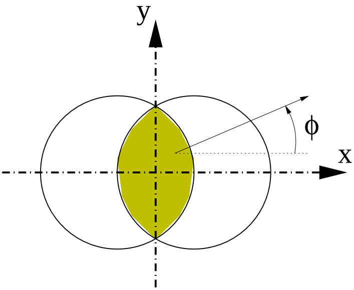


$$\frac{1}{2}m\langle \mathbf{v}_\perp^2\rangle = \frac{1}{2}m\left\langle\left(\mathbf{v}_{\perp th}+\mathbf{v}_{\perp flow}\right)^2\right\rangle = \frac{1}{2}m\langle \mathbf{v}^2_{\perp th}\rangle + \frac{1}{2}m\mathbf{v}^2_{\perp flow}$$

$$\Longrightarrow \quad T_{\rm slope} = T_{\rm fo} + \frac{1}{2}m\mathbf{v}^2_{\perp flow}$$

Radial flow gets larger going from RHIC to LHC!

Hydro predictions: elliptic flow



- In *non-central collisions* particle emission is not azimuthally-symmetric!

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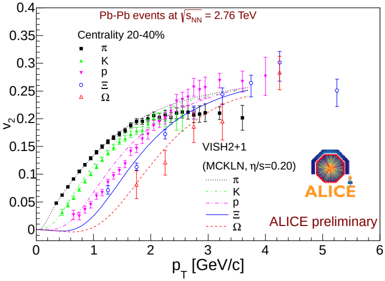


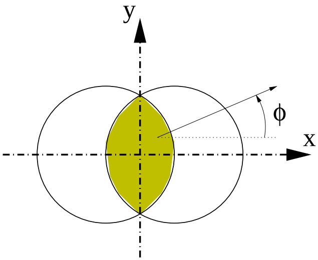
- In *non-central collisions* particle emission is not azimuthally-symmetric!
- The effect can be quantified through the *Fourier coefficient* v_2

$$\frac{dN}{d\phi} = \frac{N_0}{2\pi}\left(1 + 2v_2\cos[2(\phi - \psi_{RP})] + \dots\right)$$

$$v_2 \equiv \langle\cos[2(\phi - \psi_{RP})]\rangle$$

- $v_2(p_T) \sim 0.2$ gives a modulation 1.4 vs 0.6 for in-plane vs out-of-plane particle emission!

Elliptic flow: physical interpretation



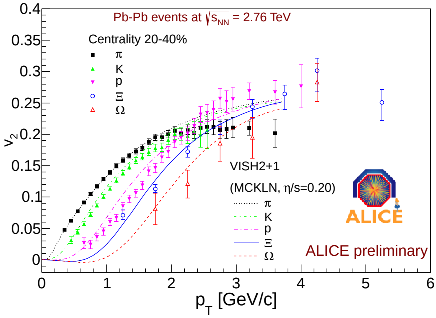
- Matter behaves like a fluid whose *expansion* is *driven by pressure gradients*

$$\frac{\partial}{\partial t}\left[(\epsilon + P)v^i\right] = -\frac{\partial P}{\partial x^i};$$

- Spatial anisotropy is converted into momentum anisotropy;
- At freeze-out particles are mostly emitted along the reaction-plane.

Elliptic flow: mass ordering

The mass ordering of v_2 is a direct consequence of the hydro expansion



- Particles emitted according to a thermal distribution $\sim \exp[-p \cdot u(x)/T_{\rm fo}]$ in the local rest-frame of the fluid-cell;
- Parametrizing the fluid velocity as

$$u^\mu \equiv \gamma_\perp(\cosh Y, \mathbf{u}_\perp, \sinh Y),$$

one gets ($v_z \equiv \tanh Y$)

$$p \cdot u = \gamma_\perp[m_\perp \cosh(y - Y) - \mathbf{p}_\perp \cdot \mathbf{u}_\perp]$$

- Dependence on m_T at the basis of *mass ordering* at fixed p_T

Initial conditions: “Bjorken” estimate

- It is useful to describe the evolution in term of the variables

$$\tau \equiv \sqrt{t^2 - z^2} \quad \text{and} \quad \eta_s \equiv \frac{1}{2} \ln \frac{t+z}{t-z}$$

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$$s \equiv \left.\frac{dS}{d\mathbf{x}_\perp dz}\right|_{z=0} = \frac{1}{\tau} \frac{dS}{d\mathbf{x}_\perp d\eta_s} \quad \longrightarrow \quad s\tau = \frac{dS}{d\mathbf{x}_\perp d\eta_s}$$

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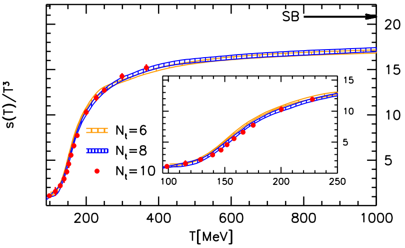
- Entropy is related to the *final multiplicity of charged particles* ($S \sim 3.6\,N$ for pions), so that (at decoupling $\eta \approx \eta_s$):

$$s_0 \approx \frac{1}{\tau_0} \frac{3.6}{\pi R_A^2} \frac{dN_{\rm ch}}{d\eta} \frac{3}{2}$$

"Bjorken" estimate: results

$$s_0 \approx \frac{1}{\tau_0} \frac{3.6}{\pi R_A^2} \frac{dN_{\rm ch}}{d\eta} \frac{3}{2}$$



- From $dN_{\rm ch}/d\eta \approx 1600$ measured by ALICE at LHC and $R_{\rm Pb} \approx 6$ fm one gets:

$$s_0 \approx (80\,{\rm fm}^{-2})/\tau_0$$

- τ_0 is found to be quite small (v_2 must develop early!):

$$0.1 \lesssim \tau_0 \lesssim 1\ {\rm fm} \longrightarrow 80 \lesssim s_0 \lesssim 800\ {\rm fm}^{-3}$$

- This should be compared with l-QCD

$$s(T=200\,{\rm MeV}) \approx 10\,{\rm fm}^{-3}$$

Initial conditions: Glauber model

Glauber model provides initial conditions for hydro. Taking as a guidance $s_0\tau_0 \approx s\tau$ one can assume the following "soft + hard" ansatz

$$s_0(\mathbf{x}) = \frac{C}{\tau_0}\left[\frac{1-\alpha}{2} n_{\text{part}}(\mathbf{x}) + \alpha n_{\text{coll}}(\mathbf{x})\right]$$

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- Optical Glauber:

$$n_{\rm part}(\mathbf{x}) = A\,\widehat{T}_A(\mathbf{x}+\mathbf{b}/2)\left\{1-[1-\widehat{T}_B(\mathbf{x}-\mathbf{b}/2)\sigma_{\rm in}^{NN}]^B\right\} +$$
$$+B\,\widehat{T}_B(\mathbf{x}-\mathbf{b}/2)\left\{1-[1-\widehat{T}_A(\mathbf{x}+\mathbf{b}/2)\sigma_{\rm in}^{NN}]^A\right\}$$
$$n_{\rm coll}(\mathbf{x}) = AB\,\sigma_{\rm in}^{NN}\,\widehat{T}_A(\mathbf{x}+\mathbf{b}/2)\widehat{T}_B(\mathbf{x}-\mathbf{b}/2)$$

