Anomalous-Magnetic-Moment Effects in the Equation of State of a Magnetized Fermion System

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Introduction

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Dirac Theory

$$\boldsymbol{\mu} = g\mu_B \boldsymbol{s}$$

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But refined experiments give $g \simeq 2.00232$

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Schwinger[‡] computed the one-loop contribution to the fermion self-energy in a weak magnetic field, this leads to an extra structure in the Dirac equation:

$$(\Pi_\mu \gamma^\mu - m + \kappa \mu_B B \Sigma_3)\psi = 0 \tag{1}$$

where κ is the AMM of the fermion, $\Sigma_3 = i\gamma_1\gamma_2$ and $\Pi_\mu = i\partial_\mu - eA_\mu$.

[‡] J. Schwinger, *Phys. Rev.* **73**, 416 (1947).
[§] B. Jancovicci, *Phys. Rev.* **187**, 2275 (1969).
[§] R. Gonzalez, A. Perez Martinez, H. Perez Rojas, M. Orsaria *Physical Review C*, **77**, 015807 (2008)

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The corresponding relativistic particle energy is

$$E^2_{l,\sigma} = [(m^2 + 2eBl)^{1/2} - \sigma\kappa\mu_B B]^2 + p_3^2, \quad l = 0, 1, 2, ..., \quad \sigma = \pm 1 \tag{2}$$

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$$\epsilon = m - \mu_B \kappa B$$

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$$\epsilon = m - \mu_B \kappa B \quad \overset{\text{if}}{\rightarrow} \quad B = \frac{m}{\mu_B \kappa} \quad \Rightarrow \quad \epsilon = 0$$

The last conclusion could lead to several problems[§], so we can not extrapolate Schwinger results to high-field values

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Introduction

The AMM term shown before is the first term in a series expansion in $B^{\ddagger}$:

$$M^{(0)} = \frac{\alpha}{2\pi} m \left[-\frac{eB}{2m^2} + \left(\frac{eB}{m^2}\right)^2 \left(\frac{4}{3}\ln\frac{m^2}{eB} - \frac{13}{18}\right) + \dots \right] \tag{3}$$

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Can we use the linear approximation for high values of the magnetic field?

fermion	p	e	u	d	s
mass(MeV)	938	0.5	5	5	150
AMM	2.79 μ_N	0.00116 μ_B	1.85 μ_N	-0.79 μ_N	-0.85 μ_N

$$\mu_N = \frac{e}{2m_p}, \quad B_c = m^2/e$$

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Motivation

It is interesting to see the effects of the AMM in the thermodynamical parameters of a magnetized fermion gas...

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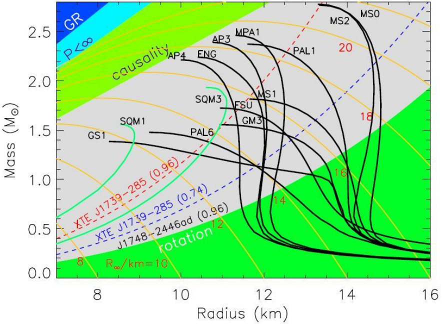

...and in the physics of Compact Objects.

Goals

1. Calculation of the fermion self-energy in a constant magnetic field (B), in a non-perturbative approach (exact in B).
2. Calculation of the AMM in this approximation.
3. Study the effects of the obtained AMM in the thermodynamical quantities of the magnetized fermion gas.

Fermion Self-energy $B \neq 0$.

- Consider a constant magnetic field in x_3–direction.
- Use the Ritus[†] transformation to diagonalize the self energy in momentum space.
- We will calculate the self-energy in the infrared limit $(p \to 0)$ for $T = 0$.

The AMM enters as a correction of the energy of particles in the calculations of the thermodynamical quantities.

[†] V. I. Ritus, *Ann.Phys.* 69 (1972) 555; *Sov. Phys. JETP* **48** (1978) 788 [Zh. Eksp. Teor. Fiz.**75** (1978) 1560].

Fermion Self-energy $B \neq 0$.

