Higgs mechanism and symmetry breaking in strong magnetic field

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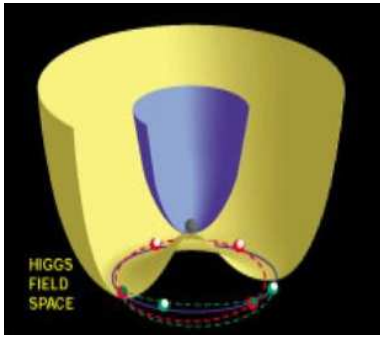
Abstract

We discuss the effect of a strong magnetic field in the behavior of the symmetry of an electrically neutral electroweak plasma. We analyze the case of a strong magnetic field and low temperatures as compared with the W rest energy. If the magnetic field is large enough, it is self-consistently maintained. Charged vector bosons play the most important role, leading only to a decrease of the symmetry breaking parameter, the symmetry restoration not being possible.

- The Standard Model of the electroweak interaction at finite temperatures predicts the existence of two phases : the symmetric and the broken one at temperatures, respectively, above and below some critical value T_c.
- We explore the possibility that $m_w 2$ would decrease, via ξ, with increasing B.
- We will conclude in the present case that there is actually a decrease of $\xi = \xi(B)$ for increasing B, but we find that the decrease is a small fraction of $\xi(0)$.
- We study our problem in the frame of quantum statistics, taking in mind possible consequences for astrophysics and cosmology. We consider the lepton sector of an electrically neutral electroweak plasma. We evaluate the variation of the symmetry breakdown parameter in the external field and examine the possibility of symmetry restoration.

Weinberg-Salam Lagrangian



$$\mathcal{L} = -\frac{1}{4}G^i_{\mu\nu}G^i_{\mu\nu} - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \bar{\psi}_L\gamma_\mu D_\mu 1\psi_L - \bar{e}_R\gamma_\mu D_\mu 2e_R - |D_\mu 3\phi|^2$$
$$-\lambda_1(\bar{\psi}_L\phi e_R + \bar{e}_R\phi^+\psi_L) - \frac{\lambda_2}{4}(\phi^+\phi - a2)2,$$

$$\phi = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \xi \end{pmatrix} + \begin{pmatrix} ih1 + h2 \\ \sigma - ih3 \end{pmatrix}\right], \qquad < 0|\phi|0 >= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \xi \end{pmatrix},$$

$$\xi = 2a$$

Effective potential

Thermodynamical properties of an electroweak plasma in a constant magnetic field can be studied if we know the effective potential associated to the system. For simplicity we obtain this potential for the leptonic sector of the plasma, in the one loop approximation, starting from the Weinberg -Salam Lagrangian:

$$V = V_e + V_w + V_h + V_z + V_A + V_\nu + V_t,$$

i	m_i	(GeV)	$\varepsilon_i \quad (n = 0, 1, 2, ..., \ \eta = \pm 1)$
e	$\lambda_1 \xi$	$5.11 \cdot 10^{-4}$	$\sqrt{p_3 2 + m_e 2 + 2eBn}$
W	$\frac{g}{2}\xi$	80.4	$\sqrt{p_3 2 + m_w 2 - eB}$
			$\sqrt{p_3 2 + m_w 2 + 2eB(n + 1/2)}$
σ	$\sqrt{\frac{\lambda_2}{2}}\xi$	> 114.4	$\sqrt{p2 + m_\sigma^2}$
Z	$\frac{1}{2}\sqrt{g2 + g'^2}\xi$	91.2	$\sqrt{p2 + m_z 2}$
A	$-$		$\sqrt{p2}$
ν	$-$		$\sqrt{p2}$

- ✔ The particle masses are related to the symmetry breaking parameter ξ.
- ✔ g, g', λ_1 and λ_2 are, respectively, the electroweak, Yukawa and Higgs scalar coupling constants.