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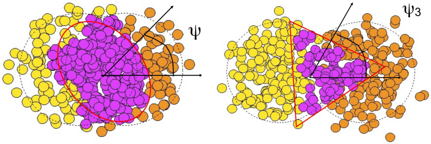
- MC-Glauber: one *counts* the number of participants/collisions within the area $\sigma_{\rm in}^{NN}$ centered at $\mathbf{x}$

$$n_{\rm part}(\mathbf{x}) = \frac{N_{\rm part}^A(\mathbf{x}) + N_{\rm part}^B(\mathbf{x})}{\sigma_{\rm in}^{NN}}, \quad n_{\rm coll}(\mathbf{x}) = \frac{N_{\rm coll}(\mathbf{x})}{\sigma_{\rm in}^{NN}}$$

Initial conditions: event-by-event fluctuations

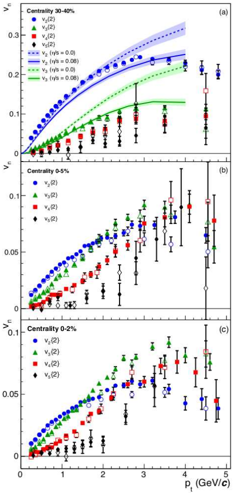
- Flow coefficients are defined as $v_n \equiv \langle\langle \cos[n(\phi - \Psi_n)] \rangle\rangle$.
- For hydro simulations with smooth initial conditions
  - $\Psi_n \equiv \Psi_{\mathrm{RP}}$ known exactly;
  - all odd-harmonics vanish.
- Real life is more complicated...



Odd harmonics appear, angles Ψ_n are not directly measured.

- Glauber-MC initial conditions mandatory to study these effects

Event-by-event fluctuations: experimental consequences



Fluctuating initial conditions giving rise to[a]:

- Non-vanishing v_2 in central collisions;
- Odd harmonics (v_3 and v_5)

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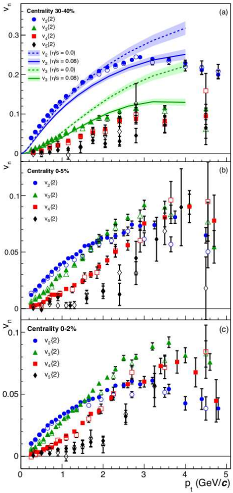
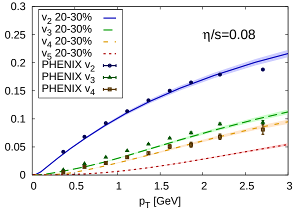


Fluctuating initial conditions giving rise to[a]:

- Non-vanishing v_2 in central collisions;
- Odd harmonics (v_3 and v_5)

Hydro can reproduce also higher harmonics[b]



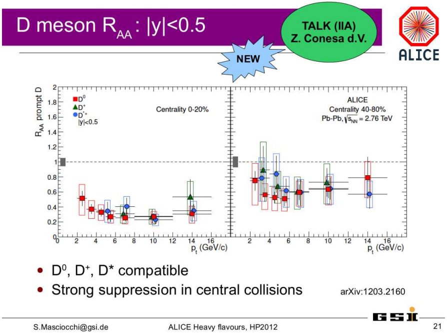
[a] ALICE, Phys.Rev.Lett. 107 (2011) 032301
[b] B: Schenke *et al.*, PRC 85, 024901 (2012)

Hard probes: outline

"External" *colored* particles produced in *hard* pQCD events (heavy quarks, high-p_T partons) allowing a *tomography of the medium*

- Experimental findings;
- Theory modeling and interpretation
  - Heavy flavor: stochastic dynamics of heavy quarks in the plasma; developing *tools allowing to describe approach to equilibrium*
  - Jet quenching: modeling of medium-induced parton branchings and modification of parton showers in a medium (angular distribution of gluon radiation, color connections...)

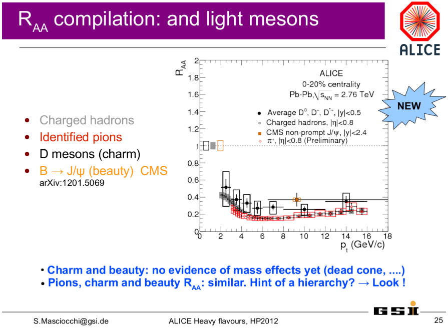
Experimental findings



$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

- Sizable *suppression* of D meson spectra;

Experimental findings



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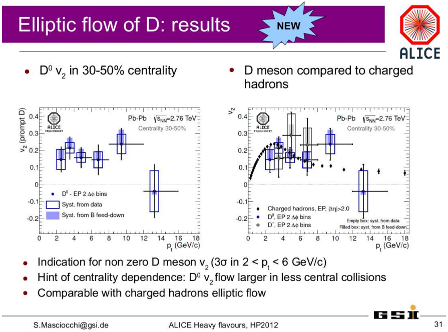

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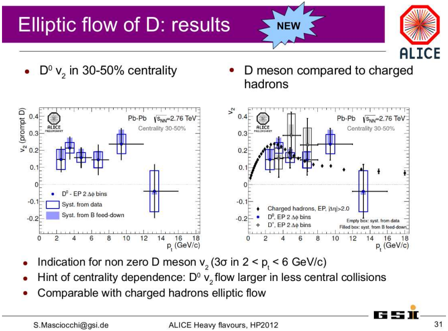

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Sizable v_2 observed for D mesons $\longrightarrow$ theoretical setup allowing to describe approach to thermalization

The Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t, \mathbf{x}, \mathbf{p})$:

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

- Total derivative along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{x}} + \mathbf{F} \frac{\partial}{\partial \mathbf{p}}$$

  Neglecting $\mathbf{x}$-dependence and mean fields: $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$

- Collision integral:

$$C[f_Q] = \int d\mathbf{k} [\underbrace{w(\mathbf{p}+\mathbf{k}, \mathbf{k}) f_Q(\mathbf{p}+\mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{k}) f_Q(\mathbf{p})}_{\text{loss term}}]$$

$w(\mathbf{p}, \mathbf{k})$: HQ transition rate $\mathbf{p} \to \mathbf{p} - \mathbf{k}$

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*[3] (Landau)

$$C[f_Q] \approx \int d\mathbf{k} \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p},\mathbf{k}) f_Q(t,\mathbf{p})]$$

[3] B. Svetitsky, PRD 37, 2484 (1988)

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The Boltzmann equation reduces to the *Fokker-Planck equation*

$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{k}\, k^i w(\mathbf{p}, \mathbf{k}) \longrightarrow A^i(\mathbf{p}) = A(p)\, p^i$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k}\, k^i k^j w(\mathbf{p}, \mathbf{k}) \longrightarrow B^{ij}(\mathbf{p}) = \hat{p}^i \hat{p}^j B_0(p) + (\delta^{ij} - \hat{p}^i \hat{p}^j) B_1(p)$$

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Problem reduced to the *evaluation of three transport coefficients*

[3] B. Svetitsky, PRD 37, 2484 (1988)

Physical interpretation

- *Ignoring the momentum dependence* of the transport coefficients $\gamma \equiv A(p)$ and $D \equiv B_0(p) = B_1(p)$ the FP equation reduces to

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- Starting from the *initial condition* $f_Q(t=0, \mathbf{p}) = \delta(\mathbf{p} - \mathbf{p}_0)$ one gets

$$f_Q(t, \mathbf{p}) = \left(\frac{\gamma}{2\pi D[1 - \exp(-2\gamma t)]} \right)^{3/2} \exp\left[-\frac{\gamma}{2D} \frac{[\mathbf{p} - \mathbf{p}_0 \exp(-\gamma t)]^2}{1 - \exp(-2\gamma t)} \right]$$

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- Asymptotically the solution *forgets about the initial condition* and tends to a thermal distribution

$$f_Q(t,\mathbf{p}) \underset{t\to\infty}{\sim} \left(\frac{\gamma}{2\pi D}\right)^{3/2} \exp\left[-\left(\frac{\gamma M_Q}{D}\right)\frac{\mathbf{p}^2}{2M_Q}\right]$$