Schwinger-Dyson equation for the fermion self-energy in the presence of a constant magnetic field gives us the self-energy:

$$\Sigma(x,x') = -ie^2\gamma^\mu G(x,x')\gamma^\nu D_{\mu\nu}(x-x'). \tag{4}$$

where the bare photon propagator is (Feynman gauge):

$$D_{\mu\nu}(x-x') = \int \frac{d^4q}{(2\pi)^4}\frac{e^{-iq\cdot(x-x')}}{q^2-i\epsilon}g_{\mu\nu}, \tag{5}$$

and the full fermion propagator obeys the equation

$$[\Pi_\mu\gamma^\mu - \Sigma(x,x')]G(x,x') = \delta^4(x-x'). \tag{6}$$

We go to momentum space by means of Ritus transformation

$$G(x,x') = \sum\!\!\!\!\!\!\!\int \frac{d^4p"}{(2\pi)^4}E_{p"}^{l"}(x)\Pi(l")\widetilde{G}^{l"}(\overline{p}")\overline{E}_{p"}^{l"}(x') \tag{7}$$

Fermion Self-energy $B \neq 0$.

The $E_p^l(x)$ functions are used for the transformation to momentum space and are defined as:

$$E_p^l(x) = E_p^+(x)\Delta(+) + E_p^-(x)\Delta(-) \quad (8)$$

where

$$\Delta(\pm) = \frac{I \pm i\gamma^1\gamma^2}{2}, \quad (9)$$

are the spin up $(+)$ and down $(-)$ projectors, and $E_p^{\pm}(x)$ are the corresponding eigenfunctions

$$E_p^+(x) = N_l e^{-i(p_0x^0+p_2x^2+p_3x^3)} D_l(\rho),$$
$$E_p^-(x) = N_{l-1} e^{-i(p_0x^0+p_2x^2+p_3x^3)} D_{l-1}(\rho) \quad (10)$$

with normalization constant $N_l = (4\pi eB)^{1/4}/\sqrt{l!}$, and $D_l(\rho)$ denoting the parabolic cylinder functions of argument $\rho = \sqrt{2eB}(x_1 - p_2/eB)$, and index given by the Landau level numbers $l = 0, 1, 2, ...$

Fermion Self-energy $B \neq 0$.

so we obtain for the self-energy in momentum space:

$$\Sigma^l(\overline{p})\Pi(l) \quad = \quad -ie^2(2eB)\Pi(l)\int \frac{d^4\widehat{q}}{(2\pi)^4}\frac{e^{-\widehat{q}_\perp^2}}{\widehat{q}^2}[L_l + L_{l+1} + L_{l-1}]. \qquad (11)$$

where we have used the notation notation $\widehat{x} = x/\sqrt{2eB}$,
$\Pi(l) = \Delta(+)\delta^{l0} + I(1-\delta^{l0})$ and the three L factors on the right hand side of the equation (11) have the explicit form

$$\begin{aligned} L_l &= \gamma_\mu^\parallel G^l(\overline{p-q})\gamma_\mu^\parallel, \\ L_{l\pm1} &= \Delta(\pm)\gamma_\mu^\perp G^{l\pm1}(\overline{p-q})\gamma_\mu^\perp\Delta(\pm), \end{aligned}$$

where

$$G^l(\overline{p}) = -\frac{\overline{p}\cdot\gamma + m}{\overline{p}^2 + m^2}, \ \gamma_\mu^\parallel = \gamma_0, \gamma_3, \ \gamma_\mu^\perp = \gamma_1, \gamma_2 \qquad (12)$$

and

$$(\overline{p-q})_l = (p^0 - q^0, 0, -\sqrt{2eBl}, p^3 - q^3).$$

Anomalous Magnetic Moment

The general structure of the self-energy is:

$$\widetilde{\Sigma}^l(\overline{p}) = Z^l_{\|}\overline{p}^{\mu}_{\|}\gamma^{\|}_{\mu} + Z^l_{\perp}\overline{p}^{\mu}_{\perp}\gamma^{\perp}_{\mu} + M^l I + iT^l\gamma^1\gamma^2 \tag{13}$$

where γ_μ are the Dirac matrixes, $Z^l_{\|}$, $Z^l_{\perp}$ are the wave function's renormalization coefficients, M^l and T^l are the mass and anomalous magnetic moment terms.