$$V_e = -\frac{eB}{4\pi^2\beta}\sum_{n=0}^{\infty}\alpha_n\int_{-\infty}^{\infty}dp_3\left[\ln\left(1+e^{-(\varepsilon_{ne}-\mu_e)\beta}\right)\left(1+e^{-(\varepsilon_{ne}+\mu_e)\beta}\right)+\beta\varepsilon_{ne}\right]$$

$$V_w = \frac{eB}{4\pi^2\beta}\int_{-\infty}^{\infty}dp_3\left[\ln\left(1-e^{-(\varepsilon_{ow}-\mu_1)\beta}\right)\left(1-e^{-(\varepsilon_{ow}+\mu_1)\beta}\right)+\beta\varepsilon_{ow}\right]+ \frac{eB}{4\pi^2\beta}\sum_{n=o}^{\infty}\beta_n\int_{-\infty}^{\infty}dp_3\left[\ln\left(1-e^{-(\varepsilon_{nw}-\mu_1)\beta}\right)\left(1-e^{-(\varepsilon_{nw}+\mu_1)\beta}\right)+\beta\varepsilon_{n\omega}\right],$$

$$V_i = \frac{S_i}{(2\pi)3\beta}\int d^3p\left[\ln\left(1-e^{-\varepsilon_i\beta}\right)+\beta\varepsilon_i\right], \qquad i = Z, A, \sigma; S_i = 3, 2, 1$$

$$V_\nu = -\frac{1}{(2\pi)^3\beta}\int_{-\infty}^{\infty}d^3p\left[\ln\left(1+e^{-(\varepsilon_\nu-\mu_2)\beta}\right)\left(1+e^{-(\varepsilon_\nu+\mu_2)\beta}\right)+\beta\varepsilon_\nu\right].$$

- By evaluating the effective potential on the mass shell

$$\partial V/\partial \xi = 0,$$

we get the thermodynamical potential $V(\xi_{\min}) = \Omega$,

$$V_t = \frac{\lambda_2}{4}\left(\frac{\xi^2}{2} - a2\right)^2 \quad \Rightarrow \quad \xi_0 = \sqrt{2}a.$$

- One can write then two other equilibrium equations. One of them is the lepton number conservation $-\partial\Omega/\partial\mu_2 = N_l = N_e + N_\nu$ (where N_i is the net density of particles (particles minus antiparticles), per unit volume) and the other $\partial\Omega/\partial\mu_1 = 0$, is the electric charge conservation, which in our simplified model is reduced to $N_e + N_w = 0$.

- The magnetization $M = -\partial\Omega/\partial B$ contains the contributions of both electrons and W bosons $M = M_e + M_w$.

- ✔ In the absence of field and at zero temperature, the effective potential coincides with the tree term $V\,|_{T=0,B=0}= V_t$, and therefore $\xi_o = \sqrt{2}a$.
- ✔ It is known that temperature modify the symmetry breaking parameter. In fact, it was pointed out long ago [A.D. Linde *Rep. Prog. Phys.* **42** 389 (1979)] that an increase of temperature decreases the symmetry breaking parameter and at some critical temperature T_c the symmetry is restored $(T_c \sim 10^{15}K)$.
- ✔ We can expect then that an intense external magnetic field also modifies the symmetry breaking parameter.
- ✔ We will restrict ourselves to the case of a strong magnetic field and/or low temperatures, when the condition $eB \gg T2$ is satisfied.

$eB \gg T2$ limit

It can be demonstrated that , in the case $eB \gg T2$, only the charged boson contribution may substantially modify the symmetry breaking parameter. For $eB \gg T2$, the average W boson population in excited Landau states is negligible small. Moreover, in that limit the Bose-Einstein distribution degenerates in a Dirac δ function and most of the W density is in the Landau ground state $n = 0$ and distributed in a very narrow interval around $p_3 = 0$.

If we only consider the contribution of the W boson sector,

$$V \approx V_t + V_{0w}^{st} + V_{0w},$$

where the first term is the statistical part and the second one is the Euler-Heisenberg vacuum term.