$\longrightarrow D = M_Q \gamma T$: Einstein *fluctuation-dissipation* relation

The challenge: addressing the experimental situation

One needs a tool, *equivalent to the Fokker-Planck equation*, but allowing to face the complexity of the experimental situation[4] in which

[4] A.B. et al., NPA 831 59 (2009) and EPJC 71 (2011) 1666
For a review: R. Rapp and H. van Hees, arXiv:0903.1096

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A proper *relativistic generalization of the Langevin equation* allows to accomplish this task

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The relativistic Langevin equation

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}},$$

with the properties of the noise encoded in

$$\langle \xi^i(\mathbf{p}_t)\xi^j(\mathbf{p}_{t'})\rangle = b^{ij}(\mathbf{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\mathbf{p}) \equiv \kappa_\parallel(p)\hat{p}^i\hat{p}^j + \kappa_\perp(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

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Transport coefficients to calculate:

- *Momentum diffusion* $\kappa_\perp \equiv \frac{1}{2}\frac{\langle \Delta p_\perp^2 \rangle}{\Delta t}$ and $\kappa_\parallel \equiv \frac{\langle \Delta p_\parallel^2 \rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D{}^{\text{Ito}}(p) = \frac{\kappa_\parallel(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial \kappa_\parallel(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_\parallel(p) - \kappa_\perp(p)}{v^2}\right]$$

fixed in order to insure approach to equilibrium (Einstein relation):
Langevin $\Leftrightarrow$ Fokker Planck with steady solution $\exp(-E_p/T)$

Langevin equation: the numerical algorithm

Update performed in the *local fluid rest-frame*:

$$\Delta\bar{p}^i_n = -\eta_D(\bar{p}_n)\bar{p}^i_n\Delta\bar{t} + \xi^i(\bar{t}_n)\Delta\bar{t} \equiv -\eta_D(\bar{p}_n)\bar{p}^i_n\Delta\bar{t} + g^{ij}(\bar{\mathbf{p}}_n)\zeta^i(\bar{t}_n)\sqrt{\Delta\bar{t}},$$

$$\Delta\bar{\mathbf{x}}_n = \bar{\mathbf{p}}_n/\bar{E}_n\Delta\bar{t}$$

with $\Delta\bar{t} = 0.02$ fm/c (*in the fluid rest-frame!*) and

$$g^{ij}(\mathbf{p}) \equiv \sqrt{\kappa_\parallel(p)}\hat{p}^i\hat{p}^j + \sqrt{\kappa_\perp(p)}(\delta^{ij} - \hat{p}^i\hat{p}^j) \quad \text{and} \quad \langle\zeta^i_n\zeta^j_{n'}\rangle = \delta^{ij}\delta_{nn'}$$

Hence one needs simply to:

- extract three independent random numbers ζ^i from a gaussian distribution with $\sigma = 1$;
- update the momentum and position of the heavy quark;
- go back to the Lab-frame: $\mathbf{x}_{n+1}$ and $\mathbf{p}_{n+1}$.

The background medium

The fields $u^\mu(x)$ and $T(x)$ are taken from the output of two longitudinally boost-invariant ("Hubble-law" longitudinal expansion $v_z = z/t$)

$$x^\mu = (\tau \cosh\eta, \mathbf{r}_\perp, \tau \sinh\eta) \quad \text{with} \quad \tau \equiv \sqrt{t^2 - z^2}$$

$$u^\mu = \gamma_\perp(\cosh\eta, \mathbf{u}_\perp, \sinh\eta) \quad \text{with} \quad \gamma_\perp \equiv \frac{1}{\sqrt{1-\mathbf{u}_\perp^2}}$$

hydro codes[6].

- $u^\mu(x)$ used to perform the update each time in the fluid rest-frame;
- $T(x)$ allows to fix at each step the value of the transport coefficients.

[6] P.F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C **62** (2000) 054909
P. Romatschke and U.Romatschke, Phys. Rev. Lett. **99** (2007) 172301

Evaluation of transport coefficients: $\kappa_{\perp}(p)$ and $\kappa_{\|}(p)$

It's the stage where the various models differ!

We account for the effect of $2 \rightarrow 2$ collisions in the medium

[7] Similar strategy for the evaluation of dE/dx in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).

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Intermediate cutoff $|t|^* \sim m_D^2$[7] *separating the contributions of*

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It's the stage where the various models differ!

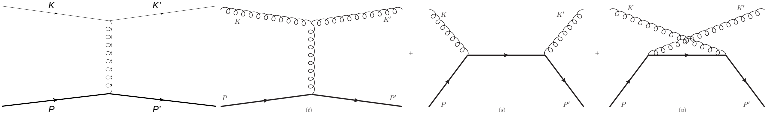
We account for the effect of $2 \to 2$ collisions in the medium

Intermediate cutoff $|t|^* \sim m_D^2$ [7] *separating the contributions of*

- hard collisions ($|t| > |t|^*$): kinetic pQCD calculation
- soft collisions ($|t| < |t|^*$): Hard Thermal Loop approximation (*resummation of medium effects*)

[7] Similar strategy for the evaluation of dE/dx in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).

$\kappa_\perp(p)$ and $\kappa_\parallel(p)$: hard contribution



$$\kappa_\perp^{g/q(\mathrm{hard})} = \frac{1}{2}\frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times (2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_\perp^2$$

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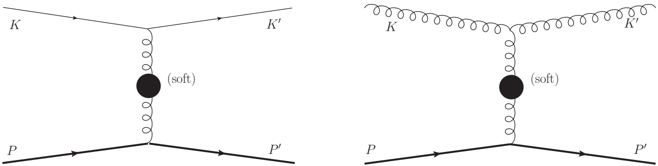
$$\times (2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_\parallel^2$$

where: $(|t| \equiv q^2 - \omega^2)$

$\kappa_\perp(p)$ and $\kappa_\parallel(p)$: soft contribution



When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.

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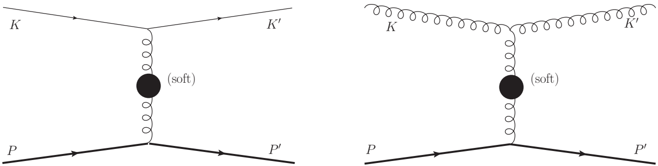

When the exchanged 4-momentum is **soft** the t-channel gluon feels the presence of the medium and requires **resummation**.
The *blob* represents the *dressed gluon propagator*, which has longitudinal and transverse components:

$$\Delta_L(z,q) = \frac{-1}{q^2 + \Pi_L(z,q)}, \quad \Delta_T(z,q) = \frac{-1}{z^2 - q^2 - \Pi_T(z,q)},$$

where *medium effects* are embedded in the HTL gluon self-energy.

Soft contribution: some comments

The resummation of the in-medium gluon self-energy prevents the appearance of soft divergences in $\kappa_{\perp/\parallel}(p)$

[8] T. Sjöstrand and P.Z. Skands, JHEP 03 (2004) 053.