Now we can compare (11) and (13) to obtain the AMM term:

$$M^0 + T^0 = \frac{e^2 m}{8\pi^2}\int d\widehat{q}^2_{\|} d\widehat{q}^2_{\perp} \frac{e^{-\widehat{q}^2_{\perp}}}{\widehat{q}^2}\left[\frac{1}{\widehat{q}^2_{\|} + \widehat{m}^2} + \frac{1}{\widehat{q}^2_{\|} + 1 + \widehat{m}^2}\right]^{\dagger}, \tag{14}$$

$$T^{l\neq 0} = -\frac{e^2 m}{16\pi^2}\int d\widehat{q}^2_{\|} d\widehat{q}^2_{\perp} \frac{e^{-\widehat{q}^2_{\perp}}}{\widehat{q}^2}\left[\frac{1}{\widehat{q}^2_{\|} + l + 1 + \widehat{m}^2} - \frac{1}{\widehat{q}^2_{\|} + l - 1 + \widehat{m}^2}\right] \tag{15}$$

These integrals must be solved numerically.

† E. Ferrer and V de la Incera, Nuclear Physics B 824, 217 (2010).

Anomalous Magnetic Moment

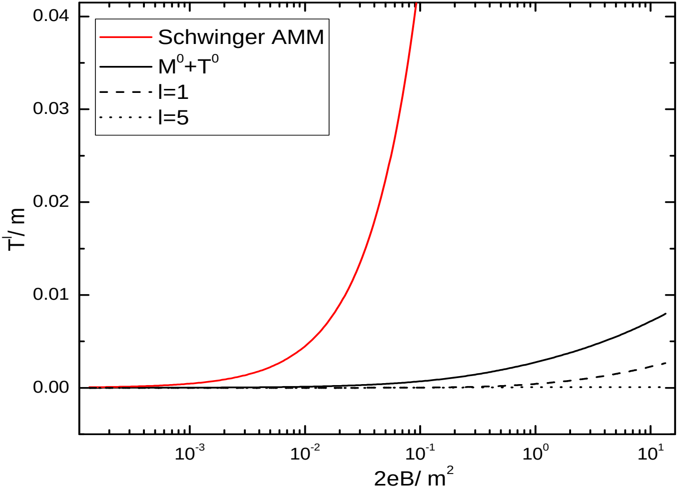

Figure: AMM for $l = 0, 1, 5$. Schwinger value is shown for comparison.

Dispersion relation of magnetized fermions

The dispersion relation is obtained from G_l^{-1} as

$$\mathrm{det}G_l^{-1} = \mathrm{det}[\overline{p}\cdot\gamma - m - M^l I - iT^l\gamma^1\gamma^2], \tag{16}$$

to give the following particle energy spectra

$$\epsilon_{\sigma,l}^2 = p_3^2 + [\sqrt{(m+M^l)^2 + 2|eB|l} + \sigma T^l]^2, \quad \sigma = \pm 1 \tag{17}$$