W bosons

$$\partial V_t/\partial\xi + \partial V_{0w}^{st}/\partial\xi + \partial V_{0w}/\partial\xi = 0,$$

- ✔ $B \to B_{cw} = m_w 2/e$:

$$\partial V_{0w}/\partial\xi \approx \frac{eB}{32\pi^2} g2\xi \ln\frac{eB}{m_w 2 - eB},$$

- ✔ $n_{ow+} = \frac{N_w}{eB}(2\pi)^2\,\delta(p_3)$:

$$\partial V_w^{st}/\partial\xi = \frac{eB}{16\pi^2} g2\xi \int_{-\infty}^{\infty} dp_3 n_{ow+}/\varepsilon_{ow} = \frac{g2}{4}\xi\frac{N_w}{\sqrt{m_w 2 - eB}},$$

$\xi 2$ vs. B

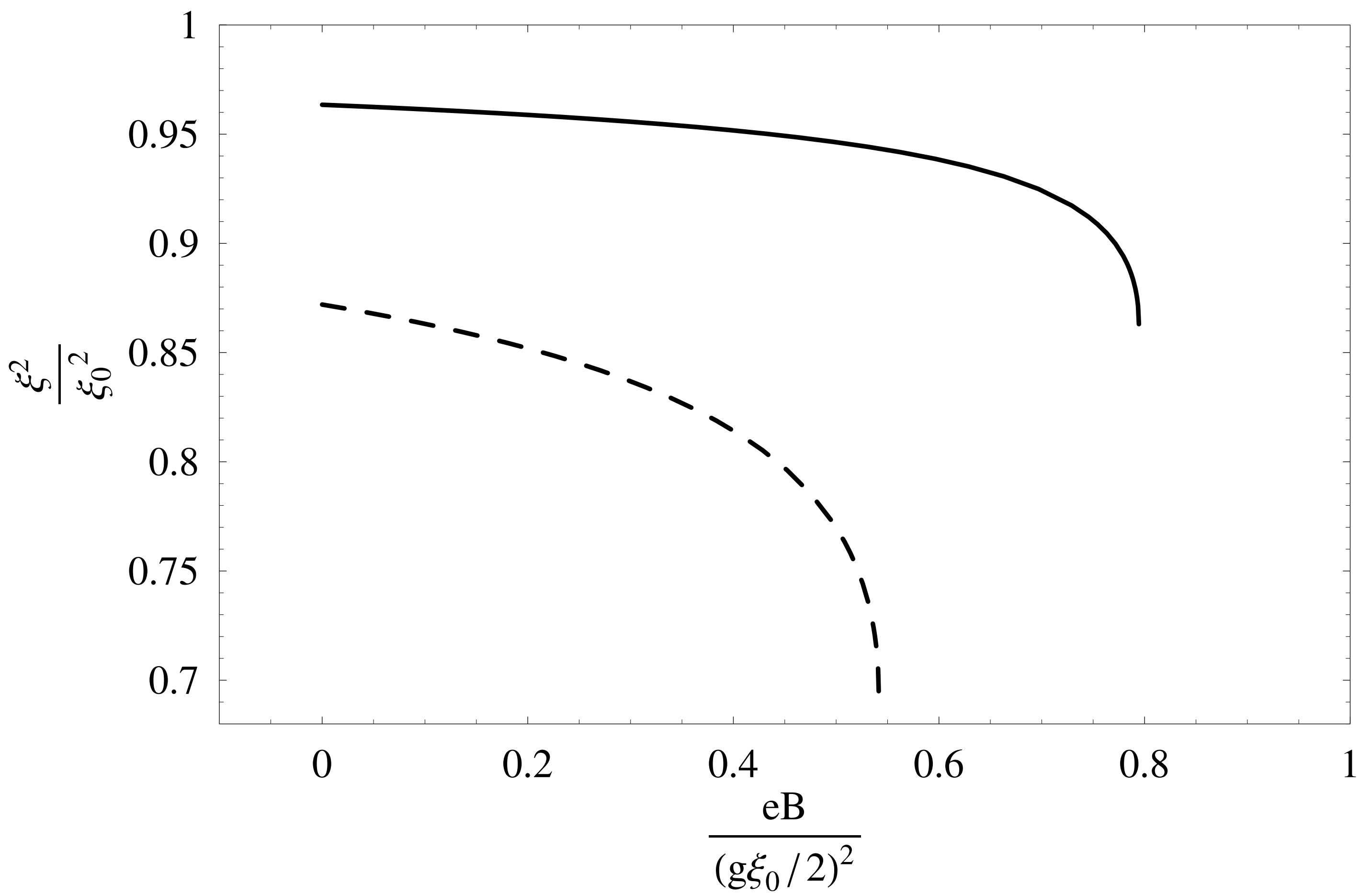


Figure 1: $\xi 2$ vs. B for $N_w = 3.0 \cdot 10^{47} cm^{-3}$ (continuous line) and for $N_w = 1.0 \cdot 10^{48} cm^{-3}$ (dashed line).