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The resummation of the in-medium gluon self-energy prevents the appearance of soft divergences in $\kappa_{\perp/\parallel}(p)$

- Dealing with MPI in *pp* collisions divergence $d\hat{\sigma}/dp_\perp^2 \sim \alpha_s^2(p_\perp^2)/p_\perp^4$ from t-channel diagrams regularized through the overall factor[8]

$$\frac{\alpha_s^2(p_\perp^2 + p_{\perp 0}^2)}{\alpha_s^2(p_T^2)} \frac{p_\perp^4}{(p_\perp^2 + p_{\perp 0}^2)^2}$$

  Physical argument: hadrons *at sufficiently large distance-scales* are neutral objects, so that scattering processes cannot involve *arbitrarily long-wavelength gluons*. $p_{\perp 0}$ is a free parameter to be tuned to data;

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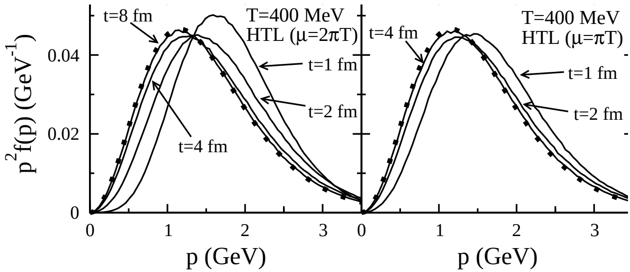
  Physical argument: hadrons *at sufficiently large distance-scales* are neutral objects, so that scattering processes cannot involve *arbitrarily long-wavelength gluons*. $p_{\perp 0}$ is a free parameter to be tuned to data;

- in thermal-QCD, at least in a weak-coupling framework, the medium correction to the tree-level gluon propagator can be calculated from first principles.

[8] T. Sjöstrand and P.Z. Skands, JHEP 03 (2004) 053.

A first check: thermalization in a static medium



For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution[9]

$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \quad \int d^3p\, f_{\rm MJ}(p) = 1$$

(Test with a sample of c quarks with $p_0 = 2$ GeV/c)

[9] A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)

HF studies: a multi-step setup

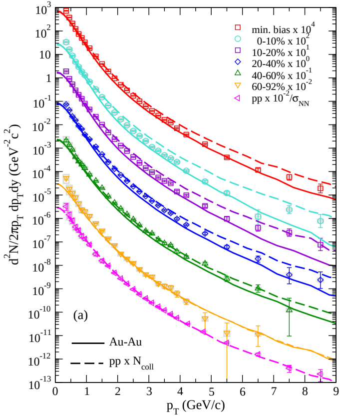
We are ready to perform numerical simulations for a realistic case!

- Initial generation of $Q\overline{Q}$ pairs (POWHEG + Parton Shower) and distribution in the transverse plane ($\widehat{T}_A(\mathbf{x}+\mathbf{b}/2)\widehat{T}_B(\mathbf{x}-\mathbf{b}/2)$);
- Langevin evolution in the QGP ($u^\mu(x)$ and $T(x)$ given by hydro);
- At T_c HQs hadronize (fragmentation with PDG branching ratios)
- and decay into electrons (PYTHIA decayer with PDG decay tables), e.g. $D \to X\nu_e e$.

NB One has first of all to *check to be able to reproduce pp results*!

Results at RHIC

Heavy-flavor electrons: invariant spectra

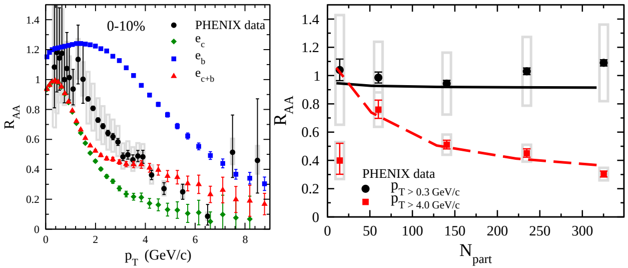


- *pp* spectrum nicely reproduced;
- Continuous curves: *AA* case after Langevin evolution[a];
- Dashed curves: *pp* result scaled by $\langle N_{\rm coll} \rangle$

[a] W.M. Alberico *et al.*, EPJC 71 (2011) 1666

Results at RHIC

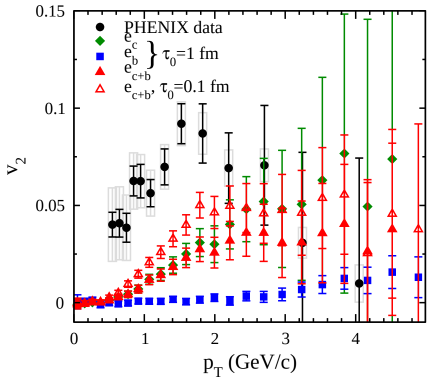
Heavy-flavor electrons: R_{AA}



- Left panel: $R_{AA}(p_T)$ in central events;
- Right panel: integrated R_{AA} vs centrality

Results at RHIC

Heavy-flavor electrons: *elliptic flow*

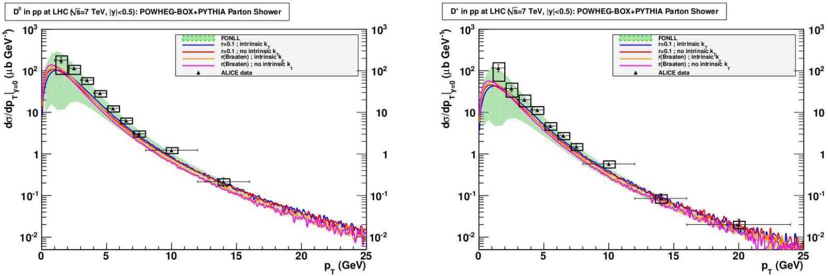


- Flow at low-p_T results underestimated;
- With a very small $\tau_0 \sim 0.1$ fm discrepancy *reduced*, but *still present*

Shortcoming of the approximations in evaluation of $\kappa_{\perp/\parallel}$? Effect of hadronization by coalescence with light quarks?

Results at LHC

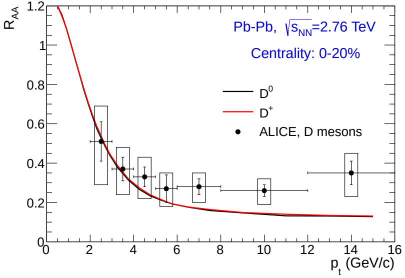
D meson spectra in *pp* collisions

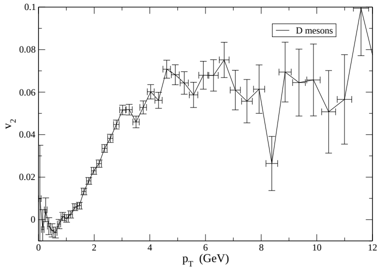


Hard production in elementary p-p collisions generated with POWHEG + PYTHIA PS: nice agreement with FONLL outcome and ALICE results

Results at LHC

D meson R_{AA} collisions





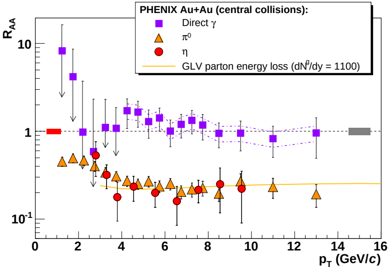
Challenge for theoretical models: reproducing both R_{AA} and v_2 [10]

[10] M. Monteno talk at "Hard Probes 2012"

Jet quenching

(in a broad sense: jet-reconstruction in AA possible only recently)

Inclusive hadron spectra: the nuclear modification factor



$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

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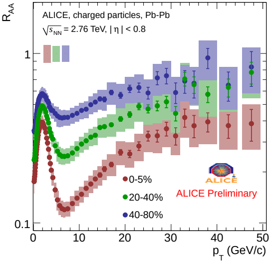

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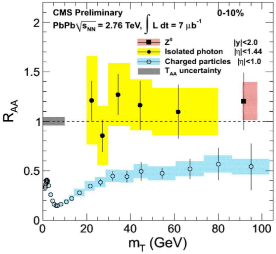

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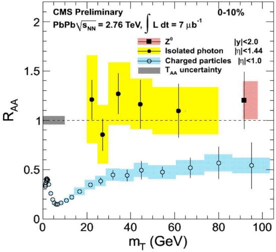

$$R_{AA} \equiv \frac{\left(dN^h/dp_T\right)^{AA}}{\langle N_{\rm coll}\rangle \left(dN^h/dp_T\right)^{pp}}$$

Hard-photon $R_{AA} \approx 1$

- supports the Glauber picture (binary-collision scaling);
- entails that quenching of inclusive hadron spectra is a *final state effect* due to in-medium energy loss.