and the LLL energy is

$$\epsilon_{1,0}^2 = p_3^2 + (m + M^0 + T^0)^2 \tag{18}$$

so the rest–energy is

$$\varepsilon^0 = m + M^0 + T^0. \tag{19}$$

Dispersion relation of magnetized fermions

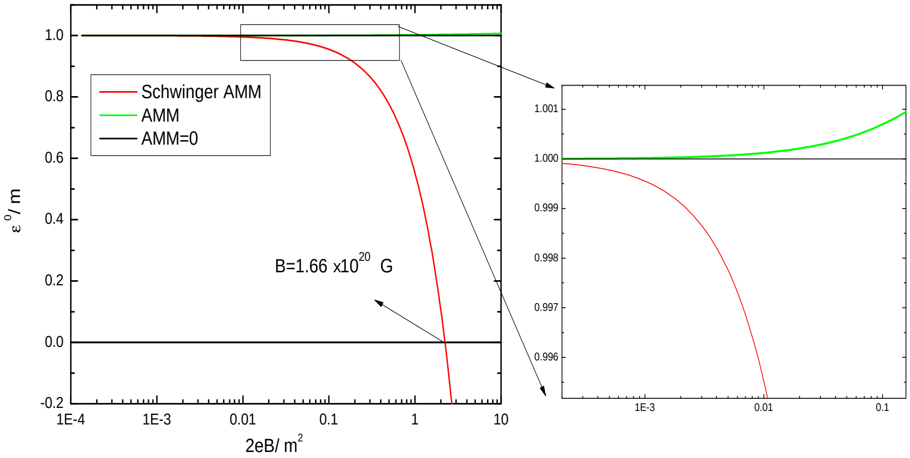

Figure: Rest energy for fermions in a constant magnetic field.

AMM effect in the thermodynamical properties

The thermodynamical potential at finite temperature and density for the magnetized system ($\Omega_f(B,\mu,T)$) including in the fermion inverse propagator the quantum contribution of the AMM is

$$\Omega_f(B,\mu,T) = -\frac{eB}{\beta}\left[\sum_{p_4}\int_{-\infty}^{\infty}\frac{dp_3}{(2\pi)^2}\ln\det\widetilde{G}_0^{-1}(\overline{p}^*) + \sum_{\sigma\pm 1}\sum_{l=1}^{\infty}\sum_{p_4}\int_{-\infty}^{\infty}\frac{dp_3}{(2\pi)^2}\ln\det\widetilde{G}_l^{-1}(\overline{p}^*)\right], \quad (20)$$

here $\overline{p}^*_\nu = (ip^4-\mu, 0, \sqrt{2eBl}, p^3)$.

We can obtain the limit $T=0$...

AMM effect in the thermodynamical properties

$$\Omega = -B\frac{em^2}{4\pi^2}\left[\Omega^0 + \sum_{l=1}^{l_{max}}\sum_{\sigma=\pm1}\left(\widetilde{\mu}\,\widetilde{p}_F^\sigma - (\varepsilon_{\sigma,l}^0)^2 \ln\frac{\widetilde{\mu}+\widetilde{p}_F^\sigma}{\varepsilon_{\sigma,l}^0}\right)\right], \quad (21)$$

where

$$\Omega^0 = \left(\widetilde{\mu}\,\widetilde{p}_F^0 - (\varepsilon_{1,0}^0)^2 \ln\frac{\widetilde{\mu}+\widetilde{p}_F^0}{\varepsilon_{1,0}^0}\right), \quad (22)$$

$$\widetilde{p}_F^\sigma = \sqrt{\widetilde{\mu}^2 - (\varepsilon_{\sigma,l}^0)^2}, \quad \widetilde{p}_F^0 = \sqrt{\widetilde{\mu}^2 - (\varepsilon_{1,0}^0)^2}, \quad \widetilde{\mu} = \frac{\mu}{m} \quad (23)$$

$$\varepsilon_{\sigma,l}^0 = \begin{cases} \sqrt{(m+M^l)^2 + 2|eB|l} + \sigma T^l, & \text{AMM}, \\ \sqrt{m^2 + 2eBl} - \sigma\kappa\mu_B B, & \text{Schwinger AMM}. \end{cases} \quad (24)$$

AMM effect in the thermodynamical properties

From Ω we can obtain the particle density $N = -(\partial\Omega/\partial\mu)$, the magnetization $M = -(\partial\Omega_f/\partial B)$, the energy density and the pressures

$$N = \frac{m^3}{2\pi^2}\frac{B}{B_c}\left(\widetilde{p}_F^0 + \sum_{l=1}^{l_{max}}\sum_{\sigma=\pm1}\widetilde{p}_F^\sigma\right), \tag{25}$$

$$M = \frac{em^2}{4\pi^2}\left\{M^0 + \sum_{l=1}^{l_{max}}\sum_{\sigma=\pm1}\widetilde{\mu}\widetilde{p}_F^\sigma - \left[(\varepsilon_{\sigma,l}^0)^2 + 2\varepsilon_{\sigma,l}^0\widetilde{\gamma}^\sigma\right]\ln\frac{\widetilde{\mu}+\widetilde{p}_F^\sigma}{\varepsilon_{\sigma,l}^0}\right\}, \tag{26}$$