Phase diagram

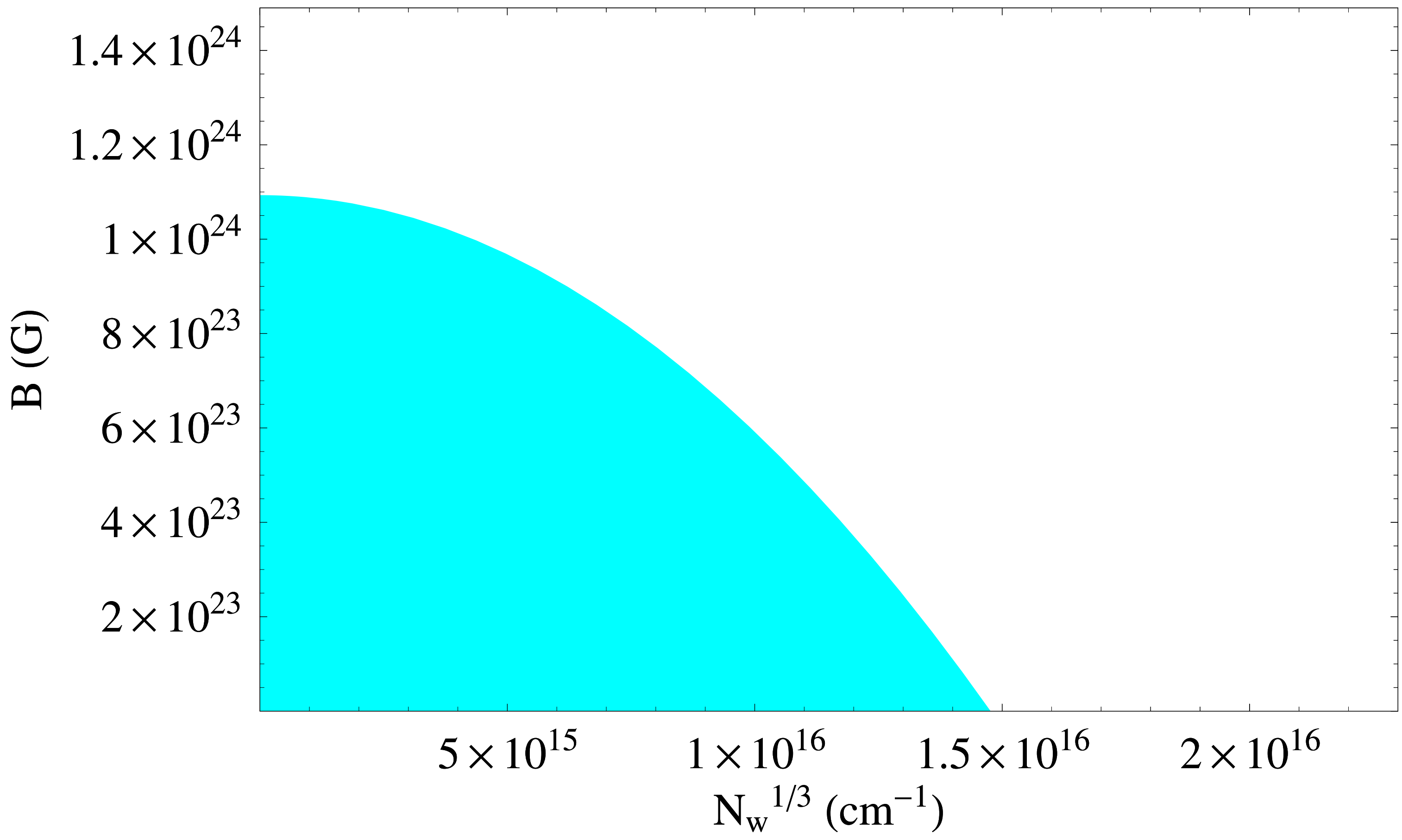


Figure 2: Phase diagram (the colored area corresponds to the broken symmetry phase).

Self-magnetization

Actually, for solving our problem, we must take into account that for sufficiently large magnetization ($M \gg H$) it can self-consistently maintain the field B. So, we can put