Some CAVEAT:

- At variance wrt e^+e^- collisions, in hadronic collisions one starts with a parton p_T-distribution ($\sim 1/p_T^\alpha$) so that inclusive hadron spectrum simply reflects *higher moments of FF*

$$\frac{dN^h}{dp_T} \sim \frac{1}{p_T^\alpha} \sum_f \int_0^1 dz\, z^{\alpha-1} D^{f\to h}(z)$$

carrying limited information on FF (but very sensitive to hard tail!)

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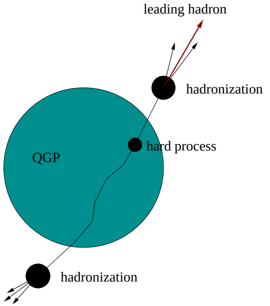
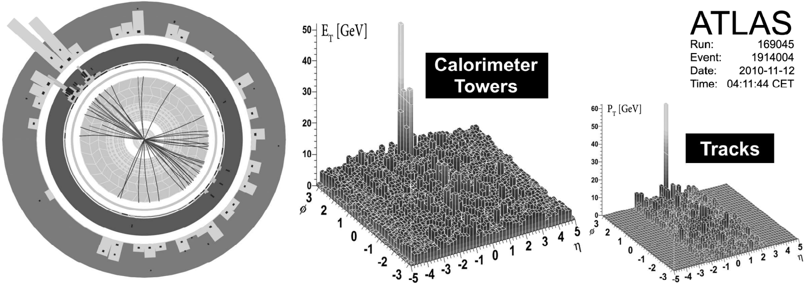
- Surface bias:



Quenched spectrum does not reflect $\langle L_{\rm QGP}\rangle$ crossed by partons distributed in the transverse plane according to $n_{\rm coll}(\mathbf{x})$ scaling, but *due to its steeply falling shape* is biased by the enhanced contribution of the ones *produced close to the surface and losing a small amount of energy*!

Di-jet imbalance at LHC: looking at the event display

An important fraction of events display a *huge mismatch* in E_T between the leading jet and its away-side partner



Possible to observe event-by-event, without any analysis!

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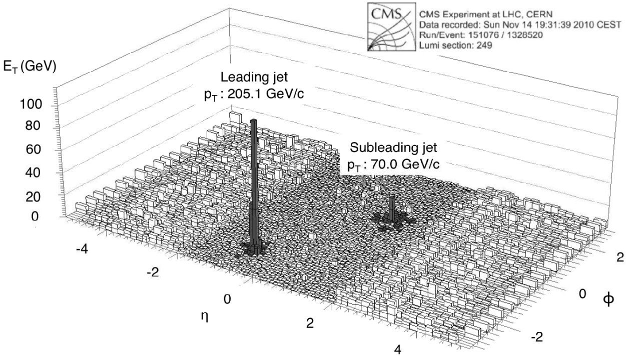

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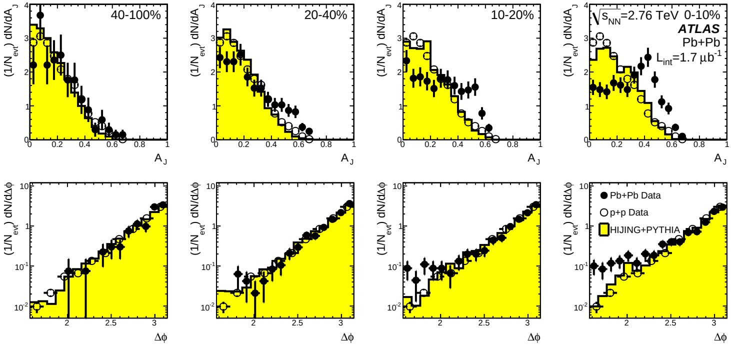
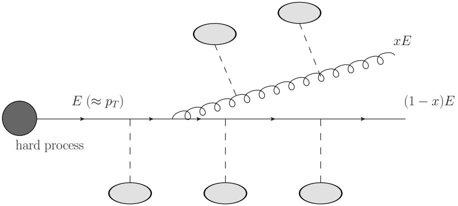
Dijet correlations: results



- Dijet asymmetry $A_j \equiv \frac{E_{T_1} - E_{T_2}}{E_{T_1} + E_{T_2}}$ enhanced wrt to p+p and increasing with centrality;
- $\Delta\phi$ distribution unchanged wrt p+p (jet pairs $\sim$ back-to-back)

Physical interpretation of the data: energy-loss *at the parton level*!



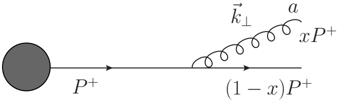
- Interaction of the high-p_T parton with the *color field of the medium* induces the radiation of (mostly) *soft* ($\omega \ll E$) and *collinear* ($k_\perp \ll \omega$) gluons;
- Radiated gluon can further re-scatter in the medium (cumulated $\mathbf{q}_\perp$ favor *decoherence* from the projectile).

The basic ingredients

- Vacuum-radiation spectrum;
- (Gunion-Bertsch) induced spectrum

Vacuum radiation by off-shell partons

A hard parton with $p_i \equiv \left[p_+, Q^2/2p_+, \mathbf{0}\right]$ loses its virtuality Q through gluon-radiation. In *light-cone coordinates*, with $p_\pm \equiv E \pm p_z/\sqrt{2}$:



$$k_g \equiv \left[xp_+, \frac{\mathbf{k}^2}{2xp_+}, \mathbf{k}\right]$$

$$p_f = \left[(1-x)p_+, \frac{\mathbf{k}^2}{2(1-x)p_+}, -\mathbf{k}\right]$$

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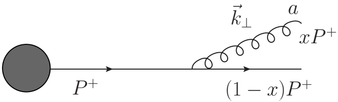

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- $k_\perp$ *vs* virtuality: $\mathbf{k}^2 = x\,(1-x)\,Q^2$;
- Radiation spectrum (our benchmark): IR and collinear divergent!

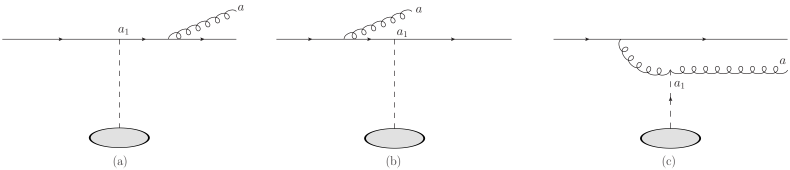
$$d\sigma^{\rm rad}_{\rm vac} = d\sigma^{\rm hard}\frac{\alpha_s}{\pi^2}C_R\frac{dk^+}{k^+}\frac{d\mathbf{k}}{\mathbf{k}^2}$$

- Time-scale (*formation time*) for gluon radiation:

$$\Delta t_{\rm rad} \sim Q^{-1}(E/Q) \sim 2\omega/\mathbf{k}^2 \quad (x \approx \omega/E)$$

Medium-induced radiation by on-shell partons

- On-shell partons propagating in a color field can radiated gluons.