$$E = \Omega + \mu N, \tag{27}$$

$$P_\parallel = -\Omega, \tag{28}$$

$$P_\perp = -\Omega - BM, \tag{29}$$

where

$$M^0 = \widetilde{\mu}\widetilde{p}_F^0 - \left[(\varepsilon_{1,0}^0)^2 + 2\varepsilon_{1,0}^0\widetilde{\gamma}^0\right]\ln\frac{\widetilde{\mu}+\widetilde{p}_F^0}{\varepsilon_{1,0}^{(0)}},$$

$$\widetilde{\gamma}^\sigma = \frac{\partial\varepsilon_{\sigma,l}^0}{\partial B}$$

AMM effect in the thermodynamical properties
Results

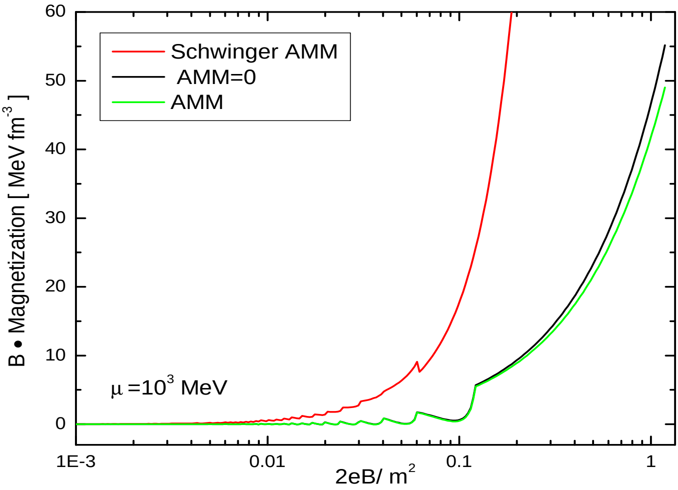

Figure: Magnetization vs B. Schwinger value and AMM$=0$ are shown for comparison.

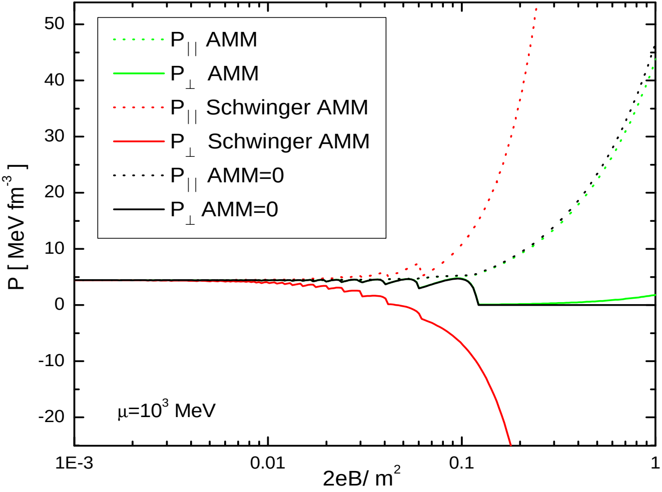

Figure: Pressures vs B. Schwinger value and AMM$=0$ are shown for comparison.

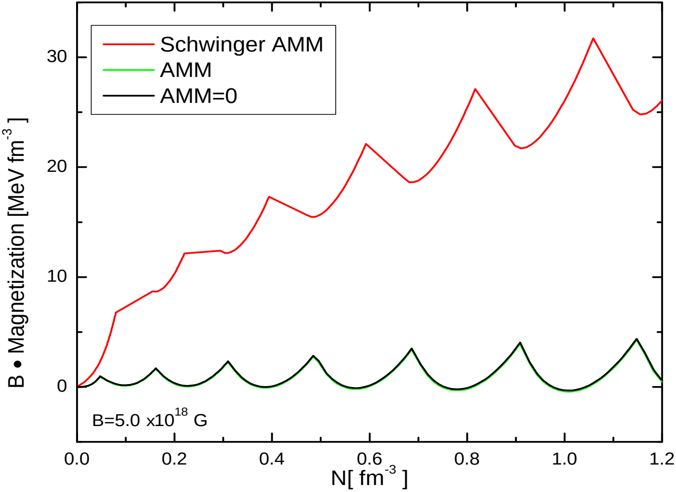

Figure: Magnetization vs density. Schwinger value and AMM$= 0$ are shown for comparison.