$$B = 4\pi \mathcal{M}_{sw} = 4\pi \frac{eN_w}{2\sqrt{m_w 2 - eB}},$$

and consider this equation together with $\partial V / \partial \xi = 0$.

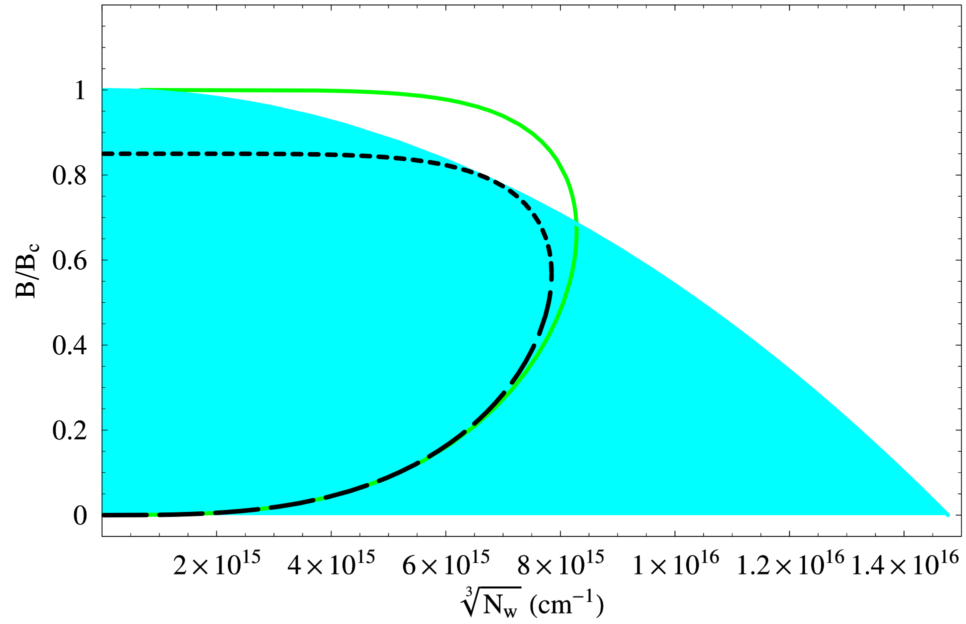


Figure 3: Self-magnetization curve (dashed lines), for $m_w(B)$.