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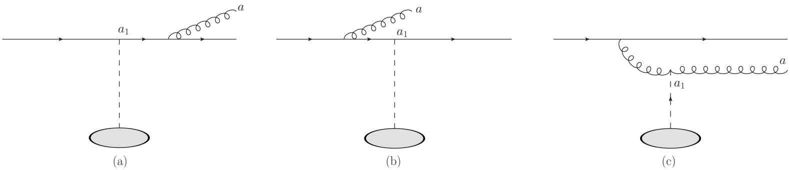

- The single-inclusive gluon spectrum: the Gunion-Bertsch result

$$x\frac{dN_g^{\rm GB}}{dxd\mathbf{k}} = C_R\frac{\alpha_s}{\pi^2}\left(\frac{L}{\lambda_g^{\rm el}}\right)\left\langle[\mathbf{K}_0-\mathbf{K}_1]^2\right\rangle = C_R\frac{\alpha_s}{\pi^2}\left(\frac{L}{\lambda_g^{\rm el}}\right)\left\langle\frac{\mathbf{q}^2}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}\right\rangle$$

where C_R is the *color charge* of the hard parton and:

$$\mathbf{K}_0\equiv\frac{\mathbf{k}}{\mathbf{k}^2},\qquad \mathbf{K}_1\equiv\frac{\mathbf{k}-\mathbf{q}}{(\mathbf{k}-\mathbf{q})^2}\qquad \text{and}\qquad \langle\dots\rangle\equiv\int d\mathbf{q}\frac{1}{\sigma^{\rm el}}\frac{d\sigma^{\rm el}}{d\mathbf{q}}$$

The induced spectrum: physical interpretation

$$\omega \frac{d\sigma^{\rm ind}}{d\omega d\mathbf{k}} = d\sigma^{\rm hard} C_R \frac{\alpha_s}{\pi^2} \left(\frac{L}{\lambda_g^{\rm el}}\right) \left\langle \left[(\mathbf{K}_0 - \mathbf{K}_1)^2 + \mathbf{K}_1^2 - \mathbf{K}_0^2\right] \left(1 - \frac{\sin(\omega_1 L)}{\omega_1 L}\right) \right\rangle$$

In the above $\omega_1 \equiv (\mathbf{k}-\mathbf{q})^2/2\omega$ and two regimes can be distinguished:

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  The full radiation spectrum can be organized as

$$d\sigma^{\rm rad}=d\sigma^{\rm GB}+d\sigma^{\rm vac}_{\rm gain}+d\sigma^{\rm vac}_{\rm loss}$$

  where

$$d\sigma^{\rm GB}=d\sigma^{\rm hard}C_R\frac{\alpha_s}{\pi^2}\left(L/\lambda_g^{\rm el}\right)\left\langle(\mathbf{K}_0-\mathbf{K}_1)^2\right\rangle(d\omega d\mathbf{k}/\omega)$$

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Gluon formation-time: physical meaning

Behavior of the induced spectrum depending on the *gluon formation-time*

$$t_{\rm form} \equiv \omega_1^{-1} = 2\omega/(\mathbf{k}-\mathbf{q})^2$$

differing from the vacuum result $t_{\rm form}^{\rm vac} \equiv 2\omega/\mathbf{k}^2$, due to the transverse **q**-kick received from the medium. Why such an expression?

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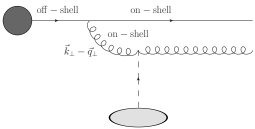
Consider for instance the $\langle \mathbf{K}_1^2 \rangle$ term, with the hard *off-shell* parton $p_i \equiv [p_+, Q^2/2p_+, \mathbf{0}]$ radiating a gluon which then scatters in the medium



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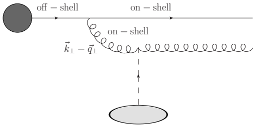

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The radiation will occur in a time set by the uncertainty principle:

$$Q^2 \sim (\mathbf{k}-\mathbf{q})^2/x \quad \longrightarrow \quad t_{\rm form} \sim Q^{-1}(E/Q) \sim 2\omega/(\mathbf{k}-\mathbf{q})^2$$

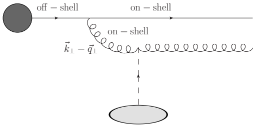
Gluon formation-time: physical meaning

Behavior of the induced spectrum depending on the *gluon formation-time*

$$t_{\rm form} \equiv \omega_1^{-1} = 2\omega/(\mathbf{k}-\mathbf{q})^2$$

differing from the vacuum result $t_{\rm form}^{\rm vac} \equiv 2\omega/\mathbf{k}^2$, due to the transverse **q**-kick received from the medium. Why such an expression?

Consider for instance the $\langle \mathbf{K}_1^2 \rangle$ term, with the hard *off-shell* parton $p_i \equiv [p_+, Q^2/2p_+, \mathbf{0}]$ radiating a gluon which then scatters in the medium



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$\longrightarrow$ if $t_{\rm form} \gtrsim L$ the process is suppressed!

Average energy loss

Integrating the lost energy ω over the inclusive gluon spectrum:

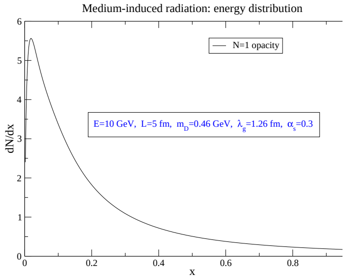
$$\langle \Delta E \rangle = \int d\omega \int d\mathbf{k}\, \omega \frac{dN_g^{\mathrm{ind}}}{d\omega d\mathbf{k}} \sim \frac{C_R \alpha_s}{4} \left(\frac{\mu_D^2}{\lambda_g^{\mathrm{el}}} \right) L^2 \ln \frac{E}{\mu_D}$$

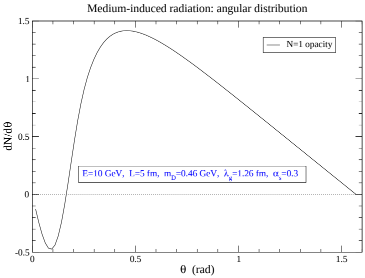
- L^2 dependence on the medium-length;
- μ_D: Debye screening mass of color interaction $\sim$ *typical momentum exchanged in a collision*;
- $\mu_D^2/\lambda_g^{\mathrm{el}}$ often replaced by the *transport coefficient* $\hat{q}$, so that

$$\langle \Delta E \rangle \sim \alpha_s \hat{q} L^2$$

$\hat{q}$: average $q_\perp^2$ acquired per unit length

Numerical results





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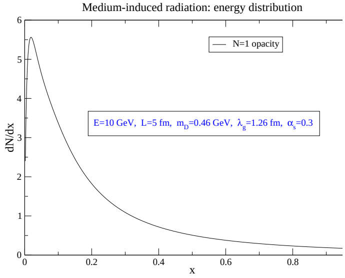
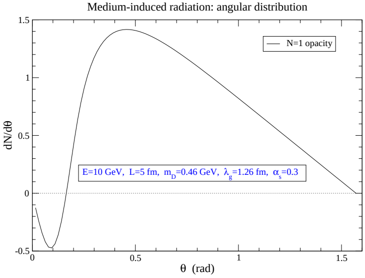


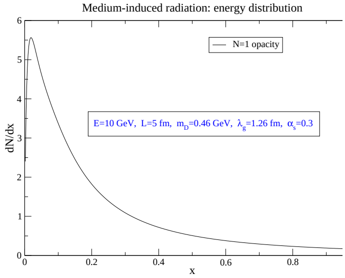


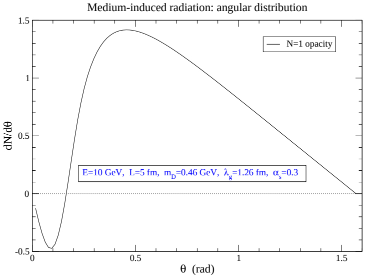
At variance with vacuum-radiation, medium induced spectrum

- Infrared safe (vanishing as $\omega \to 0$);
- Collinear safe (vanishing as $\theta \to 0$).