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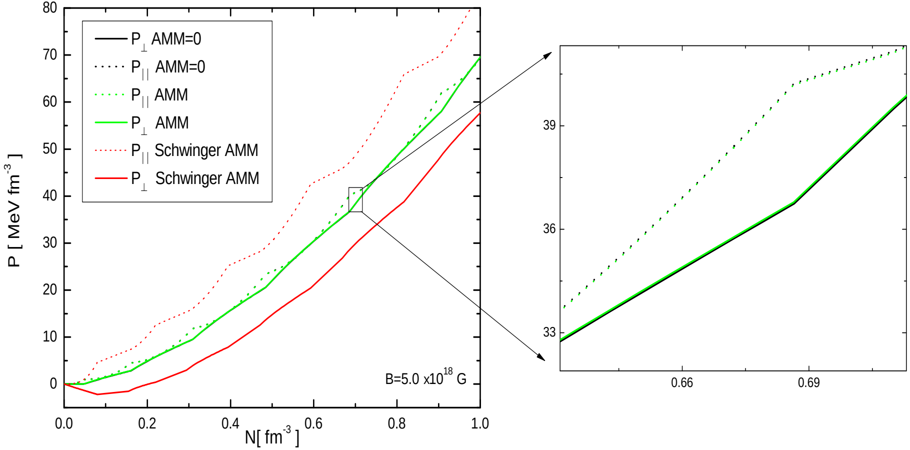

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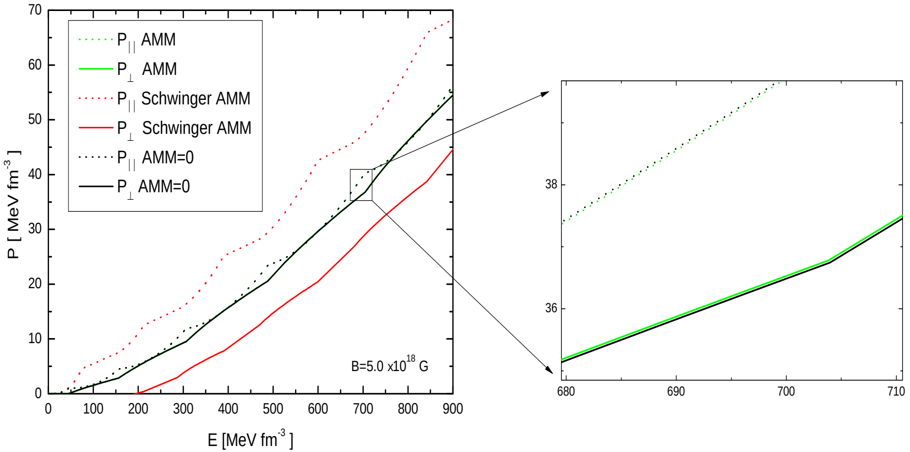

Figure: EoS, Schwinger value is shown for comparison.

Summary & Conclusions

1. The AMM for fermions in presence of constant magnetic field has been calculated at $T = 0$ in a consistent way (exact in B).
   - Schwinger AMM is valid if $\frac{B}{B_c} \lesssim 10^{-3}$
2. The effects of the AMM in the thermodynamical quantities has been studied.
   - The effects of the AMM in our approximation are negligible for $B \lesssim B_c$.
3. Our result is also applicable to other charged fermions with AMM.

Future work:

- Include $T \neq 0$.
- Include $\mu \neq 0$ in the AMM.
- Prove that our conclusions are true under stellar equilibrium condition.

Thanks

Gracias