$\xi(B)$

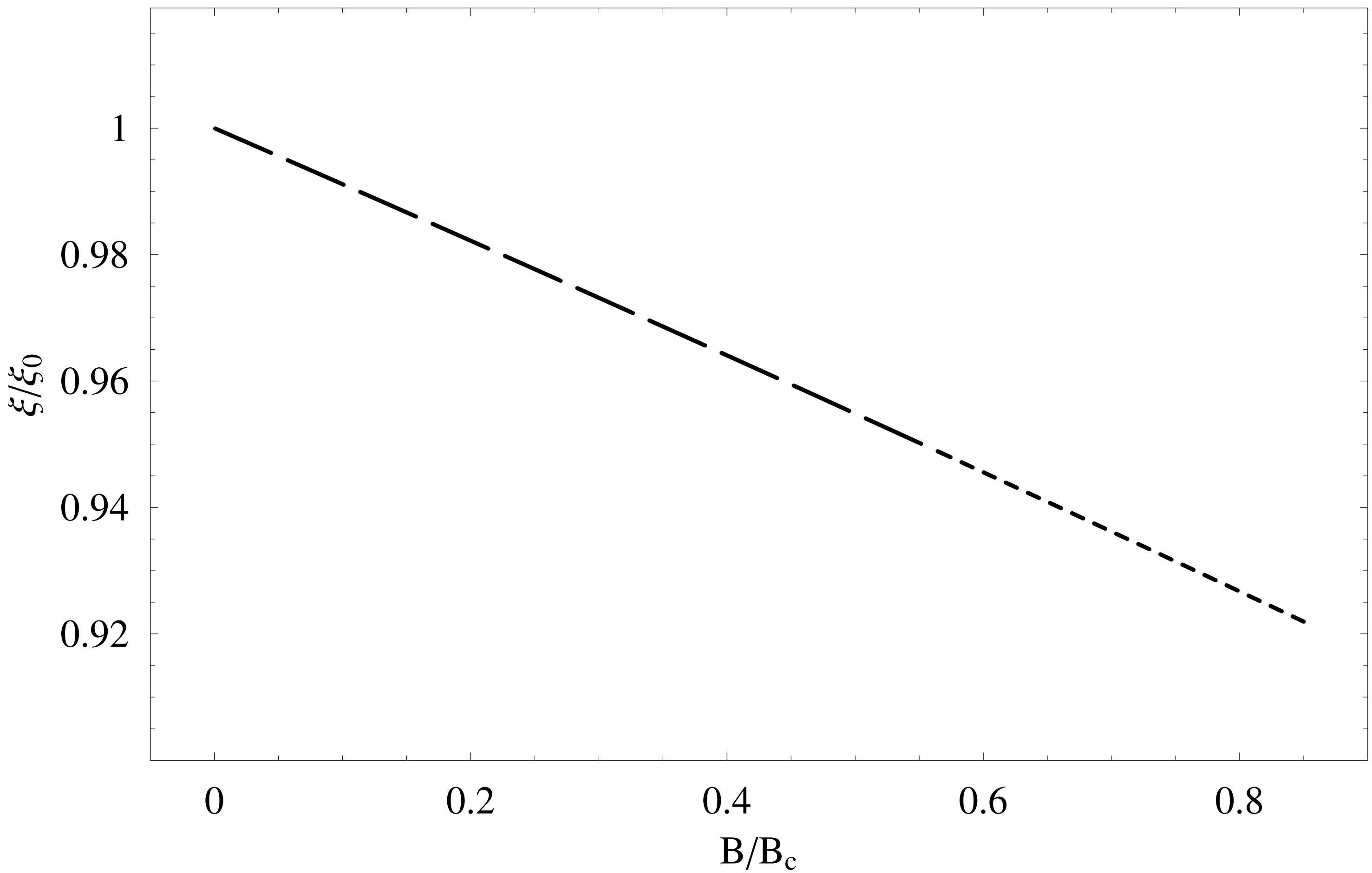


Figure 4: The symmetry breaking parameter dependence on a magnetic field $\xi(B)$.

Conclusions

1. We conclude that for a neutral electroweak plasma in a large constant magnetic field, only the charged vector particle contribution may substantially modify the symmetry breaking parameter It is assumed the system is not C symmetric, i.e. there is baryon-antibaryon asymmetry
2. For high values of the field, it is maintained self-consistently and the field never reaches its critical value B_c.
3. The symmetry breaking parameter is decreased some amount, expressed through the term $N_w > 0$ under the action of the magnetic field, and in consequence, the masses of electrons, W and Z bosons, and Higgs particles become slightly smaller than in the zero field case at zero temperature.