Depletion of gluon spectrum at small angles due to their rescattering in the medium!

Numerical results





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- Infrared safe (vanishing as $\omega \to 0$);
- Collinear safe (vanishing as $\theta \to 0$).

In general $\langle N \rangle > 1$, so that addressing *multiple gluon emission* becomes mandatory

How to address more differential observables?

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How to address more differential observables?

- So far we focused on *inclusive spectrum* of radiated gluons: a parton radiating gluons of energy ω_1 and ω_2 simply contributes twice to such a spectrum;
- A more differential information (e.g. *exclusive* one, two... gluon spectrum) is desirable in order to deal with more exclusive observables (jet fragmentation, jet-shapes...);
- Ideally one would like to *follow a full parton-shower evolution in the plasma*, described by *modified Sudakov form factors*

$$\Delta(t, t_0) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int dz \frac{\alpha_s(t', z)}{2\pi} P(z, t')\right],$$

where medium effects are included as *corrections to the DGLAP splitting functions*:

$$P(z, t) = P^{\text{vac}}(z) + \Delta P(z, t)$$

As an evolution variable one can use the parton virtuality $t \equiv Q^2$

Evaluation of modified splitting functions

- Vacuum-radiation spectrum

$$dN_g^{\rm vac} = \frac{\alpha_s}{\pi^2} C_R \frac{dk^+}{k^+} \frac{d\mathbf{k}}{\mathbf{k}^2} = \frac{\alpha_s}{2\pi} \left(\frac{2C_R}{x} \right) dx \frac{d\mathbf{k}^2}{\mathbf{k}^2}$$

allows to identify the soft limit of $P^{\rm vac}(z)$ (where $z = 1 - x$):

$$\frac{dN_g^{\rm vac}}{dz d\mathbf{k}^2} \equiv \frac{\alpha_s}{2\pi} \frac{1}{\mathbf{k}^2} P^{\rm vac}(z), \quad \longrightarrow \quad P^{\rm vac}(z) \underset{z\to1}{\simeq} \frac{2C_R}{1-z}$$

[11]Q-PYTHIA: EPJC 63 (2009) 679; Q-HERWIG: JHEP 0911 (2009) 122

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- Medium-corrections to the splitting function are then obtained through the *matching with the induced radiation spectrum*[11]:

$$\Delta P(z,t) \simeq \frac{2\pi t}{\alpha_s} \frac{dN_g^{\rm ind}}{dzdt}$$

where $\mathbf{k}^2 = z(1-z)t$.

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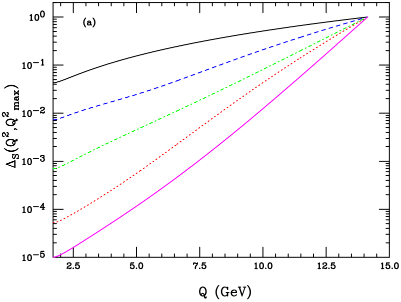
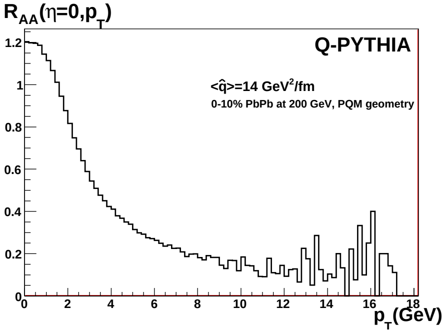
In-medium parton showers: results





Q-HERWIG Sudakov factor ($\hat{q}L_0 = 0 - 50$ GeV^2) and Q-PYTHIA R_{AA}

Some comments

- In Q-PYTHIA and Q-HERWIG the only effect of the medium enters into a modification of the splitting functions, *enhancing the probability of gluon radiation*;
- however color-exchanges with the medium can also affect[12]
  - correlations between successive gluon emissions (a.k.a. angular ordering *in the vacuum*)
  - color-flow in parton branchings

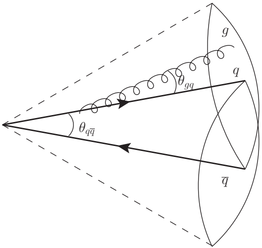
The *in-medium breaking of color-coherence* will be our next subject

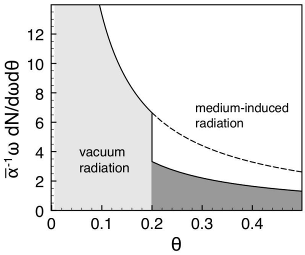
[12] A.B., arXiv:1207.4294 [hep-ph]

QCD-antenna radiation in a medium

Problem analyzed in a series of papers:
Y. Mehtar-Tani, C.A. Salgado and K. Tywoniuk, PRL 106 (2011) 122002, PLB 707 (2012) 156-159, JHEP 1204 (2012) 064...

QCD radiation in the medium: *antiangular* ordering





The total (vacuum+medium) radiation spectrum reads

$$dN^{\rm tot}_{q,\gamma^*} = \frac{\alpha_s C_F}{\pi}\frac{d\omega}{\omega}\frac{\sin\theta\, d\theta}{1-\cos\theta}\left[\theta(\cos\theta - \cos\theta_{q\bar{q}}) + \Delta_{\rm med}\theta(\cos\theta_{q\bar{q}} - \cos\theta)\right]$$

- $\Delta_{\rm med}$ from 0 (no medium effect) to 1 (complete decoherence of the $q\bar{q}$ pair, radiating as two uncorrelated color charges)
- For $\Delta_{\rm med} \to 1$ $dN^{\rm tot}_{\gamma^*} = dN^{\rm tot}_{g^*}$: pair forgets about initial color;

Medium-modification of color-flow for high-p_T probes[13]

- I will mainly focus on leading-hadron spectra...
- ...but the effects may be relevant for more differential observables (e.g. jet-fragmentation pattern)

[13] A.B, J.G.Milhano and U.A. Wiedemann, J. Phys. G G38 (2011) 124118 and Phys. Rev. C85 (2012) 031901 + arXiv:1204.4342 [hep-ph]

From partons to hadrons

The *final stage of* any *parton shower* has to be interfaced with some hadronization routine. Keeping track of color-flow one identifies *color-singlet* objects whose decay will give rise to hadrons

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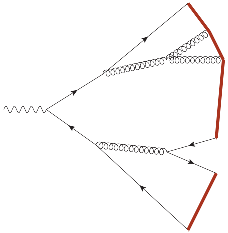

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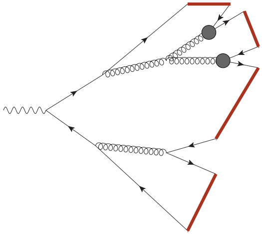
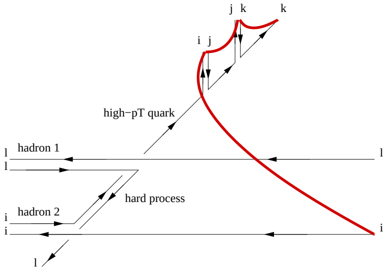


- In PYTHIA hadrons come from the fragmentation of $q\bar{q}$ *strings*, with gluons representing kinks along the string (Lund model);
- In HERWIG the shower is evolved up to a softer scale, all gluons are *forced to split* in $q\bar{q}$ pair (large-N_c!) and *singlet clusters* (usually of *low invariant mass*!) are thus identified.

Vacuum radiation: color flow (in large-N_c)



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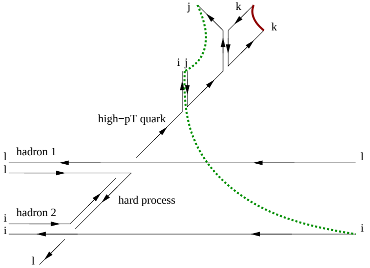
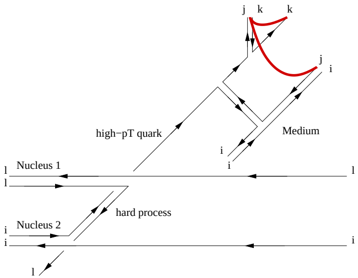


- Most of the radiated gluons in a shower remain color-connected with the projectile fragment;
- Only $g \longrightarrow q\overline{q}$ splitting can break the color connection, BUT

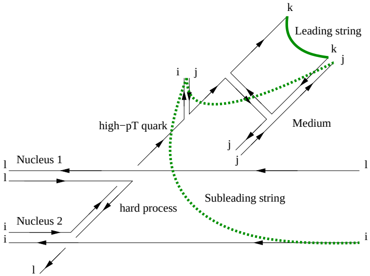
$$P_{qg} \sim \left[z^2 + (1-z)^2\right] \quad vs \quad P_{gg} \sim \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right]$$

less likely: no soft (i.e. $z \longrightarrow 1$) enhancement!

Medium-induced radiation: color-flow (+ Lund string)

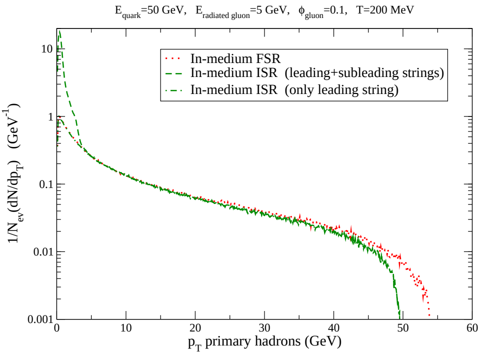


“Final State Radiation”
(gluon $\in$ leading string)
Gluon contributes to leading hadron



“Initial State Radiation”
(gluon decohered: lost!)
Gluon contributes to *enhanced soft multiplicity* from subleading string

Fragmentation function



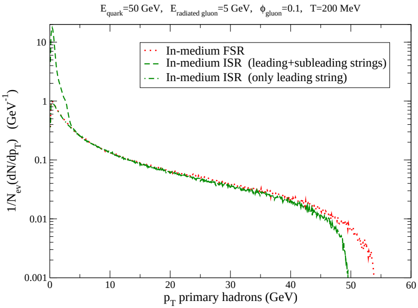
ISR characterized by:

- Depletion of hard tail of FF (gluon decohered!);
- Enhanced soft multiplicity from the subleading string

FF: higher order moments and hadron spectra

Starting from a steeply falling parton spectrum $\sim 1/p_T^n$ at the end of the shower evolution, single hadron spectrum sensitive to *higher moments* of FF:

$$dN^h/dp_T \sim \langle x^{n-1} \rangle / p_T^n$$



- Quenching of hard tail of FF affects higher moments: e.g.
  - FSR: $\langle x^6 \rangle \approx 0.078$;
  - ISR: $\langle x^6 \rangle_{\rm lead} \approx 0.052$

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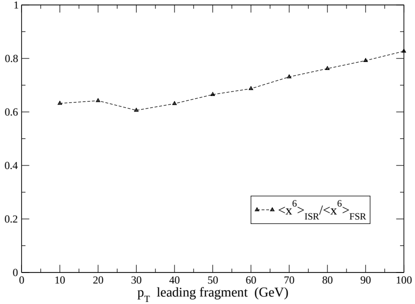
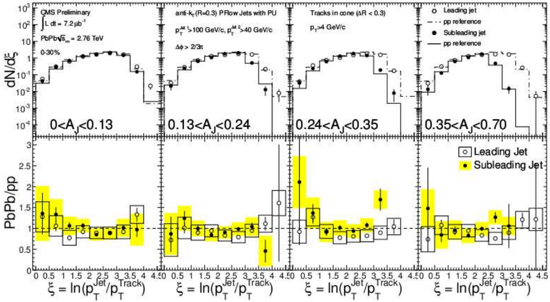


- Quenching of hard tail of FF affects higher moments: e.g.
  - FSR: $\langle x^6 \rangle \approx 0.078$;
  - ISR: $\langle x^6 \rangle_{\rm lead} \approx 0.052$
- Ratio of the two channels suggestive of the effect on the hadron spectrum

Relevance for jet observables

Some comments in the light of experimental results[14]:

- Vacuum-like fragmentation of strings of reduced energy (color-decoherence of radiated gluons), in agreement with no change of hard-FF ($p_T^{\rm track} > 4$ GeV) in Pb+Pb wrt p+p measured by CMS;



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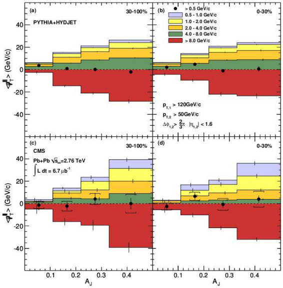

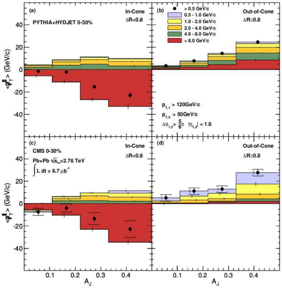

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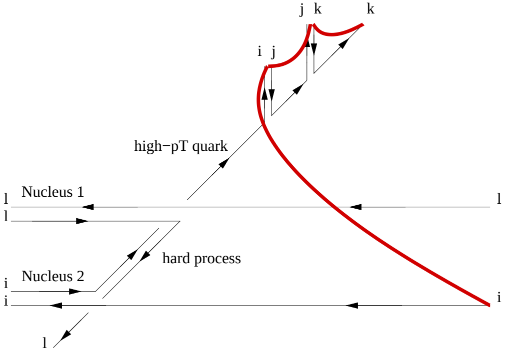
- Enhanced multiplicity of soft particles from the decay of subleading strings (decohered gluons give rise to new strings!), in agreement with CMS observations;
- Broad angular distribution of soft hadrons around the-jet axis observed by CMS remains to be explained: larger amount of partonic rescattering (i.e. higher orders in opacity) probably required.

[14] CMS PAS HIN-11-004 and PRC 84, 024906 (2011)

Relevance for info on medium properties

- Hadronization schemes developed to reproduce data from elementary collisions: a situation in which most of the radiated gluons are still color-connected with leading high-p_T fragment;



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  without accounting for the modified color-flow would result into a too hard hadron spectrum: fitting the experimental amount of quenching would require an overestimate of the energy loss at the partonic level;

- Color-decoherence of radiated gluon might contribute to reproduce the observed high-p_T suppression with milder values of the medium transport coefficients (e.g. $\hat{q}$).

Final considerations

- Heavy-ion collisions produce certainly a "dirty" environment; nevertheless the final goal is to interpret the experimental findings in terms of QCD;
- I tried to give a general overview on the subject, with the hope that some of you can find such an issue of interest and – may be – discover topics where you can give a contribution to the field: multi-disciplinary skills are welcome and necessary!
- Feel free to contact me for any question, comment, proposal...

Thank you!